ICMA 346 Project Report: The Tennessee Pterodactyls

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1 Problem Requierments and Formulation

We have a yearly budget of $50 million to sign free agents from the following table:

	Player	Position	Points	Rebounds	Assists	Minutes	Salary ($M)
1	Mack Madonna	Back court	14.7	4.4	9.3	40.3	8.2
2	Darrell Boards	Front court	12.6	10.6	2.1	34.5	6.5
3	Silk Curry	Back court	13.5	8.7	1.7	29.3	5.2
4	Ramon Dion	Back court	27.1	7.1	4.5	42.5	16.4
5	Joe Eastcoast	Back court	18.1	7.5	5.1	41.0	14.3
6	Abdul Famous	Front court	22.8	9.5	2.4	38.5	23.5
7	Hiram Grant	Front court	9.3	12.2	3.5	31.5	4.7
8	Antoine Roadman	Front court	10.2	12.6	1.8	44.4	7.1
9	Fred Westcoast	Front court	16.9	2.5	11.4	42.7	15.8
10	Magic Jordan	Back court	28.5	6.5	1.3	38.1	26.4
11	Barry Bird	Front court	24.8	8.6	6.9	42.6	19.5
12	Grant Hall	Front court	11.3	12.5	3.2	39.5	8.6

Table 1: Free Agent Information

The requirements are:

1. Sign exactly 5 players.
2. Total points per game ≥ 80.
3. Total rebounds per game ≥ 40.
4. Total assists per game ≥ 25.
5. Total minutes per game ≥ 190.
6. At most 2 front court and at most 3 back court players. As we must sign 5 players, we can simplify this to just require exactly 2 front court players, since the 3 back court player requirement will naturally follow.
7. Select the group that satisfies requirements 1-6 at minimum total salary cost.

Define a vector $\boldsymbol{x} \in \{0,1\}^{12}$, consisting of the following binary decision variables:

$$x_i = \begin{cases} 1, & \text{if player } i \text{ is signed,} \\ 0, & \text{otherwise,} \end{cases} \qquad i = 1, \ldots, 12.$$

Let $\boldsymbol{s}, \boldsymbol{p}, \boldsymbol{r}, \boldsymbol{a}, \boldsymbol{m} \in \mathbb{R}_+^{12}$ be the salary, points, rebounds, assists, minutes vectors, and $\boldsymbol{f} \in \{0,1\}^{12}$ indicate front court (1) or back court (0).

We can now formulate the integer linear programming model:

$$
\begin{aligned}
\text{minimize} \quad & z = \boldsymbol{s}^\top \boldsymbol{x} \\
\text{subject to} \quad & \mathbf{1}^\top \boldsymbol{x} = 5, \\
& \boldsymbol{p}^\top \boldsymbol{x} \geq 80, \\
& \boldsymbol{r}^\top \boldsymbol{x} \geq 40, \\
& \boldsymbol{a}^\top \boldsymbol{x} \geq 25, \\
& \boldsymbol{m}^\top \boldsymbol{x} \geq 190, \\
& \boldsymbol{f}^\top \boldsymbol{x} = 2, \\
& \boldsymbol{x}, \boldsymbol{f} \in \{0,1\}^{12}, \\
& \boldsymbol{s}, \boldsymbol{p}, \boldsymbol{r}, \boldsymbol{a}, \boldsymbol{m} \in \mathbb{R}_+^{12}.
\end{aligned}
$$

2 Binary Linear Programming Solution

2.1 Raw CPLEX Output

```
solution for: tennessee_pterodactlys
objective: 52.2
status: OPTIMAL_SOLUTION(2)
Mack Madonna=1
Ramon Dion=1
Joe Eastcoast=1
Hiram Grant=1
Grant Hall=1
```

2.2 Interpretation

We can see that we have reached a status of an optimal solution, and our objective function (minimum cost) is 52.2, with the listed five players having a binary value of 1, thus being selected. Below are two tables showing the statistics of the selected players and their aggregate stats:

	Player	**Position**	**Points**	**Rebounds**	**Assists**	**Minutes**	**Salary ($M)**
1	Mack Madonna	Back court	14.7	4.4	9.3	40.3	8.2
4	Ramon Dion	Back court	27.1	7.1	4.5	42.5	16.4
5	Joe Eastcoast	Back court	18.1	7.5	5.1	41.0	14.3
7	Hiram Grant	Front court	9.3	12.2	3.5	31.5	4.7
12	Grant Hall	Front court	11.3	12.5	3.2	39.5	8.6

Table 2: Player Stats

Statistic	**Total**
Total Points (Objective)	80.50
Total Salary	$52.20M
Total Rebounds	43.7
Total Assists	25.6
Total Minutes	194.8
Total Front Court Players	2

Table 3: Aggregate Stats

Notes: We can see that Ramon Dion is an incredibly valuable player in terms of scoring output especially when comparing him to the other highly scoring free agents. Mack Madonna, Hiram Grant, and Grant Hall are very valuable all around players with standout assists and rebound statistics respectively, with very cheap salaries in comparison to other agents, making them very cost effective. Lastly, Joe Eastcoast is just a solid jack of all trades player, with decent stats in every category.

