Algorithm Selection: A Predictive Model for Optimal Sorting

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1 Introduction

Sorting is a fundamental task that appears across various applications in computer science, from database management to data analytics and real-time processing systems. Due to its critical importance, hundreds of sorting algorithms have been developed, each with unique performance characteristics optimized for particular scenarios. Some algorithms excel at sorting nearly-sorted data, others at handling large datasets, and others at optimizing memory usage. As a consequence, no single algorithm universally outperforms all others for all problem instances. This naturally leads to the following research question:

> *Can we design a model that dynamically and intelligently selects the optimal sorting algorithm based on the characteristics of a given dataset?*

Successfully addressing this question and constructing an effective predictive model would represent a meaningful advancement. Such a model, capable of analyzing a dataset's characteristics and predicting the optimal sorting strategy based on that analysis, could deliver substantial performance improvements over current static approaches. In practical terms, this could significantly enhance efficiency in real-world scenarios, including large-scale database operations, data-intensive computations, and latency-sensitive applications, providing tangible benefits over existing sorting implementations.

2 Background and Problem Formalization

2.1 Algorithm Selection Problem

This question is an instance of the algorithm selection problem, formulated by John Rice in 1976 [2]. It is formally stated as follows:

Given:

- A problem space $\mathcal{P}$, containing all possible problem instances.
- A feature space $\mathcal{F}$, where each $f(x) \in \mathcal{F}$ represents measurable characteristics of problem $x \in \mathcal{P}$.
- An algorithm space $\mathcal{A}$, containing all applicable algorithms A_i.
- A performance space $\mathbb{R}^n$, where $p(A_i, x)$ represents the performance of algorithm $A_i \in \mathcal{A}$ on problem $x \in \mathcal{P}$.
- The final algorithm performance metric $||p||$, obtained by normalizing the raw performance measures.

Find: A selection mapping

$$S : \mathcal{F} \to \mathcal{A}$$

that maximizes performance according to some criteria.

2.2 Measures of Presortedness

Before we discuss how the defined model applies to our research question, we must talk about measures of presortedness, which are a measurement of the pre-existing order of a list. A good measure of presortedness must satisfy the following criteria:

Let m be a measure of presortedness, and X, Y be lists:

1. $m(X) = 0$ if X is *fully sorted in ascending order.*
2. If $X = [x_1, \cdots, x_n]$, $Y = [y_1, \cdots, y_n]$, and
$$x_i < x_j \iff y_i < y_j \quad \text{for all } i \text{ and } j,$$
then $m(X) = m(Y)$.
3. If $X \subseteq Y$, then $m(X) \leqslant m(Y)$.
4. If $X < Y$, then $m(XY) \leqslant m(X) + m(Y)$.
5. *For any element* a, $m(a + X) \leqslant |X| + m(X)$.

For this project, I will explicitly mention the following three measures of presortedness:

Runs [?]: The number of maximal contiguous ascending subsequences in a list X.

$$\text{Runs}(X) = |\{i : 1 \leqslant i < n \text{ and } a_{i+1} < a_i\}|\,.$$

This takes $O(n)$ time, and $O(1)$ memory.

Dis [1]: The largest distance for which an inversion exists

$$\text{Dis}(X) = \max\{j - i : 1 \leqslant i < j \leqslant n, x_i > x_j\}.$$

This takes $O(n)$ time, and $O(n)$ memory.

Mono [?]: The minimum number k such that X can be broken into k subarrays where each subarray is *monotonic*. A sequence is monotonic if it is increasing *or* decreasing.

$$\text{Mono}(X) = \min\Big\{k \in \mathbb{N} : \exists\, 1 = i_0 < i_1 < \cdots < i_k = n + 1 :$$

$$\forall j \in \{1, \ldots, k\}, (X_{i_{j-1}}, X_{i_{j-1}+1}, \ldots, X_{i_j-1}) \text{ is monotonic}\Big\}$$

This takes $O(n)$ time, and $O(1)$ memory.

Note: These are the only measures of presortedness that have a linear running time. This quality enables them to be used as features of lists for the problem of sorting.

2.3 Other List Features

Aside from the mentioned measures of presortedness, we define five other features:

1. **Size:** The total number of elements in the list,
$$\text{Size}(L) = |L|.$$
2. **Average duplicates per unique element:** For a list L with unique elements U, if $|L| = n$, then
$$\text{Avg. duplicates} = \frac{n - |U|}{|U|}.$$
3. **Shannon entropy:** With frequency function $f(u)$ for $u \in U$ and $p(u) = \frac{f(u)}{n}$,
$$H(L) = -\sum_{u \in U} p(u) \log_2 p(u).$$
4. **Categorical skewness:** For categorical data—where each unique category $u \in U$ occurs with frequency $f(u)$ and the mean μ and standard deviation σ are computed over these frequencies—the skewness is defined as
$$\gamma_1 = \frac{1}{|U|} \sum_{u \in U} \left(\frac{f(u) - \mu}{\sigma} \right)^3.$$
This measure reflects the asymmetry of the frequency distribution for categorical variables.
5. **Categorical kurtosis:** For categorical data with frequencies $f(u)$ for each $u \in U$, the kurtosis is defined by
$$\gamma_2 = \frac{1}{|U|} \sum_{u \in U} \left(\frac{f(u) - \mu}{\sigma} \right)^4.$$
This measure quantifies the tailedness of the frequency distribution for categorical variables.

2.4 Sorting Algorithm Selection Problem

In our context, the problem space $\mathcal{P}$ consists of datasets that require sorting, and an instance $x \in \mathcal{P}$ is a list to be sorted. The feature space $\mathcal{F}$ is comprised of the size, categorical skewness, categorical kurtosis, Shannon entropy and, most importantly, the three measures of presortedness Runs, Dis, and Mono.

Our algorithm space $\mathcal{A}$ includes various sorting algorithms like QuickSort, MergeSort, InsertionSort, and others. For simplicity, our performance space is defined as $\{0, 1\}$, where a value of 1 indicates that the selection algorithm has correctly identified the fastest sorter for a given problem instance. We can now more formally state our research question:

Given:

- Datasets $\mathcal{P} \ni x$, where x is a list that requires sorting.
- A feature space $\mathcal{F}$, containing of the above stated features.
- An algorithm space $\mathcal{A}$, containing all applicable sorting algorithms A_i.
- A performance space $\{0, 1\}$, where the performance metric is defined as
$$p(A_i, x) = \begin{cases} 1, & \text{if } A_i \text{ is the fastest sorter for } x, \\ 0, & \text{otherwise.} \end{cases}$$

Find: A selection mapping
$$S : \mathcal{F} \to \mathcal{A}$$
that maximizes the average number of fastest sorter selections, i.e.,
$$\max_S \frac{1}{|\mathcal{P}|} \sum_{x \in \mathcal{P}} p(S(f(x)), x).$$

3 Methodology

References

[1] Vladmir Estivill-Castro and Derick Wood. A survey of adaptive sorting algorithms. *ACM Comput. Surv.*, 24(4):441–476, December 1992.

[2] John R. Rice. The algorithm selection problem. volume 15 of *Advances in Computers*, pages 65–118. Elsevier, 1976.