REGRESSION ANALYSIS - HOMEWORK 4 SOLUTIONS

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5) Note that $\hat{\beta}_{WLS} = (X^T W X)^{-1} X^T W Y$ by the weighted least square criteria. Let Σ be the variance matrix of Y with diagonals $\Sigma_{ii} = \sigma_i^2$, where $w_i \sigma_i^2 = k$ for all i. Hence,

$$\begin{aligned}
Var(\hat{\beta}_{WLS}) &= Var\left[(X^T W X)^{-1} X^T W Y\right] \\
&= \left[(X^T W X)^{-1} X^T W\right] Var(Y) \left[(X^T W X)^{-1} X^T W\right]^T \\
&= \left[(X^T W X)^{-1} X^T W\right] \Sigma \; \left[(X^T W X)^{-1} X^T W\right]^T \\
&= (X^T W X)^{-1} X^T W \, \Sigma \, W^T X \left[(X^T W X)^{-1}\right]^T \\
&= (X^T W X)^{-1} X^T W \, \Sigma \, W^T X (X^T W^T X)^{-1} \\
&= (X^T W X)^{-1} X^T (kI) \, W^T X (X^T W^T X)^{-1} \\
&= k (X^T W X)^{-1} (X^T W^T X)(X^T W^T X)^{-1} \\
&= k (X^T W X)^{-1}.
\end{aligned}$$

`Solutions of other questions are in the following pages.`

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Date: November 3, 2023.