LOGARITHM

* Prove that

$$\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2.$$

$$L.H.S = \frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24}$$

$$= \frac{1}{\frac{\log 24}{\log 6}} + \frac{1}{\frac{\log 24}{\log 12}} + \frac{1}{\frac{\log 24}{\log 8}}$$

$$= \frac{\log 6}{\log 24} + \frac{\log 12}{\log 24} + \frac{\log 8}{\log 24}$$

$$= \frac{\log 6 + \log 12 + \log 8}{\log 24}$$

$$= \frac{\log(6 \times 8 \times 12)}{\log 24}$$

$$= \frac{\log(576)}{\log 24}$$

$$= \frac{\log 24^2}{\log 24}$$

$$= \frac{2 \log 24}{\log 24}$$

$$= 2 = R.H.S.$$

* Prove that

$$\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2.$$

$$L.H.S = \frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$$

$$= \frac{1}{\frac{\log xyz}{\log xy}} + \frac{1}{\frac{\log xyz}{\log yz}} + \frac{1}{\frac{\log xyz}{\log zx}}$$

$$= \frac{\log xy}{\log xyz} + \frac{\log yz}{\log xyz} + \frac{\log zx}{\log xyz}$$

$$= \frac{\log xy + \log yz + \log zx}{\log xyz}$$

$$= \frac{\log (xy \cdot yz \cdot zx)}{\log xyz}$$

$$= \frac{\log (xyz)^2}{\log xyz}$$

$$= \frac{2 \log xyz}{\log xyz}$$

$$= 2 = R.H.S.$$

* Prove that

$$\log_p a + \log_{p^2} a^2 + \log_{p^3} a^3 + \log_{p^4} a^4 = 4 \log_p a$$

$$L.H.S = \log_p a + \log_{p^2} a^2 + \log_{p^3} a^3 + \log_{p^4} a^4$$

$$= \frac{\log a}{\log p} + \frac{\log a^2}{\log p^2} + \frac{\log a^3}{\log p^3} + \frac{\log a^4}{\log p^4}$$

$$= \frac{\log a}{\log p} + \frac{2 \log a}{2 \log p} + \frac{3 \log a}{3 \log p} + \frac{4 \log a}{4 \log p}$$

$$= \frac{\log a}{\log p} + \frac{\log a}{\log p} + \frac{\log a}{\log p} + \frac{\log a}{\log p}$$

$$= 4 \frac{\log a}{\log p}$$

$$= 4 \log_p a = R.H.S.$$

* Prove that

$$\log\left[\sqrt{x^2+1} + x\right] + \log\left[\sqrt{x^2+1} - x\right] = 0.$$

$$L.H.S = \log\left[\sqrt{x^2+1} + x\right] + \log\left[\sqrt{x^2+1} - x\right]$$

$$= \log\left[(\sqrt{x^2+1} + x) \times (\sqrt{x^2+1} - x)\right]$$

$$= \log\left[(\sqrt{x^2+1})^2 - (x)^2\right]$$

$$= \log\left[x^2 + 1 - x^2\right]$$

$$= \log 1$$

$$= 0 = R.H.S.$$

* Prove that

$$\frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1} = 1.$$

$$L.H.S = \frac{1}{\log_x yz + 1} + \frac{1}{\log_y zx + 1} + \frac{1}{\log_z xy + 1}$$

$$= \frac{1}{\frac{\log yz}{\log x} + 1} + \frac{1}{\frac{\log zx}{\log y} + 1} + \frac{1}{\frac{\log xy}{\log z} + 1}$$

$$= \frac{1}{\frac{\log yz + \log x}{\log x}} + \frac{1}{\frac{\log zx + \log y}{\log y}} + \frac{1}{\frac{\log xy + \log z}{\log z}}$$

$$= \frac{\log x}{\log xyz} + \frac{\log y}{\log xyz} + \frac{\log z}{\log xyz}$$

$$= \frac{\log x + \log y + \log z}{\log xyz}$$

$$= \frac{\log xyz}{\log xyz}$$

$$= 1 = R.H.S.$$

* If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that $a^2+b^2 = 2ab$ or $a=b$.

$\Rightarrow \log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow 2\log\left(\frac{a+b}{2}\right) = \log(ab)$

$\Rightarrow \log\left(\frac{a+b}{2}\right)^2 = \log(ab)$

$\Rightarrow \left(\frac{a+b}{2}\right)^2 = ab$

$\Rightarrow \frac{a^2+2ab+b^2}{4} = ab$

$\Rightarrow a^2+2ab+b^2 = 4ab$

$\Rightarrow a^2+b^2 = 4ab-2ab$

$\Rightarrow a^2+b^2 = 2ab.$

$\Rightarrow a^2-2ab+b^2 = 0$

$\Rightarrow (a-b)^2 = 0$

$\Rightarrow a-b = 0$

$\Rightarrow a = b.$

* If $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that $a^2+b^2 = 6ab$ or $\frac{a}{b}+\frac{b}{a} = 6$

$\Rightarrow \log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow 2\log\left(\frac{a-b}{2}\right) = \log(ab)$

$\Rightarrow \log\left(\frac{a-b}{2}\right)^2 = \log(ab)$

$\Rightarrow \left(\frac{a-b}{2}\right)^2 = ab$

$\Rightarrow \frac{a^2-2ab+b^2}{4} = ab$

$\Rightarrow a^2-2ab+b^2 = 4ab$

$\Rightarrow a^2+b^2 = 4ab+2ab$

$\Rightarrow a^2+b^2 = 6ab.$

$\Rightarrow \frac{a^2}{ab}+\frac{b^2}{ab} = \frac{6ab}{ab}$

$\Rightarrow \frac{a}{b}+\frac{b}{a} = 6.$

* If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then prove that $x^2+y^2 = 7xy$ or $\frac{x}{y}+\frac{y}{x} = 7$

