

Quantessentials

R advice – Bayesian regressions with stan

The world of statistics is becoming more based on Bayesian techniques

The increase in the speed of computing is allowing many more Bayesian approaches to be used in practical statistics. The advantage of Bayesian statistics are many both for model building and understanding the relationships between variables.

A first step in a Bayesian direction

We have used Bayesian regressions to calculate the risk models in the UBS Portfolio Analysis System for some years, and find they have many advantages over using traditional regression. In this note we introduce how to carry out Bayesian regressions in R using the *rstan* and *rstanarm* packages.

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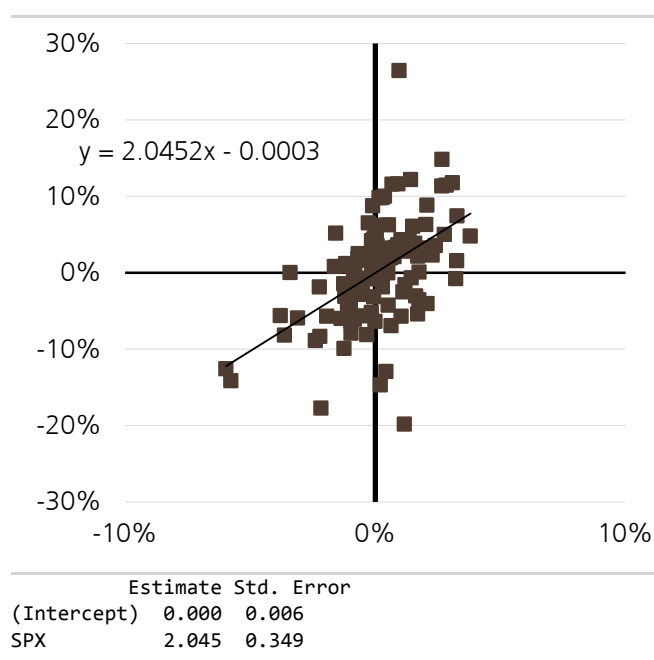
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Do you want to be a Bayesian? ¹

In a companion note ([Spotlight on...the blind reliance on Bloomberg betas, 27 March 2017](#)) we discuss using relatively simple Bayesian techniques to estimate stock betas, in particular varying the amount of shrinkage depending on the fit of the basic regression of stock returns on market returns. Here we discuss how we estimated the Bayesian regressions using R.

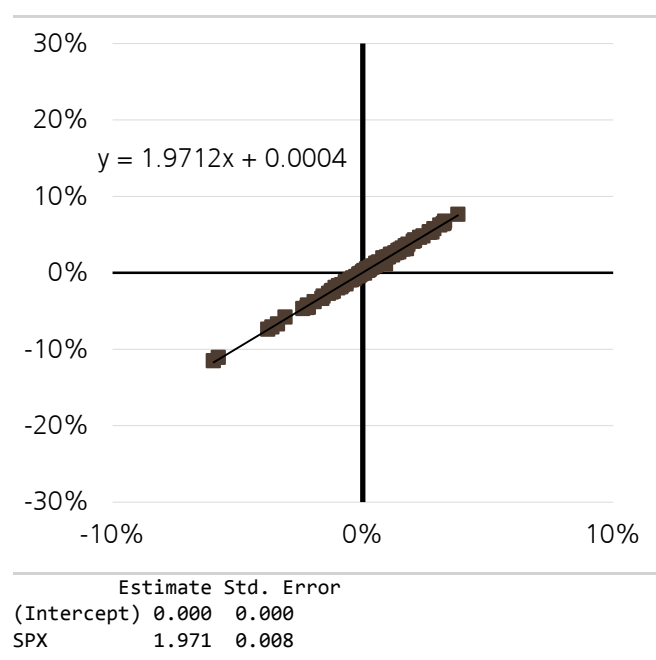
Which of these two regression results below would you have the biggest confidence in? In Figure 1 we show the regression of Qorvo's returns on the S&P 500; in Figure 2 we show the regression of the Proshares Ultra S&P500 ETF (which is a 2x geared ETF) again on the S&P 500. In both cases we obtain an answer close to 2 but for the first the standard error is quite large (0.35) where as for the second it is tiny (0.008). So if we are considering "shrinking" the betas towards 1 then perhaps we should shrink Qorvo's beta by more than the ETF's. We can formalise this idea in a Bayesian regression.