3 Answers to Case Questions

Question A. Formulate an integer linear programming model (ILP) to help the general manager and coach determine which players they should sign and solve it by using the computer.

Answer A. The ILP model, along with the five players to sign have been displayed in the section above.

Question B. Is the money provided by the owner sufficient to sign the group of players identified in (A)? If not, reformulate the model so that the available funds are a constraint and the objective is to maximize the average points of the group.

Answer B. As seen in the section above, the total salary of the selected roster is \$52.2 million, whereas the budget is \$50.0 million, thus meaning we do not have enough money to sign this combination of players. The LP reformulation and new solution are presented below.

3.1 Question B LP Reformulation

We will keep all the defined variables and vectors as before, and only change the objective function along with the constraints.

$$\begin{aligned} \text{maximize} \quad & z = \boldsymbol{p}^\top \boldsymbol{x} \\ \text{subject to} \quad & \boldsymbol{s}^\top \boldsymbol{x} \leq 50, \\ & \boldsymbol{x}, \in \{0,1\}^{12}, \\ & \boldsymbol{p}, \boldsymbol{s} \in \mathbb{R}_+^{12}. \end{aligned}$$

3.2 Raw CPLEX Output

```
solution for: tennessee_pterodactlys_B
objective: 82.7
status: OPTIMAL_SOLUTION(2)
Mack Madonna=1
Silk Curry=1
Ramon Dion=1
Joe Eastcoast=1
Hiram Grant=1
```

3.3 Interpretation

Once again, we have reached a status of an optimal solution, and our objective function (max total points) is 82.7, with five players having a binary value of 1 thus being selected. As prior, tables showing the players' statistics and the aggregate roster stats are presented below.

	Player	Position	Points	Rebounds	Assists	Minutes	Salary ($M)
1	Mack Madonna	Back court	14.7	4.4	9.3	40.3	8.2
3	Silk Curry	Back court	13.5	8.7	1.7	29.3	5.2
4	Ramon Dion	Back court	27.1	7.1	4.5	42.5	16.4
5	Joe Eastcoast	Back court	18.1	7.5	5.1	41.0	14.3
7	Hiram Grant	Front court	9.3	12.2	3.5	31.5	4.7

Table 4: Player Stats for Formulation B

Statistic	Total
Max Total Points (Objective)	82.70
Total Salary	$48.80M
Total Rebounds	39.9
Total Assists	24.1
Total Minutes	184.6
Total Front Court Players	1

Table 5: Aggregate Stats for Formulation B

Notes: We have kept approximately the same roster, except for replacing Grant Hill with Silk Curry, likely due to the difference in point scoring and salary letting us reach the budget requirements. As a result, our point scoring is slightly up, however the rest of the statistics have slightly suffered, especially the number of rebounds and minutes played. Furthermore, we now have a more unbalanced roster with 4 back court players and only 1 front court player.

4 Sensitivity Analysis

Since sensitivity analysis must be performed on the LP relaxation, we must change the binary variables to continous ones between 0 and 1. I am assuming that we should do the sensitivity analysis on the original/first LP formulation.

4.1 Raw CPLEX Output

```
solution for: tennessee_pterodactlys_LP
objective: 47.4889
status: OPTIMAL_SOLUTION(2)
Mack Madonna=1.000
Darrell Boards=0.151
Silk Curry=1.000
Ramon Dion=1.000
Joe Eastcoast=0.000
Hiram Grant=0.656
Antoine Roadman=0.647
Fred Westcoast=0.435
Barry Bird=0.111
```

4.2 Interpretation

Below is a table of the players with non-zero contribution values, along with their original raw stats, contribution fraction unaccounted for.