$\Rightarrow \log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$

$\Rightarrow 2\log\left(\frac{x+y}{3}\right) = \log(xy)$

$\Rightarrow \log\left(\frac{x+y}{3}\right)^2 = \log(xy)$

$\Rightarrow \left(\frac{x+y}{3}\right)^2 = xy$

$\Rightarrow \frac{x^2+2xy+y^2}{9} = xy$

$\Rightarrow x^2+2xy+y^2 = 9xy$

$\Rightarrow x^2+y^2 = 9xy-2xy$

$\Rightarrow x^2+y^2 = 7xy$

$\Rightarrow \frac{x^2}{xy}+\frac{y^2}{xy} = \frac{7xy}{xy}$

$\Rightarrow \frac{x}{y}+\frac{y}{x} = 7.$

FUNCTION

* If $f(x) = \log x$ then Prove that
  (i) $f(x \cdot y) = f(x) + f(y)$
  (ii) $f\left(\frac{x}{y}\right) = f(x) - f(y)$

$\Rightarrow \quad f(x) = \log x$

(i) L.H.S $= f(x \cdot y) = \log(x \cdot y)$
$= \log x + \log y$
$= f(x) + f(y) =$ R.H.S.

(ii) L.H.S. $= f\left(\frac{x}{y}\right) = \log\left(\frac{x}{y}\right)$
$= \log x - \log y$
$= f(x) - f(y) =$ R.H.S.

* If $f(x) = e^x$ then prove that
  (i) $f(x) \cdot f(y) = f(x+y)$
  (ii) $f(x) \div f(y) = f(x-y)$

$\Rightarrow \quad f(x) = e^x$
$\Rightarrow f(y) = e^y$

(i) L.H.S. $= f(x) \cdot f(y)$
$= e^x \cdot e^y$
$= e^{x+y}$
$= f(x+y) =$ R.H.S.

(ii) L.H.S. $= f(x) \div f(y)$
$= \frac{e^x}{e^y}$
$= e^{x-y}$
$= f(x-y) =$ R.H.S.

* If $f(x) = \frac{1-x}{1+x}$ then Prove that
  (i) $f(x) + f\left(\frac{1}{x}\right) = 0$
  (ii) $f(x) - f\left(\frac{1}{x}\right) = 2f(x)$
  (iii) $f(x) \cdot f(-x) = 1.$

$\Rightarrow \quad f(x) = \frac{1-x}{1+x}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}}$

$= \frac{\frac{x-1}{x}}{\frac{x+1}{x}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{x-1}{x+1}$

(i) L.H.S. $= f(x) + f\left(\frac{1}{x}\right)$
$= \frac{1-x}{1+x} + \frac{x-1}{x+1}$
$= \frac{1-x+x-1}{1+x}$
$= \frac{0}{1+x}$
$= 0 =$ R.H.S.

(ii) L.H.S $= f(x) - f\left(\frac{1}{x}\right)$
$= \frac{1-x}{1+x} - \frac{x-1}{x+1}$
$= \frac{1-x-x+1}{1+x}$
$= \frac{2-2x}{1+x}$
$= \frac{2(1-x)}{1+x}$
$= 2f(x) =$ R.H.S.

$\Rightarrow \quad f(x) = \frac{1-x}{1+x}$
$\Rightarrow f(-x) = \frac{1-(-x)}{1+(-x)} = \frac{1+x}{1-x}$

(iii) L.H.S. $= f(x) \cdot f(-x)$
$= \frac{1-x}{1+x} \cdot \frac{1+x}{1-x}$
$= 1 =$ R.H.S..

* If $f(x) = \frac{1-x}{1+x}$ then prove that
  $f\left(\frac{x+y}{1+xy}\right) = f(x) \cdot f(y)$

$\Rightarrow \quad f(x) = \frac{1-x}{1+x}$

$\Rightarrow$ L.H.S $= f\left(\frac{x+y}{1+xy}\right) = \frac{1 - \frac{x+y}{1+xy}}{1 + \frac{x+y}{1+xy}}$

$= \frac{\frac{1+xy-x-y}{1+xy}}{\frac{1+xy+x+y}{1+xy}}$

$$= \frac{1+xy-x-y}{1+xy+x+y}$$

$$= \frac{(1-x)-y(1-x)}{(1+x)+y(1+x)}$$

$$= \frac{(1-x)(1-y)}{(1+x)(1+y)}$$

$$= f(x)\cdot f(y) = R.H.S.$$

* If $f(x) = \frac{a+bx}{b+ax}$ then Prove that $f(x) \cdot f\left(\frac{1}{x}\right) = 1.$

$$\Rightarrow \quad f(x) = \frac{a+bx}{b+ax}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{a+b\left(\frac{1}{x}\right)}{b+a\left(\frac{1}{x}\right)}$$

$$= \frac{a+\frac{b}{x}}{b+\frac{a}{x}}$$

$$= \frac{\frac{ax+b}{x}}{\frac{bx+a}{x}}$$

$$= \frac{ax+b}{bx+a}$$

$$L.H.S. = f(x)\cdot f\left(\frac{1}{x}\right)$$

$$= \frac{a+bx}{b+ax} \cdot \frac{ax+b}{bx+a}$$

$$= 1 = R.H.S.$$

* If $f(x) = \log\left(\frac{1-x}{1+x}\right)$ then Prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x).$

$$\Rightarrow \quad f(x) = \log\left(\frac{1-x}{1+x}\right)$$

$$L.H.S. = f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{\frac{1+x^2-2x}{1+x^2}}{\frac{1+x^2+2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$$

$$= \log\left[\frac{(1-x)^2}{(1+x)^2}\right]$$

$$= \log\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\log\left(\frac{1-x}{1+x}\right)$$

$$= 2f(x) = R.H.S.$$

* If $f(x) = \log\left(\frac{x}{x-1}\right)$ then Prove that $f(x) + f(-x) = f(x^2)$

$$\Rightarrow \quad f(x) = \log\left(\frac{x}{x-1}\right)$$

$$\Rightarrow f(-x) = \log\left(\frac{-x}{-x-1}\right)$$

$$= \log\left[\frac{-x}{-(x+1)}\right]$$

$$\Rightarrow f(-x) = \log\left(\frac{x}{x+1}\right)$$

$$L.H.S. = f(x) + f(-x)$$

$$= \log\left(\frac{x}{x-1}\right) + \log\left(\frac{x}{x+1}\right)$$

$$= \log\left(\frac{x}{x-1} \times \frac{x}{x+1}\right)$$

$$= \log\left(\frac{x^2}{x^2-1}\right)$$

$$= f(x^2) = R.H.S.$$

* If $f(x) = \log\left(\frac{x}{x-1}\right)$ then Prove that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$

