

Home assignment № 3

Task 2.

The minimal subset of implications from which all of given are deducible is:

$$\{A \rightarrow C, C \rightarrow E, E \rightarrow B, E \rightarrow D, DB \rightarrow C\}$$

For getting $C \rightarrow D$ use 3-rd axiom: $\{C \rightarrow E, E \rightarrow D\} \Rightarrow C \rightarrow D$.

For getting $BC \rightarrow D$ use 2-nd axiom: $C \rightarrow D \Rightarrow BC \rightarrow D$.

For getting $A \rightarrow E$ use 3-rd axiom: $\{A \rightarrow C, C \rightarrow E\} \Rightarrow A \rightarrow E$.

For getting $AB \rightarrow D$ use 2-nd and 3-rd axioms.

3-rd axiom: $\{A \rightarrow E, E \rightarrow D\} \Rightarrow A \rightarrow D$;

2-nd axiom: $A \rightarrow D \Rightarrow AB \rightarrow D$.

If we remove any implication from our minimal subset then we couldn't get all needed implications.

Task 4.

N4(a)

$a'' = a \times$
 $b'' = bd \checkmark$
 $c'' = c \times$
 $d'' = d \times$
 $e'' = ce \checkmark$

$ab'' = abd \times (b'' = bd \neq ab)$
 $ac'' = acd \checkmark$
 $ad'' = abd \checkmark$
 $ae'' = M (e'' = ce \neq ae)$
 $bc'' = abd \times (b'' = bd \neq bc)$
 $bd'' = bd \times$
 $be'' = M (b'' = bd \neq be)$
 $cd'' = cd \times$
 $ce'' = ce \times$
 $de'' = cde \times (e'' = ce \neq de)$

$abc'' = abd \times (b'' = bd \neq abc)$
 $abd'' = abd \times$
 $abe'' = M \times (b'' = bd \neq abe)$
 $acd'' = abd \times (ac'' = acd \neq acd)$
 $ace'' = M \times (ac'' = abd \neq ace)$
 $ade'' = M \times (e'' = ce \neq ade)$
 $bcd'' = abd \checkmark$
 $bce'' = M \times (b'' = bd \neq bce)$
 $bde'' = M \times (e'' = ce \neq bde)$
 $cde'' = cde \times$

$abed'' = abd \times$
 $abce'' = M \times (b'' = bd \neq abce)$
 $abde'' = M \times (e'' = ce \neq abde)$
 $acde'' = M \times (ac'' = abd \neq acde)$
 $bcd e'' = M \times (bcd'' = abd \neq bcde)$
 $abcde'' = M \times$
 M''

Answer: $b \rightarrow d$
 $e \rightarrow c$
 $ac \rightarrow bd$
 $ad \rightarrow b$
 $bcd \rightarrow a$

(b)

Tree diagram showing the derivation of the answer from the initial state \emptyset .

Initial state: \emptyset
 Transitions: a , b , c , d , e
 States: a , b , c , d , e , ab , ac , ad , ae , bc , bd , cd , ce , abc , abd , abe , acd , ace , ade , bcd , bce , bde , cde , $abcde$

Analysis of the tree diagram:

- From \emptyset , transitions lead to a , b , c , d , e .
- From a , transitions lead to ab , ac , ad , ae .
- From b , transitions lead to ba , bc , bd , be .
- From c , transitions lead to ca , cb , cd , ce .
- From d , transitions lead to da , db , dc , de .
- From e , transitions lead to ea , eb , ec , ed .

Analysis of the states:

- a : $a'' = a$, $b'' = bd$, $c'' = c$, $d'' = d$
- b : $b'' = bd \Rightarrow b \rightarrow d$, $d'' = d$
- c : $c'' = c$, $e'' = ce \Rightarrow c \rightarrow e$
- d : $d'' = d$, $e'' = ce \Rightarrow d \rightarrow c$
- ab : $a'' = a$, $b'' = bd \Rightarrow ab \rightarrow d$
- ac : $a'' = a$, $c'' = c \Rightarrow ac \rightarrow bd$
- ad : $a'' = a$, $d'' = d \Rightarrow ad \rightarrow b$
- ae : $a'' = a$, $e'' = ce \Rightarrow ae \rightarrow c$
- bc : $b'' = bd \Rightarrow bc \rightarrow ad$, $c'' = c$
- bd : $b'' = bd \Rightarrow b \rightarrow d$, $d'' = d$
- cd : $c'' = c$, $d'' = d \Rightarrow cd \rightarrow b$
- ce : $c'' = c$, $e'' = ce \Rightarrow c \rightarrow e$
- abc : $a'' = a$, $b'' = bd \Rightarrow abc \rightarrow d$, $c'' = c$
- abd : $a'' = a$, $b'' = bd \Rightarrow abd \rightarrow d$, $d'' = d$
- abe : $a'' = a$, $b'' = bd \Rightarrow abe \rightarrow d$, $e'' = ce \Rightarrow abe \rightarrow c$
- acd : $a'' = a$, $c'' = c \Rightarrow acd \rightarrow bd$, $d'' = d$
- ace : $a'' = a$, $c'' = c \Rightarrow ace \rightarrow bd$, $e'' = ce \Rightarrow ace \rightarrow c$
- ade : $a'' = a$, $d'' = d \Rightarrow ade \rightarrow b$, $e'' = ce \Rightarrow ade \rightarrow c$
- bcd : $b'' = bd \Rightarrow bcd \rightarrow ad$, $c'' = c$, $d'' = d$
- bce : $b'' = bd \Rightarrow bce \rightarrow ad$, $c'' = c$, $e'' = ce \Rightarrow bce \rightarrow c$
- bde : $b'' = bd \Rightarrow bde \rightarrow ad$, $d'' = d$, $e'' = ce \Rightarrow bde \rightarrow c$
- cde : $c'' = c$, $d'' = d \Rightarrow cde \rightarrow b$, $e'' = ce \Rightarrow cde \rightarrow c$
- $abcde$: $a'' = a$, $b'' = bd \Rightarrow abcde \rightarrow d$, $c'' = c$, $d'' = d$, $e'' = ce \Rightarrow abcde \rightarrow c$

Answer: $b \rightarrow d$
 $e \rightarrow c$
 $ab \rightarrow d$
 $ac \rightarrow bd$
 $ad \rightarrow b$
 $ae \rightarrow c$
 $bc \rightarrow ad$
 $bd \rightarrow c$
 $cd \rightarrow b$
 $ce \rightarrow c$