Global Quantitative Research Monographs

Extending Volatility Targeting

Equities

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Volatility-targeting strategies

Simple volatility-targeting strategies adjust the exposure to an asset according to an estimate of historical volatility and hence exhibit ex-post an approximately flat volatility profile over time. This is effectively achieved through appropriate over- or under-weighting of the index during low- or high-volatility states respectively.

Correlation lies at the heart of diversification

Unless perfectly correlated, two assets combined in a portfolio will result in a lower total volatility. Conversely, when correlations increase, the diversification benefits diminish. Correlations tend to increase during high-volatility down-markets as shown in Figure 1, hence posing an additional challenge to the volatility-targeting strategies. Namely, how to hedge against unexpected increases in correlations?

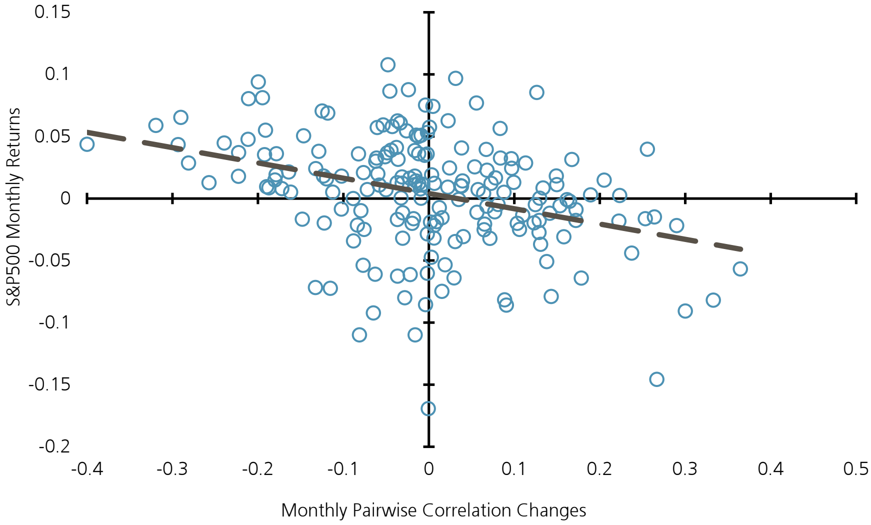
Extending the notion of volatility-targeting

We extend volatility-targeting strategies to the more generic concept of risk-adjusted strategies and try to identify more appropriate measures of risk that provide hedge against unexpected increases in pairwise correlation.

Theoretically maximum volatility

The theoretically maximum volatility of a portfolio is mathematically equal to the weighted sum of volatilities of the portfolio constituents. In other words, it is effectively equal to the volatility of the portfolio if all pairwise correlations were identically equal to one. We find that risk-adjusted strategies using this quantity as a measure of risk, not only deliver relatively larger risk-adjusted returns than constant-volatility strategies, but more importantly they achieve so with almost half the turnover. The patterns are pervasive across all universes of stocks that we test.

Figure 1: Monthly S&P500 Returns versus Pairwise Correlation Changes



Source: UBS Quantitative Research. Scatterplot between monthly returns of the S&P500 and monthly changes in the weighted pairwise correlation of S&P500 constituents. Sample period is May 1997 to June 2013.

Introduction

This research note aims to extend previous work on volatility-targeting strategies after thoroughly evaluating the effects of pairwise correlation on portfolio performance.

Revisiting volatility-targeting strategies

Volatility-targeting strategies, also known as constant-volatility strategies (*CVOL* henceforth) have been around for quite a while. They have indeed been studied in detail in our Global Quantitative Research Monograph, *Understanding Volatility Targeting* (4 October 2011). The main idea of these strategies lies in an appropriately designed adjustment for the amount of leverage employed on a particular asset, index or trading strategy. Along these lines, it is suggested to adjust the exposure to the asset of interest in such a way so that the ex-ante volatility becomes equal to a desirable target volatility level σ_{target}:

$$r_{t,t+1}^{CVOL} = \frac{\sigma_{target}}{\sigma_t} r_{t,t+1} \tag{1}$$

"Leverage effect"

Given the fact that asset returns are negatively correlated to the level of asset volatility (what is known as the "leverage effect"), this strategy achieves superior risk-adjusted returns when compared to the unadjusted asset, as it appropriately under-weights the exposure to the asset in high-volatility states and, therefore, safeguards against imminent drawdowns in the performance.

When correlations increase, diversification is scarce

In our Global Quantitative Research Monograph, *All Together Now* (19 August 2010), we also identified that in an environment of increasing pairwise correlations, diversification benefits diminish.

Building on the above evidence of (a) constant-volatility strategies and (b) increasing correlations affecting diversification potential, we first empirically show that stock market returns are not just negatively correlated with market volatility, but also that the relationship goes beyond the linear structure that the correlation coefficient can capture. In fact, we find this relationship to be concave, or in other words we find that in high-volatility states, stock market returns are more negative than what a linear return-volatility relationship would predict. The non-linearity is effectively due to the pairwise correlations that similarly increase during down markets (see Longin and Solnik 2001, Moskowitz 2003, Pollet and Wilson 2010).

