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BLOOMBERG PERFORMANCE ATTRIBUTION MODEL

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1. Introduction

The main purpose of performance attribution is to explain the relative performance of a portfolio versus its benchmark in terms of investment strategy and changes in market conditions. By assessing the contribution of each factor to the performance, an attribution model can provide portfolio managers, plan sponsors and investors with valuable information. In this document, we introduce the Bloomberg Performance Attribution Model, which has a parsimonious structure to accommodate flexible management styles of different portfolios.

There are two basic approaches to performance attribution: the sector-based approach pioneered by Brinson Fachler (1985) and Brinson Hood and Beebower (1986), and the factor-based approach proposed by Grinold and Kahn (1999).

The sector-based model, which is often referred to as the Brinson model, decomposes the active return, i.e. the return of a portfolio over the return of its benchmark, into an allocation effect, selection effect and possibly the interaction of the two effects. The factor-based approach, on the other hand, decomposes the active return into the contribution of a series of factors, each of which in turn is the product of the active exposure at the beginning of the period and the factor return over the period.

The sector-based attribution model has gained great popularity due to its intuitiveness and flexibility. However, it is not suitable for some portfolios that are managed in a quantitative fashion. For instance, if a portfolio is managed to track the Russell 3000 index with a positive tilt in the dividend yield style factor, Brinson model can't properly attribute the active return to this style factor tilt. A factor-based attribution framework is better suited to analyze the performance of such a portfolio. Of course, such a factor-based attribution model must be built upon a factor-based risk model which provides the exposures and the factor returns.

Some widely used factors, especially for fixed-income securities, are observable in the market. For instance, the yield curve factors are often represented by the actual change of yields on a set of key rate points. Such factors are often referred to as the "explicit factors" as opposed to "implicit factors", which must be obtained via regression. As long as we have the exposures to these factors computed from security pricing models, these factors can be readily analyzed within the factor-based framework without building a factor-based risk model. As a result, most commercially available fixed-income attribution systems are hybrid in the sense that the active return is explained by the contribution of some explicit factors, as well as sector allocation and security selection computed using a sector-based model on the remaining portion of the returns.

The Bloomberg Performance Attribution Model recognizes that the sector-based attribution model is a special factor-based model with the advantage of offering flexible configurations on the fly. Therefore, we have the factor-based model as the generic framework and offer four different model specifications that can be used for different types of portfolios and management styles. We will try to accommodate more flexible specifications continuously under the guidance of the framework going forward.

This document describes the Bloomberg Performance Attribution Model, the four specifications that are currently supported, and examples of their applications. It is organized as follows. Section 2 introduces the general framework of the model. Section 3 outlines the different model specifications we currently support. Section 4 shows a variety of examples to illustrate the application of each specification. Finally, in Section 5 we conclude with a brief discussion of future development.

2. Generic Model Framework

2.1 Brinson Attribution

As a background, we first introduce the popular Brinson Attribution model. The basic idea is to use some hypothetical "notional portfolios" to help decompose the active return: the Active Asset Allocation Fund and the Active Stock Selection Fund. They are the weighted average of portfolio sector weights and benchmark sector returns, and benchmark sector weights and portfolio sector returns. $w_s^{P/B}$ is the sector weight and $R_s^{P/B}$ is the sector return in the portfolio/benchmark.

	Portfolio Sector Returns	Benchmark Sector Returns
Portfolio Sector Weights	(IV) Portfolio $\sum_s w_s^P R_s^P$	(II) Active Asset Allocation Fund $\sum_s w_s^P R_s^B$
Benchmark Sector Weights	(III) Active Stock Selection Fund $\sum_s w_s^B R_s^P$	(I) Benchmark $\sum_s w_s^B R_s^B$

Table 1: Brinson Model Notional Portfolios

With these notional portfolios, the active return can be decomposed into three attribution effects as shown in the table below. The Asset Allocation effect is the return difference between the Active Asset Allocation Fund and the Benchmark. It measures the contribution from the active weights if the portfolio sector returns are identical to those in the benchmark. Similarly, the Stock Selection effect measure the contribution from the return difference between the portfolio and the benchmark for a given sector weight in the benchmark. Finally, an interaction effect is needed to keep the equality and it measures the combined effect of the active weight decision and the active sector return difference.

Attribute	Notional Portfolios	Single-period Algebraic Result
Asset Allocation	A=II-I	$\sum_s (w_s^P - w_s^B) R_s^B$
Stock Selection	S=III-1	$\sum_s w_s^B (R_s^P - R_s^B)$
Interaction	I=IV-III-II+I	$\sum_s (w_s^P - w_s^B)(R_s^P - R_s^B)$
Total	T=IV-I	$\sum_s w_s^P R_s^P - \sum_s w_s^B R_s^B$

Table 2: Brinson Model Attributes

The interaction can be combined into allocation attribute or selection attribute based on the order of the decision making process. For instance, if a fund is managed top down with sector allocation decision made first and stock selection in each sector done subsequently. The total active return can be decomposed into allocation and selection effects as follows. It basically wrap the interaction term into the Stock Selection effect. Since this model is equally valid for a fixed income portfolio, we refer to the Stock Selection effect as Security Selection effect instead.

$$\sum_s w_s^P R_s^P - \sum_s w_s^B R_s^B = \sum_s (w_s^P - w_s^B) R_s^B + \sum_s \boldsymbol{w_s^P} (R_s^P - R_s^B) \qquad (1)$$

This is often referred to as the “absolute mode” of the Brinson attribution. The implicit assumption here is that the portfolio can take “unlimited” leverage. If a sector has a positive return, the sector should be over-weighted. However, in reality even if a portfolio manager predicts that a sector is going to have a positive return, she may not overweight the sector if she thinks that the index is going to perform well and this particular sector is actually going to under-perform the index. In other words, under the budge/leverage constraint, the decision is made on the relative performance rather than absolute performance. That is why the Brinson attribution is often seen in its “relative mode” as follows:

$$\sum_s w_s^P R_s^P - \sum_s w_s^B R_s^B = \sum_s (w_s^P - w_s^B)(R_s^B - \boldsymbol{R^B}) + \sum_s w_s^B (R_s^P - R_s^B) \qquad (2)$$

Compared to the “absolute mode”, the seemingly only difference is that the Asset Allocation effect is computed on the sector return excess of the overall benchmark return. However, we would like to point out that mathematically there should be an additional term $\sum_s (w_s^P - w_s^B) R^B$ to keep the equality. This term is zero if and only if the sum of weights in the portfolio equals to that in the benchmark. When this condition does not hold, we do need to explicitly show this additional term.

