VPMP POLYTECHNIC, GANDHINAGAR

ENGINEERING MATHEMATICS

COURSE CODE: - 4320002(CE/EE/EC)

ASSIGNMENT

UNIT-I

MARKS-16

MATRICES

MATRICES

- **MQC'S**

1) Order of $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ is _____.

2) Order of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is _____.

3) Order of $\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$ is _____.

4) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then$A^T =$ _____.

5) If $A = \begin{bmatrix} 1 & -3 & 4 \\ -2 & 1 & 2 \end{bmatrix}$ then$A^T =$ _____.

6) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then$A^T =$ _____.

7) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$ then$A^T =$ _____.

8) If A is non-singular matrix then ______.

9) If $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ then $3A =$ _____.

10) If $\begin{bmatrix} 3 & 2 \\ x-1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & y+1 \\ 4 & 5 \end{bmatrix}$ then $(x, y) =$ _____.

11) Find x and y if $\begin{bmatrix} x+y & 3 \\ -7 & x-y \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 2 \end{bmatrix}$.

12) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} =$ _____.

13) If $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ then $2A + 3B =$ _____.

14) If $A = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$ then $2A - 3I =$ _____.

15) If $A_{3\times4}$ and $B_{4\times4}$ are the matrices then the order of AB is ________.

16) If $A_{2\times3}$ and $B_{3\times4}$ are the matrices then the order of $(AB)^T$ is ________.

17) If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ then AB = _______.

18) If $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ then AB = _______.

19) If $A = \begin{bmatrix} 2 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ then BA = _______.

20) If $A = \begin{bmatrix} -7 & 6 \\ 5 & -2 \end{bmatrix}$ then $AI =$ _____.

21) If $\begin{bmatrix} 0 & x & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [4]$ then x = _______.

22) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $A^2 =$ _____.

23) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $adjA =$ _____.

24) If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ then $adjA =$ _____.

25) If $A = \begin{bmatrix} -8 & 4 \\ -6 & 3 \end{bmatrix}$ then $A^{-1} =$ _____.

26) If $A = \begin{bmatrix} sin\theta & cos\theta \\ -cos\theta & sin\theta \end{bmatrix}$ then $A^{-1} =$ _____.

27) If $AB = I$ then matrix B =_______.

➤ **EXAMPLES** **(Based on Addition , subtraction and equality)**

1) If $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 3 & 5 \\ -1 & 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 7 \\ 2 & 3 & -4 \end{bmatrix}$ then find $2A + 3B$.

2) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ then find $3A - 2B$.

3) If $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 4 & 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 4 & 3 \\ 3 & 5 & 4 \end{bmatrix}$ abd $C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 4 & 5 & 6 \end{bmatrix}$ then find $3A + 2B - 4C$.

4) If $A = \begin{bmatrix} 2 & 3 & 6 \\ -1 & 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & -8 \\ 2 & 4 & -2 \end{bmatrix}$ abd $C = \begin{bmatrix} 1 & 3 & -3 \\ 1 & 4 & 1 \end{bmatrix}$ then prove that $2A + 3B - 4C = 0$.

5) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ then find matrix X from $X + A + B = 0$.

6) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 2 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 & 3 \\ -2 & 3 & 2 \end{bmatrix}$ then find matrix X from $3(X + B) + 5A = 0$.

7) If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ then prove that $(A + B)^T = A^T + B^T$.

8) If $M = \begin{bmatrix} -2 & 3 & 8 \\ 5 & -7 & 9 \\ 1 & -4 & 6 \end{bmatrix}$, $N = \begin{bmatrix} 15 & -6 & 2 \\ 11 & 4 & 7 \\ 13 & 5 & 6 \end{bmatrix}$ then prove that $(M + N)^T = M^T + N^T$.

9) If $3A - 2B = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}$ and $B - 4A = \begin{bmatrix} -1 & 2 \\ -4 & 4 \end{bmatrix}$ then find A and B.

10) If $\begin{bmatrix} a + 2b & 3c + 2d \\ 2a - b & c - d \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 3 & 5 \end{bmatrix}$ then find the values of a, b, c and d.

➤ **EXAMPLES** **(Based on multiplication)**

1) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$ find AB or BA whichever exist.

2) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find AB and BA.

3) If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ then show that $A^2 = A$.

4) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then prove that $A^2 - 5A - 2I = 0$.

5) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 4A + 7I = 0$.

6) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then prove that $A^2 - 4A - 5I = 0$.

7) If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ then prove that $A^4 = I$.

8) If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$ then find $(AB)^T$.

9) If $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$ then prove that $(AB)^T = B^T . A^T$.

10) From equation $\begin{bmatrix} x & 3 \\ y & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$ find value of $x\ and\ y$.

