Unit-1 Determinant & Function - 16 Marks

* Determinant

$a_1 x + b_1 y = c_1$ $\qquad$ $2x + 3y = 1$

$a_2 x + b_2 y = c_2$ $\qquad$ $x - y = 4$

$$\frac{x}{\begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}}$$

* 2x2 Determinant

Leading element

$$D = \begin{array}{c} \\ \end{array}\begin{vmatrix} \boxed{a_1} & b_1 \\ a_2 & b_2 \end{vmatrix} \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array}$$

$\qquad\quad \downarrow C_1 \quad \downarrow C_2$

Subsidiary diagonal $\qquad$ Principal diagonal

*Solution of 2x2 Determinant

$$* \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2$$

$\underset{\text{Row}}{\underset{\uparrow}{2}} \times \underset{\text{column}}{\underset{\uparrow}{2}}$ is order of Determinant.

* why determinant is always square type ?

2x2, 3x3, 4x4 .....

* Solve the following Determinents.

* $\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$ $\qquad$ * $\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix}$ $\qquad$ * $\begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix}$

* $\begin{vmatrix} 5 & -8 \\ 9 & 2 \end{vmatrix}$ $\qquad$ * $\begin{vmatrix} 5 & 4 \\ 1 & -2 \end{vmatrix}$ $\qquad$ * $\begin{vmatrix} -3 & 1 \\ -4 & 2 \end{vmatrix}$

* $\begin{vmatrix} -2 & 5 \\ -3 & 2 \end{vmatrix}$ $\qquad$ * $\begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$ $\qquad$ * $\begin{vmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{vmatrix}$

* If $\begin{vmatrix} x & 1 \\ 4 & 2 \end{vmatrix} = 0$ then $x =$ ______

* If $\begin{vmatrix} x & 3 \\ -2 & 2 \end{vmatrix} = 2$ then $x =$ ______

* If $\begin{vmatrix} x & -3 \\ y & 3 \end{vmatrix} = 9$ then $x + y =$ ______

* If $\begin{vmatrix} 2 & x \\ -3 & 5 \end{vmatrix} = 13$ then $x =$ ______

* If $\begin{vmatrix} x-5 & 4 \\ 6 & x+5 \end{vmatrix} = 0$ then $x =$ ____.

* If $\begin{vmatrix} x-1 & 2 \\ 4 & x+1 \end{vmatrix} = 0$ then $x =$ ____.

* 3X3 Determinent

$2x - y + z = 1$
$x + 3y - 2z = 2$
$2x - y + 4z = 3$

$\Rightarrow \dfrac{x}{|\ \ |} = \dfrac{-y}{|\ \ |} = \dfrac{z}{|\ \ |} = \dfrac{1}{|\ \ |}$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$\underset{\text{Row}}{3} \times \underset{\text{column}}{3} \Rightarrow$ order of Determinent

* solution of 3X3 Determinent (by Row or column method)

$$D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$$

* $D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & -2 \\ 4 & 2 & 1 \end{vmatrix} = (-12)$

* $D = \begin{vmatrix} 3 & 7 & 3 \\ 2 & -5 & 2 \\ 1 & 8 & 1 \end{vmatrix} = (0)$

* $D = \begin{vmatrix} 2 & 6 & -3 \\ 1 & 2 & 0 \\ 3 & 4 & -2 \end{vmatrix} = (10)$

* $D = \begin{vmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ -5 & 4 & 5 \end{vmatrix} = (-50)$

* $D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 0 \end{vmatrix} = (2)$

* $D = \begin{vmatrix} 3 & -2 & -2 \\ 1 & 4 & 0 \\ -2 & 3 & -1 \end{vmatrix} = (-36)$

* $D = \begin{vmatrix} 1 & 7 & -3 \\ -4 & 6 & 2 \\ 2 & -5 & 3 \end{vmatrix} = (116)$

* $\begin{vmatrix} 1 & 1 & 2 \\ 3 & 5 & -1 \\ 2 & 2 & 4 \end{vmatrix} (R_1 = R_3)$ * $\begin{vmatrix} 2 & -2 & 1 \\ 4 & 7 & 2 \\ 6 & 0 & 3 \end{vmatrix} (C_1 = C_3)$ * $\begin{vmatrix} a & b & c \\ x & y & z \\ a & b & c \end{vmatrix} (R_1 = R_3)$

* any two row or any two column are same then answer of Determinant is zero.