2.2 Generic Model Framework

Our generic model starts with the basic factor model framework in which the total return of a security n in a given currency can be written as the sum of K return components plus a residual term; each component is simply the product of a factor and the exposure to that factor.

$$r_{nt} = \sum_{k=1}^{K} \beta_{nkt} * F_{kt} + \epsilon_{nt} \tag{3}$$

In equation (3)

r_{nt} is security n return over period t,

β_{nkt} is a pre-defined factor exposure of security n to a factor k, at beginning of period t with K factors in total,

F_{kt} is a factor return[1], to a factor k over period t,

ϵ_{nt} is residual return of security n at time t that is not explained by the above factors.

The active return of a portfolio over its benchmark can therefore be written as:

$$AR \equiv \sum_{K=1}^{K} \left(\sum_{n=1}^{N} (w_n^P - w_n^B) \beta_{nk} \right) * F_k + \sum_{n=1}^{N} (w_n^P - w_n^B) * \epsilon_n \tag{4}$$

where $w_n^{P/B}$ is the weight of security n in the portfolio/benchmark at the beginning of the period. The sub-script for the time period is dropped in the notation here and after to simplify notation.

The active return is simply the sum of a few return components, such as the return due to base yield curve change, the return due to volatility change, and the return due to change in credit spread. Ex-ante, a portfolio manager's task is to set the desired active exposure to each of these factors based on her views of how each factor return would fare over the given investment horizon. Ex-post, the portfolio manager can review the contribution to active return from each factor and adjust the exposures as necessary.

The sector-based model can be viewed as a special case of the factor-based model in which the factors are simply defined as the group level averages of the benchmark returns

[1] Note, here return should not be interpreted literarily. Depending on the factor, it can be a change in spread or change in yield, rather than the "return" as the percentage change in value.

and the factor exposure of security n to a group is simply an indicator whether the security n belongs to a group. The nice feature of this simple "factor model" is that the factors are constructed on the fly and therefore they can be tailored to each portfolio without the work of constructing a custom risk factor model. On the flip side, the factors are restricted to be weighted average returns, rather than with more sophisticated structure as in a risk factor model. Taking a simple example, if there is a data error in one of the security returns in a benchmark sector, the sector return from a good risk model would remove or alleviate the impact of this outlier. However, in a sector-based model, it can cause erroneous attribution results.

Assume that a portfolio is managed explicitly on a few factors and the residual portion of the active return is managed along a sector structure with S groups. Following the sector-based approach, the active residual return can therefore be further decomposed into Asset Allocation, Security Selection and Interaction effects. If we prefer to combine the interaction with selection, we will have two terms as follows:

$$\sum_{s=1}^{S}(w_s^P - w_s^B)\, R_{\varepsilon,s}^B + \sum_{s=1}^{S} w_s^P \left(R_{\varepsilon,s}^P - R_{\varepsilon,s}^B\right) \tag{5}$$

Recognizing the link between the sector-based model and the factor-based model helps us to extend the traditional sector model: the factors do not have to be the sector returns, but rather they can be factors such as sector spread change. In that case, the decomposition is generalized to equation (6).

$$\sum_{s=1}^{S}(w_s^P \beta_s^P - w_s^B \beta_s^B)\, F_{\varepsilon,s}^B + \sum_{s=1}^{S} w_s^P \beta_s^P \left(F_{\varepsilon,s}^P - F_{\varepsilon,s}^B\right) \tag{6}$$

Furthermore, the factors can be defined relative to a *Hurdle Return*, F^H. By doing so, the active residual return has the following attribution terms.

$$\sum_{s=1}^{S}(w_s^P \beta_s^P - w_s^B \beta_s^B)\left(F_{\varepsilon,s}^B - F^H\right) + \sum_{s=1}^{S} w_s^P \beta_s^P \left(F_{\varepsilon,s}^P - F_{\varepsilon,s}^B\right) + \sum_{s=1}^{S}(w_s^P \beta_s^P - w_s^B \beta_s^B)\, F^H \tag{7}$$

The last term is referred to as the "leverage effect". In the case of a simple sector allocation as in Equation (5), once we add the *Hurdle Return*, the last additional term is simply: $\sum_{s=1}^{S}(w_s^P - w_s^B)\, F^H$, if the portfolio does not take any leverage, i.e. $\sum_{s=1}^{S} w_s^P = \sum_{s=1}^{S} w_s^B = 100\%$, this term reduces to zero. However, when the portfolio does take leverage, this term measures the contribution of the leverage to the active return. It is worth mentioning that when the exposure is not "market value weight", but rather some other measure such as contribution to spread duration, the "leverage effect" really measures the contribution from the portfolio level spread duration decision based on a view of the overall spread movement. However, for simplicity and consistency, we still label the effect as "leverage".

It is easy to see that (6) can be viewed as a special case with F^H set to zero. The specification with zero *Hurdle Return* is the absolute mode and the specification with non-zero *Hurdle Return* is the relative mode.

3. Model Specifications

The general framework introduced in Section 2 can be parameterized differently to produce different model specifications. With the current release, we offer four different specifications, which can be used for portfolios managed with various management styles.

3.1 Total Local Return Model

This is often referred to as the "equity attribution model". It can be viewed as a one factor model with the factor being the currency return. The total return in base currency $\widetilde{R_n}$ is the sum of the currency return X_n and the residual term, which is simply the total local return.

$$\widetilde{R_n} = X_n + R_n^{\varepsilon} \tag{8}$$

Note that the currency return X_n is the sum of the relative change in foreign exchange rate over the period and its cross-term with the local return. The contribution of currency return for a given group is given by

$$\sum_{n \in s} (w_n^P - w_n^B) X_n \tag{9}$$

Currently, the active currency returns are aggregated using the same grouping assigned by the user, which is used to attribute the active local return into Allocation Effect and Selection Effect.

