

Unit-4 Co-ordinate Geometry - 14 Marks

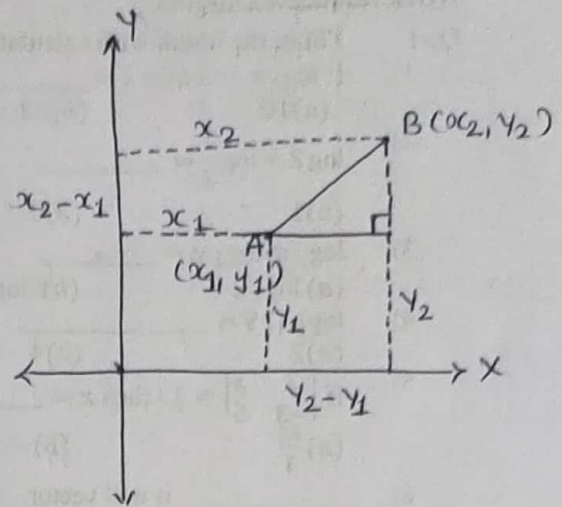
* Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ then

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



* Find the distance between the points $(4, 5)$ and $(7, 3)$.

* Find the distance between the points $(1, 3)$ and $(0, -4)$.

* Find the distance between the points $(1, 1)$ and $(2, -1)$.

* Find the distance between the points $(7, -5)$ and $(3, -2)$.

* Find the distance between the points $(-1, 2)$ and $(-7, 6)$.

* $d[(3, 2), (-1, 1)] = \underline{\hspace{2cm}}$

* If P is the mid point of a line segment AB for the points $A(-2, -1)$ and $B(4, 3)$ then find P .

* If the mid point of line segment AB is $(1, 1)$ and $B(4, 3)$ then find co-ordinates of A .

* If the distance between the point $(5, 7)$ and $(-3, m)$ is 10 then find the value of m .

* Co-linearity

$$D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If $D=0$ then given three vertices are co-linear.

$(3, 2)$ $(5, 4)$ $(7, 6)$

$(1, 0)$ $(0, 1)$ $(-1, 2)$

$(-3, -2)$ $(5, 2)$ $(9, 4)$

→ Using Distance Formula

$$AB = AC + BC$$

$$AC = AB + BC$$

$$BC = AB + AC$$

* Prove that $(3,2)$, $(5,4)$ and $(7,6)$ are co-linear.

* prove that $(1,0)$, $(0,1)$ and $(-1,2)$ are co-linear.

* prove that $(-3,-2)$, $(5,2)$ and $(9,4)$ are co-linear.

* prove that $(a,b+c)$, $(b,c+a)$, and $(c,a+b)$ are co-linear.

* If three point $(-k,1)$, $(k,3)$ and $(6,5)$ are co-linear then find value of k .

* Generalized equation of line $ax + by + c = 0$

→ x-intercept = $-\frac{c}{a}$

→ y-intercept = $-\frac{c}{b}$

* To find slope (m)

→ Given two points (x_1, y_1) & (x_2, y_2) .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

→ Given angle θ then $m = \tan \theta$

→ Given line $ax + by + c = 0$ then $m = -\frac{a}{b}$.

* Find the slope of line passing through the point $(3, 2)$ and $(1, 4)$.

* Find the slope of line passing through the point $(8, 5)$ and $(1, -2)$.

* Find the slope of line passing through the point $(1, 3)$ and $(4, -5)$.

* Find slope and intercepts of the line $4x + 3y - 7 = 0$.

* Find slope and intercepts of the line (i) $2x - 3y + 5 = 0$

(ii) $3x + 5 = 0$

(iii) $2y - 3x + 4 = 0$

* Find slope and intercepts of line (i) $2x + y - 8 = 0$

(ii) $2x - 5y + 3 = 0$ (iii) $2x + 3y - 4 = 0$

* Find slope of a line $(\cos \alpha)x + (\sin \alpha)y = 5$.

* To find Equation of line:-

→ Given two points (x_1, y_1) & (x_2, y_2) then eqⁿ $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$.