* solve $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$= \begin{vmatrix} a+b+c & b+c & 1 \\ b+c+a & c+a & 1 \\ c+a+b & a+b & 1 \end{vmatrix} \quad (C_1 + C_2$

$= (a+b+c) \begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix}$

$= (a+b+c)(0) \quad (\because C_1 = C_3)$

$= 0$

* solve $\begin{vmatrix} x & x+a & a \\ y & y+b & b \\ z & z+c & c \end{vmatrix}$

$= \begin{vmatrix} x & x+a & x+a \\ y & y+b & y+b \\ z & z+c & z+c \end{vmatrix} \quad (C_3 + C_1)$

$= 0 \quad (C_2 = C_3)$

* If $\begin{vmatrix} x-2 & 2 & 2 \\ -1 & x & -2 \\ 2 & 0 & 4 \end{vmatrix} = 0$ then find value of x.
$(x = 0$ OR $x = 3)$

* Sarru's Method

$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{matrix} a_1 & \boxed{b_1} \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}$

$= [a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3] - [b_1 a_2 c_3 + a_1 c_2 b_3 + c_1 b_2 a_3]$

* $D = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 0 & 5 \\ -3 & 2 & 4 \end{vmatrix} = (-121)$

* $D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = (0)$

* Function

* Let A and B be two non-empty sets. It every element of A is related to a unique element of B, then the relation is called the Function from A to B.

→ It is denoted as $f : A \to B$, $\forall\, x \in A \;\exists$ a unique element of B, say y, such that

$$\boxed{y = f(x)}$$

→ $f : A \to B$.
↑ Domain ↖ Co-Domain.

→ $y = f(x)$ is the relation for independent variable $x \in A$ and dependent variable $x \in B$.

* $f : A \to B$, $f(x) = x + 1$. find $f(1)$

$f(1) = 1 + 1 = 2$

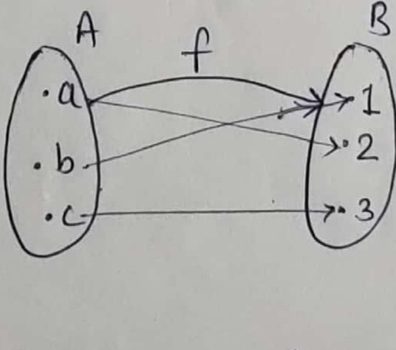
* $f : A \to B$ $f(x) = x + 1$

$A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5, 6\}$

$f(1) = 2$

$f(2) = 3$

$f(3) = 4$



$x_1 \neq x_2$
$\Rightarrow f(x_1) \neq f(x_2)$

* $f(1) = 2$
$f(a) = 2$
$\Rightarrow f(1) = f(a)$
$\therefore a = 1$

$\Rightarrow$ $f(x_1) = f(x_2)$
$x_1 = x_2$.

* Types of Function

* one - one function $\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

* Many - one function $\Rightarrow$ It function is not one-one then its called Many-one function.

* odd function : $\Rightarrow f(-x) = -f(x)$.

$f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

x^3, is an odd function.

* even function $\Rightarrow f(-x) = f(x)$

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$

x^2 is an even function.

* If $f(x) = \log x$ then $f(1) =$ ____

* If $f(x) = \log x$ then $f(x) - f(y) =$ ____

* If $f(x) = \log x$ then $f(x) + f(y) =$ ____

* If $f(x) = \log \frac{1}{x}$ then $f(100) =$ ____

* If $f(x) = x^2 - 1$ then $f(-1) =$ ____

* If $f(x) = x^3 - 1$ then $f(2) + f(-3) =$ ____

* If $f(x) = 2^x - x^2$ then $f(2) =$ ____

* If $f(x) = 2^x - \log_2 x$ then $f(2) =$ ____

* If $f(x) = \log_2 x$ then $f(4) =$ ____

* If $f(x) = \log(e^x)$ then $f(0) =$ ____

* If $f(x) = \log(e^x)$ then $f(-1) =$ ____

* If $f(x) = \dfrac{a^x + a^{-x}}{2}$ is ____ function. (odd, even)

* If $f(x) = \log x$ then prove that (i) $f(xy) = f(x) + f(y)$
(ii) $f\left(\frac{x}{y}\right) = f(x) - f(y)$ (iii) $f(x^2) = 2f(x)$.

* If $f(x) = e^x$ then prove that (i) $f(x) \cdot f(y) = f(x+y)$
(ii) $\frac{f(x)}{f(y)} = f(x-y)$.

* If $f(x) = a^x$ then prove that (i) $f(x+y) = f(x) \cdot f(y)$
(ii) $f(x-y) = \frac{f(x)}{f(y)}$.