Alternatively, it makes sense to aggregate the contribution of currency factors by a currency grouping. Furthermore, Assume the expected currency return to be the difference between the risk free rates of the base currency r_0 and that of the local currency r_n, we can subtract that cash rate differential to get the unexpected currency return: $X_n - (r_0 - r_n)$. In this case, the contribution from a given currency c is given by

$$\sum_{n \in c} (w_n^P - w_n^B)\big(X_n - (r_0 - r_n)\big) \tag{10}$$

This analysis of the currency decision is in line with the work by Karnosky and Singer (1994), and it will be available at a future date.

The total local return (or the total local return over risk free rate when adjusted currency return is used) is decomposed into allocation, selection and potentially interaction using the standard sector-based model in Equation (7) with exposures set to 1, factors set to be the benchmark local returns at group levels and the hurdle return to be the average benchmark local returns. Note that since the sum of weights of the portfolio equals to that of the benchmark, the leverage effect is zero. So the Allocation and Selection are given by the following formula, where the interaction term is suppressed.

$$\sum_{s=1}^{S} (w_s^P - w_s^B)\left(R_{\varepsilon,s}^B - R_{\varepsilon}^B\right) + \sum_{s=1}^{S} w_s^P \left(R_{\varepsilon,s}^P - R_{\varepsilon,s}^B\right) \tag{11}$$

If a group is empty in the benchmark, we replace $R_{\varepsilon,s}^B$ with$R_{\varepsilon,s}^P$. This implies that such an out-of-index bet contributes only to the Allocation Effect, and it has zero contribution to the Selection Effect.

As mentioned in previous section, this relative simple decomposition holds if and only if the sum of the weights of the portfolio equals to that of the benchmark. In the case they differ, the additional leverage effect need to be accounted for ($\sum_{s=1}^{S} (w_s^P - w_s^B)\, R_\varepsilon^B$).

In the case of a multi-layer grouping, the attribution can be applied recursively on the Selection term $w_s^P\left(R_{\varepsilon,s}^P - R_{\varepsilon,s}^B\right)$ by simply using the relative weights. For details, please refer to the white paper on nested attribution.

Finally, at the very bottom layer of the grouping, we can use the same technique to attribute the total selection from a given bottom group into contributions from each security or position by treating each security or position as a sub-group by itself. Assume that s is the one of bottom group and a security in the group, n, has a contribution given by the following formula.

$$w_s^P \left(\frac{w_n^P}{w_s^P} - \frac{w_n^B}{w_s^B}\right)\left(R_{\varepsilon,n}^B - R_{\varepsilon,s}^B\right) + w_n^P\left(R_{\varepsilon,n}^P - R_{\varepsilon,n}^B\right) \tag{12}$$

In general, the total local return of the same security is the same in the portfolio and the benchmark. In such case, the second term is reduced to zero. The first term is referred to as the *Security Level Selection Effect*. In the case that different pricing, and therefore different returns, are allowed between a portfolio and its benchmark, the second term can be non-zero for securities with different returns and is referred to as the *Pricing Difference*. By construction, the sum of contributions from the union of all securities in a given group in the portfolio and the benchmark equals to the total Selection Effect of the immediate parent group.

3.2 Excess Return Model (MV weighted)

Under this specification, the total return of a security is decomposed into the currency return, the curve carry return, the curve change return and the residual return, which is often referred to as the excess return, as in Equation (13). The reason that we offer this model specification is that with the increased popularity of interest rate derivatives, yield curve bets are managed separately from credit bets in most fixed-income portfolios. Furthermore, major fixed-income index providers decompose index returns into yield curve returns and excess returns.

$$\widetilde{R_n} = X_n + \sum_{k=1}^{K} (\omega_{nk} y_{nk} \Delta t) + \sum_{k=1}^{K} (-\delta_{nk} \Delta y_{nk}) + .5\gamma_n \overline{\Delta y_n}^2 + R_n^{\varepsilon} \qquad (13)$$

In this equation, the first summation term is the curve carry return, where ω_{nk} and $y_{nk}$are the weight and yield level on k-th maturity point respectively, and Δt is the change in time. The second summation is the first-order curve change return, where δ_{nk} and Δy_{nk} are the key rate duration and the change in yield at the k-th key rate point. The second-order curve change return is captured by the term immediately after it, where γ_n is the option adjusted convexity; and $\overline{\Delta y_n}$ is the average change across all key rate points. Finally, the excess return is the residual return in this decomposition.

The contributions of the curve carry and curve change factors to the active return can be computed by first aggregating the exposures over all securities and then taking the difference to get the net exposure and finally multiplying the net exposure by the appropriate factor. For instance, the contribution from the 5 year curve change factor is given by the following, where the summation is simply the net 5 year key rate duration.

$$-\sum_{n=1}^{N} \left(w_n^P \delta_{n5y}^P - w_n^B \delta_{n5y}^B \right) \Delta y_{n5y} \qquad (14)$$

For a single currency portfolio, the factor, Δy_{n5y} is the change of 5 year yield on the base curve. When a portfolio contains securities denominated in multiple currencies, we compute the curve change returns for each currency (curve) separately.

Furthermore, we offer clients the choice to define another curve change factor — the parallel shift, which is often defined as the average of yield change along the term structure. By doing so, all other curve change factors become additional yield change over the parallel shift at each key rate point. Assume that all securities in a given currency c share the same base curve and denote the contribution to key rate duration in this currency by $\delta_{c,k}^{P/B}$, then the contribution from the parallel shift and additional tilts are given separately by:

Parallel curve shift effect is computed as:

$$-\sum_{k=1}^{K}\left(\delta_{c,k}^{P}-\delta_{c,k}^{B}\right)\overline{\Delta y_c} \tag{15}$$

Additional curve change effect at k-th point is:

$$-\left(\delta_{c,k}^{P}-\delta_{c,k}^{B}\right)\left(\Delta y_{c,k}-\overline{\Delta y_c}\right) \tag{16}$$

where $\overline{\Delta y_c}$ is the parallel shift defined by the user. It can be an actual change at a given point on the curve (1, 3 and 10 year), or a weighted average of the changes using key rate duration contributions in the benchmark as weights.

The excess return is attributed to Allocation and Selection in the same fashion as in the previous section.

3.3 Excess Return Model (SD weighted)

This model specification shares the same factor model structure: the total base currency return is decomposed into the currency return, the curve carry return, the curve change return and the excess return as in Equation (13).