11) Evaluate $\begin{bmatrix} 2 & 1 & -1 \\ 4 & -5 & 6 \\ -3 & 7 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -6 & 4 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}$.

- **<u>EXAMPLES</u> (Based on adjoint and inverse)**

1) If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then prove that $adjA = A$.

2) If $A = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$ then find A^{-1}.

3) If $A = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$ then find A^{-1}.

4) Find matrix X such that $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}.X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

5) If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}\ and\ A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ then find $(AB)^{-1}$.

6) Find A^{-1} if exists for $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

7) Find inverse of matrix $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$.

8) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ then find A^{-1}.

9) If $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$ then find A^{-1}.

10) If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ then find A^{-1}.

- **<u>EXAMPLES</u> (Based on simultaneous linear equation)**

1) Solve the equation $3x - y = 1\ and\ x + 2y = 5$ using matrix method.
2) Solve the equation $2x + 5y = 7\ and\ 8x - 3y = 5$ using matrix method.
3) Solve the equation $2x + 3y = 1\ and\ y - 4x = 2$ using matrix method.
4) Solve the equation $2x - 3y = -5\ and\ 3x + y = 9$ using matrix method.
5) Solve the equation $3x + 2y = 7\ and\ 11x - 4y = 3$ using matrix method.
6) Solve the equation $3x + 2y = 5\ and\ 2x - y = 1$ using matrix method.
7) Solve the equation $2x + 3y = 6xy\ and\ x - y = xy$ using matrix method.

UNIT-II

MARKS-16

DIFFERENTIATION

AND

ITS APPLICATION

DIFFERENTIATION

- **MCQ'S**

1) $\frac{\mathrm{d}}{\mathrm{dx}}(\sin^2 x + \cos^2 x) =$ _____ .
2) $If\ f(x) = 2^x\ then\ f'(0) =$ ________ .
3) $\frac{d}{dx}(x\ logx) =$ ______ .
4) $If\ f(x) = e^{2x}\ then\ f'(x) =$ _______ .
5) $\frac{d}{dx}\log(cosx) =$ _______ .
6) $\frac{\mathrm{d}}{\mathrm{dx}}(\sin^{-1}x + \cos^{-1}x) =$ _____ .
7) $\frac{\mathrm{d}}{\mathrm{dx}}(x^x) =$ _______ .
8) $x^2 + y^2 = 29\ then\ \frac{dy}{dx}\ at\ the\ point\ (2,5) =$ ______ .
9) $\frac{d}{dx}(cotx) =$ ________
10) $\frac{d(logsinx)}{dx} =$ _______ .
11) $\frac{d\left(\frac{u}{v}\right)}{dx} =$ _____ .
12) $\frac{d\left(e^{-logx}\right)}{dx} =$ _________ .
13) $If\ x = sec\theta + tan\theta\ and\ y = sec\theta - tan\theta\ then\ \frac{dy}{dx} =$ ________ .
14) $If\ f(x) = log\sqrt{x^2+1}\ then\ f'(0) =$ ________ .
15) $If\ x = at\ and\ y = \frac{a}{t} then\ \frac{dy}{dx} =$ _________.
16) $If\ \frac{d}{dx}(x^n) = nx^{n-1}\ then\ \frac{d}{dx}(y^4) =$ ________ .
17) $\frac{d}{dx}(secx) =$ _________ .
18) $If\ x = cos\theta\ and\ y = sin\theta\ then\ \frac{dy}{dx} =$ _________ .
19) $If\ x^2 + y^2 = 1\ then\ \frac{dy}{dx} =$ _________ .
20) $If\ y = sin^{99}\left(\frac{\pi}{2}\right)\ then\ \frac{dy}{dx} =$ ______ .
21) $If\ y = cotx\ then\ \frac{dy}{dx} =$ ________ .
22) $If\ y = e^x\ then\ \frac{d^2y}{dx^2} =$ _________ .
23) $\frac{d}{dx}(x^2 + 2^x + 2^2) =$ _______ .
24) $\frac{d}{dx}(\sqrt{x}) =$ _________ .
25) $If\ f(x) = sinx\ then\ f'\left(\frac{\pi}{2}\right) =$ _______ .
26) $\frac{d}{dx}(3sinx - 4sin^3x) =$_________ .
27) $\frac{d}{dx}(a^x) =$ __________ .
28) $\frac{\mathrm{d}}{\mathrm{dx}}(\tan^{-1}x + \cot^{-1}x) =$ _________ .

29) $\frac{d}{dx}(x^2 + 2x + 7) =$ _________ .

30) $\frac{d}{dx}(tanx) =$ _________ .

