Quantessentials

R advice – Rollin' Rollin' Rollin'

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Calculating rolling means and other statistics

Resisting the temptatation to call out "Rawhide!" this month's addition to the Quantessentials series looks at the various implementations of a simple and common operation – calculating rolling means and other statistics from time series. There are a few packages which offer this functionality and we highlight the ones that we prefer and those that we find less attractive.

Turning ggplots interactive

ggplot2 is a great graphics library but sometimes adding in a little bit of interactivity for inclusion in a web page (or Shiny application) in our view would improve things a bit. The ggiraph package makes this very straightforward. We give a simple example.

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Calculating rolling statistics

A very common operation that we all need to do is calculate various statistics over rolling windows along data. These could be univariate operations (e.g. moving averages, rolling standard deviations) or multivariate (e.g. rolling correlations).

As is common in R there are a number of packages which offer functions to do this. In this note we will compare them[1] for both speed and convenience.

The list of packages we will investigate is

- caTools
- RcppRoll
- roll
- TTR
- zoo

We can also (at least for some of the operations) use the *filter* function from the *stats* rather than the *dplyr* package.

There is also the rollply package which does allow rolling window calculations but in our view is not in a very convenient form.

We look at two very basic operations – rolling means and rolling correlations – in order to capture the efficiency of all the packages on a univariate and a multivariate calculation. For rolling means we analyse the calculation on both a single time series and a matrix of time series.

The *roll* package is interesting as it uses the RcppParallel package to implement its various functions. This means that it can use all the cores on your machine in order to speed up its calculations. The downside of this is if you use all the cores it does make doing anything else on the machine basically impossible.

Rolling means on a single time series

The first example we will run is looking at calculating a simple moving average on a single time series.

For this simple case, as can be seen in Figure 1 below, the *RcppRoll* package seems the quickest but there is little between *caTools*, *RcppRoll* and *roll*. The main message from this analysis is to avoid the *zoo* rolling mean function.

[1] We have attempted to find all the packages which offer this functionality, but it is quite possible that we have missed some.

Figure 1: Calculating a simple moving average

```
require (caTools)
require (RcppRoll)
require (roll)
require (TTR)
require (zoo)

require (microbenchmark)
require (RcppParallel)

x <- runif(1e5)
k <- 50

x1 <- caTools::runmean(x, k, align='right', endrule='NA')
x2 <- RcppRoll::roll_mean(x, k, fill=NA, align='right')
x3 <- drop(roll::roll_mean(as.matrix(x), k))
x4 <- TTR::SMA (x, k)
x5 <- zoo::rollmean(x, k, align='right', na.pad=T)
x6 <- stats::filter (x, rep (1/k, k), sides= 1, circular = FALSE)@.Data

all.equal(x1, x2)
[1] TRUE
all.equal(x1, x3)
[1] TRUE
all.equal(x1, x4)
[1] TRUE
all.equal(x1, x5)
[1] TRUE
all.equal(x1, x6)
[1] TRUE

microbenchmark(caTools::runmean(x, k, align='right', endrule='NA'),
        RcppRoll::roll_mean(x, k, fill=NA, align='right'),
        drop (roll::roll_mean(as.matrix(x), k)),
        TTR::SMA (x, k),
        zoo::rollmean(x, k, align='right', na.pad=T),
        stats::filter (x, rep (1/k, k), sides= 1, circular = FALSE)@.Data)

Unit: milliseconds
                  expr        min         lq       mean     median         uq       max neval  cld
       caTools::runmean   2.625196   2.970995   5.381337   3.210291   4.750618  42.22544   100 ab
    RcppRoll::roll_mean   2.657341   2.845263   3.415133   2.939299   3.152158  35.00600   100 a
        roll::roll_mean   3.041597   3.324606   4.964298   3.511627   4.390847  39.702089  100 ab
              TTR::SMA   3.150054   3.525747   7.130375   5.206827   5.807996  45.99679   100  b
         zoo::rollmean 107.517016 110.611339 120.639898 112.835003 127.388942 164.60013   100    d
         stats::filter  14.157064  14.829437  17.672127  16.079693  17.013444  55.16454   100   c
```

Source: UBS. Analysis run in the standard implementation of R.