The treatment of the currency and curve factors are the same as in the market value weighted excess return model. The only difference is in the attribution of the excess return: although both use the same generic decomposition, the MV (market value) weighted version has the exposures set to 1 and the factor set to excess return, while the SD (spread duration) weighted version has the exposure set to spread duration contribution and the factor set to change of spread[2]. Denote spread duration of a sector s by $SD_s^{P/B}$ and the implied change of spread by $\Delta S_s^{P/B}$, the active excess return can be attributed into the following terms. The last term is the "leverage effect". As mentioned in previous section, if the portfolio and benchmark hold only cash bonds, this term is

[2] Strictly speaking, it is the implied change of spread. It is computed as the negative of excess return divided by the spread duration at the beginning of the period.

nonzero if the spread duration of the portfolio is different than that of the benchmark. It measures the contribution of the spread duration decision at the portfolio level. If the portfolio manager anticipate an overall tightening of the spread, it would be appropriate for him to take a long position in spread duration. The *hurdle return* can be set to zero, in that case, the allocation decision for a given sector is taken solely based on the view of the spread movement of that sector.

$$\sum_{s=1}^{S} -\left(w_s^P SD_s^P - w_s^B SD_s^B\right)\left(\Delta S_{\varepsilon,s}^B - \Delta S^H\right) + \sum_{s=1}^{S} -w_s^P SD_s^P \left(\Delta S_{\varepsilon,s}^P - \Delta S_{\varepsilon,s}^B\right) + \sum_{s=1}^{S} -\left(w_s^P SD_s^P - w_s^B SD_s^B\right)\Delta S^H \tag{17}$$

The interpretation of the Allocation and Selection terms is very similar to that in the previous section. In the case the hurdle level is set to zero, if a sector's spread widens, a decision to long the sector would generate negative Allocation Effect and a decision to short the sector would generate positive Allocation Effect. For the Selection Effect, assuming that the portfolio has positive spread duration bet in a given sector, if the spread in the portfolio tightens more than that in the benchmark, it would generate a positive Selection Effect. If the spread in the portfolio widens more than that in the benchmark, the Selection Effect would be negative.

3.4 Spread Return Model

Under this specification, the total return of a security is decomposed into the currency return, the curve carry return, the curve change return, the spread carry return, and the residual term. The residual term is primarily the spread change return. Denote the option-adjusted spread by S_n, the total return in base currency is decomposed into the following terms.

$$\widetilde{R_n} = X_n + \sum_{k=1}^{K}(\omega_{nk} y_{nk} \Delta t) + \left(\sum_{k=1}^{K}(-\delta_{nk}\Delta y_{nk}) + .5\gamma_n \Delta y_n^2\right) + S_n \Delta t + R_n^{\varepsilon} \tag{18}$$

Currency and curve components are handled the same way as described before. The newly added spread carry is computed simply as the spread multiplied by the change in time for each security. We attribute both the active spread carry return and active spread change return into Allocation and Selection using the general framework.

For the spread carry component, the exposure is set to 1 and the factor is set to be the spread carry return, this gives rise to decomposition into Allocation and Selection described in the total return model with total return replaced by spread carry return.

For the spread change component, the exposure is set to spread duration contribution and the factor is set to be the implied spread change (the residual term in equation (18) divided by the spread duration at the beginning of the period). Therefore, this gives rise to the same decomposition into Allocation and Selection as described in spread duration weighted excess return model.

For a generic period of time that is longer than one day, the total active return over the period is defined as the geometrically compounded portfolio return minus the geometrically compounded benchmark return. Even though the active return over each day can be attributed into a few additive terms as described in this section, we cannot simply geometrically compound these attribution terms. To solve this problem, we adopt the linking method by Carino (1999) to adjust all the attribution effects, such that the sum of all the attribution effects equals to the total active return for the period[3].

4. Examples

In this section, we construct a few examples to show how each model specification can be used. A proper model specification and grouping scheme should be chosen based on how a portfolio is managed: how many distinct decisions (factors) are used, what is the grouping and the decision factor used for allocation and selection decisions.

4.1 Total Return Decomposition

In this sub-section, we decompose the total return of a security into a few return components over a period of one day as in Equation (18), which is the most detailed decomposition currently available.

Assume that the security is denominated in EUR and the base currency of the portfolio is USD. To compute each return component, we need the following information: the key rate durations, option-adjusted convexity and option-adjusted spread of the security at the beginning of the period, the fx rates, and the yield curves at both the beginning and the end of the period. The details are given in Table 3. The returns are all in basis points.

The currency return is computed as the relative change in exchange rate plus its cross term with the total local return. Over the day, the euro appreciates relative to the dollar; therefore, the currency return is positive.

The curve change return at each key rate point is computed as the negative key rate duration multiplied by the yield change, and the curve change due to the convexity term is half of the convexity multiplied by the square of the average curve change. This security has most of its curve change return from the 5 year point because its exposure to this point is 3.5 yr, which accounts for about more than half its total curve exposure, and

[3] For more details on Carino linking, please refer to the appendix.

the curve widened by 10 bps on that point, which is greater or equal to the curve shifts on other points.

Total Base Return					**16.78**	
Currency Return	FX t0	FX t1			Currency Return	
	1.451	1.453			**13.79**	
Total Local Return	Dirty Price t0	Dirty Price t1	Cashflow		Total Local Return	
	100.35	100.38	0.00		**2.99**	
Maturity(yr)	Yield t0(%)	Yield t1 (%)	KRD/OAC(yr)	wgt.%	Curve Change Return	Curve Carry Return
0.5	2.30	2.40	0.00	0.00	0.00	0.00
1	2.50	2.60	0.30	0.17	-3.00	0.12
2	3.10	3.18	0.55	0.16	-4.40	0.14
3	3.30	3.35	0.80	0.15	-4.00	0.14
5	3.60	3.70	3.50	0.41	-35.00	0.40
7	3.70	3.75	1.30	0.11	-6.50	0.11
10	3.80	3.89	0.00	0.00	0.00	0.00
20	3.80	3.90	0.00	0.00	0.00	0.00
30	3.80	3.90	0.00	0.00	0.00	0.00
Convexity	3.32	3.41	0.90		0.33	
Curve Return	3.32	3.41	6.45	1.00	**-52.57**	**0.90**
Spread Return	OAS t0	OAS t1			Spread Change Return	Spread Carry Return
	57.00	49.00			**54.50**	**0.16**

Table 3: Total Base Return Decomposition

To compute the curve carry return, we first compute the percentage weight on each maturity point, and then simply multiply this weight by the initial yield level and the change in time. This security has most of its cash flow (about 41%) coming from the 5 year point, and that contributes 0.4 bps to curve carry return.

