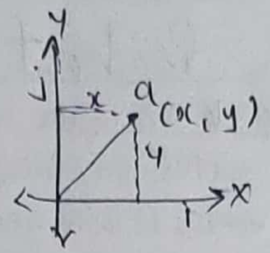
Unit-3 Vectors - 14 Marks

* Vector → Direction + Value (Displacement, flow of fluid)

  Scalar → only value. (Mass, work, Energy, Frequency, Electric charge)

* Co-ordinate form

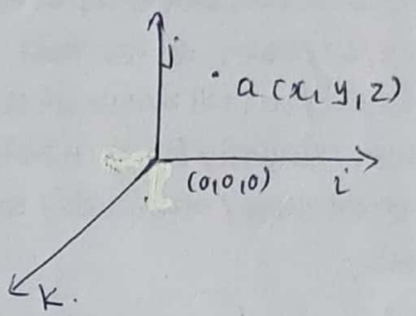
  $a(x, y)$

  vector form

  $\bar{a} = xi + yj$

* $a(x, y, z) \Rightarrow \bar{a} = xi + yj + zk$

* $a(1, -1, 3) \Rightarrow \bar{a} = i - j + 3k$

  $b(5, 2, -3) \Rightarrow \bar{b} = 5i + 2j - 3k$

* $\bar{a} = 3i - j + 5k \Rightarrow a(3, -1, 5)$
* $\bar{b} = 4i - k \Rightarrow b(4, 0, -1)$
* $\bar{c} = 3i - 2k + j \Rightarrow c(3, 1, -2)$

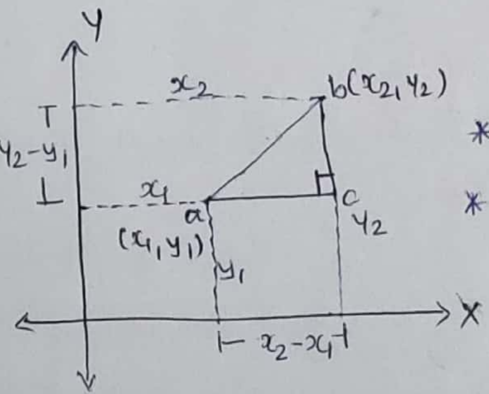
* Modulus

  $|-3| = 3$

  $|-4| = 4$

  $ab^2 = ac^2 + bc^2$

  $= (x_2 - x_1)^2 + (y_2 - y_1)^2$

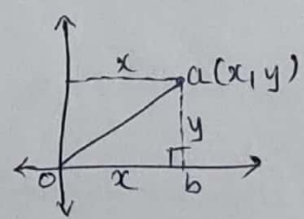
  

* $|\bar{x}| \geq 0$
* $|\bar{x}| = 0 \Leftrightarrow \bar{x} = \bar{0}$

→ $oa^2 = ab^2 + ob^2$

$= x^2 + y^2$

$\therefore oa = \sqrt{x^2 + y^2}$.



→ $a(x, y, z) \Rightarrow |\bar{a}| = \sqrt{x^2 + y^2 + z^2}$

* If $\bar{a} = i - 2j + k$ then $|\bar{a}| =$ ___
* If $\bar{a} = 2i - j + 4k$ then $|\bar{a}| =$ ___
* If $\bar{b} = i - 3k + j$ then $|\bar{b}| =$ ___
* If $\bar{c} = (1, 0, 0)$ then $|\bar{c}| =$ ___
* If $\bar{a}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ then $|\bar{a}| =$ ___
* $|3i - 4j + 5k| =$ ___
* $|2i + 4j + 4k| =$ ___

* If $\bar{x} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ then $|\bar{x}| =$ ____

* If $\bar{c} = (\sin\theta, \cos\theta)$ then $|\bar{c}| =$ ____

* **Unit Vector** :- If Modulus of vector is unit then given vector is called unit vector. $|\bar{a}| = 1$.

* **Unit vector of $\bar{a}$** :- $\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

* If $\bar{a} = i - 3k + j$ then find unit vector of $\bar{a}$.

$\hat{a} = \left(\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}\right)$

→ For verification :- $\sqrt{x^2+y^2+z^2} = 1$.

* If $\bar{x} = 3i - j + 4k$ then find unit vector of $\bar{x}$.

$\hat{x} = \left(\frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)$

* **Addition & substraction of vector**

→ $\bar{a} = (x_1, y_1, z_1)$ $\bar{b} = (x_2, y_2, z_2)$

$\bar{a} \pm \bar{b} = (x_1, y_1, z_1) \pm (x_2, y_2, z_2)$

$= (x_1 \pm x_2, y_1 \pm y_2, z_1 \pm z_2)$

* If $\bar{a} = i - 3j + 5k$ and $\bar{b} = 4i + j - 2k$ then find (i) $\bar{a} + \bar{b}$ (ii) $\bar{a} - \bar{b}$ (iii) $2\bar{a} + 3\bar{b}$ (iv) $3\bar{a} - 2\bar{b}$.