$$\Rightarrow \quad f(x) = \log\left(\frac{x}{x-1}\right)$$

$$\Rightarrow f(a+1) = \log\left(\frac{a+1}{a+1-1}\right)$$

$$\Rightarrow f(a+1) = \log\left(\frac{a+1}{a}\right)$$

$$\Rightarrow f(a) = \log\left(\frac{a}{a-1}\right)$$

$$L.H.S. = f(a+1) + f(a)$$

$$= \log\left(\frac{a+1}{a}\right) + \log\left(\frac{a}{a-1}\right)$$

$$= \log\left(\frac{a+1}{a} \times \frac{a}{a-1}\right)$$

$$= \log\left(\frac{a+1}{a-1}\right) = R.H.S.$$

* If $f(x) = a^x$ then prove that $f(x+1) - f(x) = (a-1) \cdot f(x)$

$\Rightarrow f(x) = a^x$

$\Rightarrow f(x+1) = a^{x+1}$

$L.H.S. = f(x+1) - f(x)$

$= a^{x+1} - a^x$

$= a^x \cdot a^1 - a^x$

$= a^x (a-1)$

$= (a-1) \cdot f(x) = R.H.S.$

DETERMINANT

* If $\begin{vmatrix} x-1 & 2 & 1 \\ x & 1 & x+1 \\ 1 & 1 & 0 \end{vmatrix} = 4$ then find 'x'.

$\Rightarrow (x-1)\begin{vmatrix} 1 & x+1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} x & x+1 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} x & 1 \\ 1 & 1 \end{vmatrix} = 4$

$\Rightarrow (x-1)(0-(x+1)) - 2(0-(x+1)) + 1(x-1) = 4$

$\Rightarrow -(x-1)(x+1) + 2(x+1) + x - 1 = 4$

$\Rightarrow -(x^2-1) + 2x + 2 + x - 1 = 4$

$\Rightarrow -x^2 + 1 + 3x + 1 - 4 = 0$

$\Rightarrow -x^2 + 3x - 2 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\Rightarrow x-2 = 0$ or $x-1 = 0$

$\Rightarrow x = 2$ or $x = 1$.

* If $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$ then find 'x'.

$\Rightarrow (x-2)\begin{vmatrix} x & -2 \\ 0 & 4 \end{vmatrix} - 2\begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} -1 & x \\ 2 & 0 \end{vmatrix} = 0$

$\Rightarrow (x-2)(4x-0) - 2(-4+4) + 2(0-2x) = 0$

$\Rightarrow 4x^2 - 8x - 4x = 0$

$\Rightarrow 4x^2 - 12x = 0$

$\Rightarrow 4x(x-3) = 0$

$\Rightarrow 4x = 0$ or $x-3 = 0$

$\Rightarrow x = 0$ or $x = 3$.

* If $\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$ then prove that $a = b$ or $a = -2b$.

$\Rightarrow a\begin{vmatrix} a & b \\ b & a \end{vmatrix} - b\begin{vmatrix} b & b \\ b & a \end{vmatrix} + b\begin{vmatrix} b & a \\ b & b \end{vmatrix} = 0$

$\Rightarrow a(a^2-b^2) - b(ab-b^2) + b(b^2-ab) = 0$

$\Rightarrow a[(a-b)(a+b)] - b^2(a-b) - b^2(a-b) = 0$

$\Rightarrow (a-b)[a(a+b) - b^2 - b^2] = 0$

$\Rightarrow (a-b)[a^2 + ab - b^2 - b^2] = 0$

$\Rightarrow (a-b)[a^2 - b^2 + ab - b^2] = 0$

$\Rightarrow (a-b)[(a-b)(a+b) + b(a-b)] = 0$

$\Rightarrow (a-b) \cdot (a-b)[a+b+b] = 0$

$\Rightarrow (a-b)^2 \cdot (a+2b) = 0$

$\Rightarrow (a-b)^2 = 0$ or $a+2b = 0$

$\Rightarrow a-b = 0$ or $a = -2b$

$\Rightarrow a = b$ or $a = -2b$.

LIMIT

* Evaluate:- $\lim\limits_{x\to 1} \dfrac{x^2-4x+3}{x^2+2x-3}$

$$= \lim_{x\to 1} \frac{(x-1)(x-3)}{(x-1)(x+3)}, \quad x \neq 1$$

$$= \lim_{x\to 1} \frac{x-3}{x+3}$$

$$= \frac{1-3}{1+3}$$

$$= -\frac{2}{4}$$

$$= -\frac{1}{2}.$$

* Evaluate:- $\lim\limits_{x\to 1} \dfrac{x^3-x^2+x-1}{x^2-1}$

$x^3-x^2+x-1 = (x-1)(x^2+1)$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 1 & -1 \\ & 0 & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & \underline{0} \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to 1} \frac{(x-1)(x^2+1)}{(x-1)(x+1)}, \quad x \neq 1$$

$$= \lim_{x\to 1} \frac{x^2+1}{x+1}$$

$$= \frac{1+1}{1+1} = \frac{2}{2} = 1.$$

* Evaluate:- $\lim\limits_{x\to -2} \dfrac{x^3+2x^2+x+2}{x^2+x-2}$

$x^3+2x^2+x-2 = (x+2)(x^2+1)$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & 0 & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & \underline{0} \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to -2} \frac{(x+2)(x^2+1)}{(x+2)(x-1)}, \quad x \neq -2$$

$$= \lim_{x\to -2} \frac{x^2+1}{x-1}$$

$$= \frac{4+1}{-2-1}$$

$$= \frac{5}{-3} = -\frac{5}{3}.$$

* Evaluate:- $\lim\limits_{x\to 2} \dfrac{x^3-2x^2+x-2}{x^2-x-2}$

$x^3-2x^2+x-2 = (x-2)(x^2+1)$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 1 & -2 \\ & 0 & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & \underline{0} \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+1)}{(x-2)(x+1)}, \quad x \neq 2$$