Finally, the spread carry return is computed as the initial option adjusted spread (OAS) multiplied by the change in time and it is 0.16 bps in this example. The spread change return, which is the residual term in this specification, is therefore 54 bps. If we take the change in OAS and multiply it by the negative spread duration, we get 51.6 bps. This means that the first order approximation works reasonably well in this case.

4.2 Total Return Attribution Example

The total return model is often used for an equity portfolio with a sector grouping. However, a high-yield portfolio manager may find it the best model specification for her portfolios as well.

Assume that an equity portfolio is benchmarked against a popular equity index. With the help of hindsight, we construct a portfolio as of the end of October 2010. We overweight the Financials sector by 3.66% and underweight the Health Care sector by the same percentage. Within each sector, we keep the relative weights of all the stocks in the portfolio identical to those in the benchmark. In other words, we employ a pure sector allocation strategy and make no stock selection within each sector.

We run this sample equity portfolio through the total return attribution model with average benchmark return as the hurdle return for 1 day. The result is exactly as expected: the total active return is 0.29% and it is all from sector Allocation. The sector details are given in Table 4. Returns are in percentage. This example and examples followed are constructed for illustration purpose and the returns are exaggerated for 1 day.

Sector	% Port	% Bench	+/-	Total Return (Port)	Total Return (Bench)	Allocation	Selection
Utilities	1.35	1.35	0.00	0.75	0.75	0.00	0.00
Materials	2.53	2.53	0.00	5.93	5.93	0.00	0.00
Telecommunication Services	4.08	4.08	0.00	2.45	2.45	0.00	0.00
Consumer Discretionary	8.03	8.03	0.00	3.49	3.49	0.00	0.00
industrials	9.22	9.22	0.00	3.47	3.47	0.00	0.00
Health Care	7.44	11.10	-3.66	-0.18	-0.18	0.14	0.00
Energy	12.99	12.99	0.00	5.16	5.16	0.00	0.00
Financials	18.26	14.60	3.66	7.81	7.81	0.15	0.00
Consumer Staples	13.92	13.92	0.00	1.65	1.65	0.00	0.00
Information Technology	22.18	22.18	0.00	3.52	3.52	0.00	0.00
Total	100.00	100.00	0.00	3.95	3.66	0.29	0.00

Table 4: Total Return Attribution of the Equity Portfolio

By construction, the sector returns are identical between the portfolio and the benchmark, and the portfolio has an active weight only in the Health Care and Financials sectors. Therefore, all the selection effects and allocation effects for sectors other than these two are zero. For the Health Care sector, the total return over the period is -0.18%, which is lower than the average return of the benchmark at 3.66%. Therefore, the underweight of this sector is a good allocation decision and contributes 0.14% to the active return. The Financials sector, on the contrary, has a return of 7.81% over the period, which is higher than the average return of 3.66%; therefore, an overweight of such a sector is a good allocation decision and contributes 0.15% to the active return. When the hurdle return is set to the average benchmark return, the results are very intuitive: overweight in a sector with superior return or underweight in a sector with inferior return gives rise to positive Allocation Effect, and underweight in a sector with superior return or overweight in a sector with inferior return gives rise to negative Allocation Effect.

4.3 Excess Return (MV weighted) Attribution Example

This model is typically used for a fixed-income portfolio in which the curve return is managed separately from the excess return. The curve return is mainly comprised of the return due to the yield curve change and is usually managed at the portfolio level. The excess return is often managed using a combination of sector allocation and security selection strategies.

We construct an investment grade (IG) corporate portfolio against the Bank of America Merrill Lynch corporate index (C0A0) as of the beginning of 2010. We keep all sectors market value neutral against the benchmark. If we run a total return model specification on this portfolio for the month of January 2010, the Allocation Effect is close to zero and almost all the active return is attributed to Selection (Table 5). Note that the returns are all in bps in this table and all subsequent tables. In particular, most of the Selection Effect comes from the Financial sector because the total return of the sector is 367 bps in the portfolio and 207 bps in the benchmark. Since it is hard to replicate a monthly attribution report, we have reported hypothetical results as if the returns are from a 1 day run throughout the example of the credit portfolio.

Sector	% Port	% Bench	+/-	Total Return (Port)	Total Return (Bench)	Allocation	Selection
Basic Materials	4.41	4.41	0.00	103	162	0.00	-2.60
Communications	11.78	11.82	-0.04	141	164	0.01	-2.71
Consumer, Cyclical	4.08	4.05	0.03	168	161	-0.01	0.29
Consumer, Non-cyclical	12.92	12.92	0.00	283	201	0.00	10.59
Diversified	0.50	0.50	0.00	123	146	0.00	-0.12
Energy	10.70	10.69	0.01	325	188	0.00	14.66
Financial	37.41	37.34	0.07	367	207	0.01	59.86
Funds	0.00	0.17	-0.17		337	-0.24	0.00
Government	0.19	0.19	0.00	255	222	0.00	0.06
Industrial	6.46	6.46	0.00	239	218	0.00	1.36
Technology	2.46	2.45	0.01	279	218	0.00	1.50
Utilities	9.09	9.00	0.09	313	204	0.01	9.91
Total	100.00	100.00	0.00	288	196	-0.22	92.80

Table 5: Total Return Attribution of the Credit Portfolio

If the curve component is managed at the portfolio level and the sector allocation and security selection decisions are made based on the excess returns, the results can be quite different. We run the same portfolio for the same time period using the excess return model with market value weighting. For the curve return, we analyze the result by the key rate points along the term structure and for the excess return we attribute it into Allocation and Selection using a sector grouping. The results on the excess return are given in Table 6.

Compared to the results from the total return specification shown in Table 5, Selection Effect drops substantially from 93 bps to 62 bps. In particular, the contribution from the Financial sector drops by 26 bps. This means that part of the Selection Effect in the total return model actually comes from yield curve management.

