

Proof:

Let \mathbf{v}_1 and \mathbf{v}_2 be two edge vectors of a face on a 3D object. The normal vector \mathbf{n} is given by:

$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2$$

Let us apply a scaling transformation with scaling factors (s_x, s_y, s_z) . The scaled vectors \mathbf{v}'_1 and \mathbf{v}'_2 become:

$$\mathbf{v}'_1 = (s_x v_{1x}, s_y v_{1y}, s_z v_{1z})$$

$$\mathbf{v}'_2 = (s_x v_{2x}, s_y v_{2y}, s_z v_{2z})$$

The new normal \mathbf{n}' after scaling is:

$$\mathbf{n}' = \mathbf{v}'_1 \times \mathbf{v}'_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ s_x v_{1x} & s_y v_{1y} & s_z v_{1z} \\ s_x v_{2x} & s_y v_{2y} & s_z v_{2z} \end{vmatrix}$$

Expanding this determinant, we obtain:

$$\mathbf{n}' = (s_y s_z (v_{1y} v_{2z} - v_{1z} v_{2y}), s_x s_z (v_{1z} v_{2x} - v_{1x} v_{2z}), s_x s_y (v_{1x} v_{2y} - v_{1y} v_{2x}))$$

It is clear that $\mathbf{n}' \neq \mathbf{n}$, therefore scaling does not preserve the normals of all edges.