Figure 1: Qorvo beta calculation



Source: UBS. Stock used for example purposes only

Figure 2: 2x ETF beta calculation



Source: UBS

Kruschke (2015) says that "Bayesian data analysis has two foundational ideas. The first idea is that Bayesian inference is reallocation of credibility across possibilities. The second foundational idea is that the possibilities, over which we allocate credibility, are parameter values in meaningful mathematical models". What does this mean in our case? What do we know about betas? The first thing is that across all stocks in an index they are likely to have a mean of 1, and that most betas fall between around 0.4 and 1.6. So without any data we might decide that our initial allocation of credibility (or probability) could be expressed as saying betas are distributed as $N(1, 0.3)$. This is called our prior distribution. We then introduce the data (in this case the stock and market returns) and these then change this distribution of credibility as shown in Figure 5 below. The purple line is our view of

¹ One feels there is a whole song with this title to the tune of "Do you want to build a snowman?" from Frozen.

the world with no information (our "prior" view); the black line is the distribution of the data and the blue line is our "reallocated" (or "posterior") credibility for the beta of Qorvo.

This is putting into words what Bayes rule does. We are not going to attempt to introduce the intricacies of Bayesian techniques in this note; just how we ran the regression. Kruschke (2015) and McElreath (2016) are good introductions to the topic, and Gelman et al (2013) is a good reference.

We note that our prior could be more sophisticated. We might know the industry in which the stock lies – if it's a technology stock we might expect the mean to be greater than one; if it's a utility then it would probably be less than one. This topic was discussed at our 2015 London Quantitative Conference by Mathijs Cosemans where he presented his paper "Estimating Security Betas Using Prior Information Based on Firm Fundamentals" (Cosemans et al, 2016). There is a summary of his paper available [here](#).

The CRAN Task View for Bayesian Inference (at <https://CRAN.R-project.org/view=Bayesian>) is rather long to say the least. For our problem of estimating betas using a Bayesian regression we looked at two related packages; [rstan](#) and [rstanarm](#)². As always with R there are other approaches which can also be used.

The easiest approach is to use *rstanarm* which has functions which are designed to replace the standard R functions: in this case we could replace *lm* with *stan_lm* or *stan_glm*. *rstan* takes a little more work to get started but has the advantage of flexibility. We begin by running the usual linear regressions.

Figure 3: Basic linear regressions

```
twoStocks <- read.csv ("two stocks.csv") ## Contains the index too

summary (lm (SSO ~ SPX, data = twoStocks)) ## SSO is 2x geared ETF

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0003540  0.0001392   2.544   0.0125 *
SPX          1.9712264  0.0079144 249.068  <2e-16 ***

## QRVO is the stock
summary (qrvoLM <- lm (QRVO ~ SPX, data = twoStocks))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0002968  0.0061417  -0.048   0.962
SPX          2.0452491  0.3492801   5.856 5.95e-08 ***
```

Source: UBS

As already mentioned, the *rstanarm* package is designed to simply replace the *lm* call above. The *stan_lm* function uses an odd way of expressing the priors and so it turns out to be easier to use the *stan_glm* function.

Our model is

² We also wanted to use the *bugs* and *jags* based packages, *rjags* and *runjags* but these need the installation of the *WinBUGS* and *jags* programs which we could not access from within UBS.

$$r_{stock} \sim N(\beta_1 + \beta_2 r_{market}, \sigma)$$

Hence we need priors for the three parameters. For the slope (the parameter in which we are primarily interested) we are using an informative prior of a normal distribution with a mean of 1 and a standard deviation of 0.3. For the intercept and the standard deviation term we will use an uninformative prior - a very broad distribution - effectively saying we know little about their values. The prior for the standard deviation term is taken from Gelman (2006) – and is the default for *stan_glm* - which is a Cauchy distribution.