$$= \lim_{x\to 2} \frac{x^2+1}{x+1}$$

$$= \frac{4+1}{2+1}$$

$$= \frac{5}{3}.$$

* Evaluate:- $\lim\limits_{x\to 2} \dfrac{x^3-x^2-5x+6}{x^2-5x+6}$

$x^3-x^2-5x+6 = (x-2)(x^2+x-3)$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -5 & 6 \\ & 0 & 2 & 2 & -6 \\ \hline & 1 & 1 & -3 & 0/\!/ \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+x-3)}{(x-2)(x-3)}, \quad x \neq 2$$

$$= \lim_{x\to 2} \frac{x^2+x-3}{x-3}$$

$$= \frac{4+2-3}{2-3}$$

$$= \frac{3}{-1} = -3.$$

* Evaluate:- $\lim\limits_{x\to -1} \dfrac{2x^3+5x^2+4x+1}{3x^3+5x^2+x-1}$

$2x^3+5x^2+4x+1 = (x+1)(2x^2+3x+1)$

$$\begin{array}{r|rrrr} -1 & 2 & 5 & 4 & 1 \\ & 0 & -2 & -3 & -1 \\ \hline & 2 & 3 & 1 & 0/\!/ \\ & x^2 & x & 1 & \end{array}$$

$3x^3+5x^2+x-1 = (x+1)(3x^2+2x-1)$

$$\begin{array}{r|rrrr} -1 & 3 & 5 & 1 & -1 \\ & 0 & -3 & -2 & 1 \\ \hline & 3 & 2 & -1 & 0/\!/ \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to -1} \frac{(x+1)(2x^2+3x+1)}{(x+1)(3x^2+2x-1)}, \quad x \neq -1$$

$$= \lim_{x\to -1} \frac{2x^2+3x+1}{3x^2+2x-1}$$

$$= \lim_{x\to -1} \frac{(x+1)(2x+1)}{(x+1)(3x-1)}, \quad x \neq -1$$

$$= \lim_{x\to -1} \frac{2x+1}{3x-1}$$

$$= \frac{2(-1)+1}{3(-1)-1} = \frac{-2+1}{-3-1} = \frac{-1}{-4} = \frac{1}{4}.$$

* Evaluate:- $\lim\limits_{x\to 1} \dfrac{x^3-3x+2}{7x^2-6x-1}$

$x^3-3x+2 = (x-1)(x^2+x-2)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0/\!/ \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to 1} \frac{(x-1)(x^2+x-2)}{(x-1)(7x+1)}, \quad x \neq 1$$

$$= \lim_{x\to 1} \frac{x^2+x-2}{7x+1}$$

$$= \frac{1+1-2}{7+1} = \frac{0}{8} = 0.$$

* Evaluate:- $\lim_{x\to 2} \frac{x^3-6x^2+11x-6}{x^3-8}$

$x^3-6x^2+11x-6 = (x-2)(x^2-4x+3)$

$$\begin{array}{c|cccc} 2 & 1 & -6 & 11 & -6 \\ & 0 & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & 0 \\ & x^2 & x & 1 & \end{array}$$

$x^3-8 = (x-2)(x^2+2x+4)$

$$\begin{array}{c|cccc} 2 & 1 & 0 & 0 & -8 \\ & 0 & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \\ & x^2 & x & 1 & \end{array}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2-4x+3)}{(x-2)(x^2+2x+4)}, x\neq 2$$

$$= \lim_{x\to 2} \frac{x^2-4x+3}{x^2+2x+4}$$

$$= \frac{4-8+3}{4+4+4} = \frac{-1}{12}.$$

* Evaluate:- $\lim_{x\to 0} \frac{\sqrt{9+x}-3}{x}$

$$= \lim_{x\to 0} \frac{\sqrt{9+x}-3}{x} \times \frac{\sqrt{9+x}+3}{\sqrt{9+x}+3}$$

$$= \lim_{x\to 0} \frac{(\sqrt{9+x})^2-(3)^2}{x\cdot(\sqrt{9+x}+3)}$$

$$= \lim_{x\to 0} \frac{9+x-9}{x\cdot(\sqrt{9+x}+3)}$$

$$= \lim_{x\to 0} \frac{x}{x\cdot(\sqrt{9+x}+3)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{9+x}+3}$$

$$= \frac{1}{\sqrt{9+0}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}.$$

* Evaluate:- $\lim_{x\to a} \frac{\sqrt{2a-x}-\sqrt{x}}{a-x}$

$$= \lim_{x\to a} \frac{\sqrt{2a-x}-\sqrt{x}}{a-x} \times \frac{\sqrt{2a-x}+\sqrt{x}}{\sqrt{2a-x}+\sqrt{x}}$$

$$= \lim_{x\to a} \frac{(\sqrt{2a-x})^2-(\sqrt{x})^2}{(a-x)(\sqrt{2a-x}+\sqrt{x})}$$