	Player	Contrib	Pos	Pts	Rbds	Asts	Mins	Salary ($M)
1	Mack Madonna	1.000	Back court	14.7	4.4	9.3	40.3	8.2
2	Darrell Boards	0.151	Front court	12.6	10.6	2.1	34.5	6.5
3	Silk Curry	1.000	Back court	13.5	8.7	1.7	29.3	5.2
4	Ramon Dion	1.000	Back court	27.1	7.1	4.5	42.5	16.4
7	Hiram Grant	0.656	Front court	9.3	12.2	3.5	31.5	4.7
8	Antoine Roadman	0.647	Front court	10.2	12.6	1.8	44.4	7.1
9	Fred Westcoast	0.435	Front court	16.9	2.5	11.4	42.7	15.8
11	Barry Bird	0.111	Front court	24.8	8.6	6.9	42.6	19.5

Table 6: Raw Player Stats for LP Relaxation

Below is a table of the players with the same logic, except that all of their values are scaled by their contribution, so (player stat) * (player contribution).

	Player	Contrib	Pos	Pts	Rbds	Asts	Mins	Salary ($M)
1	Mack Madonna	1.000	Back court	14.70	4.40	9.30	40.30	8.20
2	Darrell Boards	0.151	Front court	1.90	1.60	0.32	5.21	0.98
3	Silk Curry	1.000	Back court	13.50	8.70	1.70	29.30	5.20
4	Ramon Dion	1.000	Back court	27.10	7.10	4.50	42.50	16.40
7	Hiram Grant	0.656	Front court	6.10	8.00	2.30	20.67	3.08
8	Antoine Roadman	0.647	Front court	6.60	8.15	1.16	28.72	4.60
9	Fred Westcoast	0.435	Front court	7.35	1.09	4.96	18.57	6.87
11	Barry Bird	0.111	Front court	2.75	0.95	0.77	4.73	2.16

Table 7: Contribution Accounted Player Stats for LP Relaxation

Below is the aggregate statistics for the LP relaxation. Note that since we are allowed to use fractions of players, we are now able to perfectly reach the required constraints allowing us to fully minimize the cost and reach below the allocated $50.00 million budget.

Statistic	Total
Total Salary (Objective)	$47.49M
Total Points	80.00
Total Rebounds	40.00
Total Assists	25.00
Total Minutes	190.00
Total Front Court Players	2.00
Total Number of Players	5.00

Table 8: Aggregated Stats for LP Relaxation

4.3 Objective Coefficient Sensitivity Analysis (LP Relaxation)

The following sensitivity ranges indicate how much each player's salary can change before the current optimal basis of the LP relaxation solution changes. These ranges are critical for understanding the robustness of our player selection decisions under salary fluctuations.

Player	**Current Salary ($M)**	**Lower Bound ($M)**	**Upper Bound ($M)**	**Allowable Decrease**	**Allowable Increase**
Mack Madonna	8.20	$-\infty$	13.90	∞	5.70
Darrell Boards	6.50	5.29	7.04	1.21	0.54
Silk Curry	5.20	$-\infty$	6.60	∞	1.40
Ramon Dion	16.40	$-\infty$	20.48	∞	4.08
Joe Eastcoast	14.30	12.90	22.31	1.40	8.01
Abdul Famous	23.50	14.51	$+\infty$	8.99	∞
Hiram Grant	4.70	3.98	6.38	0.72	1.68
Antoine Roadman	7.10	4.37	8.73	2.73	1.63
Fred Westcoast	15.80	5.45	17.71	10.35	1.91
Magic Jordan	26.40	18.39	$+\infty$	8.01	∞
Barry Bird	19.50	17.91	22.90	1.59	3.40
Grant Hall	8.60	7.69	$+\infty$	0.91	∞

Table 9: Salary Sensitivity Ranges for Players

4.4 Interpretation of Sensitivity Analysis

4.4.1 Value for Money Players

Players with lower bounds of $-\infty$ (Mack Madonna, Silk Curry, and Ramon Dion) represent exceptional value propositions. These players have maximum contribution values (1.00). Due to this, a salary decrease wouldn't increase their contribution rating, it is already at its maximum (but would make them *even more* value for money). Their performance-to-cost ratios are so favorable that they remain optimal full selections even with significant salary inreases. As an example, even if Mack Madonna's salary was up to $5.70 million dollars higher, he would still be selected with a contribution of 1.00, therefore signed.

4.4.2 Overpriced Players

Players with upper bounds of $+\infty$ (Abdul Famous, Magic Jordan, and Grant Hall) are players which were not selected by the model. This means that their salary is currently too high to justify their statistics, thus explaining the infinite increase: it doesn't matter if their salary increases, they are already not picked. It would take a salary decrease for them to have a non-zero contribution value. As an example, Magic Jordan would have to have an $8.01 million dollar salary decrease to even be considered for contributing.