Figure 4: Bayesian regressions with rstanarm

```
library (rstan)
library (coda)
library (rstanarm)

ssoStan = stan_glm(SSO ~ SPX, data = twoStocks,
  family = gaussian(),
  prior = normal(1, 0.3, autoscale = FALSE),
  prior_intercept = normal(0, 10, autoscale = FALSE),
  prior_aux = cauchy (0, 2.5), chains = 1,
  cores = 4, seed = 1)
summary (ssoStan)
```

Estimates:

	mean	sd	2.5%	25%	50%	75%	97.5%
(Intercept)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPX	2.0	0.0	2.0	2.0	2.0	2.0	2.0
sigma	0.0	0.0	0.0	0.0	0.0	0.0	0.0
mean_PPD	0.0	0.0	0.0	0.0	0.0	0.0	0.0
log-posterior	511.6	1.2	508.3	511.2	512.0	512.5	512.9

```
qrvoStan <- stan_glm(QRVO ~ SPX, data = twoStocks,
  family = gaussian(),
  prior = normal(1, 0.3, autoscale = FALSE),
  prior_intercept = normal(0, 10, autoscale = FALSE),
  prior_aux = cauchy (0, 2.5), chains = 1,
  cores = 4, seed = 1)
summary (qrvoStan)
```

Estimates:

	mean	sd	2.5%	25%	50%	75%	97.5%
(Intercept)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPX	1.4	0.2	1.0	1.3	1.4	1.6	1.9
sigma	0.1	0.0	0.1	0.1	0.1	0.1	0.1
mean_PPD	0.0	0.0	0.0	0.0	0.0	0.0	0.0
log-posterior	127.9	1.2	124.9	127.3	128.2	128.8	129.3

```
# McElreath suggests not using 0.95 but another number to save
# confusion with traditional significance levels
posterior_interval (qrvoStan, prob = 0.89)
```

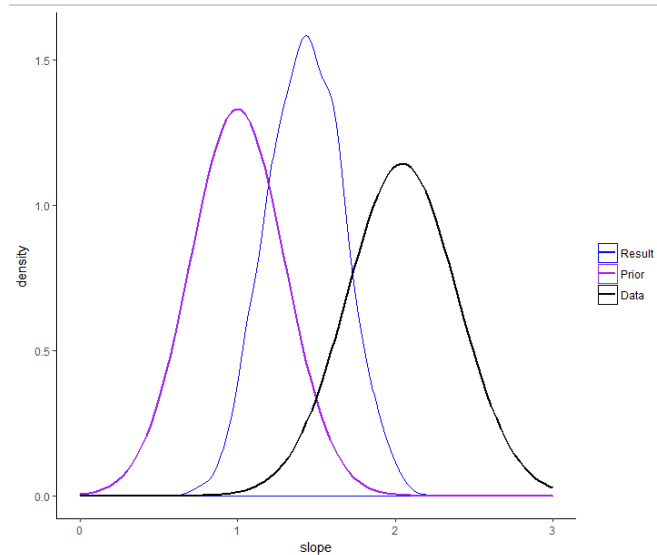
	5.5%	94.5%
(Intercept)	-0.009780303	0.01129120
SPX	1.068053097	1.79135148
sigma	0.056798832	0.07129282

Source: UBS

For the ETF the Bayesian regression leaves the estimated beta unchanged – the estimate of the slope from the regression is so "confident" it overrides any