$$= \lim_{x\to a} \frac{2a-x-x}{(a-x)(\sqrt{2a-x}+\sqrt{x})}$$

$$= \lim_{x\to a} \frac{2a-2x}{(a-x)(\sqrt{2a-x}+\sqrt{x})}$$

$$= \lim_{x\to a} \frac{2(a-x)}{(a-x)(\sqrt{2a-x}+\sqrt{x})}$$

$$= \lim_{x\to a} \frac{2}{\sqrt{2a-x}+\sqrt{x}}$$

$$= \frac{2}{\sqrt{2a-a}+\sqrt{a}}$$

$$= \frac{2}{\sqrt{a}+\sqrt{a}}$$

$$= \frac{2}{2\sqrt{a}} = \frac{1}{\sqrt{a}}.$$

* Evaluate:- $\lim_{x\to 0} \frac{3\sin x - \sin 3x}{x^3}$

$$= \lim_{x\to 0} \frac{3\sin x - (3\sin x - 4\sin^3 x)}{x^3}$$

$(\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$

$$= \lim_{x\to 0} \frac{3\sin x - 3\sin x + 4\sin^3 x}{x^3}$$

$$= \lim_{x\to 0} \frac{4\sin^3 x}{x^3}$$

$$= 4 \lim_{x\to 0} \frac{\sin^3 x}{x^3}$$

$$= 4(1) = 4.$$

* Evaluate:- $\lim_{x\to 0} \frac{e^x+\sin x-1}{x}$

$$= \lim_{x\to 0} \frac{(e^x-1)+\sin x}{x}$$

$$= \lim_{x\to 0} \frac{e^x-1}{x} + \lim_{x\to 0} \frac{\sin x}{x}$$

$$= \log_e e + 1$$

$$= 1+1 = 2.$$

* Evaluate:- $\lim_{x\to 0} \frac{e^x+\sin 2x-1}{x}$

$$= \lim_{x\to 0} \frac{(e^x-1)+\sin 2x}{x}$$

$$= \lim_{x\to 0} \frac{e^x - 1}{x} + \lim_{x\to 0} \frac{\sin 2x}{x}$$

$$= \log_e e + \lim_{x\to 0} \frac{\sin 2x}{2x} \times 2$$

$$= 1 + 1.(2)$$

$$= 1+2 = 3.$$

* Evaluate:- $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

$$= \lim_{x\to 0} \frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2.(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2.(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2} \cdot \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$= 1 \cdot \frac{1}{1+\cos(0)}$$

$$= \frac{1}{1+1} = \frac{1}{2}.$$

* Evaluate:- $\lim_{x\to\infty} \sqrt{x^2+x} - x$

$$= \lim_{x\to\infty} \sqrt{x^2+x} - x \times \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+x}\right)^2 - (x)^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2\left(\frac{x^2}{x^2}+\frac{x}{x^2}\right)}+x}$$

$$= \lim_{x\to\infty} \frac{x}{x\sqrt{1+\frac{1}{x}}+x}$$

$$= \lim_{x\to\infty} \frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{1+1} = \frac{1}{2}.$$

* Evaluate:- $\lim_{n\to\infty} \sqrt{n^2+n+1} - n$

$$= \lim_{n\to\infty} \sqrt{n^2+n+1} - n \times \frac{\sqrt{n^2+n+1}+n}{\sqrt{n^2+n+1}+n}$$

$$= \lim_{n\to\infty} \frac{\left(\sqrt{n^2+n+1}\right)^2 - (n)^2}{\sqrt{n^2+n+1}+n}$$

$$= \lim_{n\to\infty} \frac{n^2+n+1-n^2}{\sqrt{n^2+n+1}+n}$$

$$= \lim_{n\to\infty} \frac{n+1}{\sqrt{n^2+n+1}+n}$$

$$= \lim_{n\to\infty} \frac{n\left(\frac{n}{n}+\frac{1}{n}\right)}{\sqrt{n^2\left(\frac{n^2}{n^2}+\frac{n}{n^2}+\frac{1}{n^2}\right)}+n}$$

$$= \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n.\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+n}$$

$$= \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+1\right)}$$

$$= \frac{1+0}{\sqrt{1+0+0}+1}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}.$$

VECTOR

* If $\bar{a} = j + k - i$, $\bar{b} = 2i + j - 3k$ then find $|2\bar{a} + 3\bar{b}|$.

$\Rightarrow \bar{a} = j + k - i = -i + j + k = (-1, 1, 1)$

$\bar{b} = 2i + j - 3k = (2, 1, -3)$.

$\Rightarrow 2\bar{a} + 3\bar{b} = 2(-1, 1, 1) + 3(2, 1, -3)$

$= (-2, 2, 2) + (6, 3, -9)$

$= (-2+6,\ 2+3,\ 2-9)$

$= (4, 5, -7)$

$\Rightarrow |2\bar{a} + 3\bar{b}| = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(4)^2 + (5)^2 + (-7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90} = 3\sqrt{10}$.

* If $\bar{a} = 3i - 2j + k$, $\bar{b} = 2i - 4j - 3k$ and $\bar{c} = -i + 2j + 2k$ then find $|2\bar{a} - 3\bar{b} - 5\bar{c}|$.

$\Rightarrow \bar{a} = (3, -2, 1)$, $\bar{b} = (2, -4, -3)$, $\bar{c} = (-1, 2, 2)$

$\Rightarrow 2\bar{a} - 3\bar{b} - 5\bar{c} = 2(3, -2, 1) - 3(2, -4, -3) - 5(-1, 2, 2)$

$= (6, -4, 2) - (6, -12, -9) - (-5, 10, 10)$

$= (6-6+5,\ -4+12-10,\ 2+9-10)$

$= (5, -2, 1)$

$\Rightarrow |2\bar{a} - 3\bar{b} - 5\bar{c}| = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(5)^2 + (-2)^2 + (1)^2}$

$= \sqrt{25 + 4 + 1}$

$= \sqrt{30}$.

* If $\bar{a} = i - 2j + k$, $\bar{b} = 2i + j + 3k$ and $\bar{c} = -i + 2j - 3k$ then find $|3\bar{a} - 2\bar{b} + \bar{c}|$.

$\Rightarrow \bar{a} = (1, -2, 1)$, $\bar{b} = (2, 1, 3)$, $\bar{c} = (-1, 2, -3)$.

$\Rightarrow 3\bar{a} - 2\bar{b} + \bar{c} = 3(1, -2, 1) - 2(2, 1, 3) + (-1, 2, -3)$

$= (3, -6, 3) - (4, 2, 6) + (-1, 2, -3)$

$= (3-4-1,\ -6-2+2,\ 3-6-3)$

$= (-2, -6, -6)$

$\Rightarrow |3\bar{a} - 2\bar{b} + \bar{c}| = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(-2)^2 + (-6)^2 + (-6)^2}$

$= \sqrt{4 + 36 + 36}$

$= \sqrt{76}$.