* If $f(x) = a^x$ then prove that $f(x+1) - f(x) = (a-1) f(x)$

* If $f(x) = 4^x$ then prove that $f(x+1) - f(x) = 3f(x)$.

* If $f(x) = \frac{1}{x+1}$ then prove that $f(x) + f\left(\frac{1}{x}\right) = 1$

* If $f(x) = \frac{ax+b}{bx+a}$ then show that $f(x) \cdot f\left(\frac{1}{x}\right) = 1$.

* If $f(x) = \frac{1-x}{1+x}$ then P. T. (i) $f(x) + f\left(\frac{1}{x}\right) = 0$
(ii) $f(x) - f\left(\frac{1}{x}\right) = 2 \cdot f(x)$ (iii) $f(x) \cdot f(-x) = 1$.

* If $f(x) = \log\left(\frac{x}{x-1}\right)$ then P. T. $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$

* If $f(x) = \log\left(\frac{x-1}{x}\right)$ then P.T. $f(x) + f(-x) = f(x^2)$.

* If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then P. T. $f(x) + f(-x) = 0$

* If $f(x) = \log\left(\frac{1-x}{1+x}\right)$ then P. T. $f\left(\frac{2x}{1+x^2}\right) = 2 \cdot f(x)$.

* If $f(x) = \frac{1+x}{1-x}$ then P.T. $f\left(\frac{x+y}{1+xy}\right) = f(x) \cdot f(y)$.

* If $f(x) = \log_2 x$ and $g(x) = x^4$ then find $f(g(2))$.

* If $f(x) = \frac{1+x}{1-x}$ then P.T. $x f(f(x)) + 1 = 0$

* If $f(x) = \tan x$ then prove that

(i) $f(x+y) = \dfrac{f(x)+f(y)}{1-f(x)f(y)}$

(ii) $f(2x) = \dfrac{2f(x)}{1-(f(x))^2}$

* If $f(x) = \sin x$ then show that $2f(x) \cdot f(\frac{\pi}{2}+x) = f(2x)$.

* If $f(x) = \dfrac{x+3}{4x-5}$ and $t = \dfrac{3+5x}{4x-1}$ then prove that $x = f(t)$.

* Index & Indices Rules

$\rightarrow a^x \cdot a^y = a^{x+y}$

$\rightarrow \frac{a^x}{a^y} = a^{x-y}$

$\rightarrow (a^m)^n = a^{m \cdot n}$

$\rightarrow \log x + \log y = \log (xy)$

$\rightarrow \log x - \log y = \log \left(\frac{x}{y}\right)$

$\rightarrow \log x^n = n \cdot \log x$

$\rightarrow \log_y x = \frac{\log x}{\log y}$

* Values of Logarithm

$\log 1 = 0$, $\log_a a = 1$

$a^{\log_a y} = y$

* $a^x = n \iff \log_a n = x$

Exponential form — Logarithmic form.

* $2^3 = 8$
* $3^4 = 81$
* $9^{3/2} = 27$
* $10^{-2} = 0.01$
* $(-2)^3 = -8$

* $\log_2 32 = 5$
* $\log_5 125 = 3$
* $\log_{10} 0.001 = 3$
* $\log_{12} \frac{1}{144} = 2$

* $\log_{15} 1 =$ ____
* $\log 1 \cdot \log 2 \cdot \log 3 \cdots \log 2000 =$ ____
* $\log_2 8 =$ ____
* $\log_2 64 =$ ____
* $\log_5 125 =$ ____
* $\log_a \frac{1}{a} =$ ____
* $\log_3 \frac{1}{27} =$ ____
* $\log_2 \frac{1}{8} =$ ____
* $\log_{10} 0.001 =$ ____
* $\log_b a \times \log_a b =$ ____
* $\log(\tan\theta) + \log(\cot\theta) =$ ____
* $\log 2 + \log \frac{1}{2} =$ ____
* $\log a + \log \frac{1}{a} =$ ____
* $\log m - \log n =$ ____
* $\log 32 \div \log 16 =$ ____
* $\log 27 \div \log 9 =$ ____
* $\log 32 - \log 16 =$ ____
* If $\log_7 x = 1$ then $x =$ ____
* If $\log_a 32 = 5$ then $a =$ ____

* $a^{\log_a b} =$ ____
* $2^{-\log_2 3} =$ ____
* $1024^{\log_2 m} =$ ____
* If $\log x + \log 2x = \log 18$ then $x =$ ____
* $\log_{10}(x+1) + \log_{10}(x-1) = \log_{10} 3$ then $x =$ ____
* $2^{\log_4 9}$
* $9^{\log_3 2}$
* $16^{\log_4 5}$
* $1024^{\log_2 m}$
* $\sqrt{3}^{\log_3 81}$

* $\log\left(\frac{x^2}{yz}\right)+\log\left(\frac{y^2}{zx}\right)+\log\left(\frac{z^2}{xy}\right)$.

