REGRESSION ANALYSIS - HOMEWORK 2 SOLUTIONS

ALAITTIN KIRTISOGLU

1) Based on the least square criteria the objective function is $Q(\hat{\beta}_1) = \sum_{i=1}^{n}(y_i - \hat{\beta}_1 x_i)^2$. Set the derivative of S to 0.

$$\frac{dS}{d\hat{\beta}_1} = -2\sum_{i=1}^{n}(y_i - \hat{\beta}_1 x_i)x_i = 0 \tag{1}$$

Find the solution $\hat{\beta}_1$ of the normal equation in (1) as $\hat{\beta}_1 = \left(\sum_{i=1}^{n} x_i y_i\right) / \left(\sum_{i=1}^{n} x_i^2\right)$.

3.1) Note that $\bar{\hat{y}} = \bar{y}$, and $\sum_i e_i \hat{y}_i = 0$ and $\sum_i e_i = 0$.

$$Cor(Y,\hat{Y})^2 = \frac{\left[\sum_{i=1}^{n}(y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})\right]^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2 \sum_{i=1}^{n}(\hat{y}_i - \bar{\hat{y}})^2} = \frac{\left[\sum_{i=1}^{n}(y_i - \hat{y}_i + \hat{y}_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})\right]^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2 \sum_{i=1}^{n}(\hat{y}_i - \bar{\hat{y}})^2} = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} = \frac{SSR}{SST} = R^2$$

3.2) Observe that if the correlation between X and Y is negative, then $\hat{\beta}_1 < 0$, and so

. $Cor(Y, \hat{\beta}_1 X) = -Cor(Y, X) = |Cor(Y, X)|$. Hence, we have

. $Cor(Y, \hat{Y}) = Cor(Y, \hat{\beta}_0 + \hat{\beta}_1 X) = Cor(Y, \hat{\beta}_0) + Cor(Y, \hat{\beta}_1 X) = 0 + Cor(Y, \hat{\beta}_1 X) = |Cor(Y, X)|$.

4) $(t^\star)^2 = \dfrac{\hat{\beta}_1^{\,2}}{s.e(\hat{\beta}_1)} = \dfrac{(n-2)\hat{\beta}_1^{\,2}\sum_{i=1}^{n}(x_i - \bar{x})^2}{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} = \dfrac{\sum_{i=1}^{n}(\hat{\beta}_1 x_i - \hat{\beta}_1 \bar{x})^2}{SSE}(n-2) = \dfrac{SSR}{SSE}(n-2) = F^\star$. Because,

. $\sum_{i=1}^{n}(\hat{\beta}_1 x_i - \hat{\beta}_1 \bar{x})^2 = \sum_{i=1}^{n}(\hat{\beta}_1 x_i + \hat{\beta}_0 - \hat{\beta}_0 - \hat{\beta}_1 \bar{x})^2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$.

Note: MLR part will not be graded.

5) $F^\star = \dfrac{SSE_{red} - SSE_{full}}{df_{red} - df_{full}} \dfrac{df_{full}}{SSE_{full}} = \dfrac{38460756 - 22657938}{3} \dfrac{88}{22657938} = 20.45858$.

. $F_{critical} = F(3, 88, 0.05) = 2.708186$. Since $F^\star > F_{critical}$, the hypothesis is rejected.

Questions 2 is in the following page.

Illinois Institute of Technology, Applied Mathematics, Chicago, IL, USA

Email address: akirtisoglu@hawk.iit.edu

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