Unit - 2. Trigonometry - 14 Marks

* **Unit circle**

$x^2 + y^2 = 1$

* convert Degree into Radian form

* 60° * 135° * 75° * 30° * 45°

* 330° * 105°

* convert Radian into Degree form

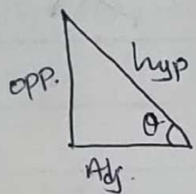
* $\frac{\pi}{15}$ * $\frac{\pi}{30}$ * $\frac{11\pi}{6}$ * $\frac{7\pi}{4}$

* $\frac{7\pi}{6}$ * $\frac{\pi}{12}$ * $\frac{2\pi}{9}$ * $\frac{3\pi}{20}$

* $\sin\theta = \frac{\text{opp. s.}}{\text{hyp.}}$

* $\cos\theta = \frac{\text{Adj. s.}}{\text{hyp}}$

* $\tan\theta = \frac{\text{opp. s}}{\text{Adj. s.}}$



* $\sin\theta = \frac{1}{\csc\theta}$ * $\csc\theta = \frac{1}{\sin\theta}$

* $\cos\theta = \frac{1}{\sec\theta}$ * $\sec\theta = \frac{1}{\cos\theta}$

* $\tan\theta = \frac{\sin\theta}{\cos\theta}$ * $\cot\theta = \frac{\cos\theta}{\sin\theta}$

* $\sin\theta \cdot \csc\theta = 1$

* $\cos\theta \cdot \sec\theta = 1$

* $\tan\theta \cdot \cot\theta = 1$

* $\sin^2\theta + \cos^2\theta = 1$

* $\sec^2\theta - \tan^2\theta = 1$

* $\csc^2\theta - \cot^2\theta = 1$

*

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* P.T. $\dfrac{\sin(\pi+\theta)}{\sin(2\pi-\theta)} + \dfrac{\tan(\frac{\pi}{2}+\theta)}{\cot(\pi-\theta)} + \dfrac{\cos(2\pi+\theta)}{\sin(\frac{\pi}{2}+\theta)} = 3$

* P.T. $\dfrac{\sin(\frac{\pi}{2}-\theta)}{\cos(\pi-\theta)} + \dfrac{\tan(\frac{\pi}{2}+\theta)}{\cot(\pi+\theta)} + \dfrac{\operatorname{cosec}(\frac{\pi}{2}+\theta)}{\sec(\pi+\theta)} = -3.$

* Evaluate $\dfrac{\sin(\theta-\frac{3\pi}{2})}{\cos(\theta-2\pi)} + \dfrac{\sec(\frac{3\pi}{2}+\theta)}{\operatorname{cosec}(\pi+\theta)} + \dfrac{\cot(\frac{\pi}{2}+\theta)}{\tan(2\pi+\theta)}$ (-1)

* Evaluate $\dfrac{\sin(\theta-\frac{\pi}{2})}{\cos(\theta-\pi)} + \dfrac{\tan(\frac{\pi}{2}+\theta)}{\cot(\pi+\theta)} + \dfrac{\operatorname{cosec}(\frac{\pi}{2}+\theta)}{\sec(\pi+\theta)}$ (-1)

* P.T. $\dfrac{\sin(\pi-A)}{\tan(\pi+A)} \cdot \dfrac{\cot(\frac{\pi}{2}-A)}{\tan(\frac{\pi}{2}+A)} \cdot \dfrac{\cos(2\pi-A)}{\sin(-A)} = \sin A$

* Simplify $\dfrac{\sin(180°-\theta)\cos(270-\theta)\operatorname{cosec}(90°+\theta)}{\sec(270°+\theta)\cot(90°+\theta)\tan(360°+\theta)}$ $(\sin\theta\cos\theta)$

* P.T. $\cos\frac{19\pi}{6} \cdot \sin\frac{17\pi}{6} - \sin\frac{11\pi}{6}\cos\frac{13\pi}{6} = 0$

* P.T. $\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2$

* P.T. $\tan\frac{\pi}{20}\tan\frac{3\pi}{20}\tan\frac{5\pi}{20}\tan\frac{7\pi}{20}\tan\frac{9\pi}{20} = 1$

* P.T. $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20} = 1$

* P.T. $\sin 407 \cdot \sin 133 + \sin 317 \cdot \sin 223 = 1$

* P.T. $\tan 225 \cdot \cot 405 + \tan 485 \cdot \cot 315 = 0$

* FOR ΔABC P. T.

(i) $\sin(B+C) = \sin A$

(ii) $\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$

(iii) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

* Compound angle

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

* P.T. $\tan 50 = \tan 40^\circ + 2\tan 10^\circ$

* $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

* P.T. $\frac{\sin(B-C)}{\sin B\sin C} + \frac{\sin(C-A)}{\sin C\sin A} + \frac{\sin(A-B)}{\sin A\sin B} = 0.$

* FOR ΔABC P. T. $\tan A + \tan B + \tan C = \tan A\cdot\tan B\cdot\tan C$

* P. T. $\tan 5A - \tan 3A - \tan 2A = \tan 5A\ \tan 3A\ \tan 2A$

* P. T. $\tan 55 = \frac{\cos 10 + \sin 10}{\cos 10 - \sin 10}$ * $\tan 57 = \frac{\cos 12 + \sin 12}{\cos 12 - \sin 12}$

* P. T. $\tan 20^\circ + \tan 25^\circ + \tan 20^\circ\tan 25^\circ = 1$.   * $\tan 66^\circ =$

* P. T. $(1 + \tan 25^\circ)(1 + \tan 20^\circ) = 2$

* P. T. $\tan 10^\circ + \tan 35^\circ + \tan 10^\circ\tan 35^\circ = 1$.