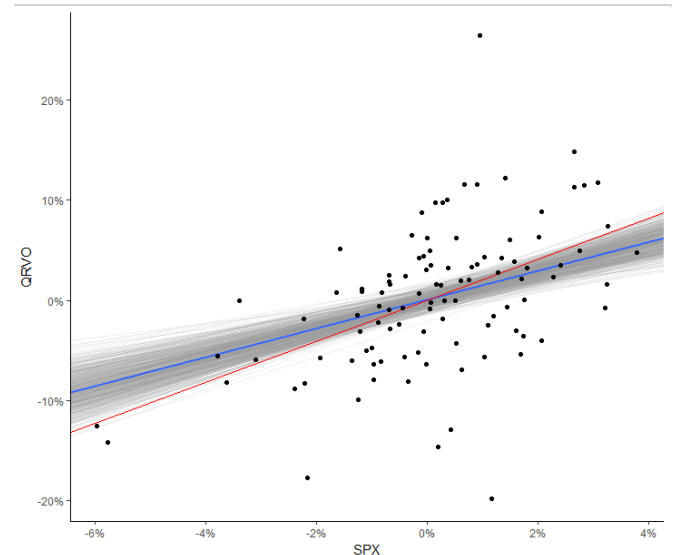
shrinkage. The estimate of the beta for QRVO however falls to around 1.4 and as we see from the bottom of Figure 4 it lies between 1.03 and 1.78.

Figure 5: Distribution of betas for QRVO



Source: UBS

Figure 6: Bayesian regression for QRVO



Source: UBS. The red line shows the result from a simple regression, the blue line is the median estimate from the Bayesian regression.

Using *rstan* directly involves writing a small model specification file. The advantage of this approach is you can very explicitly set out all your assumptions about the model; the downside is that you have to learn the syntax – not that tricky but another potential source of error. In Figure 7 we show the file we used in this case³. The first part of the *stan* file sets out the variables we are using – here the stock and the market returns. The second the parameters of the model; and the third the model itself. The priors are identical to those used above. We save the specification in a file called "simpleRegression.stan".

Figure 7: Stan model file – simpleRegression.stan

```
data {
  int<lower=0> N;
  vector[N] stock;
  vector[N] spx;
}
parameters {
  vector[2] beta;
  real<lower=0> sigma;
}
model {
  beta[1] ~ normal (0, 10);
  beta[2] ~ normal (1, 0.3); // prior for slope
  sigma ~ cauchy (0, 2.5);
  stock ~ normal(beta[1] + beta[2] * spx, sigma);
}
```

Source: UBS

³ Very much inspired by the example files available at <https://github.com/stan-dev/example-models/wiki>.

After we have created the specification we run the model as shown in Figure 8. The first time running any particular model is very slow as *rstan* builds some C++ code on the fly and then compiles it (this has already been done for *rstanarm*). This only needs to be done once. You will note we create the variables with the same names as in the .stan file above.

Figure 8: Running the regression using rstan

```
stock <- twoStocks$SSO
spx <- twoStocks$SPX
N <- length(spx)

data.list.1 <- c("N", "stock", "spx")
sso_res <- stan(file = "simpleRegression.stan",
               data = data.list.1, iter = 1000)
sso_res
```

Inference for Stan model: simpleRegression.
 4 chains, each with iter=1000; warmup=500; thin=1;
 post-warmup draws per chain=500, total post-warmup draws=2000.

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
beta[1]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2000	1.00
beta[2]	1.97	0.00	0.01	1.96	1.97	1.97	1.98	1.99	890	1.01
sigma	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	818	1.01
lp__	612.50	0.05	1.11	609.58	611.98	612.81	613.30	613.77	605	1.00

Samples were drawn using NUTS(diag_e) at Tue Mar 07 18:07:55 2017.
 For each parameter, n_eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor on split chains (at
 convergence, Rhat=1).

```
stock <- twoStocks$QRVO

qrvo_res <- stan(file = "simpleRegression.stan", data = data.list.1, iter = 1000)
qrvo_res
```

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
beta[1]	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.01	2000	1
beta[2]	1.43	0.01	0.23	0.97	1.26	1.43	1.59	1.88	2000	1
sigma	0.06	0.00	0.00	0.06	0.06	0.06	0.07	0.07	2000	1
lp__	228.75	0.04	1.21	225.54	228.19	229.07	229.65	230.15	933	1