There are two things to note in the code. The first is for the *roll* package the x variable has to be converted to a matrix and in order to return a vector one has to convert it back via the *drop* function[2]. Also the *filter* function returns an object of the *ts* class.

In the example above we showed the checks that all the functions returned the same answer. We ran this for the other examples below and as they all passed we don't report it.

[2] One could also use the matrix extraction [, 1] but this is consistently slower than drop.

When running the same analysis in Microsoft R Open all the timings were (probably surprisingly) slower except for *filter*. On our Linux machine the results were similar but the *roll* function just beat the others.

The forecast package contains an *ma* function but it only creates centred moving averages and so is awkward to use in financial situations where one wants the moving average to be aligned to the right.

Figure 2: Example of ma in the forecast package

```
require (forecast)

x <- 1:10
forecast::ma (x, 5)
[1] NA NA  3  4  5  6  7  8 NA NA
```

Source: UBS

Rolling means on a matrix

The second test we carried out was calculating the rolling means for a matrix. For this test we dropped the *zoo* and *filter* functions for being too slow.

Figure 3: Calculating the rolling means of a matrix (Windows)

```
x <- matrix (runif(1e6), ncol = 100)
k <- 50

microbenchmark(caTools::runmean(x, k, align='right', endrule='NA'),
        RcppRoll::roll_mean(x, k, fill=NA, align='right'),
        roll::roll_mean(x, k),
        apply (x, 2, TTR::SMA, n = k))

Unit: milliseconds
                expr      min       lq     mean   median       uq      max neval   cld
     caTools::runmean 17.79262 19.70894 22.71246 20.41766 22.33714 55.11166   100 a
  RcppRoll::roll_mean 29.22925 30.70784 32.91910 31.76747 33.10816 65.91198   100    d
      roll::roll_mean 22.51575 23.09423 24.71792 23.80792 25.31474 57.23273   100 ab
      apply(TTR::SMA) 53.65456 56.47564 61.51094 58.09603 61.65903 97.25929   100     e
```

Source: UBS

The results we achieved are similar to those above. The *caTools* package is just the quickest but the function in the *roll* package is comparable. *RcppRoll*, the quickest in the univariate case, is now around 33% slower than the fastest options.

roll is slow if it can't use all the cores of the machine

As we mentioned above, the *roll* package uses the *RcppParallel* package to speed up some calculations. On our test machine the default number of threads was 8, but reducing this to 4 made little difference to the speed. Dropping the number of cores to 1 however increased the times for the roll::roll_mean function by around a factor of 4.

On our Linux machine (which has 20 cores) the benefit to using the parallel enabled *roll* came through much more, as can be seen in Figure 4 below. The *roll* function beat *caTools* by around a factor of two.

Figure 4: Calculating the rolling means of a matrix (Linux)

```
Unit: milliseconds
                expr      min       lq      mean   median       uq      max neval cld
    caTools::runmean 36.51636 40.20384  54.63619 41.78244 45.62528 169.4807   100  b
  RcppRoll::roll_mean 39.79583 41.43693  49.71411 43.21604 44.18452 152.0087   100  b
      roll::roll_mean 14.20528 15.60131  18.75688 16.41521 17.59046 122.9053   100 a
     apply(TTR::SMA) 86.52237 91.13151 106.89582 93.00480 97.44380 301.6138   100   c
```

Source: UBS

Rolling correlations

The conclusion from our final test, calculating a rolling correlation matrix, is a very simple one as only the *roll* package offers this functionality. However there is an option in the package for whether the operations are run in parallel by rows or columns.