Sector	% Port	% Bench	+/-	Excess Return (Port)	Excess Return (Bench)	Allocation	Selection
Basic Materials	4.41	4.41	0.00	-108	-23	0.00	-3.74
Communications	11.78	11.82	-0.04	37	-18	0.02	6.51
Consumer, Cyclical	4.08	4.05	0.03	99	-4	-0.01	4.16
Consumer, Non-cyclical	12.92	12.92	0.00	63	19	0.00	5.68
Diversified	0.50	0.50	0.00	-70	-22	0.00	-0.24
Energy	10.70	10.69	0.01	90	-4	0.00	10.08
Financial	37.41	37.34	0.07	139	46	0.02	34.60
Funds	0.00	0.17	-0.17		191	-0.29	0.00
Government	0.19	0.19	0.00	71	72	0.00	0.00
Industrial	6.46	6.46	0.00	-13	32	0.00	-2.91
Technology	2.46	2.45	0.01	74	50	0.00	0.58
Utilities	9.09	9.00	0.09	91	12	-0.01	7.15
Total	100.00	100.00	0.00	82	21	-0.27	61.87

Table 6: MV Weighted Excess Return Attribution of the Credit Portfolio

Since the curve factors are common for all securities denominated in the same currency, it is usually managed at the portfolio level, rather than at the sector level. Depending upon the view, a portfolio manager can set up the interest rate strategy completely separately from the excess return management by using futures or interest rate swaps.

In this example, there is only one currency; therefore the curve return analysis is relatively simple. The details of the curve active return are provided in Table 7. The total contribution from active curve management is 30.50 bps; most of this is from curve change (active curve positioning with yield curve shift), and a much smaller portion, 5.87 bps, is from curve carry (active yield positioning with the elapse of time). The table has two parts. The first part displays the relative curve positioning measured by key rate duration. Combined with the actual change experienced by the yield curve, this gives rise to the active curve change return. This portfolio is primarily long duration on the longer end of the curve and slightly short on the shorter end of the curve. We set the parallel shift to the actual change at the 5 year point. The curve tightened by 36 bps on this point and the portfolio's overall long position gives rise to 46 bps in active curve change return.

The contribution from additional movement at each key rate point over the parallel shift can be computed as the net exposure (net key rate duration) multiplied by the additional movement. For instance, this portfolio loses 18 bps from the 20 year point because the 20 year point tightened less than the parallel shift and the portfolio is long by almost a year on this point. Similarly, the portfolio loses about 13 bps on the 10 year point.

Maturity Points	Yield Change (bps)	Dur./OAC (Port)	Dur./OAC (Bench)	+/-	Curve Change	Yield Level (%)	% Port	% Bench	+/-	Curve Carry	Total Curve
Totals		**7.54**	**6.18**	**1.36**	**27.78**		**99.98**	**99.68**	**0.30**	**3.42**	**31.20**
Parallel	-0.36				49.10						49.10
0.5	-0.05	0.01	0.02	-0.01	0.31	0.41	2.38	3.34	-0.96	-0.03	0.28
1	-0.14	0.05	0.08	-0.03	0.66	0.92	5.04	7.79	-2.75	-0.21	0.45
2	-0.32	0.14	0.20	-0.06	0.24	1.21	7.68	9.82	-2.14	-0.21	0.03
3	-0.36	0.32	0.54	-0.22	0.00	1.85	10.16	18.38	-8.22	-1.25	-1.25
5	-0.36	0.46	0.74	-0.28	0.00	2.79	8.67	15.46	-6.79	-1.56	-1.56
7	-0.32	1.30	1.13	0.17	-0.68	3.36	20.99	17.61	3.38	0.93	0.25
10	-0.26	2.41	1.18	1.23	-12.30	3.84	28.36	13.85	14.51	4.58	-7.72
20	-0.16	2.04	1.11	0.93	-18.60	4.40	13.01	7.59	5.42	1.96	-16.64
30	-0.13	0.81	1.18	-0.37	8.51	4.49	3.69	5.84	-2.15	-0.79	7.72
Convexity	-0.23	0.95	0.75	0.20	0.54						0.54

Table 7: Active Curve Return Analysis

The second section provides information on the curve positioning measured by percentage weight on each point along the curve and computes the active curve carry return. Our sample portfolio is over-weighted on the long end of the curve, which has higher yield; therefore, the portfolio has a small curve carry advantage of 3.42 bps.

4.4 Excess Return (SD weighted) Attribution Example

In the previous sub-section, we attribute the active excess return using the market value weights as exposures. This is appropriate if the investment decisions are how to over- or underweight each sector (the sector allocation) and over- or underweight each security within each sector. These decisions are made based on the prediction of the relative excess return of sectors and the excess returns of securities relative to their sector. A portfolio manager would overweight sectors or securities with higher predicted excess returns and underweight sectors or securities with lower predicted excess returns. However, for some portfolios, the investment decision is based not only on the weights of each sector but also on the spread duration of each sector. To better fit the management style of such portfolios, we introduce the spread duration weighted excess return model. In this model, the factor is the spread change and the exposure is the spread duration contribution, which is simply defined as the product of the weight and the spread duration. Under this model, the asset allocator and security picker evaluate sectors and securities based on their predictions of spread movement of sectors and securities. They would long sectors or securities with good predicted spread movement (spread to tighten) and short sectors or securities with bad predicted spread movement (spread to widen). In order to long a sector, one can use the combination of weight and spread duration by either overweighting the sector or focusing on the bonds with longer spread durations, or both.

When making the spread duration allocation decisions, there are potentially two approaches. One is that the portfolio manager has an assessment of the credit spread movement as a whole and she first makes a decision in terms of the overall spread duration positioning. If she thinks that the credit spread is going to widen, she will short spread duration to outperform the benchmark, and vice versa. Once this decision is made, it puts a constraint on sector spread duration allocation in the sense that the spread duration contributions of all the sectors should add up to the overall spread duration target. Under this constraint, the sector allocator would long a sector only if she predicts that this sector is going to outperform the predicted market-wide spread change. Therefore, the additional contribution from sector spread duration tilt is computed as the negative of net spread duration contribution to the sector and the sector spread movement over the market-wide spread movement. To analyze this portfolio, we should use the relative mode SD weighted excess return model, in which the *Hurdle Return* is set to be the average spread change of the benchmark.