4.4.3 Average Value Players

Players with finite bounds on both sides (Darrell Boards, Joe Eastcoast, Hiram Grant, Antoine Roadman, Fred Westcoast, and Barry Bird) represent marginal decisions in the current solution. These players have moderate contribution values, and their selection is sensitive to salary changes in both directions. As an example, if Barry Bird's salary dropped by $1.59 million, his contribution would increase. On the other hand, if his salary increases by $3.40 million, his contibution would decrease.

4.5 RHS and Shadow Price Sensitivity Analysis (LP Relaxation)

The sensitivity analysis for the right-hand side (RHS) values provides crucial insights into how changes in constraint requirements affect the optimal solution. Below are the sensitivity ranges for each constraint's RHS value, along with their shadow prices, in the context of the LP relaxation solution. These ranges indicate how much a constraint's RHS can change before the optimal basis of the LP solution changes, while the shadow price represents the change in the optimal objective value for a one-unit increase in the RHS.

4.5.1 Sensitivity Analysis Results

Constraint	Current RHS	Shadow Price	Lower Bound	Upper Bound	Width
Five Players	5.00	-11.7277	5.00	5.05	0.05
Points	80.00	0.7010	78.57	91.51	12.94
Rebounds	40.00	0.1700	38.96	40.48	1.52
Assists	25.00	0.6340	23.10	25.42	2.32
Minutes	190.00	0.2154	182.81	193.76	10.95
Front Court	2.00	-1.1693	1.82	2.00	0.18

Table 10: RHS Sensitivity Ranges and Shadow Prices for Constraints

4.5.2 Interpretation of Results

Roster Allocation Requirements (Negative Shadow Prices)
Constraints with negative shadow prices represent structural requirements where relaxing the constraint would decrease the total cost:

Five Players Constraint (Most Critical)

- Currently at its lower bound (5.00), indicating this constraint is actively limiting the solution
- Extremely narrow sensitivity range (0.05), making it highly sensitive to changes
- Shadow price of -$11.7277 million suggests that allowing a sixth player would dramatically reduce costs
- This represents the most significant opportunity for cost reduction, though it requires changing the fundamental roster structure

Front Court Constraint (Structurally Important)

- Currently at its upper bound (2.00), indicating maximum utilization of front court slots
- Narrow sensitivity range (0.18), showing moderate sensitivity
- Shadow price of -$1.1693 million indicates that requiring an additional front court player would reduce costs
- This suggests the current front court requirement may be artificially constraining optimal roster construction

Performance Requirements (Positive Shadow Prices)
Constraints with positive shadow prices represent performance standards where increasing requirements would increase costs:

Points Constraint (Highest Value)

- Shadow price of $0.7010 million per additional point required

- Substantial upper bound allowance (11.51 points), providing flexibility for increased requirements
- Limited lower bound flexibility (1.43 points), suggesting current requirement is near optimal minimum
- This represents the most expensive performance metric to improve

Assists Constraint (High Value)

- Shadow price of $0.6340 million per additional assist required
- Moderate sensitivity range (2.32), with limited upward flexibility
- Second-highest impact on cost among performance metrics
- Increasing assist requirements would significantly impact roster costs

Minutes Constraint (Moderate Impact)

- Shadow price of $0.2154 million per additional minute required
- Good flexibility in both directions (range of 10.95 minutes)
- Relatively low cost impact compared to other performance metrics
- Provides reasonable room for adjustment without major cost implications

Rebounds Constraint (Least Constraining)

- Lowest shadow price of $0.1700 million per additional rebound required
- Small sensitivity range (1.52), but lowest cost impact
- Represents the most cost-effective performance metric to improve
- Current requirement appears well-calibrated to available talent

5 Insights from Sensitivity Analysis

The sensitivity analysis reveals several critical insights about the optimal roster construction strategy and provides valuable guidance for decision-making under uncertainty. By examining both objective coefficient and RHS sensitivity ranges, we can identify key vulnerabilities, opportunities, and strategic priorities.

5.1 Player Value Tiers and Selection Strategy

The objective coefficient sensitivity analysis reveals three distinct tiers of players, each requiring different management approaches:

5.1.1 Elite Value Players (Infinite Decrease Tolerance)

Mack Madonna, Silk Curry, and Ramon Dion represent the foundation of any optimal roster. These players demonstrate exceptional value propositions with maximum contribution values (1.00) and infinite tolerance for salary decreases. Their performance-to-cost ratios are so favorable that they remain essential selections even with substantial salary increases. This tier provides the greatest stability and should be prioritized in contract negotiations.