→ (i) $\bar{a} + \bar{b} = (5, -2, 3)$ (iii) $2\bar{a} + 3\bar{b} = (14, -3, 4)$

(ii) $\bar{a} - \bar{b} = (-3, -4, 7)$ (iv) $3\bar{a} - 2\bar{b} = (-5, -11, 19)$

* If $\bar{a} = j + k - i$ and $\bar{b} = 2i + j - 3k$ then find the value of $|2\bar{a} + 3\bar{b}|$. $= 3\sqrt{10}$

* If $\bar{a} = (3, -1, -4)$, $\bar{b} = (-2, 4, -3)$ and $\bar{c} = (-1, 2, -5)$ then find $|\bar{a} + 2\bar{b} - \bar{c}|$. $= \sqrt{50}$

* If $\bar{a} = i - 2j + k$, $\bar{b} = 2i + j + 3k$ and $\bar{c} = -i + 2j - 3k$ then find $|2\bar{a} - 3\bar{b} + \bar{c}|$. $= 5\sqrt{6} = \sqrt{150}$

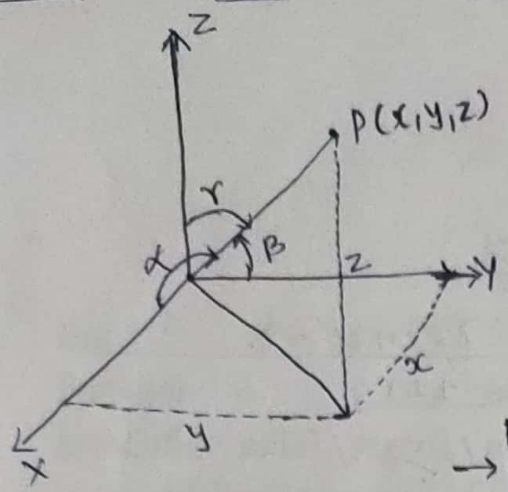
* If $\bar{a} = (3, -1, -4)$, $\bar{b} = (-2, 4, -3)$ and $\bar{c} = (-1, 2, -1)$ then find $|3\bar{a} - 2\bar{b} + 4\bar{c}|$. $= \sqrt{190}$

* If $\bar{a} = i - 2j + 4k$, $\bar{b} = -3i + j - 4k$ and $\bar{c} = i + 2j - 4k$ then find $|5\bar{a} + 3\bar{b} + 2\bar{c}|$. $= \sqrt{30}$

* If $\bar{a} = i + 2j + k$, $\bar{b} = 2i - 3j + k$ and $\bar{c} = -2i - j + 5k$ then find $|2\bar{a} + 3\bar{b} - \bar{c}|$.

* If $\bar{a} = 3i - 2j + k$, $\bar{b} = 2i - 4j - 3k$ and $\bar{c} = -i + 2j + 2k$ then find $|2\bar{a} - 3\bar{b} - 5\bar{c}| = \sqrt{30}$

* Direction cosine of vector :-



$$l = \cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$m = \cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$n = \cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}.$$

$$\rightarrow \boxed{l^2+m^2+n^2=1}$$

* If $\bar{a} = 3i - j - 4k$, $\bar{b} = -2i + 4j - 3k$ and $\bar{c} = i + 2j - k$ then find the direction cosine of vector $3\bar{a} - 2\bar{b} + 4\bar{c}$.

* If $\bar{a} = 3i - j - 4k$, $\bar{b} = -2i + 4j - 3k$ and $\bar{c} = -i + 2j - 5k$ then find the direction cosine of $\bar{a} + 2\bar{b} - \bar{c}$.

* Multiplication of vectors

* Dot product $(\bar{a} \cdot \bar{b})$ Scalar product
* Cross product $(\bar{a} \times \bar{b})$ Vector product
* Dot product $\bar{a} = (x_1, y_1, z_1)$, $\bar{b} = (x_2, y_2, z_2)$

$$\bar{a} \cdot \bar{b} = (x_1, y_1, z_1) \cdot (x_2, y_2, z_2)$$
$$= x_1 x_2 + y_1 y_2 + z_1 z_2.$$

* If $\bar{a} = i - 2j + k$ and $\bar{b} = 3i + j + 4k$ then find (i) $\bar{a} \cdot \bar{b}$ (ii) $\bar{a} \cdot (\bar{a} + \bar{b})$. (iii) $\bar{a} \cdot (\bar{a} - \bar{b})$.

