Kirill Zuev

Home assignment № 3

Task 1.

For (G^2, M, I) takes place $A \to B \iff A' \subseteq B' \iff$

$$\{\{g,h\} \in G^2 \mid \forall m \in A : m(g) = m(h)\} \subseteq \{\{g,h\} \in G^2 \mid \forall m \in B : m(g) = m(h)\}$$

$\iff \forall m \in A : m(g) = m(h) \Rightarrow \forall m \in B : m(g) = m(h) \iff$ For (G, M, W, J) takes place $A \to B$.

Task 2.

The minimal subset of implications from which all of given are deducible is:

$$\{A \to C,\ C \to E,\ E \to B,\ E \to D,\ DB \to C\}$$

For getting $C \to D$ use 3-rd axiom: $\{C \to E,\ E \to D\} \Rightarrow C \to D$.

For getting $BC \to D$ use 2-nd axiom: $C \to D \Rightarrow BC \to D$.

For getting $A \to E$ use 3-rd axiom: $\{A \to C,\ C \to E\} \Rightarrow A \to E$.

For getting $AB \to D$ use 2-nd and 3-rd axioms.
3-rd axiom: $\{A \to E,\ E \to D\} \Rightarrow A \to D$;
2-nd axiom: $A \to D \Rightarrow AB \to D$.

If we remove any implication from our minimal subset then we couldn't get all needed implications.

Task 3.

I wrote a Haskell program (*CloseByOne.hs*) including Close-by-One algorithm for any formal concept inputed as a matrix.

Task 4.

You can find solution on the next page :)

№4 (a)

$a'' = a$ ×

$b'' = bd$ ✓

$c'' = c$ ×

$d'' = d$ ×

$e'' = ce$ ✓

$ab'' = abd$ × $(b'' = bd \not\subset ab)$

$ac'' = abcd$ ✓

$ad'' = abd$ ✓

$ae'' = M$ $(e'' = ce \not\subset ae)$

$bc'' = abcd$ × $(b'' = bd \not\subset bc)$

$bd'' = bd$ ×

$be'' = M$ $(b'' = bd \not\subset be)$

$cd'' = cd$ ×

$ce'' = ce$ ×

$de'' = cde$ × $(e'' = ce \not\subset de)$

$abc'' = abcd$ × $(b'' = bd \not\subset abc)$

$abd'' = abd$ ×

$abe'' = M$ × $(b'' = bd \not\subset abe)$

$acd'' = abcd$ × $(ac'' = abcd \not\subset acd)$

$ace'' = M$ × $(ac'' = abcd \not\subset ace)$

$ade'' = M$ × $(e'' = ce \not\subset ade)$

$bcd'' = abcd$ ✓

$bce'' = M$ × $(b'' = bd \not\subset bce)$

$bde'' = M$ × $(e'' = ce \not\subset bde)$

$cde'' = cde$ ×

$abcd'' = abcd$ ×

$abce'' = M$ × $(b'' = bd \not\subset abce)$

$abde'' = M$ × $(e'' = ce \not\subset abde)$

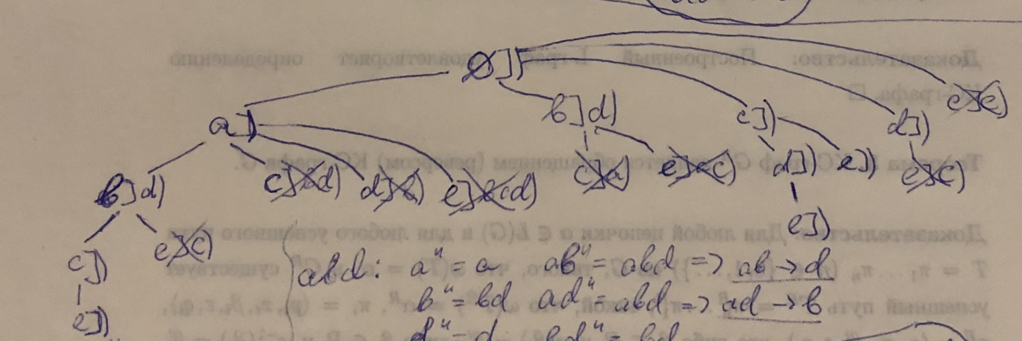
$acde'' = M$ × $(ac'' = abcd \not\subset acde)$

$bcde'' = M$ × $(bcd'' = abcd \not\subset bcde)$

$abcde'' = M$ ×
$= M''$

Answer: $b \to d$, $e \to c$, $ac \to bd$, $ad \to b$, $bcd \to a$

(b)



abd: $a'' = a$, $b'' = bd$, $d'' = d$; $ab'' = abd \Rightarrow ab \to d$; $ad'' = abd \Rightarrow ad \to b$; $bd'' = bd$

bd: $b'' = bd \Rightarrow b \to d$; $d'' = d$

cd: $c'' = c$; $d'' = d$

ce: $c'' = c$; $e'' = ce \Rightarrow e \to c$

cde: $c'' = c$; $d'' = d$; $e'' = ce$; $cd'' = cd$; $ce'' = ce$; $de'' = cde \Rightarrow de \to c$

$abcd$: $a'' = a$; $b'' = bd$; $c'' = c$; $d'' = d$; $ac'' = abcd \Rightarrow ac \to bd$; $bc'' = abcd \Rightarrow bc \to ad$

$abcde$: $ae'' = abcde \Rightarrow ae \to bcd$; $be'' = abcde \Rightarrow be \to acd$

Answer: $b \to d$, $e \to c$, $ab \to d$, $ad \to b$, $de \to c$, $ac \to bd$, $bc \to ad$, $ae \to bcd$, $be \to acd$