31) $\frac{\mathrm{d}}{\mathrm{dx}}(\tan^2 \mathrm{x} - \sec^2 \mathrm{x}) =$________ .

32) $If\ y = 1{,}00{,}000\ then\ \frac{dy}{dx} =$ _________ .

33) $If\ y = tanx\ then\ \frac{dy}{dx} =$ _________ .

34) $If\ y = e^x + 4\ \ then\ \frac{d^2y}{dx^2} =$ __________ .

35) $\frac{d}{dx}(cosecx) =$ ___________ .

36) $\frac{d}{dx}(x^n) =$ _______ .

- **EXAMPLES** **(Based on Definition)**

1) Using definition of derivative, find the derivative of x^2 with respect to x.
2) Find the derivative of x^3 using definition.
3) Using first principle of differentiation, differentiate $\sqrt{x}$ with respect to x.
4) Differentiate a^x using definition.
5) Find derivative of e^x using definition.
6) Find derivative of sinx using definition.
7) Differentiate f(x) = cosx with respect to x by using definition of derivative.
8) Differentiate x^2+2x-1 with respect to x by using first principle.
9) Differentiate $y = 2x^2+3$ using definition.

- **EXAMPLES** **(Based on Addition , subtraction , Multiplication , division ,Chain rule)**

1) Find $\frac{d}{dx}(x^3.logx)$.
2) $If\ y = e^x.sinx\ then\ find\ \frac{dy}{dx}$.
3) $If\ y = x^2.tanx\ then\ find\ \frac{dy}{dx}$.
4) Find $\frac{dy}{dx}$ when (I) $y = e^x.secx$ (II) $y = \frac{logx}{x}$.
5) For $y = \frac{sinx}{x}$ find$\frac{dy}{dx}$.
6) Find the derivative of $\frac{logx}{x}$ at x=1.
7) If $y = \frac{x^2-1}{x^2+1}\ then\ find\ \frac{dy}{dx}$.
8) If $y = \frac{1+sinx}{1-sinx}\ then\ find\ \frac{dy}{dx}$.
9) Find $\frac{dy}{dx}$ if $y = \frac{a+bsinx}{b+asinx}$.
10) Find $\frac{dy}{dx}$ if $y = \frac{3+4sinx}{4+3sinx}$.
11) If $y = \frac{1+tanx}{1-tanx}\ then\ find\ \frac{dy}{dx}$.
12) If $y = \log(tanx) + cosx + x$ then find $\frac{dy}{dx}$.
13) Find $\frac{dy}{dx}$ for $y = \log(cos2x)$.
14) If $y = x^3.tanx + (tanx)^3$ then find $\frac{dy}{dx}$.
15) Find $\frac{dy}{dx}$ if $y = \log(cosecx - cotx)$.
16) Find $\frac{dy}{dx}$ if $y = \log(secx + tanx)$.
17) If $y = e^{3x}.cos2x$ then find $\frac{dy}{dx}$.
18) If $y = e^{sinx}.sec2x$ then find $\frac{dy}{dx}$.
19) If $y = x^3.\sin(logx)$ then find $\frac{dy}{dx}$.
20) If $y = \frac{\sin(logx)}{x}$ then find $\frac{dy}{dx}$.
21) Find $\frac{dy}{dx}$ from $y = \log\left(\frac{sinx}{1+cosx}\right)$.
22) If $y = log\left(x + \sqrt{x^2+1}\right)$ then find $\frac{dy}{dx}$.

23) If $y = log\sqrt{\frac{a+x}{a-x}}$ then find $\frac{dy}{dx}$.

- **EXAMPLES (Based on taking log on both side)**

1) If $y = x^x$ then find $\frac{dy}{dx}$.

2) If $y = x^{sinx}$ then find $\frac{dy}{dx}$.

3) If $y = sinx^x$ then find $\frac{dy}{dx}$.

4) If $y = (sinx)^{tanx}$ then find $\frac{dy}{dx}$.

5) If $y = (sinx)^{cosx}$ then find $\frac{dy}{dx}$.

6) If $y = x^x + (sinx)^x$ then find $\frac{dy}{dx}$.

- **EXAMPLES (Based on Implicit function)**

1) If $x^2 + y^2 = xy$ then find $\frac{dy}{dx}$.

2) If $x^3 + y^3 = 3xy$ then find $\frac{dy}{dx}$.

3) If $x^3 + y^3 = 3axy$ then find $\frac{dy}{dx}$.

4) If $x + y = sin(xy)$ then find $\frac{dy}{dx}$.

5) If $y = sin(x + y)$ then find $\frac{dy}{dx}$.

6) If $y = cos(x + y)$ then find $\frac{dy}{dx}$.