* $(2, 1)$ $(-1, 3)$

* $(2, 3)$ $(3, -1)$

* $(1, 6)$ $(-2, 5)$

→ Given point (x_1, y_1) and slope (m) then eqⁿ $(-2, -3)$, $3/2$

$y - y_1 = m(x - x_1)$ (point-slope equation)

→ Given slope (m) and intercept (c) on y-axis, then eqⁿ

$y = mx + c$.

- * Find the eqⁿ of line passing through the points $(2, 1)$ & $(-1, 3)$.
($2x + 3y - 7 = 0$)
- * Find the eqⁿ of line passing through the points $(2, 3)$ & $(3, -1)$.
($4x + y - 11 = 0$)
- * Find the eqⁿ of line passing through the points $(1, 6)$ & $(-2, 5)$.
- * Find the eqⁿ of line passing through the point $(-2, -3)$ and having slope $\frac{3}{2}$.
($3x - 2y = 0$)
- * Find the eqⁿ of line passing through the point $(2, 5)$ and having slope $-\frac{1}{2}$.

* Angle between two lines:-

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\therefore \theta = \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|,$$

where m_1 = slope of first line

m_2 = slope of second line

θ = Angle between two lines

- * The angle between two line is 45° . If the slope of first line is $\frac{2}{3}$. Find slope of other line.
($m_2 = 5$ or $-\frac{1}{5}$)
- * Find the angle between two straight line $x + y = 0$ and $x - y = 0$.
($\theta = \frac{\pi}{2}$)
- * Find the angle between two straight line $\sqrt{3}x - y + 1 = 0$ and $x - \sqrt{3}y + 2 = 0$.
($\theta = \frac{\pi}{6}$)
- * Find the angle between two straight line $x + y + 1 = 0$ and $2x + 3y + 4 = 0$.
($\theta = \frac{\pi}{4}$)

* Condition of two parallel line $m_1 = m_2$

* Condition of two perpendicular line $m_1 \cdot m_2 = -1$

* Prove that line $3x + 2y + 1 = 0$ and $6x + 4y + 3 = 0$ are parallel.

* Prove that line $7x + y - 1 = 0$ and $3x - 21y + 2 = 0$ are perpendicular.

* Check the line $3x + 2y + 1 = 0$ and $2x - 3y + 7 = 0$ are parallel or perpendicular.

* If two lines $3x + 4my + 8 = 0$ and $3my - 9x + 10 = 0$ are perpendicular to each other then find value of m .

* If two lines $5x - py = 3$ and $2x + 3y = 4$ are perpendicular to each other then find value of p .

* If two lines $3mx - 2my - 10 = 0$ and $(5m + 2)x - 4my - 28 = 0$ are parallel to each other then find value of m . ($m = 2$)

$(3, 5) \quad (-1, 2) \quad (1, -2) \quad (5, 6) \quad (-1, 1)$

* General eqⁿ of line parallel to line $ax + by + c = 0$ is $ax + by + k = 0$

* General eqⁿ of line perpendicular to line $ax + by + c = 0$ is

$$bx - ay + k = 0$$

+	+	-
-	-	-
+	-	+
-	+	+

* Find the equation of line parallel to the line $2x + y - 1 = 0$ and passing through the point $(4, 5)$. ($2x + y - 13 = 0$)

* Find the equation of line parallel to the line $4x - 3y + 7 = 0$ and passing through the point $(4, 3)$. ($4x - 3y - 7 = 0$)

* Find the equation of line perpendicular to the line $4x - y + 5 = 0$ and passing through the point $(1, -2)$. ($x + 4y + 7 = 0$)

* Find the equation of line perpendicular to the line $x - 3y + 3 = 0$ and passing through the point $(-1, 2)$. ($3x + y + 1 = 0$)

* Find the equation of line perpendicular to the line $-3x + 4y + 7 = 0$ and passing through the point $(4, 3)$. ($4x + 3y - 25 = 0$)

* Find the equation of line perpendicular to the line $3x - 2y + 4 = 0$ and passing through the point $(1, 3)$. ($2x + 3y - 11 = 0$)