* If $\bar{a} = (3, -1, -4)$, $\bar{b} = (-2, 4, -3)$ and $\bar{c} = (-1, 2, -1)$ then find $|3\bar{a} - 2\bar{b} + 4\bar{c}|$.

$\Rightarrow \bar{a} = (3, -1, -4)$, $\bar{b} = (-2, 4, -3)$, $\bar{c} = (-1, 2, -1)$.

$\Rightarrow 3\bar{a} - 2\bar{b} + 4\bar{c} = 3(3, -1, -4) - 2(-2, 4, -3) + 4(-1, 2, -1)$

$= (9, -3, -12) - (-4, 8, -6) + (-4, 8, -4)$

$= (9+4-4,\ -3-8+8,\ -12+6-4)$

$= (9, -3, -10)$

$\Rightarrow |3\bar{a} - 2\bar{b} + 4\bar{c}| = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(9)^2 + (-3)^2 + (-10)^2}$

$= \sqrt{81 + 9 + 100}$

$= \sqrt{190}$.

* If $\bar{x} = (-4, 9, 6)$, $\bar{y} = (0, 7, 10)$ and $\bar{z} = (-1, 6, 6)$ then prove that $(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = 0$.

$\Rightarrow \bar{x} - \bar{z} = (-4, 9, 6) - (-1, 6, 6)$

$= (-4+1,\ 9-6,\ 6-6)$

$= (-3, 3, 0)$

$\Rightarrow \bar{y} - \bar{z} = (0, 7, 10) - (-1, 6, 6)$

$= (0+1,\ 7-6,\ 10-6)$

$= (1, 1, 4)$

L.H.S $= (\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z})$

$= (-3, 3, 0) \cdot (1, 1, 4)$

$= -3 + 3 + 0$

$= 0 =$ R.H.S.

* If $2i - 3j + 5k$ and $Ri - 6j - 8k$ are perpendicular to each other then find the value of 'R'.

$\Rightarrow \bar{a} = (2, -3, 5)$, $\bar{b} = (R, -6, -8)$

$\Rightarrow$ Given that $\bar{a} \perp \bar{b}$, so $\bar{a}\cdot\bar{b} = 0$

$\Rightarrow (2,-3,5)\cdot(R,-6,-8) = 0$

$\Rightarrow 2R + 18 - 40 = 0.$

$\Rightarrow 2R - 22 = 0$

$\Rightarrow 2R = 22$

$\Rightarrow R = \frac{22}{2}$

$\Rightarrow R = 11.$

* For what value of 'p' the vectors $2i+3j-k$ and $pi-j+3k$ are perpendicular to each other?

$\Rightarrow$ Here $\bar{a} = (2,3,-1)$, $\bar{b} = (p,-1,3)$

Given that $\bar{a} \perp \bar{b}$, so $\bar{a}\cdot\bar{b} = 0$

$\Rightarrow (2,3,-1)\cdot(p,-1,3) = 0$

$\Rightarrow 2p - 3 - 3 = 0$

$\Rightarrow 2p - 6 = 0$

$\Rightarrow 2p = 6$

$\Rightarrow p = \frac{6}{2}$

$\Rightarrow p = 3.$

* Find the angle between two vectors $(1,2,3)$ and $(-2,3,1)$.

$\Rightarrow \bar{a} = (1,2,3)$, $\bar{b} = (-2,3,1)$

$\bar{a}\cdot\bar{b} = (1,2,3)\cdot(-2,3,1)$

$= -2 + 6 + 3$

$= 7.$

$|\bar{a}| = \sqrt{x^2+y^2+z^2} = \sqrt{1+4+9} = \sqrt{14}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2} = \sqrt{4+9+1} = \sqrt{14}$

$\Rightarrow \cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$

$= \frac{7}{\sqrt{14}\cdot\sqrt{14}}$

$= \frac{7}{14}$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right).$

* Prove that the angle between two vectors $i+j-k$ and $2i-2j+k$ is $\sin^{-1}\left(\sqrt{\frac{26}{27}}\right)$.

$\Rightarrow \bar{a} = (1,1,-1)$, $\bar{b} = (2,-2,1)$

$\bar{a}\cdot\bar{b} = (1,1,-1)\cdot(2,-2,1)$

$= 2-2-1$

$= -1.$

$|\bar{a}| = \sqrt{x^2+y^2+z^2} = \sqrt{1+1+1} = \sqrt{3}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2} = \sqrt{4+4+1} = \sqrt{9}$

$\Rightarrow \cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$

$= \frac{-1}{\sqrt{3}\cdot\sqrt{9}} = \frac{-1}{\sqrt{27}}.$

Now, $\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{-1}{\sqrt{27}}\right)^2$

$= 1 - \frac{1}{27}$

$= \frac{27-1}{27}$

$\Rightarrow \sin^2\theta = \frac{26}{27}$

$\Rightarrow \sin\theta = \sqrt{\frac{26}{27}}$

$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\frac{26}{27}}\right).$

* Prove that the angle between two vectors $i+2j$ and $i+j+3k$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$.

$\Rightarrow \bar{a} = (1,2,0)$, $\bar{b} = (1,1,3)$

$\bar{a}\cdot\bar{b} = (1,2,0)\cdot(1,1,3)$

$= 1+2+0$

$= 3.$

$|\bar{a}| = \sqrt{x^2+y^2+z^2} = \sqrt{1+4+0} = \sqrt{5}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2} = \sqrt{1+1+9} = \sqrt{11}.$

$\Rightarrow \cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$

$= \frac{3}{\sqrt{5}\cdot\sqrt{11}} = \frac{3}{\sqrt{55}}.$

Now, $\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{3}{\sqrt{55}}\right)^2$

$= 1 - \frac{9}{55}$

$= \frac{55-9}{55}$

$\Rightarrow \sin^2\theta = \frac{46}{55}$

$\Rightarrow \sin\theta = \sqrt{\frac{46}{55}}$

$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$.

* Prove that the angle between two vectors $i+2j-3k$ and $2i+j-k$ is $\sin^{-1}\left(\sqrt{\frac{35}{84}}\right)$.