7) If $xsiny + ysinx = 5$ then find $\frac{dy}{dx}$.

8) If $ax^2 + 2hxy + by^2 = 0$ then find $\frac{dy}{dx}$.

- **EXAMPLES (Based on Parametric function)**

1) If $x = at^2$ and $y = 2at$ then find $\frac{dy}{dx}$.

2) If $x = asin\theta$ and $y = a(1 + cos\theta)$ then find $\frac{dy}{dx}$.

3) If $x = a(\theta + sin\theta)$ and $y = a(1 - cos\theta)$ then find $\frac{dy}{dx}$.

4) If $x = a(1 + cos\theta)$ and $y = b(\theta + sin\theta)$ then find $\frac{dy}{dx}$.

5) If $x = sec\theta + tan\theta$ and $y = sec\theta - tan\theta$ then find $\frac{dy}{dx}$.

6) If $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{1}{2}\left(t - \frac{1}{t}\right)$ then find $\frac{dy}{dx}$.

7) If $x = e^{2t}cost$ and $y = e^{2t}sint$ then find $\frac{dy}{dx}$.

- **EXAMPLES** **(Based on Successive differentiation)**

1) If $y = log(sinx)$ then prove that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0.$

2) If $y = log\left(e^{sinx}\right)$ then prove that $\frac{dy}{dx} - cosx = 0.$

3) If $y = e^{2x}$ then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$

4) If $x = at^2\ and\ y = 2at$ then prove that $yy_{2+}y_1^2 = 0.$

5) If $y = 2e^{3x} + 3e^{-2x}$ then prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$

6) If $y = Acospt + Bsinpt$ then prove that $\frac{d^2y}{dt^2} + p^2y = 0.$

- **EXAMPLES** **(Based on application of differentiation Maximum and Minimum)**

1) Find maximum and minimum values of $f(x) = x^3 - 3x + 11.$

2) Find maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 5.$

3) Find maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 36x + 10.$

4) Find maximum and minimum values of $f(x) = x^3 + x^2 - x.$

5) Find maximum and minimum values of $f(x) = 3x^3 - 4x^2 - x + 5.$

6) Find maxima and minima of $f(x) = x + \frac{1}{x}.$

- **EXAMPLES** **(Based on application of differentiation Velocity and Acceleration)**

1) The equation of motion of particle is $s = t^3 - 6t^2 + 8t - 4$ then find velocity and acceleration of the moving particle at $t = 3\ second.$

2) The distance of a moving particle is given by $s = t^3 - 3t^2 + 4t + 3$. Find velocity and acceleration at $t = 2\ second.$

3) If the law of motion is $s = t^3 + 6t^2 + 3t + 5$ then find velocity and acceleration at $t = 3second.$

4) The equation of motion of particle is $s = t^3 - 6t^2 + 9t$ then find velocity and acceleration at $t = 3\ second.$

5) The motion of particle is given by $s = t^3 - 6t^2 + 9t + 6$ then find velocity when acceleration is zero.

6) The equation of motion of particle is $s = t^3 - 5t^2 + 3t$. When particle comes to rest? Find acceleration at that time.

7) A particle has motion of $s = 2t^3 + 3t^2 - 12t + 6$. Find velocity and acceleration at $t = 2\ second.$ Also find acceleration when particle comes to rest.

8) The equation of motion of particle is $s = t^3 - 6t^2 + 9t$ then find $t\ and\ s\ at\ v = 0.$

9) The equation of motion of particle is $s = t^3 - 6t^2 + 9t + 6$ where s is in meter and t is in second. (I) Find $v\ \ and\ a\ when\ t = 2second.$ (II) Find s when particle change its direction.

10) The equation of motion of particle is $s = t^3 + 3t\ (t > 0).$
(I) Find velocity and acceleration at $t = 3second.$
(II) When do velocity and acceleration become equal?