Figure 5: Calculating rolling correlations

```
x <- matrix (runif(1e6), ncol = 100)
k <- 50

res <- microbenchmark(roll::roll_cor (x, k, parallel_for = "rows"),
       roll::roll_cor (x, k, parallel_for = "cols"), times = 10)

Unit: seconds
                                  expr      min       lq     mean   median       uq      max neval cld
roll_cor(x, k, parallel_for = "rows") 18.73922 18.80316 18.86287 18.85362 18.91406 18.99667    10  a
roll_cor(x, k, parallel_for = "cols") 21.85780 21.94201 22.19310 22.19057 22.45358 22.59075    10   b
```

Source: UBS

We see that the default option, rows, is consistently quicker. Do note that the units for the time in this case are seconds rather than the milliseconds in the other examples.

Conclusion

Based on this analysis, the choice seems to come down to a two way tie on Windows. *caTools* is marginally quicker for most single variable operations, but the *roll* package is a very close second and has the advantage of being able to calculate rolling correlation- and covariance matrices.

caTools and _roll_ are most preferred …

The least preferred choice which seems very clear is not to use the functions from the *zoo* package. These functions are the only ones written just in R – all the others use either C or Rcpp to speed up the calculations.

… and _zoo_ is least preferred

Making interactive plots from ggplot

There is a useful package which has recently turned up on cran. This is called ggiraph. It is a way of turning ggplot charts into interactive ones.

For example, let's suppose we have run a backtest[3] where we have separated our data into 10 deciles and calculated their performance. As an example:

Figure 6: Plotting the results from a backtest

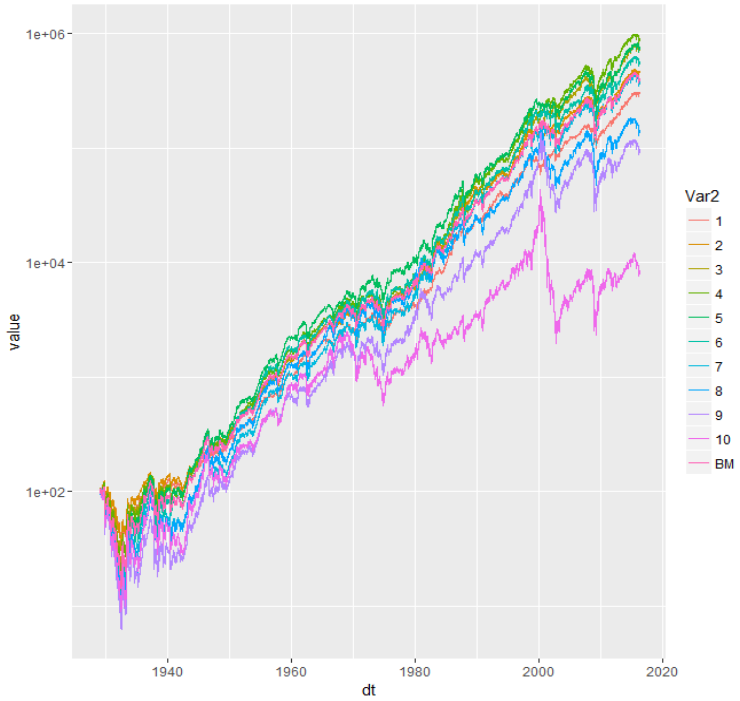
```
require (ggplot2)
require (reshape2)
require (lubridate)

head (res$index)
                 1        2         3         4        5        6         7         8        9       10        BM
1928-12-31 100.0000 100.0000 100.00000 100.00000 100.0000 100.0000 100.00000 100.00000 100.0000 100.0000 100.00000
1929-01-02 101.2988 101.2782 101.50666 101.12464 101.6292 102.4313 101.61263 102.71568 101.4855 100.8295 101.59274
1929-01-03 101.4897 101.7485 101.79646 101.36420 101.7696 102.7129 101.50452 102.20886 101.9097 101.8416 101.83305
1929-01-04 101.8353 101.4698 101.83368 101.22549 102.0951 102.6396 101.41820 101.95304 102.0261 102.1575 101.86504
1929-01-05 101.6046 101.1697 101.54181 100.46877 101.0026 101.6976 100.27906 100.77526 102.0891 101.5164 101.21179
1929-01-07 100.4996 100.2130  99.96206  99.23583  99.0019  99.9472  98.87468  98.77548 101.2601 100.3144  99.80419

# Convert into long format
index.df = melt (res$index)

# Convert to a date
index.df$dt = ymd (index.df$Var1)

# Plot the chart
ggplot (index.df, aes (x = dt, y = value, group = Var2, color = Var2)) + geom_line() +
               scale_y_log10()
```