Strategic Implications:

- These players represent the core of the optimal roster and should be secured at current or higher salaries
- Their robust value propositions provide insurance against salary inflation
- Focus retention efforts on these players as they offer the best long-term value

5.1.2 Marginal Value Players (Finite Sensitivity Ranges)

Darrell Boards, Joe Eastcoast, Hiram Grant, Antoine Roadman, Fred Westcoast, and Barry Bird occupy the middle tier, with selection decisions sensitive to salary fluctuations in both directions. These players represent tactical flexibility but also vulnerability to market changes. Their finite sensitivity ranges indicate that their roster spots are contestable and require active management.

Strategic Implications:

- These players require careful salary monitoring and negotiation
- Their roster positions are vulnerable to market salary increases
- Consider developing contingency plans for players with narrow sensitivity ranges
- Focus on securing favorable long-term contracts to minimize salary volatility risk

5.1.3 Overpriced Players (Infinite Increase Tolerance)

Abdul Famous, Magic Jordan, and Grant Hall currently represent poor value propositions, with infinite tolerance for salary increases indicating they are already priced out of consideration. However, their finite decrease requirements suggest potential value if salaries decline sufficiently.

Strategic Implications:

- Monitor these players for salary reductions that could make them viable
- Consider these players as potential mid-season acquisitions if market conditions change
- Use their current exclusion to benchmark acceptable salary levels for similar player profiles

5.2 Constraint Management and Flexibility

The RHS sensitivity analysis reveals critical insights about constraint management and potential rule modifications:

5.2.1 Structural Constraints Create Artificial Limitations

The five-player constraint emerges as the most significant cost driver, with a shadow price of -$11.73 million and extremely narrow sensitivity range (0.05). This suggests that the current roster size limitation is artificially constraining optimal solutions and represents the largest opportunity for cost reduction.

Similarly, the front court constraint shows a shadow price of -$1.17 million with a narrow range (0.18), indicating that positional requirements may be suboptimal from a cost-efficiency perspective.

Strategic Implications:

- Advocate to sign more than 5 free agents
- Consider the true cost of positional requirements when evaluating roster construction
- These constraints represent the highest-value areas for rule negotiation or modification

5.2.2 Performance Constraints Show Varying Cost Sensitivity

The analysis reveals a clear hierarchy of performance constraint costs:

1. **Points ($0.70M per unit):** Most expensive to improve, but offers substantial upward flexibility
2. **Assists ($0.63M per unit):** High cost with limited flexibility

3. **Minutes ($0.22M per unit):** Moderate cost with good flexibility
4. **Rebounds ($0.17M per unit):** Least expensive to improve

Strategic Implications:

- Focus performance improvements on rebounding for maximum cost efficiency
- Points requirements should be carefully calibrated given their high cost
- Minutes allocation provides good tactical flexibility without major cost implications
- Assists requirements represent a high-cost, low-flexibility constraint

5.3 Risk Management and Robustness

The sensitivity analysis identifies several risk factors that require active management:

5.3.1 High-Risk Constraints

Constraints with narrow sensitivity ranges pose the greatest risk for basis changes:

- Five Players constraint (0.05 range) - Extremely sensitive
- Front Court constraint (0.18 range) - Highly sensitive
- Rebounds constraint (1.52 range) - Moderately sensitive

5.3.2 Player Selection Vulnerabilities

Players with narrow salary tolerance ranges represent selection vulnerabilities:

- Darrell Boards (decrease: $1.21M, increase: $0.54M)
- Hiram Grant (decrease: $0.72M, increase: $1.68M)
- Grant Hall (decrease: $0.91M, infinite increase)

5.4 Optimization Opportunities

The sensitivity analysis reveals several optimization opportunities:

5.4.1 Immediate Cost Reduction Potential

- Roster size flexibility could reduce costs by $11.73M per additional player
- Front court requirement relaxation could reduce costs by $1.17M
- These represent the highest-value structural modifications

5.4.2 Performance Enhancement Strategy

- Focus on rebounding improvements for maximum cost efficiency
- Leverage points constraint flexibility for strategic performance increases
- Use minutes allocation as a tactical adjustment tool

5.4.3 Contract Negotiation Priorities

- Prioritize long-term contracts for elite value players
- Monitor marginal players for salary movement opportunities
- Track overpriced players for potential value emergence