* If $\bar{x} = (1, -2, 3)$ and $\bar{y} = (-2, 3, 1)$ then find $(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y})$.

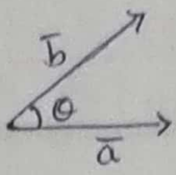
* If $\bar{x} = (1, -2, 3)$ and $\bar{y} = (1, 2, -2)$ then find $(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y})$.

* If $\bar{a} = (-4, 9, 6)$, $\bar{b} = (0, 7, 10)$ and $\bar{c} = (-1, 6, 6)$ then P.T. $(\bar{a} - \bar{c}) \cdot (\bar{b} - \bar{c}) = 0$.

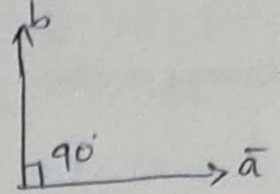
* $\bar{x} \cdot \bar{x} \geq 0$ * $\bar{x} \cdot (\bar{y} + \bar{z}) = \bar{x} \cdot \bar{y} + \bar{x} \cdot \bar{z}$
* $\bar{x} \cdot \bar{x} = 0 \Leftrightarrow \bar{x} = \bar{0}$
* $\bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x}$

* Angle between two Vectors

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$$



$$\therefore \theta = \cos^{-1}\left(\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}\right)$$



→ If $\bar{a}\cdot\bar{b}=0$ then $\cos\theta = 0$

$\theta = \cos^{-1}0 = 90^\circ$

* If $\bar{a}\cdot\bar{b}=0$ then $\bar{a}$ and $\bar{b}$ are perpendicular to each other.
* If $\bar{a}$ & $\bar{b}$ are perpendicular to each other then $\bar{a}\cdot\bar{b}=0$.
* $\sin^2\theta + \cos^2\theta = 1$

* If $\bar{x} = (1,1,1)$ and $\bar{y} = (2,-1,-1)$ then P.T. $\bar{x}$ and $\bar{y}$ are perpendicular to each other.
* If $\bar{x} = (1,-2,-3)$ and $\bar{y} = (2,P,4)$ then For what value of 'p'. $\bar{x}$ and $\bar{y}$ are perpendicular to each other. $(P=-5)$.
* Find x, If $\bar{a} = (2,-3,5)$ and $\bar{b} = (x,-6,-8)$ are perpendicular to each other. $(x=11)$.
* If $2i+3j+k$ and $pi-j-3k$ are perpendicular to each other then find 'p'. $(p=3)$.
* If $(m,2m,4)$ and $(m,-3,2)$ are perpendicular to each other then find m. $(m=4$ or $m=2)$

* Find the angle between two Vectors $(1,2,3)$ and $(-2,3,1)$.
* Find the angle between vectors $(1,2,4)$ and $(3,1,2)$.
* prove that the angle between two vectors $i+j-k$ and $2i-2j+k$ is $\sin^{-1}\sqrt{\frac{26}{27}}$.
* show that the angle between two vectors $i+2j$ and $i+j+3k$ is $\sin^{-1}\sqrt{\frac{46}{55}}$
* prove that the angle between two vectors $i+2j-3k$ and $2i+j-k$ is $\sin^{-1}\sqrt{\frac{35}{84}}$.
* P.T. the angle between two vectors $3i+j+2k$ and $2i-2j+4k$ is $\sin^{-1}\frac{2}{\sqrt{7}}$

* **Cross product** $(\bar{a} \times \bar{b})$

$\bar{a} = (x_1, y_1, z_1)$, $\bar{b} = (x_2, y_2, z_2)$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = i \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} - j \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} + k \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

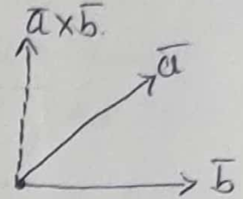
* $\bar{x} \times \bar{y} = -(\bar{y} \times \bar{x})$
* $\bar{x} \times \bar{x} = \bar{0}$
* $\bar{x} \times (\bar{y} + \bar{z}) = \bar{x} \times \bar{y} + \bar{x} \times \bar{z}$

* $|i| = |j| = |k| = 1$
* $i \cdot i = j \cdot j = k \cdot k = 1$
* $i \cdot j = j \cdot k = k \cdot i = 0$

* $i \times j = k$, $j \times k = i$, $k \times i = j$
* $i \times i = 0$, $j \times j = 0$, $k \times k = 0$

