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Home assignment № 1

Task 1.

U is a finite set, $f : U \to U$ and f is a surjective function. It means that

$$\forall y \in U \ \exists x \in U : f(x) = y$$

Let f is **not** an injective function. Then $\exists x_1 \neq x_2 : f(x_1) = f(x_2)$. And also $\exists y' \ \nexists x : f(x) = y'$. But U is finite set and $\forall y \in U \ \exists x \in U : f(x) = y$. Therefore,

$$\forall x_1, x_2 : f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

It means that f is also an injective function.

Task 2.

a) Asymmetric and transitive: 4231

Computate this number by a Haskell program *TransitiveAsymmetricRelations.hs*.

b) Antisymmetric and antireflexive: 3^{10}

Consider the relation matrix P. Diagonal elements p_{ii} $(i = 1 \ldots 5)$ must be always 0. For non-diagonal elements:

$$\forall i, j = 1 \ldots 5, \ i \neq j : (p_{ij} = 1 \wedge p_{ji} = 0) \vee (p_{ij} = 1 \wedge p_{ji} = 0) \vee (p_{ij} = 0 \wedge p_{ji} = 0)$$

There are 10 pairs of different set elements and all of them have 3 of 4 variants of values. Therefore, we have a result: 3^{10}.

Task 3.

In Task 2 I wrote a Haskell program (*TransitiveAsymmetricRelations.hs*) including a function *isTransitive*. This function get a relation matrix P $(n \times n)$, square it $(O(n^3))$ and check that result of multiplication is including on original relation matrix $P^2 \subseteq P$ $(O(n^2))$. As result we have a complexity: $O(n^3 + n^2) = O(n^3)$.

Task 4.

Another my Haskell program (*TopologicalSort.hs*) include topological sort algorithm with two types of input data: list of edges (a) and adjacency matrix (b).

a) List of edges complexity: $O(n + m)$

b) Adjacency matrix complexity: $O(n^2)$

where n — number of vertices, m — number of edges.

Task 5.

$R : (x_1, y_1)R(x_2, y_2) \leftrightarrow x_1 \leq x_2, y_1 \leq y_2$.

R is reflexive.
$\forall (x, y) \in Z^2 : x \leq x,\ y \leq y$, because $\leq$ is reflexive $\Rightarrow (x, y)R(x, y)$.

R is transitive.
$\forall (x_1, y_1), (x_2, y_2), (x_3, y_3) : (x_1, y_1)R(x_2, y_2), (x_2, y_2)R(x_3, y_3) \Rightarrow x_1 \leq x_2,\ y_1 \leq y_2,\ x_2 \leq x_3,\ y_2 \leq y_3$. $\leq$ is transitive $\Rightarrow x_1 \leq x_3,\ y_1 \leq y_3 \Rightarrow (x_1, y_1)R(x_3, y_3)$.

R is antisymmetric.
$\forall (x_1, y_1), (x_2, y_2) : (x_1, y_1)R(x_2, y_2), (x_2, y_2)R(x_1, y_1) \Rightarrow x_1 \leq x_2,\ y_1 \leq y_2,\ x_2 \leq x_1,\ y_2 \leq y_1$. $\leq$ is antisymmetric $\Rightarrow x_1 = x_2,\ y_1 = y_2 \Rightarrow (x_1, y_1) = (x_2, y_2)$.

It means that R is a partial order.

1) $A_1 = (x, y) \mid x \leq 3,\ y \leq 4$
$\nexists min,\ max = (3, 4)$.

2) $A_2 = (x, y) \mid x^2 + y^2 \leq 4$
$min = (-2, -2),\ max = (2, 2)$.