The other approach is that the portfolio manager does not think that there is a clear trend for the overall credit spread movement, but rather each sector may move in different directions. Under this scenario, the portfolio manager makes the decision to long or short a sector solely based on her prediction of the spread movement of that sector, potentially subject to some risk budget. This strategy can be analyzed with the absolute mode by setting the *Hurdle Return* to zero. As mentioned in Section 2, only the absolute mode is currently available, the relative mode will be added at a later date.

Sector	% Port	% Bench	SD Port	SD Bench	SD Con (Port)	SD Con (Bench)	Spread Chg (Port)	Spread Chg (Bench)	Allocation	Selection
Basic Materials	4.41	4.41	6.20	6.13	0.27	0.27	17	4	-0.01	-3.73
Communications	11.78	11.82	2.67	6.63	0.31	0.78	-14	3	1.28	5.23
Consumer, Cyclical	4.08	4.05	1.93	5.97	0.08	0.24	-51	1	0.10	4.07
Consumer, Non-cyclical	12.92	12.92	6.32	6.36	0.82	0.82	-10	-3	-0.01	5.69
Diversified	0.50	0.50	4.86	5.03	0.02	0.03	14	4	0.00	-0.25
Energy	10.70	10.69	6.70	6.79	0.72	0.73	-13	1	0.01	10.08
Financial	37.41	37.34	9.93	5.26	3.71	1.96	-14	-9	15.43	19.19
Funds	0.00	0.17		4.04	0.00	0.01		-47	-0.32	0.00
Government	0.19	0.19	4.46	3.93	0.01	0.01	-16	-18	0.02	-0.02
Industrial	6.46	6.46	6.33	6.37	0.41	0.41	2	-5	-0.01	-2.89
Technology	2.46	2.45	5.44	5.33	0.13	0.13	-14	-9	0.03	0.55
Utilities	9.09	9.00	7.17	7.19	0.65	0.65	-13	-2	0.01	7.15
Total	100.00	100.00	7.14	6.03	7.14	6.03	-12	-3	16.52	45.08

Table 8: SD Weighted Excess Return Attribution

We continue with the previous example and analyze the same portfolio using this model specification. The result from the absolute mode is presented in Table 8. Compared to Table 6, where almost all the active excess return is attributed to Selection, the analysis with SD weight shows that 15 bps of the active return actually comes from sector Allocation. Even though all the sectors are market value neutral, the portfolio clearly takes a long position in the Financial sector and a short position in the Communications and Consumer, Cyclical sector. The long position in the Financial sector gives rise to a 15 bps contribution to Allocation because the sector spread tightened by 9 bps. At the same time, the contribution from the Financial sector to Selection is reduced from 35 bps to 19 bps. This example highlights the importance of matching the model specification with investment management process. The same portfolio could be managed differently and therefore different model specifications should be used for attribution analysis.

4.5 Spread Return Attribution Example

In general, the excess return is mostly the return due to credit spread movement. However, under certain market conditions, the spread carry return can contribute significantly to the excess return as well, especially over a relatively long period of time. In addition, the spread carry and spread change bets can be made separately: to earn higher carry, a portfolio manager could overweight sectors with higher spreads. At the same time, if she predicts that the spreads of these sectors are going to widen, she may decide to short these sectors by picking bonds with very short spread duration, or use credit derivatives such as CDS to achieve the desired spread duration allocation. To analyze such a portfolio correctly, we need to separate the spread carry return from the spread change return. For spread carry return we analyze it using market value weight as the exposure, while for spread change return we analyze it using contribution to spread duration as the exposure. Both components are attributed into Allocation and Selection effects.

We continue with our previous example. The results are presented in Table 9. As expected, the spread carry allocation is close to zero because all sectors are roughly market value neutral. However, we do see that over a month, spread carry contributes about 5 bps to Selection Effect. Taking the Financial sector as an example, the sector has significant spread advantage over other sectors in the benchmark. Even though our sample portfolio is long in this sector, it has done so by focusing on the longer part of the credit spread curve with neutral market value weight. Therefore, there is almost zero contribution to Allocation from spread carry decision. However, the portfolio picks higher spread bonds within the Financial sector (325 bps versus 205 bps), and it contributes 3.75 bps to the spread carry Selection effect. Financial sector spread tightened 6 bps, and the portfolio long the sector significantly, therefore this generate a positive Allocation effect of 9.7 bps from Spread Change. In addition, the portfolio spread tightened more than that of the benchmark, this contributes to a positive Selection effect of 21.1 bps from the spread change.

Sector	% Port	% Bench	OAS (Port)	OAS (Bench)	SD Con (Port)	SD Con (Bench)	Spread Chg (Port)	Spread Chg (Bench)	S Carry Allocation	S Carry Selection	S Chg Allocation	S Chg Selection
Basic Materials	4.41	4.41	232	156	0.27	0.27	21	6	0.00	0.28	-0.02	-4.00
Communications	11.78	11.82	88	142	0.31	0.78	-11	5	0.00	-0.53	2.12	4.93
Consumer, Cyclical	4.08	4.05	201	139	0.08	0.24	-42	3	0.00	0.21	0.41	3.54
Consumer, Non-cyclical	12.92	12.92	153	107	0.82	0.82	-8	-2	0.00	0.50	-0.01	5.18
Diversified	0.50	0.50	134	146	0.02	0.03	17	7	0.00	0.00	0.01	-0.24
Energy	10.70	10.69	178	154	0.72	0.73	-11	2	0.00	0.22	0.02	9.84
Financial	37.41	37.34	325	205	3.71	1.96	-11	-6	0.01	3.75	9.74	21.12
Funds	0.00	0.17		322	0.00	0.01		-41	-0.05	0.00	-0.28	0.00
Government	0.19	0.19	48	49	0.01	0.01	-15	-17	0.00	0.00	0.02	-0.02
Industrial	6.46	6.46	161	123	0.41	0.41	4	-3	0.00	0.21	-0.01	-3.10
Technology	2.46	2.45	185	89	0.13	0.13	-11	-8	0.00	0.20	0.03	0.36
Utilities	9.09	9.00	171	141	0.65	0.65	-11	0	0.01	0.23	0.00	6.92
Total	100.00	100.00	221	160	7.14	6.03	-9	-1	-0.02	5.05	12.03	44.54

Table 9: Spread Return Attribution

In this example, the decision to focus on the longer end of the credit curve contributes to the higher average spread in the Financial sector in the portfolio. In other words, the decision of the weight allocation and spread duration positioning are not totally separate. However, by using credit derivatives, the two decisions could be well separated and this gives portfolio managers more freedom to take best positions in all factors to generate outperformance.