UNIT-III

MARKS-14

INTEGRATION

AND

ITS APPLICATION

INTEGRATION

- **MCQ'S**

1) $\int x^3\,dx =$ _____ $+ c.$
2) $\int x^4\,dx =$ _____ $+ c.$
3) $\int x^7\,dx =$ _____ $+ c.$
4) $\int \frac{1}{x^2}\,dx =$ _____ $+ c.$
5) $\int logx\,dx =$ _____ $+ c.$
6) $\int sinx\,dx =$ _____ $+ c.$
7) $\int cosx\,dx =$ _____ $+ c.$
8) $\int (\sin^2 x + \cos^2 x)\,dx =$ _____ $+ c.$
9) $\int \tan^2 x\,dx =$ _____ $+ c.$
10) $\int tanx\,dx =$ _____ $+ c.$
11) $\int e^{xloga}\,dx =$ _____ $+ c.$
12) $\int e^{-\log(secx)}\,dx =$ _____ $+ c.$
13) $\int \frac{1}{1+x^2}\,dx =$ _____ $+ c.$
14) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx =$ _____ $+ c.$
15) $\int \frac{1}{x^2+25}\,dx =$ _____ $+ c.$
16) $\int \frac{1}{\sqrt{1-x^2}}\,dx =$ _____ $+ c.$
17) $\int \cos(ax+b)\,dx =$ _____ $+ c.$
18) $\int \left(\sqrt{1+sin2x}\right)dx =$ _____ $+ c.$
19) $\int_0^1 e^x\,dx =$ _____ $+ c.$
20) $\int_0^1 x\,dx =$ _____ $+ c.$
21) $\int_2^5 x^3\,dx =$ _____ $+ c.$
22) $\int_{-1}^1 x^3\,dx =$ _____ $+ c.$
23) $\int_1^e \frac{dx}{x} =$ _____ $+ c.$
24) $\int_{-1}^1 (x^2+1)\,dx =$ _____ $+ c.$
25) $\int_1^e logx\,dx =$ _____ $+ c.$
26) $\int \boldsymbol{e^x}(\tan x + \sec^2 x)\,dx =$ _____ $+ c.$
27) $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx =$ _____ $+ c.$
28) Area covered by the curve $x^2 + y^2 = 4$ is _______.

- **<u>EXAMPLES</u> (Based on Addition and subtraction)**

1) Evaluate $\int 4x^2 + 3x + 9\, dx.$

2) Evaluate $\int \left(\frac{2x^2-3x-11}{x}\right) dx.$

3) Evaluate $\int \left(\frac{3x^2+2x-5}{x}\right) dx.$

4) Evaluate $\int \left(\frac{x^3+5x^2+4x+1}{x^2}\right) dx.$

5) Evaluate $\int \frac{x^2+5x+6}{x^2+2x}\, dx.$

6) Evaluate $\int \left(x+\frac{1}{x}\right)^2 dx.$

7) Evaluate $\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx.$

8) Evaluate $\int \frac{x^4+x^2+1}{x^2+1}\, dx.$

9) Evaluate $\int \frac{x^2+4x+1}{x^3+x}\, dx.$

10) Evaluate $\int \frac{2+3sinx}{cos^2x}\, dx.$

11) Evaluate $\int \frac{4+3cosx}{sin^2x}\, dx.$

12) Evaluate $\int \frac{sin^3x+cos^3x}{sin^2x.cos^2x}\, dx.$

13) Evaluate $\int \frac{1}{sin^2x.cos^2x}\, dx.$

14) Evaluate $\int sec^2x.\,cosec^2x\, dx.$

15) Evaluate $\int \frac{cos2x}{sin^2x.cos^2x}\, dx.$

16) Evaluate $\int \frac{dx}{1-sinx}.$

17) Evaluate $\int \frac{cosx}{1+cosx}\, dx.$

- **<u>EXAMPLES</u> (Based on integration by parts)**

1) Evaluate $\int x.\,e^x\, dx.$

2) Evaluate $\int sinx.\,x\, dx.$

3) Evaluate $\int x.\,cosx\, dx.$

4) Evaluate $\int x.\,logx\, dx.$

5) Evaluate $\int x.\,e^{mx}\, dx.$

6) Evaluate $\int x.\,e^{3x}\, dx.$

7) Evaluate $\int x^2.\,logx\, dx.$

8) Evaluate $\int x^2.\,e^x\, dx.$

9) Evaluate $\int x.\tan^{-1}x\, dx.$

10) Evaluate $\int logx\, dx.$

- **<u>EXAMPLES</u> (Based on substitution method)**

1) Evaluate $\int e^{sinx}.cosx\,dx$.
2) Evaluate $\int e^{cosx}.sinx\,dx$.
3) Evaluate $\int e^{tanx}.sec^2x\,dx$.
4) Evaluate $\int sin^5x.cosx\,dx$.
5) Evaluate $\int cosx.\sqrt{sinx}\,dx$.
6) Evaluate $\int \frac{sin(logx)}{x}dx$.
7) Evaluate $\int \frac{(logx)^5}{x}dx$.
8) Evaluate $\int \frac{2\tan^{-1}x}{1+x^2}dx$.
9) Evaluate $\int \frac{e^x(x+1)}{sin^2(xe^x)}dx$.
10) Evaluate $\int \frac{e^x(x+1)}{cos^2(xe^x)}dx$.
11) Evaluate $\int 2x.e^{x^2}\,dx$.
12) Evaluate $\int \frac{x^2}{1+x^6}dx$.
13) Evaluate $\int 2x.(x^2+8)^8\,dx$.
14) Evaluate $\int(2x+1)\sqrt{x^2+x+9}\,dx$.