* Find the equation of line perpendicular to the line $y - 4x + 1 = 0$ and passing through the point $(2, 1)$. ($x + 4y - 6 = 0$)

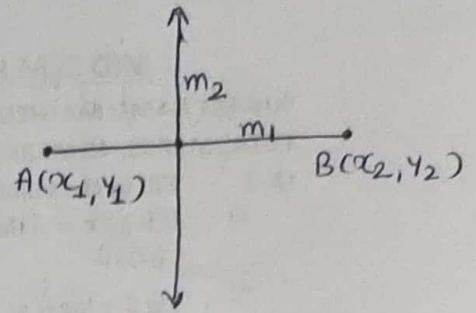
* Perpendicular Bisector

→ Mid point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

→ slope $\frac{y_2 - y_1}{x_2 - x_1}$

→ \perp^{lar} line slope

→ point-slope equation $y - y_1 = m(x - x_1)$.



* Find the equation of line which is perpendicular bisector of line joining points $(8, -2)$ and $(6, 4)$. $(x - 3y - 4 = 0)$

* Find the equation of line which is perpendicular bisector of line joining points $(3, 5)$ and $(1, 1)$. $(x + 2y - 8 = 0)$

* Find the equation of line which is perpendicular bisector of line joining points $(-1, 2)$ and $(1, -2)$. $(x - 2y = 0)$

* Find the equation of line which is perpendicular bisector of line joining points $(5, 6)$ and $(-1, 1)$. $(12x + 10y - 59 = 0)$

CIRCLE

$$CP = r$$

$$\therefore CP^2 = r^2$$

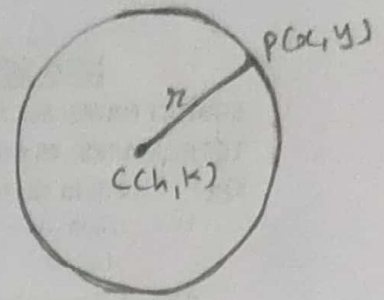
$$\therefore (x-h)^2 + (y-k)^2 = r^2$$

Centre point $C(h, k)$ and radius r
then eqn of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

→ If centre point $C(0, 0)$ and radius r then eqn of circle is

$$x^2 + y^2 = r^2$$



* Find equation of circle having centre $(6, 7)$ and radius 5.

$$(x^2 + y^2 - 12x - 14y + 60 = 0)$$

* Find equation of circle having centre $(-2, 5)$ and radius 4.

$$(x^2 + y^2 + 4x - 10y + 13 = 0)$$

* Find the equation of circle having centre $(-4, 3)$ and tangent to x-axis.

$$(x^2 + y^2 + 4x - 6y + 4 = 0) \quad (r=3)$$

* Find the equation of circle having centre $(3, -4)$ and tangent to y-axis.

$$(x^2 + y^2 - 6x + 8y + 16 = 0) \quad (r=3)$$

* Find the equation of circle having centre $(1, 1)$ and passing through $(-2, 4)$.

$$(x^2 + y^2 - 2x - 2y - 16 = 0)$$

* Find the equation of circle having centre $(4, 3)$ and passing through $(7, -2)$.

$$(x^2 + y^2 - 8x - 6y - 9 = 0)$$

* Find the equation of circle having centre $(2, 3)$ and passing through $(3, 4)$.

$$(x^2 + y^2 - 4x - 6y + 11 = 0)$$

* Find the equation of circle having centre $(3, 4)$ and passing through origin.