5. Summary and Conclusion

This document compares four performance attribution model specifications which are currently offered via the Bloomberg Portfolio & Risk Analytics function, PORT. The model can be summarized as follows: The total return of a security in portfolio base currency can be decomposed as the sum of return components; each in turn is expressed as the product of an exposure to a factor and the factor return. The return of a portfolio is simply the weighted sum of the security level returns. Similarly, the active return of a portfolio over its benchmark can be written as the sum of active return components, each being the product of the net exposure to a factor and the factor return itself.

The typical Brinson model can be viewed as a special case of this general model. For instance, the Brinson model with a sector grouping is equivalent to a factor model with unit exposures and factors being the realized benchmark sector returns. Therefore, the contribution from a factor is equivalent to the allocation effect for the corresponding sector in the Brinson model.

The structure of the general framework offers a great level of flexibility. Depending on the nature of a given factor, we can either simply aggregate that component up or apply an on-the-fly further breakdown into contributions from more detailed factors for which we do not yet have a risk model. For instance, if a client uses a different sector scheme than GICS, which is the classification used in our US equity factor model, she may decide to use the total return model with her sector scheme rather than using a pure factor-based attribution model based on factor returns obtained in our risk model. In addition, this approach puts factor-based attribution and the traditional Brinson attribution into one unified framework.

Going forward, we will offer more specifications under the generic framework. With the development of our suite of risk models in PORT, we will be able to offer a pure factor-based attribution model that is consistent with the risk models. In the short run, we will refine the treatment of the currency return to reflect the cash rate differential between currencies, and we will provide additional asset-specific factors to better accommodate certain asset classes.

Bibliography

Brinson, G., and N. Fachler, 1985, Measuring non-US equity portfolio performance, *Journal of Portfolio Management* pp. 73-76

Brinson, G. P., L.R. Hood, and G. L. Beebower, 1986, Determinants of portfolio performance, *Financial Analysts Journal* pp. 39-44.

Carino, D., 1999, Combining attribution effects over time, *Journal of Performance Measurement* pp. 5-14.

Grinold, R., and R. Kahn, 1999, Active Portfolio Management: A Quantitative Approach for Producing Superior Returns and Controlling Risk.

Karnosky, D. S., and B. D. Singer, 1994, *Global Asset Management and Performance Attribution* (The Research Foundation of the Institute of Chartered Financial Analysts).

Appendix I: Multiple Period Attribution Carino Linking

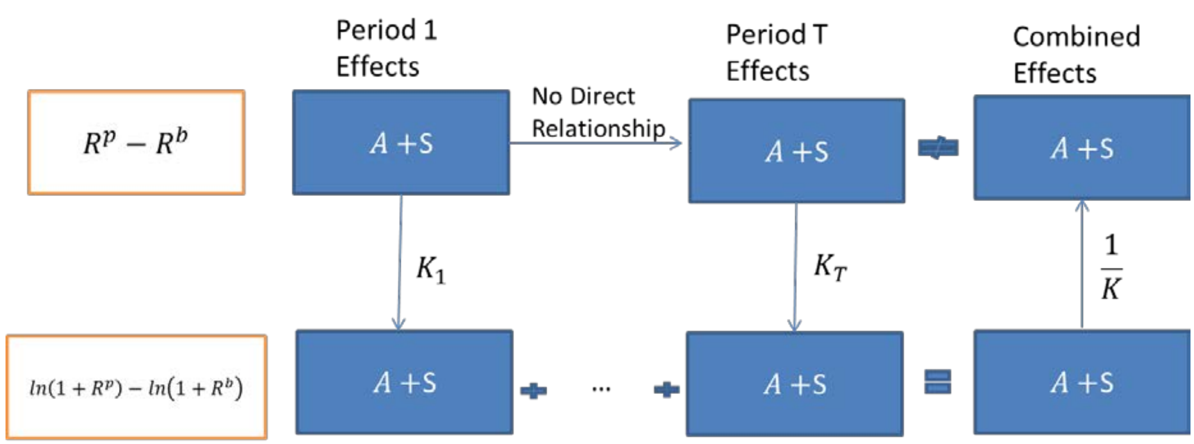
For each period, the active return can be decomposed into some additive attribution effects. However, when multiple-period is concerned, the attrition effects over multiple-period cannot be obtained by simply summing or compounding single effects. Let's assume that there are T days and we run attribution by sector without interaction. Each day t, the active return is the sum of S allocation effects and S selection effects. The T-day active return is simply defined as the difference of the geometrically compounded portfolio return and benchmark return.

$$AR = \prod_{t=1}^{T}(1 + R_t^P) - \prod_{t=1}^{T}(1 + R_t^B)$$

Each day, the active return is the sum of the Total Allocation Effect and Total Selection Effect.

$$R_t^P - R_t^B = A_t + S_t = \sum_{s=1}^{S}\left(w_{s,t}^P - w_{s,t}^P\right)\left(R_{s,t}^B - R_t^B\right) + \sum_{s=1}^{S} w_{s,t}^P\left(R_{s,t}^P - R_{s,t}^B\right)$$

However, it is easy to verify that the sum of these effects does not equal to the active return. The question is how to construct the attribution effects such that they carry the same economic meaning and add up to the active return. Carino (1999), among others, provides a simple solution to such problems. Recognizing that log returns are additive over time, Carino linking methodology first converts each single period effect by a coefficient (K_t) such that the adjusted sum is a log return rather than simple linear return. Then we add these adjusted effects over time, this naturally equal to the log return over the period. Finally, we convert the log return back to linear return by dividing the sum by another coefficient that is computed using the compounded return (K).



$$K_t = \frac{ln(1 + R_t^P) - ln(1 + R_t^B)}{R_t^P - R_t^B} \qquad K = \frac{ln(1 + R^P) - ln(1 + R^B)}{R^P - R^B}$$