$\Rightarrow \bar{a} = (1,2,-3)$, $\bar{b} = (2,1,-1)$

$\bar{a}\cdot\bar{b} = (1,2,-3)\cdot(2,1,-1)$

$= 2+2+3$

$= 7$

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+1+1}$

$= \sqrt{6}$

$\Rightarrow \cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$

$= \frac{7}{\sqrt{14}\cdot\sqrt{6}} = \frac{7}{\sqrt{84}}$.

Now, $\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{7}{\sqrt{84}}\right)^2$

$= 1 - \frac{49}{84}$

$= \frac{84-49}{84}$

$\Rightarrow \sin^2\theta = \frac{35}{84}$

$\Rightarrow \sin\theta = \sqrt{\frac{35}{84}}$

$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\frac{35}{84}}\right)$.

* Prove that the angle between two vectors $3i+j+2k$ and $2i-2j+4k$ is $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$.

$\Rightarrow \bar{a} = (3,1,2)$, $\bar{b} = (2,-2,4)$

$\bar{a}\cdot\bar{b} = (3,1,2)\cdot(2,-2,4)$

$= 6-2+8$

$= 12$.

$|\bar{a}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

$|\bar{b}| = \sqrt{x^2+y^2+z^2}$

$= \sqrt{4+4+16}$

$= \sqrt{24}$

$\Rightarrow \cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$

$= \frac{12}{\sqrt{14}\cdot\sqrt{24}} = \frac{12}{\sqrt{336}}$.

Now, $\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{12}{\sqrt{336}}\right)^2$

$= 1 - \frac{144}{336}$

$= \frac{336-144}{336}$

$= \frac{192}{336} = \frac{4\times48}{7\times48}$

$\Rightarrow \sin^2\theta = \frac{4}{7}$

$\Rightarrow \sin\theta = \frac{2}{\sqrt{7}}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$.

* A Particle moves from the point $3i-2j+k$ to the point $i+3j-4k$ under the effect of constant forces $i-j+k$, $i+j-3k$ and $4i+5j-6k$. Find the work done.

$\Rightarrow F_1 = (1,-1,1)$ $\qquad d_1 = (3,-2,1)$

$F_2 = (1,1,-3)$ $\qquad d_2 = (1,3,-4)$

$F_3 = (4,5,-6)$

$\Rightarrow F = F_1 + F_2 + F_3$

$= (1,-1,1) + (1,1,-3) + (4,5,-6)$

$= (1+1+4, -1+1+5, 1-3-6)$

$= (6,5,-8)$.

$\Rightarrow d = d_2 - d_1$

$= (1,3,-4) - (3,-2,1)$

$= (1-3,\ 3+2,\ -4-1)$

$= (-2,5,-5)$

$\Rightarrow W = F \cdot d$

$= (6,5,-8) \cdot (-2,5,-5)$

$= -12 + 25 + 40$

$= 53$ unit.

* Forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and particle moves from the point $2i + 3j + k$ to the point $5i + 2j + 3k$ under the effect of these forces. Find the work done.

$\Rightarrow F_1 = (3,-1,2)$ $\quad d_1 = (2,3,1)$

$F_2 = (1,3,-1)$ $\quad d_2 = (5,2,3)$

$\Rightarrow F = F_1 + F_2$

$= (3,-1,2) + (1,3,-1)$

$= (3+1,\ -1+3,\ 2-1)$

$= (4,2,1)$

$\Rightarrow d = d_2 - d_1$

$= (5,2,3) - (2,3,1)$

$= (5-2,\ 2-3,\ 3-1)$

$= (3,-1,2)$

$\Rightarrow W = F \cdot d$

$= (4,2,1) \cdot (3,-1,2)$

$= 12 - 2 + 2$

$= 12$ unit.

* A particle moves from $(-1,2,1)$ to $(2,3,-1)$ under the effect of the forces $(1,2,1)$ and $(2,-1,0)$. Find work done.

$\Rightarrow F_1 = (1,2,1)$ $\quad d_1 = (-1,2,1)$

$F_2 = (2,-1,0)$ $\quad d_2 = (2,3,-1)$

$\Rightarrow F = F_1 + F_2$

$= (1,2,1) + (2,-1,0)$

$= (1+2,\ 2-1,\ 1+0)$

$= (3,1,1)$

$\Rightarrow d = d_2 - d_1$

$= (2,3,-1) - (-1,2,1)$

$= (2+1,\ 3-2,\ -1-1)$

$= (3,1,-2)$

$\Rightarrow W = F \cdot d$

$= (3,1,1) \cdot (3,1,-2)$

$= 9 + 1 - 2$

$= 8$ unit.

* Forces $(1,2,3)$, $(-1,2,3)$ and $(-1,2,-3)$ act on a particle and particle moves from the point $(0,1,-2)$ to the point $(-1,3,2)$ under the effect of these forces. Find work done.

$\Rightarrow F_1 = (1,2,3)$ $\quad d_1 = (0,1,-2)$

$F_2 = (-1,2,3)$ $\quad d_2 = (-1,3,2)$

$F_3 = (-1,2,-3)$

$\Rightarrow F = F_1 + F_2 + F_3$

$= (1,2,3) + (-1,2,3) + (-1,2,-3)$

$= (1-1-1,\ 2+2+2,\ 3+3-3)$

$= (-1,6,3)$

$\Rightarrow d = d_2 - d_1$

$= (-1,3,2) - (0,1,-2)$

$= (-1-0,\ 3-1,\ 2+2)$

$= (-1,2,4)$

$\Rightarrow W = F \cdot d$

$= (-1,6,3) \cdot (-1,2,4)$

$= 1 + 12 + 12$

$= 25$ unit.

CIRCLE

* Find the equation of tangent and normal to the circle $x^2+y^2-6x+10y+21=0$ at $(1,-2)$.