$$(x^2 + y^2 - 6x - 8y = 0)$$

* Centre $(-2, 5)$ passing through the intersection of lines $2x + y - 3 = 0$ and $x - 3y + 2 = 0$

$$x = 1, y = 1, r^2 = 25 \quad x^2 + y^2 + 4x - 12y + 4 = 0$$

* General equation of circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\text{centre} = (-g, -f) \quad \text{radius } r = \sqrt{g^2 + f^2 - c}$$

$$\rightarrow x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore x^2 + 2gx + g^2 + y^2 + 2fy + f^2 + c - g^2 - f^2 = 0$$

$$\therefore (x+g)^2 + (y+f)^2 = g^2 + f^2 - c$$

$$\therefore (x - (-g))^2 + (y - (-f))^2 = g^2 + f^2 - c$$

$$\text{comparing with } (x-h)^2 + (y-k)^2 = r^2$$

$$\text{then we have } C(h, k) = (-g, -f) \text{ and } r^2 = g^2 + f^2 - c$$

$$r^2 = g^2 + f^2 - c$$

* Find centre and radius of circle $x^2 + y^2 - 2x + 4y - 1 = 0$

$$(\text{centre } (1, -2), r = \sqrt{6})$$

* Find centre and radius of circle $x^2 + y^2 - 4x - 6y - 4 = 0$

$$(\text{centre } (2, 3), r = \sqrt{17})$$

* Find centre and radius of circle $4x^2 + 4y^2 + 8x - 12y - 3 = 0$

$$(\text{centre } (-1, 3/2), r = 2)$$

* Find centre and radius of circle $2x^2 + 2y^2 + 4x + 6y - 7 = 0$

$$(\text{centre } (-1, -3/2), r = \sqrt{24}/2)$$

* Find centre and radius of circle $36x^2 + 36y^2 + 24x - 36y - 23 = 0$

$$(\text{centre } (-1/3, 1/2), r = 1)$$

* If radius of a circle $x^2 + y^2 - 4x - 8y + k = 0$ is 4 unit.

then find value of k.

$$(k = 4)$$

* If radius of a circle $2x^2 + 2y^2 - 4x - 8y + k = 0$ is 4 unit.

then find value of k.

$$(k = -22)$$

* If radius of a circle $x^2 + y^2 - 4x - 4y + k = 0$ is 4 unit

then find value of k.

$$(k = -8)$$

* Find the eqn of circle having centre $(-2, 5)$ and passing through the intersection of lines $2x + y - 3 = 0$ and $x - 3y + 2 = 0$.

$$(x = 1, y = 1, r^2 = 25)$$

$$x^2 + y^2 + 4x - 12y + 4 = 0$$

* Equation of tangent and Normal to the circle $x^2 + y^2 = r^2$

Equation of tangent $xx_1 + yy_1 = r^2$

Equation of Normal $\frac{x}{x_1} = \frac{y}{y_1}$

* Equation of tangent and Normal to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

→ Equation of tangent

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$$

→ Equation of Normal

$$\frac{x - x_1}{x_1 + g} = \frac{y - y_1}{y_1 + f}$$

* Find the eqⁿ of tangent and Normal to the circle $x^2 + y^2 = 52$ at point $(-6, 4)$. [Tangent $6x - 4y + 52 = 0$
 $2x + 3y = 0$]

* Find the eqⁿ of tangent and Normal to the circle $x^2 + y^2 - 6x + 10y + 21 = 0$ at point $(1, -2)$.

$$\begin{aligned} \text{[Tangent } 2x - 3y - 8 &= 0 \\ \text{Normal } 3x + 2y + 1 &= 0] \end{aligned}$$

* Find the eqⁿ of tangent and Normal to the circle $x^2 + y^2 - 4x + 2y + 3 = 0$ at point $(1, -2)$.
[Tangent $x + y + 1 = 0$, Normal $x - y - 3 = 0$]

* Find the eqⁿ of tangent and Normal to the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ at point $(-2, 2)$.
[Tangent $3x - 4y + 14 = 0$, Normal $4x + 3y + 2 = 0$]

* Find the eqⁿ of tangent and Normal to the circle $x^2 + y^2 - 2x - 7 = 0$ at point $(2, 3)$.
[Tangent $x + y - 5 = 0$, Normal $x - y + 1 = 0$]

* Find the eqⁿ of tangent and Normal to the circle $2x^2 + 2y^2 + 3x - 4y + 1 = 0$ at point $(-1, 2)$.
[Tangent $x - 4y + 9 = 0$, Normal $4x + y + 2 = 0$]