Source: UBS

We see that the answers are identical to those from *rstanarm*. To generate Figure 6 we use the code below

Figure 9: Producing the chart above

```
beta.post.1 <- extract(qrvo_res, "beta")$beta
colnames(beta.post.1) = c("intercept", "slope")
beta.mean.1 <- colMeans(beta.post.1)

alpha_level <- .15
col_draw <- "grey60"
col_median <- "#3366FF"

# Plot a random sample of rows as gray semi-transparent lines
ggplot(twoStocks) +
  aes(x = SPX, y = QRVO) +
  geom_abline(aes(intercept = intercept, slope = slope),
             data = sample_n(as.data.frame(beta.post.1), 500),
             color = col_draw, alpha = alpha_level) +
  # Plot the median values in blue
  geom_abline(intercept = median(beta.post.1[, 1]),
             slope = median(beta.post.1[, 2]),
             size = 1, color = col_median) +
  # Results from the lm call
  geom_abline(intercept = coef(qrvoLM)[1],
             slope = coef(qrvoLM)[2], colour = "red") +
  geom_point() +
  scale_y_continuous(labels = scales::percent) +
  scale_x_continuous(labels = scales::percent) +
  theme_classic()
```

Source: UBS

Suppose one wanted to run a Fama French regression, then one would have to specify priors on all the regression parameters. Figure 10 below shows part of the stan file that could do this (where in the Fama-French case K would be 3). This shows one can use loops within the stan file.

Figure 10: stan file (part) for multiple linear regression

```
data {
  int<lower=0> K; // number of predictors
}
parameters {
  vector[4] beta;
  real<lower=0> sigma;
}
model {
  beta[1] ~ normal(0,10); //prior for the intercept
  beta[2] ~ normal(1, 0.3); // prior for the market term
  for(i in 2:K)
    beta[i] ~ normal(0,1); //prior for the other slopes
}
```

Source: UBS

This has been a very basic introduction to running Bayesian regressions – we will return to the topic in a future Quantessentials looking at some of the diagnostics available and how to write more general models.

Appendix

R environment

We used R version 3.3.1 for our tests. The files were written to and from our local hard drive in order to lessen any delays caused by our network. It is running on an Intel i7-6700 running at 3.4GHz.

We show below the version numbers of the packages we used.

Figure 11: Package versions used

Package	Version	Package	Version
rstan	2.14.1	rstanarm	2.14.1
coda	0.18.1		

Source: UBS

References

Cosemans, M., R. Frehen, P. C. Schotman & R. Bauer (2016) Estimating Security Betas Using Prior Information Based on Firm Fundamentals, *Review of Financial Studies* 29(4) 1072-1112.

Gelman, A. (2006) Prior distributions for variance parameters in hierarchical models, *Bayesian Analysis*, 1(3) 515-533.

Gelman, A., J.B. Carlin, H.S. Stern, D.B. Dunson, A. Vehtari & D.B. Rubin (2013) *Bayesian Data Analysis*, Third Edition. Chapman & Hall/CRC Texts in Statistical Science.

Kruschke, John K. (2015) *Doing Bayesian Data Analysis: A Tutorial with R, JAGS and Stan*. Academic Press / Elsevier.

McElreath, Richard (2016) *Statistical Rethinking: A Bayesian Course with Examples in R and Stan*. Chapman & Hall / CRC Texts in Statistical Science.

Research Publications

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Neutral	FSR is between -6% and 6% of the MRA.	39%	27%
Sell	FSR is > 6% below the MRA.	15%	16%
Short-Term Rating	Definition	Coverage ³	IB Services ⁴
Buy	Stock price expected to rise within three months from the time the rating was assigned because of a specific catalyst or event.	<1%	<1%
Sell	Stock price expected to fall within three months from the time the rating was assigned because of a specific catalyst or event.	<1%	<1%

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