* $\log\left(\frac{a-b}{b-c}\right)+\log\left(\frac{b-c}{c-a}\right)+\log\left(\frac{c-a}{a-b}\right)$.

* $\log\left(\frac{25}{32}\right)+\log\left(\frac{225}{64}\right)+\log\left(\frac{128}{25}\right)+\log\left(\frac{32}{450}\right)$.

* $\log\left(\frac{9}{14}\right)-\log\left(\frac{15}{16}\right)+\log\left(\frac{35}{24}\right)$.

* P.T. $\log\left(\frac{75}{16}\right)-2\log\left(\frac{5}{9}\right)+\log\left(\frac{32}{243}\right)=\log 2$

* P.T. $\log\left(x+\sqrt{x^2-1}\right)+\log\left(x-\sqrt{x^2-1}\right)=0$.

* P.T. $\log\left(\sqrt{x^2+1}+x\right)+\log\left(\sqrt{x^2+1}-x\right)=0$.

P.T. $2\log\left(\frac{6}{7}\right)+\frac{1}{2}\log\left(\frac{81}{16}\right)-\log\left(\frac{27}{196}\right)=\log 12$

Simplify $\log 2+16\log\left(\frac{16}{15}\right)+12\log\left(\frac{25}{24}\right)+7\log\left(\frac{81}{80}\right)$.

* Change of base

* P.T. $\frac{1}{\log_3 6}+\frac{1}{\log_2 6}=1$

* P.T. $\frac{1}{\log_5 15}+\frac{1}{\log_3 15}=1$

* P.T. $\frac{1}{\log_{24} 12}+\frac{1}{\log_8 12}+\frac{1}{\log_9 12}=3$.

* P.T. $\frac{1}{\log_6 24}+\frac{1}{\log_{12} 24}+\frac{1}{\log_8 24}=2$

* P.T. $\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}=1$

* P.T. $\frac{1}{\log_{12} 60}+\frac{1}{\log_{30} 60}+\frac{1}{\log_{10} 60}=2$

* P.T. $\frac{1}{\log_{xy} xyz}+\frac{1}{\log_{yz} xyz}+\frac{1}{\log_{zx} xyz}=2$

* P.T. $\log_m x+\log_{m^2} x^2+\log_{m^3} x^3+\log_{m^4} x^4=4\cdot\log_m x$

* P.T. $\log_b a\cdot\log_c b\cdot\log_a c=1$

* P.T. $\log_x y\cdot\log_y z\cdot\log_z \omega=\log_x \omega$

* $\log_y x^2\cdot\log_z y^3\cdot\log_x z^4=24$

* $\log_{\sqrt{q}} p^2\cdot\log_{\sqrt{r}} q^2\cdot\log_{\sqrt{p}} r^2=64$

* $\log_{b^3} a^2\cdot\log_{a^3} b^2\cdot\log_{a^3} c^2=\frac{8}{27}$

* $\frac{1}{\log_x yz+1}+\frac{1}{\log_y zx+1}+\frac{1}{\log_z xy+1}=1$

* If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then P.T. $a^2+b^2 = 2ab$.

* If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then P.T. $a = b$.

* If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then P.T. $x^2+y^2 = 7xy$

* If $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then P.T. $a^2+b^2 = 6ab$

  OR $\frac{a}{b} + \frac{b}{a} = 6$.

* If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then P.T. $\frac{x}{y} + \frac{y}{x} = 7$.

* If $\frac{4\log 3 \times \log x}{\log 9} = \log 27$ then find the value of x.

* If $\frac{\log x \times \log 16}{\log 32} = \log 256$ then find the value of x.

* If $\frac{2\log_5^{x} + \log_5^{9}}{\log_5 3x} = \log_5^{x}$ then find the value of x.

* Solve $\log x^3 - \log 25 = \log x$

* Simplify $\log_3 84 - \log_3 28 - 3^{\log_3 1}$

* Solve $\log x + \log(x-5) = \log 6$

* Solve $\log_2(x+5) + \log_2(x-2) = 3$

* Solve $\log_2(\log_3(\log_2^{x})) = 1$.

$\log\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = \log 2 - \log x + \log\left(\frac{1}{y}\right)$ then P.T. $x = y$