$\Rightarrow 2g=-6 \quad | \quad 2f=10 \quad | \quad c=21$

$\Rightarrow g=-3 \quad | \quad f=5 \quad | \quad c=21.$

$(x_1,y_1)=(1,-2)$

Tangent : $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$\Rightarrow x(1)+y(-2)-3(x+1)+5(y-2)+21=0$

$\Rightarrow x-2y-3x-3+5y-10+21=0$

$\Rightarrow -2x+3y+8=0$

$\Rightarrow 2x-3y-8=0.$

Normal : $\frac{x-x_1}{x_1+g}=\frac{y-y_1}{y_1+f}$

$\Rightarrow \frac{x-1}{1-3}=\frac{y+2}{-2+5}$

$\Rightarrow \frac{x-1}{-2}=\frac{y+2}{3}$

$\Rightarrow 3x-3=-2y-4$

$\Rightarrow 3x+2y-3+4=0$

$\Rightarrow 3x+2y+1=0.$

* Find the equation of tangent and normal to the circle $x^2+y^2+6x-8y+17=0$ at $(-1,2)$.

$\Rightarrow 2g=6 \quad | \quad 2f=-8 \quad | \quad c=17$

$\Rightarrow g=3 \quad | \quad f=-4 \quad | \quad c=17$

$(x_1,y_1)=(-1,2)$

Tangent : $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$\Rightarrow x(-1)+y(2)+3(x-1)-4(y+2)+17=0$

$\Rightarrow -x+2y+3x-3-4y-8+17=0$

$\Rightarrow 2x-2y+6=0$

$\Rightarrow x-y+3=0.$

Normal : $\frac{x-x_1}{x_1+g}=\frac{y-y_1}{y_1+f}$

$\Rightarrow \frac{x+1}{-1+3}=\frac{y-2}{2-4}$

$\Rightarrow \frac{x+1}{2}=\frac{y-2}{-2}$

$\Rightarrow -2x-2=2y-4$

$\Rightarrow 2x+2y-4+2=0$

$\Rightarrow 2x+2y-2=0$

$\Rightarrow x+y-1=0.$

* Find the equation of tangent and normal to the circle $x^2+y^2-2x+4y-20=0$ at $(-2,2)$.

$\Rightarrow 2g=-2 \quad | \quad 2f=4 \quad | \quad c=-20$

$\Rightarrow g=-1 \quad | \quad f=2 \quad | \quad c=-20$

$(x_1,y_1)=(-2,2).$

Tangent : $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0.$

$\Rightarrow x(-2)+y(2)-1(x-2)+2(y+2)-20=0$

$\Rightarrow -2x+2y-x+2+2y+4-20=0$

$\Rightarrow -3x+4y-14=0$

$\Rightarrow 3x-4y+14=0.$

Normal : $\frac{x-x_1}{x_1+g}=\frac{y-y_1}{y_1+f}$

$\Rightarrow \frac{x+2}{-2-1}=\frac{y-2}{2+2}$

$\Rightarrow \frac{x+2}{-3}=\frac{y-2}{4}$

$\Rightarrow 4x+8=-3y+6$

$\Rightarrow 4x+3y+8-6=0$

$\Rightarrow 4x+3y+2=0.$

* Find the equation of tangent and normal to the circle $x^2+y^2-2x-7=0$ at $(2,3)$.

$\Rightarrow 2g=-2 \quad | \quad 2f=0 \quad | \quad c=-7$

$\Rightarrow g=-1 \quad | \quad f=0 \quad | \quad c=-7$

$(x_1,y_1)=(2,3)$

Tangent : $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$\Rightarrow x(2)+y(3)-1(x+2)+0(y+3)-7=0$

$\Rightarrow 2x+3y-x-2-7=0$

$\Rightarrow x+3y-9=0.$

Normal : $\frac{x-x_1}{x_1+g}=\frac{y-y_1}{y_1+f}$

$\Rightarrow \frac{x-2}{2-1}=\frac{y-3}{3+0}$

$\Rightarrow \frac{x-2}{1}=\frac{y-3}{3}$

$\Rightarrow 3x-6=y-3$

$\Rightarrow 3x-y-6+3=0$

$\Rightarrow 3x-y-3=0.$

* Find centre and radius of the circle
$x^2+y^2-2x+4y-1=0$

$\Rightarrow x^2+y^2-2x+4y-1=0$

$\Rightarrow 2g=-2$	$2f=4$	$c=-1$
$\Rightarrow g=-1$	$f=2$	$c=-1$

$\Rightarrow$ centre $(-g,-f) = (1,-2)$

$\Rightarrow$ radius $(r) = \sqrt{g^2+f^2-c}$

$= \sqrt{(-1)^2+(2)^2-(-1)}$

$= \sqrt{1+4+1}$

$= \sqrt{6}.$

* Find centre and radius of the circle
$4x^2+4y^2+8x-12y-3=0.$

$\Rightarrow 4x^2+4y^2+8x-12y-3=0$

$\Rightarrow x^2+y^2+2x-3y-\frac{3}{4}=0.$

$\Rightarrow 2g=2$	$2f=-3$	$c=-\frac{3}{4}$
$\Rightarrow g=1$	$f=-\frac{3}{2}$	$c=-\frac{3}{4}$

centre $(-g,-f) = (-1, 3/2)$

radius $(r) = \sqrt{g^2+f^2-c}$

$= \sqrt{1+\frac{9}{4}+\frac{3}{4}}$

$= \sqrt{\frac{4+9+3}{4}}$

$= \sqrt{\frac{16}{4}} = \sqrt{4} = 2.$

* If the radius of the circle
$2x^2+2y^2-4x-8y+k=0$ is 4 unit
then find 'k'.

$\Rightarrow 2x^2+2y^2-4x-8y+k=0$

$\Rightarrow x^2+y^2-2x-4y+\frac{k}{2}=0.$

$\Rightarrow 2g=-2$	$2f=-4$	$c=\frac{k}{2}$
$\Rightarrow g=-1$	$f=-2$	$c=\frac{k}{2}.$

radius $(r) = 4$ unit.

$\Rightarrow r^2 = g^2+f^2-c$

$\Rightarrow (4)^2 = (-1)^2+(-2)^2-\frac{k}{2}$

$\Rightarrow 16 = 1+4-\frac{k}{2}$

$\Rightarrow \frac{k}{2} = 5-16$

$\Rightarrow \frac{k}{2} = -11$

$\Rightarrow k = -22.$