ICMA 346 Project Report: The Tennessee Pterodactyls

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1 Problem Requierments and Formulation

We have a yearly budget of $50 million to sign free agents from the following table:

	Player	Position	Points	Rebounds	Assists	Minutes	Salary ($M)
1	Mack Madonna	Back court	14.7	4.4	9.3	40.3	8.2
2	Darrell Boards	Front court	12.6	10.6	2.1	34.5	6.5
3	Silk Curry	Back court	13.5	8.7	1.7	29.3	5.2
4	Ramon Dion	Back court	27.1	7.1	4.5	42.5	16.4
5	Joe Eastcoast	Back court	18.1	7.5	5.1	41.0	14.3
6	Abdul Famous	Front court	22.8	9.5	2.4	38.5	23.5
7	Hiram Grant	Front court	9.3	12.2	3.5	31.5	4.7
8	Antoine Roadman	Front court	10.2	12.6	1.8	44.4	7.1
9	Fred Westcoast	Front court	16.9	2.5	11.4	42.7	15.8
10	Magic Jordan	Back court	28.5	6.5	1.3	38.1	26.4
11	Barry Bird	Front court	24.8	8.6	6.9	42.6	19.5
12	Grant Hall	Front court	11.3	12.5	3.2	39.5	8.6

Table 1: Free Agent Information

The requirements are:

1. Sign exactly 5 players.
2. Total points per game ≥ 80.
3. Total rebounds per game ≥ 40.
4. Total assists per game ≥ 25.
5. Total minutes per game ≥ 190.
6. At most 2 front court and at most 3 back court players. As we must sign 5 players, we can simplify this to just require exactly 2 front court players.
7. Select the group that satisfies requirements 1-6 at minimum total salary cost.

Define a vector $\boldsymbol{x} \in \{0,1\}^{12}$, consisting of the following binary decision variables:

$$x_i = \begin{cases} 1, & \text{if player } i \text{ is signed,} \\ 0, & \text{otherwise,} \end{cases} \qquad i = 1, \ldots, 12.$$

Let $\boldsymbol{s}, \boldsymbol{p}, \boldsymbol{r}, \boldsymbol{a}, \boldsymbol{m} \in \mathbb{R}_+^{12}$ be the salary, points, rebounds, assists, minutes vectors, and $\boldsymbol{f} \in \{0,1\}^{12}$ indicate front court (1) or back court (0).

We can now formulate the integer linear programming model:

$$\begin{aligned}
\text{minimize} \quad & z = \boldsymbol{s}^\top \boldsymbol{x} \\
\text{subject to} \quad & \mathbf{1}^\top \boldsymbol{x} = 5, \\
& \boldsymbol{p}^\top \boldsymbol{x} \geq 80, \\
& \boldsymbol{r}^\top \boldsymbol{x} \geq 40, \\
& \boldsymbol{a}^\top \boldsymbol{x} \geq 25, \\
& \boldsymbol{m}^\top \boldsymbol{x} \geq 190, \\
& \boldsymbol{f}^\top \boldsymbol{x} = 2, \\
& \boldsymbol{x}, \boldsymbol{f} \in \{0,1\}^{12}, \\
& \boldsymbol{s}, \boldsymbol{p}, \boldsymbol{r}, \boldsymbol{a}, \boldsymbol{m} \in \mathbb{R}_+^{12}.
\end{aligned}$$

2 Linear Programming Solution

2.1 Raw CPLEX Output

```
solution for: tennessee_pterodactlys
objective: 52.2
status: OPTIMAL_SOLUTION(2)
Mack Madonna=1
Ramon Dion=1
Joe Eastcoast=1
Hiram Grant=1
Grant Hall=1
```

2.2 Interpretation

We can see that we have reached a status of an optimal solution, and our objective function (minimum cost) is 52.2, with the listed five players having a binary value of 1, thus being selected. Below are two tables showing the statistics of the selected players and their aggregate stats:

	Player	**Position**	**Points**	**Rebounds**	**Assists**	**Minutes**	**Salary ($M)**
1	Mack Madonna	Back court	14.7	4.4	9.3	40.3	8.2
4	Ramon Dion	Back court	27.1	7.1	4.5	42.5	16.4
5	Joe Eastcoast	Back court	18.1	7.5	5.1	41.0	14.3
7	Hiram Grant	Front court	9.3	12.2	3.5	31.5	4.7
12	Grant Hall	Front court	11.3	12.5	3.2	39.5	8.6

Table 2: Player Stats

Statistic	**Total**
Total Points (Objective)	80.50
Total Salary	$52.20M
Total Rebounds	43.7
Total Assists	25.6
Total Minutes	194.8
Total Front Court Players	2

Table 3: Aggregate Stats

Notes: We can see that Ramon Dion is an incredibly valuable player in terms of scoring output especially when comparing him to the other highly scoring free agents. Mack Madonna, Hiram Grant, and Grant Hall are very valuable all around players with standout assists and rebound statistics respectively, with very cheap salaries in comparison to other agents, making them very cost effective.

Joe Eastcoast is a decent jack of all trades player with a farily expensive salary in comparison, likely meaning he was selected specifically to fulfill the constraints, despite not being as cost-effective as the other players. Due to this, he seems to be the weak link in this roster in terms of value for money, so another player could be a better choice if we are willing to relax some constraints.

3 Answers to Case Questions

Question A. Formulate an integer linear programming model (ILP) to help the general manager and coach determine which players they should sign and solve it by using the computer.

Answer A. The ILP model, along with the five players to sign have been displayed in the section above.

Question B. Is the money provided by the owner sufficient to sign the group of players identified in (A)? If not, reformulate the model so that the available funds are a constraint and the objective is to maximize the average points of the group.

Answer B. As seen in the section above, the total salary of the selected roster is \$52.2 million, whereas the budget is \$50.0 million, thus meaning we do not have enough money to sign this combination of players. The LP reformulation and new solution are presented below.

3.1 Question B LP Reformulation

We will keep all the defined variables and vectors as before, and only change the objective function along with the constraints.

$$\begin{aligned} \text{maximize} \quad & z = \boldsymbol{p}^\top \boldsymbol{x} \\ \text{subject to} \quad & \boldsymbol{s}^\top \boldsymbol{x} \leq 50, \\ & \boldsymbol{x}, \in \{0,1\}^{12}, \\ & \boldsymbol{p}, \boldsymbol{s} \in \mathbb{R}_+^{12}. \end{aligned}$$

3.2 Raw CPLEX Output

```
solution for: tennessee_pterodactlys_B
objective: 82.7
status: OPTIMAL_SOLUTION(2)
Mack Madonna=1
Silk Curry=1
Ramon Dion=1
Joe Eastcoast=1
Hiram Grant=1
```

3.3 Interpretation

Once again, we have reached a status of an optimal solution, and our objective function (max total points) is 82.7, with five players having a binary value of 1 thus being selected. As prior, tables showing the players' statistics and the aggregate roster stats are presented below.

	Player	Position	Points	Rebounds	Assists	Minutes	Salary ($M)
1	Mack Madonna	Back court	14.7	4.4	9.3	40.3	8.2
3	Silk Curry	Back court	13.5	8.7	1.7	29.3	5.2
4	Ramon Dion	Back court	27.1	7.1	4.5	42.5	16.4
5	Joe Eastcoast	Back court	18.1	7.5	5.1	41.0	14.3
7	Hiram Grant	Front court	9.3	12.2	3.5	31.5	4.7

Table 4: Player Stats for Formulation B

Statistic	Total
Max Total Points (Objective)	82.70
Total Salary	$48.80M
Total Rebounds	39.9
Total Assists	24.1
Total Minutes	184.6
Total Front Court Players	1

Table 5: Aggregate Stats for Formulation B

Notes: We have kept approximately the same roster, except for replacing Grant Hill with Silk Curry, likely due to the difference in point scoring and salary letting us reach the budget requirements. As a result, our point scoring is slightly up, however the rest of the statistics have slightly suffered, especially the number of rebounds and minutes played. Furthermore, we now have a more unbalanced roster with 4 back court players and only 1 front court player.

4 Sensitivity Analysis

5 Insights from Sensitivity Analysis