* If $\bar{a} = i - j + 3k$ and $\bar{b} = 3i + 2j - k$ then find (i) $\bar{a} \times \bar{b}$ (ii) $\bar{b} \times \bar{a}$.
* If $\bar{a} = 2i - j$ and $\bar{b} = i + 3j - 2k$ then find (i) $|\bar{a} \times \bar{b}|$ (ii) $|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})|$.
* If $\bar{a} = (2, -3, -1)$ and $\bar{b} = (1, 4, -3)$ then find $|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})|$.
* Simplify. $(10i + 2j + 3k) \cdot [(i - 2j + 2k) \times (3i - 2j - 2k)]$

Box product $\bar{a} \cdot (\bar{b} \times \bar{c}) = [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$



* **Unit vector perpendicular to both vectors** :-

- U. V. P. to both $\bar{a}$ & $\bar{b}$ $= \pm \dfrac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}$ $\begin{cases} \bar{a} \perp (\bar{a} \times \bar{b}) \\ \bar{b} \perp (\bar{a} \times \bar{b}). \end{cases}$

* If $\bar{x} = 3i - j + 2k$ and $\bar{y} = 2i + j - k$ then find unit vector perpendicular to both vectors $\bar{x}$ and $\bar{y}$. $\dfrac{(-1, 7, 5)}{\sqrt{75}}$
* Find the unit vector perpendicular to both vectors $\bar{a} = (5, 7, -2)$ and $\bar{b} = (3, 1, -2)$. $\dfrac{(-12, 4, -16)}{\sqrt{416}}$.
* Find the unit vector perpendicular to both vectors $\bar{a} = (1, 2, 3)$ and $\bar{b} = (-2, 1, -2)$. $\dfrac{(-7, -4, 5)}{\sqrt{90}}$
* Find the unit vector perpendicular to both vectors $\bar{a} = (3, 1, 2)$ and $\bar{b} = (2, -2, 4)$
* If $\bar{a} = 2i - 3j + 4k$ and $\bar{b} = i - j + k$ then find unit vector perpendicular to $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$. $\dfrac{(-2, -4, -2)}{\sqrt{24}}$
* If $\bar{x} = (1, 1, 1)$ and $\bar{y} = (2, -1, -1)$ then P. T. $\bar{x}$ is perpendicular to $\bar{y}$. Also find unit vector perpendicular to both $\bar{x}$ and $\bar{y}$.

* **Work done by force** :- (W)

$\boxed{W = F \cdot d}$, where F = Total force $F_1 + F_2 + F_3 + \dots$

d = Displacement $d_2 - d_1$

* The forces $3i - 2j + k$ and $-i - j + 2k$ act on a particle and particle moves from the point $(2, 2, -3)$ to the point $(-1, 2, 4)$ under the effect of these forces. Find the work done. ($W = 15$ Units)

* The constant forces (1,2,3) and (3,1,2) act on a particle and particle moves from the point (0,1,−2) to the point (5,1,2). Find the work done. (w 40 units)

* A particle moves from (−1,2,1) to (2,3,−1) under the effect of the forces (1,2,1) and (2,−1,0). Find work done. (w = 8 units)

* The constant forces (1,2,3), (−1,2,3) and (−1,2,−3) act on a particle and under the effect of these forces particle move to the point (−1,3,2) from the point (0,1,−2). Find work done.

(w = 25 units).

* The constant forces (1,2,3) and (3,1,1) act on a particle and particle moves from the point (0,1,−2) to the point (5,1,2). Find the work done. w = 36 unit.

* The constant forces (1,−1,1) (1,1,−3) and (4,5,−6) act on a particle and particle moves from the point (3,−2,1) to the point (1,3,−4). Find work done. (w = 53 units)

* <u>Moment of force</u> :-

→ The moment of a force $\bar{F}$ about a point $A(\bar{a})$ is a vector which is given by $\overrightarrow{AP} \times \bar{F}$, where $P(\bar{p})$ is a point on the line of force $\bar{F}$.

→ Magnitude of the moment of a force $\bar{F}$ about a point $A(\bar{a})$ on a particle $P(\bar{p}) = |\overrightarrow{AP} \times \bar{F}|$.

passing through the point
acting at the point } P.   about the point → A

→ $AP = OP - OA^{(\bar{p}-\bar{a})} \rightarrow \overrightarrow{AP} \times \bar{F}$

* A force $\bar{F} = 2i + j + k$ is acting at the point $(-3,2,1)^{P}$. Find the magnitude of the moment of force about the point $(2,1,2)^{A}$.

$[\overrightarrow{AP} \times \bar{F} = (2,3,-7)$, $|\overrightarrow{AP} \times \bar{F}| = \sqrt{62}]$

* A force $3i - j + 2k$ is acting at the point (1,2,−1). Find the moment of force about the point (3,0,1)

$[\overrightarrow{AP} \times \bar{F} = (2, -2, -4)$