- **<u>EXAMPLES</u> (Based on partial fraction)**

1) Evaluate $\int \frac{dx}{x^2-3x+2}$.
2) Evaluate $\int \frac{x}{(x+1)(x+2)}dx$.
3) Evaluate $\int \frac{x+3}{(x-1)(x-2)}dx$.
4) Evaluate $\int \frac{2x+1}{(x+1)(x-3)}dx$.
5) Evaluate $\int \frac{2x+3}{(x-1)(x+2)}dx$.

- **<u>EXAMPLES</u> (Based on definite integration)**

1) Evaluate $\int_1^3(x^2+x+1)\,dx$.
2) Evaluate $\int_1^2(x^2+4x+1)\,dx$.
3) Evaluate $\int_1^3(2x^2+5x+1)\,dx$.
4) Evaluate $\int_1^2\frac{x^3-1}{x-1}dx$.
5) Evaluate $\int_{-1}^1\frac{x^3-8}{x-2}dx$.
6) Evaluate $\int_0^1\frac{2}{1+x^2}dx$.
7) Evaluate $\int_1^3\frac{2x}{1+x^2}dx$.
8) Evaluate $\int_0^2\frac{x^2}{1+x^3}dx$.
9) Evaluate $\int_{-1}^2\frac{x}{x^2+3}dx$.

10) Evaluate $\int_1^e \frac{(logx)^2}{x} dx$.

11) Evaluate $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$.

12) Evaluate $\int_0^1 \frac{x}{x+1} dx$.

13) Evaluate $\int_{-4}^{-3} \frac{x}{7+x} dx$.

➢ **<u>EXAMPLES</u> (Based on definite integration for continuous function)**

1) Evaluate $\int_0^{\frac{\pi}{2}} \frac{sinx}{sinx+cosx} dx$.

2) Evaluate $\int_0^{\frac{\pi}{2}} \frac{tanx}{tanx+cotx} dx$.

3) Evaluate $\int_0^{\frac{\pi}{2}} \frac{secx}{secx+cosecx} dx$.

4) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{tanx}}{\sqrt{tanx}+\sqrt{cotx}} dx$.

5) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{secx}}{\sqrt{secx}+\sqrt{cosecx}} dx$.

6) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{sinx}}{\sqrt[3]{sinx}+\sqrt[3]{cosx}} dx$.

7) Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}} dx$.

8) Evaluate $\int_0^{\frac{\pi}{2}} log(tanx)\, dx$.

9) Evaluate $\int_0^{\frac{\pi}{2}} log(cotx)\, dx$.

10) Evaluate $\int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} dx$.

11) Evaluate $\int_0^7 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}} dx$.

➢ **<u>EXAMPLES</u> (Based on application of integration area and volume)**

1) Find the area bounded by the curve $x^2 + y^2 = a^2$.
2) Show that the area enclosed between the parabola $y = x^2$ and lines $x = 2, x = 3\ and\ x - axis$ is $\frac{19}{3}\ sq.unit$.
3) Find the area of region bounded by $y = 2x^2, x - axis$ and lines $x = 5$.
4) Find the area of region bounded by $y = 3x^2, x = 2, x = 3\ and\ x - axis$.
5) Find the area of region bounded by $y = 4x^2, x = 1, x = 2$.
6) Find the area of region bounded by the curves $y = x^2\ and\ y = x$.
7) Find the area of region bounded by the curves $y = x^2\ and\ the\ line\ y = x + 2$.
8) Find the area of region bounded by the curves $y^2 = 4x\ and\ x = 2$.
9) Find the area of region bounded by the curves $y = x^2 - 7x + 10\ and\ x - axis$.
10) Find the area bounded by the curves $x + y = 1\ and\ the\ axes$.
11) Find the volume of a sphere of radius r by method of integration.

UNIT-IV

MARKS-12

DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATIONS

- **MCQ'S**

1) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{dy}{dx}\right)^2 - 5y = 0$ are ______ respectively.

2) The order of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{d^2y}{dx^2}\right)^4 + xsiny = 0$ is______.

3) Order of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = xy$ is given by_______.

4) The order of the differential equation $x\frac{d^2y}{dx^2} - 5\left(\frac{dy}{dx}\right)^3 - 2y = 14$ is_______.

5) The degree of the differential equation $x^2\frac{dy}{dx} + sin\left(\frac{d^2y}{dx^2}\right) = 0$ is________.

6) Order of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6 = 0$ is_______.