Source: UBS

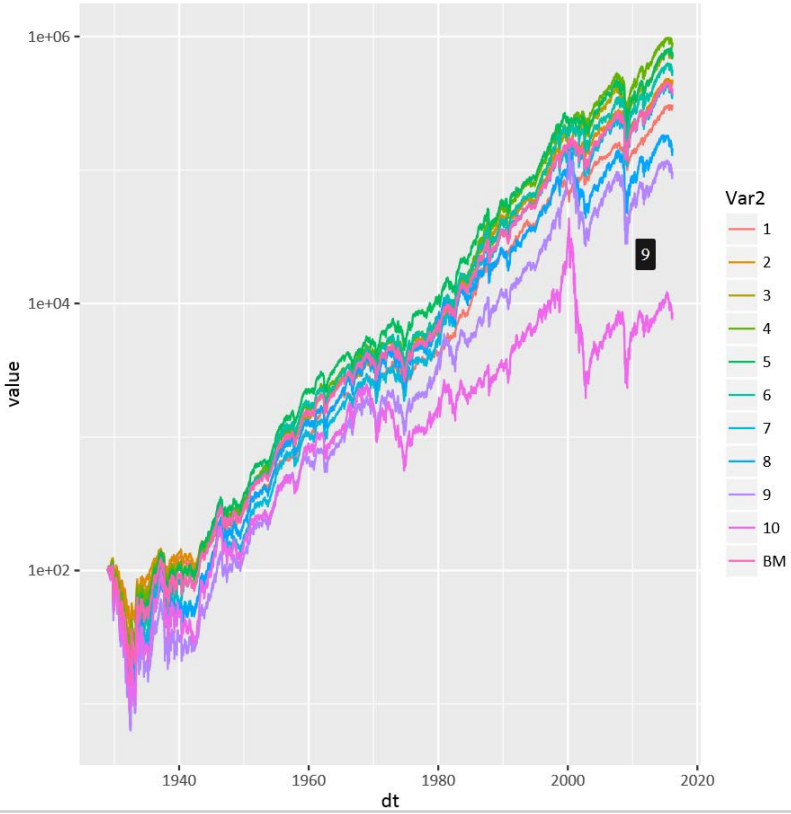
[3] The data is taken from our recent note Do low-volatility stocks have interest rate risk?

The problem comes in identifying which line is which. A similar problem can occur with a scatter plot – which point is which. A simple change to the code to use the ggiraph package solves this.

Figure 7: Example of the ggiraph interactive chart

```
h <- ggplot (index.df, aes (x = dt, y = value, group = Var2, tooltip = Var2, color = Var2)) +
                geom_line_interactive() + scale_y_log10()

ggiraph(code = {print(h)})
```



Source: UBS

The only changes are replacing `geom_line ()` with `geom_line_interactive ()` and adding a *tooltip* option in the `aes ()` function. The charts can be put into a Shiny application. If you hover over one of the lines with your mouse then the text in the *tooltip* variable appears. There are many other libraries which produce interactive graphics (all using JavaScript, we believe) which we intend to investigate in a future note.

Appendix

R environment

We used two versions of R in our tests. Both are based on R version 3.3.1. The first is the standard implementation from cran; the second is the Microsoft R Open version which calls the Intel MKL library and hence parallelises various operations. Both are running on an Intel i7-6700 running at 3.4GHz.

Our Linux machine is a virtual machine with 20 CPUs / cores running at 2.5GHz.

We show below the version numbers of the packages we compared.

Figure 8: Package versions tested

Package	Version	Package	Version
caTools	1.71.1	roll	1.0.6
forecast	6.1	rollply	0.5.0
RcppParallel	4.3.30	TTR	0.23.1
RcppRoll	0.2.2	zoo	1.0.6

Source: UBS

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