# Homework 2 - Questions

1. Determine whether the natural cubic spline that interpolates the table

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| y | 1 | 1 | 0 | 10 |

   is or is not the function

$$f(x) = \begin{cases} 1 + x - x^3 & x \in [0, 1] \\ 1 - 2(x-1) - 3(x-1)^2 + 4(x-1)^3 & x \in [1, 2] \\ 4(x-2) + 9(x-2)^2 - 3(x-2)^3 & x \in [2, 3]. \end{cases}$$

2. Find the natural cubic spline function whose knots are $-1, 0$, and 1 and that takes the values $S(-1) = 13, S(0) = 7, S(1) = 9$.

3. Show how to use Richardson extrapolation if $L = \phi(h) + a_1 h + a_3 h^3 + a_5 h^5 + ....$

4. Using Taylor series expansions, derive the error term for the formula $f''(x) \approx \frac{1}{h^2}[f(x) - 2f(x+h) + f(x+2h)]$.

5. **Coding Project**

   Using the US population Census data, do the following:

   (a) Determine the interpolation polynomial for these data.

   (b) Determine a cubic spline for these data.

   (c) Using both results, to answer the questions: what is the estimated US population on Jan. 1st, 2005? Which estimate do you think makes more sense?