7) The order of the differential equation $\frac{d^2y}{dx^2} = \left(3 + \frac{dy}{dx}\right)^3$ is_______.

8) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{dy}{dx}\right)^2 - 9y = 0$ are ______ and______respectively.

9) For the differential equation $\frac{dy}{dx} + Py = Q$, Integrating factor is________.

10) Integrating factor of $\frac{dy}{dx} + \frac{2y}{x} = e^x$ is______.

11) The integrating factor of the equation $\frac{dy}{dx} = ytanx + e^x$ is________.

12) The integrating factor of the equation $\frac{dy}{dx} + y = 3x$ is________.

13) The integrating factor of the equation $\frac{dy}{dx} + \frac{y}{x} = x$ is________.

14) The integrating factor of the equation $\frac{dy}{dx} + 2y = e^x$ is________.

15) The integrating factor of the equation $\frac{dy}{dx} + ytanx = cosx$ is________.

➢ **EXAMPLES** (Based on Separable variable)

1) Find the order and degree of $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho\left(\frac{d^2y}{dx^2}\right)^2$.
2) Find the differential equation for $y = a\sin(x+b)$, where a and b are arbitrary constants.
3) Form the differential equation whose general solution is $y = Acosx + Bsinx$.
4) Solve $\frac{dy}{dx} = \frac{y}{x}$.
5) Solve the differential equation $x.dy + y.dx = 0$.
6) Solve $x(1+y^2)dx - y(1+x^2)dy = 0$.
7) Solve $(1+x^2)dx = (1+y^2)dy$.
8) Solve $(1+x^2)dy - (1+y^2)dx = 0$.
9) Solve $\frac{dy}{dx} + x^2.e^{-y} = 0$.
10) Solve $x\frac{dy}{dx} + coty = 0$.
11) Solve the differential equation $tany\ dx + tanx.sec^2y\ dy = 0$.
12) Solve the differential equation $sec^2x.tany\ dx + sec^2y.tanx\ dy = 0$.
13) Solve $sec^2x.tany\ dx + sec^2y.tanx\ dy = 0\ where\ y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$.
14) Solve $x.cos^2y\ dx = y.cos^2x\ dy$.

➢ **EXAMPLES** (Based on Linear differential equation)

1) Solve $\frac{dy}{dx} + \frac{2y}{x} = sinx$.
2) Solve $\frac{dy}{dx} + 2y = e^x$.
3) Solve $\frac{dy}{dx} + ytanx = cosx$.
4) Solve $\frac{dy}{dx} + ytanx = secx$.
5) Solve $\frac{dy}{dx} + ycotx = cosx$.
6) Solve $\frac{dy}{dx} + ytanx = sec^2x$.
7) Solve $x\ \frac{dy}{dx} - y = x^2$.
8) Solve $cosx\ \frac{dy}{dx} + y = sinx$.
9) Solve $cos^2x\ \frac{dy}{dx} + y = tanx$.
10) Solve $x\ logx\frac{dy}{dx} + y = logx^2$.
11) Solve $(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$.
12) Solve $x\ \frac{dy}{dx} + 2y = logx$.
13) Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^3}$.

UNIT-V

MARKS-12

COMPLEX NUMBERS

COMPLEX NUMBER

- **MCQ'S**

1) $i^8 = \_\_\_\_\_$.
2) $i^9 = \_\_\_\_\_$.
3) $i + i^2 + i^3 + i^4 = \_\_\_\_\_\_$.
4) $\sqrt{-4} = \_\_\_\_\_\_$.
5) $\sqrt{-9 + 0i} = \_\_\_\_\_\_$.
6) If $z = 5 - 2i$ $then$ $\bar{z} = \_\_\_\_\_$.
7) If $z = 3i - 2$ $then$ $\bar{z} = \_\_\_\_\_$.
8) $z + \bar{z} = \_\_\_\_\_$.
9) For $z \in c, z - \bar{z} = \_\_\_\_\_$.
10) If $\bar{z} = cos\theta + isin\theta$ then $z + \bar{z} = \_\_\_\_\_$.
11) If $\overline{|z|} = 16, then\ |z| = \_\_\_\_$.
12) If $z = 3 - 4i\ then\ |z| = \_\_\_\_$.
13) $|(3 - 4i)^2| = \_\_\_\_$.
14) If $z = \frac{3}{5} - \frac{4}{5}i\ then\ |z| = \_\_\_\_$.
15) If $z_1 = 2 + 2i$ and $z_2 = -3 - 2i\ then\ |z_1 + z_2| = \_\_\_\_$.
16) An argument of $1 + i = \_\_\_\_$.
17) Amplitude of $1 - \sqrt{3}i$ is ______.
18) If $z = -5i\ then\ \arg(z) = \_\_\_\_\_$.
19) $\arg(35) = \_\_\_\_\_$.
20) $\arg(-1) = \_\_\_\_\_$.
21) $(2 + 3i)(3 - 2i) = \_\_\_\_\_$.
22) $\frac{1-i}{1+i} = \_\_\_\_\_$.
23) $(1 + i)^2 = \_\_\_\_$.
24) Inverse of the number $3 + 4i$ is _______.
25) $(1 + i)^{-1} = \_\_\_\_$.
26) Inverse of the number $5 - 4i$ is _______.
27) Inverse of the number i is _______.
28) If $3x + 2yi = 6 + 4i\ then\ (x, y) = \_\_\_\_$.
29) If $x + 4iy = xi + y + 3\ then\ (x, y) = \_\_\_\_$.
30) $[cos\theta + isin\theta]^4 + [cos\theta + isin\theta]^{-4} = \_\_\_\_\_$.
31) If $z = cos\theta + isin\theta\ then\ z^3 + \frac{1}{z^3} = \_\_\_\_$.

