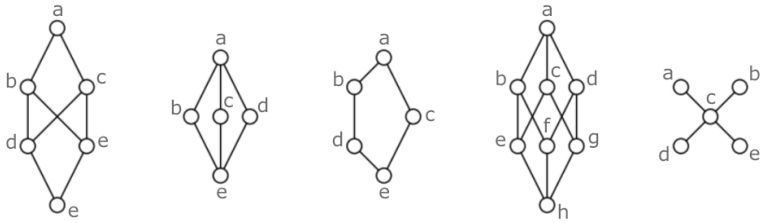
Kirill Zuev

Home assignment № 2

Task 1.

Mark the names of air companies with g_i $(i = 1, \ldots, 13)$ and the names of regions with m_j $(j = 1, \ldots, 9)$. I wrote a Haskell program *FormalConcepts.hs* to find all formal concepts:

1) $A = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9, g_{10}, g_{11}, g_{12}, g_{13}\},\ B = \varnothing$
2) $A = \{g_1, g_2, g_3, g_5, g_6, g_7, g_9, g_{10}, g_{11}, g_{12}, g_{13}\},\ B = \{m_2\}$
3) $A = \{g_1, g_2, g_3, g_4, g_5, g_7, g_9, g_{10}, g_{11}, g_{12}, g_{13}\},\ B = \{m_4\}$
4) $A = \{g_1, g_2, g_3, g_5, g_7, g_8, g_9, g_{10}, g_{11}, g_{12}, g_{13}\},\ B = \{m_9\}$
5) $A = \{g_1, g_7, g_8, g_9, g_{11}, g_{12}, g_{13}\},\ B = \{m_1, m_9\}$
6) $A = \{g_1, g_5, g_7, g_8, g_{10}, g_{12}\},\ B = \{m_3, m_9\}$
7) $A = \{g_1, g_2, g_3, g_5, g_7, g_9, g_{10}, g_{11}, g_{12}, g_{13}\},\ B = \{m_2, m_4, m_9\}$
8) $A = \{g_1, g_7, g_9, g_{11}, g_{12}, g_{13}\},\ B = \{m_1, m_2, m_4, m_9\}$
9) $A = \{g_1, g_5, g_7, g_{10}, g_{12}\},\ B = \{m_2, m_3, m_4, m_9\}$
10) $A = \{g_1, g_5, g_7, g_{10}\},\ B = \{m_2, m_3, m_4, m_5, m_9\}$
11) $A = \{g_5, g_7, g_9, g_{10}, g_{13}\},\ B = \{m_2, m_4, m_6, m_9\}$
12) $A = \{g_7, g_9, g_13\},\ B = \{m_1, m_2, m_4, m_6, m_9\}$
13) $A = \{g_5, g_7, g_{10}\},\ B = \{m_2, m_3, m_4, m_5, m_6, m_9\}$
14) $A = \{g_1, g_7, g_8, g_{12}, g_{13}\},\ B = \{m_1, m_7, m_9\}$
15) $A = \{g_1, g_7, g_8, g_{12}\},\ B = \{m_1, m_3, m_7, m_9\}$
16) $A = \{g_1, g_7, g_{12}, g_{13}\},\ B = \{m_1, m_2, m_4, m_7, m_9\}$
17) $A = \{g_1, g_7, g_{12}\},\ B = \{m_1, m_2, m_3, m_4, m_7, m_9\}$
18) $A = \{g_1, g_7\},\ B = \{m_1, m_2, m_3, m_4, m_5, m_7, m_9\}$
19) $A = \{g_7, g_{13}\},\ B = \{m_1, m_2, m_4, m_6, m_7, m_9\}$
20) $A = \{g_7\},\ B = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_9\}$
21) $A = \{g_1, g_8, g_{11}, g_{12}\},\ B = \{m_1, m_8, m_9\}$
22) $A = \{g_1, g_{11}, g_{12}\},\ B = \{m_1, m_2, m_4, m_8, m_9\}$
23) $A = \{g_1, g_8, g_{12}\},\ B = \{m_1, m_3, m_7, m_8, m_9\}$
24) $A = \{g_1, g_{12}\},\ B = \{m_1, m_2, m_3, m_4, m_7, m_8, m_9\}$
25) $A = \{g_1\},\ B = \{m_1, m_2, m_3, m_4, m_5, m_7, m_8, m_9\}$
26) $A = \varnothing,\ B = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9\}$



Task 2.

(i) $\nexists d \vee e \Rightarrow$ It is not a lattice.

(ii) Diamond is a lattice. And it is complete because all finite lattices are complete.
$a \vee x = a$ $(x = b, \dots e)$, $b \vee c = b \vee d = c \vee d = a$, $e \vee x = x$ $(x = b, \dots, d)$
$e \wedge x = e$ $(x = a, \dots d)$, $b \wedge c = b \wedge d = c \wedge d = e$, $a \wedge x = x$ $(x = b, \dots, d)$

(iii) Pentagon is a lattice. And it is complete because all finite lattices are complete.
$a \vee x = a$ $(x = b, \dots e)$, $b \vee c = c \vee d = a$, $b \vee d = b$, $e \vee x = x$ $(x = b, \dots, d)$
$e \wedge x = e$ $(x = a, \dots d)$, $b \wedge c = c \wedge d = e$, $b \wedge d = d$, $a \wedge x = x$ $(x = b, \dots, d)$

(iv) Boolean cube is a lattice because all pairs have supremum and infimum. And it is complete because all finite lattices are complete.

(v) $\nexists a \vee b \Rightarrow$ It is not a lattice.

Task 3.

$(L, \leq)$ is a lattice with infimum and supremum defined as usual.
$\forall x, y \in L$:

a) $x \vee x = sup\{x, x\} = sup\{x\} = x$

b) $x \vee (x \wedge y) = sup\{x, inf\{x, y\}\}$
$y \leq x \Rightarrow sup\{x, inf\{x, y\}\} = sup\{x, y\} = x$
$x \leq y \Rightarrow sup\{x, inf\{x, y\}\} = sup\{x, x\} = x$

c) $x \vee y = sup\{x, y\} = sup\{y, x\} = y \vee x$