* $\sin 2\theta = 2\sin\theta\cos\theta$ * Multiple - Submultiple angle

$\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$= 2\sin\theta\cos\theta$

$\rightarrow \sin 2\theta = \frac{2\sin\theta\cos\theta}{1} = \frac{2\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta} = \frac{2\frac{\sin\theta}{\cos\theta}}{\tan^2\theta+1}$

$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

* $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$

$\rightarrow \cos 2\theta = \cos^2\theta - (1-\cos^2\theta) = 2\cos^2\theta - 1$

$\rightarrow \cos 2\theta = 1-\sin^2\theta - \sin^2\theta = 1-2\sin^2\theta$

$\rightarrow \cos 2\theta = \frac{\cos^2\theta - \sin^2\theta}{1} = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

dividing $\cos^2\theta$

* $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\rightarrow \tan 2\theta = \tan(\theta+\theta) = \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta} = \frac{2\tan\theta}{1-\tan^2\theta}$

* $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\sin 3\theta = \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= (2\sin\theta\cos\theta)\cos\theta + (1-2\sin^2\theta)\cdot\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$\therefore \sin 3\theta = 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

* $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

L.H.S $= \cos 3\theta = \cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta).\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta.$

$=$ R.H.S

* $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

* $\frac{\theta}{2}$ Formula

* P.T. $\dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = \tan A$, $\dfrac{1+\sin\theta - \cos\theta}{1+\sin\theta+\cos\theta} = \tan\frac{\theta}{2}$

* P.T. $\dfrac{\sin A + \sin 2A}{1+\cos A + \cos 2A} = \tan A$

$\dfrac{\sin A + 2\sin A\cos A}{1+\cos A + 2\cos^2 A - 1}$

$\dfrac{\sin A(1+2\cos A)}{\cos A(1+2\cos A)} = \tan A$

* If $\tan\theta = -\frac{3}{4}$ then find $\sin 2\theta$, $\cos 2\theta$.

* $S + S = 2SC$

$S - S = 2CS$

$C + C = 2CC$

$C - C = -2SS$

* $2\sin A \cos B = \sin(A+B) + \sin(A-B)$.

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$-2\sin A \sin B = +\cos(A+B) - \cos(A-B)$

* $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$.

* P.T. $\frac{\sin\theta + \sin 2\theta + \sin 3\theta}{\cos\theta + \cos 2\theta + \cos 3\theta} = \tan 2\theta$

* P.T. $\frac{\cos A + \cos 3A + \cos 5A}{\sin A + \sin 3A + \sin 5A} = \cot 3A$

* P.T. $\frac{\cos 4\theta + 2\cos 5\theta + \cos 6\theta}{\sin 4\theta + 2\sin 5\theta + \sin 6\theta} = \cot 5\theta$

* Inverse Trigonometry Function

* P.T. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Let $\sin^{-1}x = \theta$

$\therefore x = \sin\theta$

we know that $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

$\therefore x = \cos\left(\frac{\pi}{2} - \theta\right)$

$\therefore \cos^{-1}x = \frac{\pi}{2} - \theta$

$\therefore \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

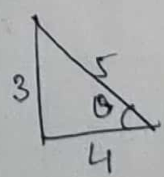
* P.T. $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$
* P.T. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$.

* $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$

$= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy > 1$

$= \frac{\pi}{2}$, if $xy = 1$

* P.T. $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$
* P.T. $\tan^{-1}\left(\frac{5}{7}\right) + \tan^{-1}\left(\frac{1}{6}\right) = \frac{\pi}{4}$
* P.T. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \tan^{-1}\left(\frac{1}{2}\right)$
* P.T. $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$
* P.T. $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$
* P.T. $2\tan^{-1}\left(\frac{2}{3}\right) = \tan^{-1}\left(\frac{12}{5}\right)$
* P.T. $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{2}$.

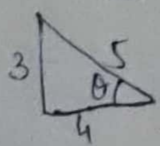


$\sin^{-1}\left(\frac{3}{5}\right) = \theta$

$\sin\theta = \frac{3}{5}$

$\tan\theta = \frac{3}{4}$

* P.T. $\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{2}$



$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

* $y = \sin x$, $0 \le x \le 2\pi$ $\quad$ $y = \sin x$, $0 \le x \le \pi$

* $y = \cos x$, $0 \le x \le 2\pi$ $\quad$ $y = \cos x$, $0 \le x \le \pi$

* $y = \sin x$, $-\pi/2 \le x \le \pi/2$

* $y = \cos x$, $-\pi/2 \le x \le \pi/2$

* $y = 2\sin x$, $0 \le x \le \pi$

* $y = 2\cos x$, $0 \le x \le \pi$

* $y = \sin \frac{x}{2}$, $0 \le x \le 2\pi$

* $y = \cos \frac{x}{2}$, $0 \le x \le 2\pi$