Assignment

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Q1 mean $= \theta_1$ (Normal distribution)
variance $= \theta_2$

max likelihood estimate function

$$L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_2)^2}{2\theta^2}}$$

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = \frac{-n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i \theta_j)^2$$

for θ_1

$$\frac{\partial}{\partial\theta_1} \ln L(\;) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

R.H.S $= 0$

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\sum (x_i - \theta_1) = 0$$

$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$ so MLE of $\theta_1 \rightarrow$ sample mean

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for θ_2

$$\frac{\partial}{\partial\theta_2} \ln L(\,) = \frac{-H}{2\theta_2} + \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

R.H.S = 0

$$\frac{-n}{2\theta_2} = \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n} \Sigma (x_i - \hat{\theta}_1)^2$$

so MLE for $\theta_2 \longrightarrow$ sample variance

Q2 Bernoulli distribution

parameter $\rightarrow \theta \in \Theta = (0,1)$ unknown
$\rightarrow m$ (known +ve Z)

MLE

$$L(\theta \mid x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} P(x_i = x_i/\theta)$$

$$P(X_i = x_i/\theta) = \theta^{x_i}(1-\theta)^{m-x_i}$$

Taking log

$$\ln L(\theta(x_1, x_2 \ldots x_n) = \sum_{x_i=1}^{n} \ln\left(\theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{m} \left(x_i \ln\theta \; (m - x_i) \ln(1-\theta)\right)$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = mn - \sum_{i=1}^{n} \frac{x_i}{1-\theta}$$

$$\therefore \quad \theta = \sum_{i=1}^{n} \frac{x_i}{n.m}$$

so max likelihood estimate of θ is :

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n.m}$$