FALL 2023 MATH 484/564 HOMEWORK #1

- Due: September 18, Monday, 11:59PM
- How to submit: via Blackboard. If you have multiple files, upload a zipped file
- Submission link will disappear after 48 hours
- Homework solution is not required to be typed, but must be legible.

Problem 1 Derive intercept $\hat{\beta}_0$ and slope $\hat{\beta}_1$ for the estimated regression line $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$ using the least square criterion.

Problem 2 Prove the following properties of the regression line $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$.

1) Show that the regression line has only one degree of freedom.

2) Show that $\sum_{i=1}^{n} (\hat{y}_i - \bar{y}) = 0$.

3) Define $e_i = y_i - \hat{y}_i$. Show that $\sum_{i=1}^{n} e_i = 0$.

4) Show that $\sum_{i=1}^{n} x_i e_i = 0$.

5) Show that $\sum_{i=1}^{n} \hat{y}_i e_i = 0$.

6) Show that SST=SSE+SSR for Simple Linear Regression (SLR).

Problem 3 The purity of oxygen (Y) produced by a fractional distillation process is thought to be related to the percentage of hydrocarbons (X) in the main condenser of the processing unit. 25 samples were measured and shown in the attached data sheet.

1) Fit a simple linear regression model to the data, find the coefficients $\hat{\beta}_0$ and $\hat{\beta}_1$ in equation $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$.
2) Test the hypothesis $H_0 : \beta_1 = 0$ against the alternative $H_A : \beta_1 \neq 0$, and conclude if there is significant linear relationship between the purity of oxygen and the percentage of hydrocarbons.
3) Calculate the coefficient of determination, r^2.
4) Find a 95% confidence interval on the slope.
5) Find a 95% confidence interval on the mean purity when the hydrocarbon percentage is 1.05.
6) What is the correlation coefficient r between Y and X?
7) Test the hypothesis: $H_0 : \rho = 0$ against $H_A : \rho \neq 0$ using a t-test based on the correlation coefficient r computed from the previous step.

Problem 4 Show that the sample correlation coefficient r between X and Y is a value between -1 and 1, i.e., $-1 \leq r \leq 1$.

Problem 5 Exercise 2.10 from the TEXT.