- **EXAMPLES** **(Based on $x + iy$ or $a + ib$ form, conjugate, modulus and inverse)**

1) Express into $x + iy$ form $\frac{5+2i}{2+3i}$.

2) Express $\frac{4+2i}{(3+2i)(5-3i)}$ in $a + ib$ form.

3) Express the complex number $\frac{1+7i}{(2-i)^2}$ in the form of $x + iy. x, y \in R$.

4) If $\frac{(1+i)^2}{3+i} = x + iy$, then find the value of $x + y$.

5) If $z = \frac{3+7i}{1-i}$ then find its conjugate complex and modulus.

6) Find complex conjugate and modulus of $z = \frac{1-i}{1+i}$.

7) Find $x, y \in R$ from the equation $(2x - y) + 2yi = 6 + 4i$.

8) Find $x, y \in R$ from the equation $(3x - 7) + 2iy = 5y + (5 + x)i$.

9) Find the inverse of complex number $\frac{2+3i}{4-3i}$.

- **EXAMPLES** **(Based on square root, modulus, argument or amplitude and polar form)**

1) Find square root of $7 + 24i$.

2) Find square root of $5 - 12i$.

3) Find square root of $3 - 4i$.

4) Find square root of $3 + 4\sqrt{10}i$.

5) For $z = 1 + i$ find $\overline{|z|}$ $and\ arg(z)$.

6) Find modulus and amplitude of $\frac{1+i}{1-i}$.

7) Find the modulus and principle argument of $z = \sqrt{3} + i$ and express z into polar form.

8) Express the following complex number in polar form also find modulus and principle argument $-1 + \sqrt{3}\, i$.

9) Convert $1 - \sqrt{3}\, i$ into polar form.

- **EXAMPLES** **(Based on De Moivre's Theorem and some special case)**

1) If $z = cis\theta$ then show that $z^n + \frac{1}{z^n} = 2cosn\theta$ and $z^n - \frac{1}{z^n} = 2isinn\theta$.

2) Simplify $\frac{cos6\theta + isin6\theta}{cos2\theta + isin2\theta}$.

3) Simplify $\left(\frac{cos3\theta + isin3\theta}{cos\theta - isin\theta}\right)^2$.

4) Simplify $\frac{(cos3\theta + isin3\theta)^{-4}\,(cos\theta - isin\theta)^5}{(cos2\theta - isin2\theta)^6\,(cos13\theta + isin13\theta)}$.

5) Simplify $\frac{(cos2\theta + isin2\theta)^{-3}\,(cos3\theta - isin3\theta)^2}{(cos2\theta - isin2\theta)^{-7}\,(cos5\theta - isin5\theta)^3}$.

6) Prove that $\frac{1+cos\theta+isin\theta}{1-cos\theta+isin\theta} = -ie^{i\theta}cot\frac{\theta}{2}$.

7) Prove that $\left(\sqrt{3} + i\right)^n + \left(\sqrt{3} - i\right)^n = 2^{n+1}cos\frac{n\pi}{6}$.

8) If $\alpha + i\beta = \frac{1}{a+ib}$ then prove that $(\alpha^2 + \beta^2)(a^2 + b^2) = 1$.

9) If $z = 3 - 2i$ then prove that $z^2 - 6z + 13 = 0$. Find the value of $z^4 - 4z^3 + 6z^2 - 4z + 17$.

10) If $z = -3 + \sqrt{2}i$ then find the value of $z^4 + 5z^3 + 8z^2 + 7z + 4$.

11) If $z = x + iy$ and $|3z| = |z - 4|$ then prove that $x^2 + y^2 + x = 2$.

12) Find cube-roots of unity.