Volatility-targeting strategies are under-hedged when correlations unexpectedly increase

It is therefore critical to alter the volatility-targeting strategies to adjust the exposure on the index in such a way that safeguards against not only imminent increases in volatility but also increases in pairwise correlation of the constituents.

Generalising the concept of risk-adjustment

We therefore extend volatility-targeting strategies to the more generic concept of risk-adjusted strategy (RAS) defined as:

$$r_{t,t+1}^{RAS} = \frac{\lambda}{RISK_t} r_{t,t+1} \tag{2}$$

where $RISK_t$ is an ex-ante estimate of risk of the asset/strategy available at time t and λ is a risk-control constant that is appropriately determined, so that the overall level of strategy volatility is within desirable levels.

In what follows, we aim to investigate different choices for the $RISK_t$ variable. In short, we find that the theoretically maximum volatility of an index, defined as the weighted sum of volatilities of the constituents, results in a strategy with relatively larger risk-adjusted returns, but more importantly with almost half the turnover of a constant-volatility strategy. Quite a remarkable improvement!

Empirical Patterns

Diversification Dynamics

We start this research note by first looking into the time-series patterns of average correlation and therefore diversification potential. This analysis is conducted using a dataset of daily returns and index weights for the S&P500 constituents and the index itself over the period between May 1997 and June 2013.

Consider a portfolio or index of N securities with volatilities $\sigma_1, \sigma_2, \dots, \sigma_N$ and weights $w_1, w_2, \dots, w_N$. The portfolio volatility, σ_P, is given by:

$$\sigma_P^2 = \sum_{j=1}^{N} w_j^2 \sigma_j^2 + 2 \sum_{j=1}^{N} \sum_{i>j}^{N} w_i w_j \sigma_{i,j} \tag{3}$$

$$= \sum_{j=1}^{N} w_j^2 \sigma_j^2 + 2 \sum_{j=1}^{N} \sum_{i>j}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j} \tag{4}$$

where $\sigma_{i,j}$ and $\rho_{i,j}$ denote the covariance and correlation between assets i and j.

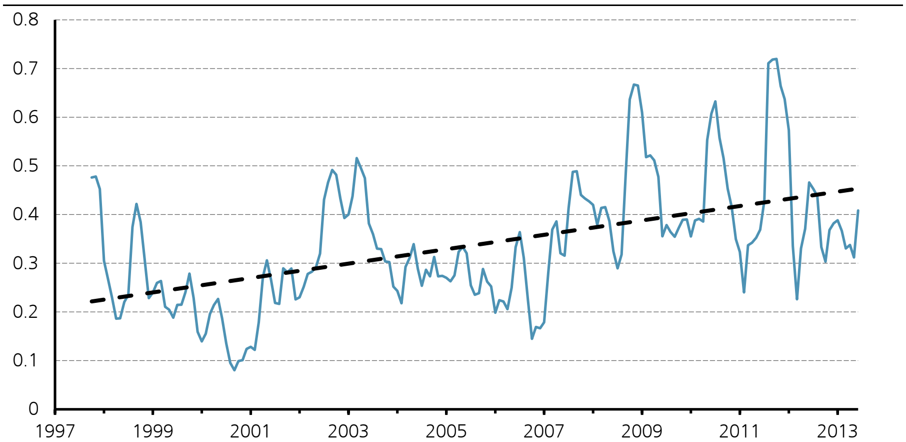
Sum of off-diagonal elements of correlation matrix

The standard measure of average pairwise correlation is the weighted sum of the off-diagonal elements of the correlation matrix:

$$\bar{\rho} = \frac{2 \sum_{j=1}^{N} \sum_{i>j}^{N} w_i w_j \rho_{i,j}}{1 - \sum_{j=1}^{N} w_j^2} \tag{5}$$

Using this definition, we calculate the market-cap weighted pairwise realised correlation of the S&P500 constituents at the end of each month using a 3-month rolling window of estimation[1]. The time-series of correlation is shown in Figure 2.

Figure 2: Average Pairwise Correlation of S&P500 Constituents



Source: UBS Quantitative Research. Monthly estimates of weighted pairwise correlation of the S&P500 constituents over 3-month windows of daily returns.

Average pairwise correlation is far from constant over time

Evidently, the average pairwise correlation of the S&P500 constituents is far from constant. Over the last 15 years, it has been fluctuating between low levels of around 0.1 and high levels of 0.7. This empirical observation is of paramount importance, because the main objective of portfolio construction is diversification; and it is obvious that when correlations increase, the diversification benefits

[1] Using 1-month or 6-month windows does not affect the qualitative features of the analysis. Expectedly, smaller windows give more noisy estimates, whereas longer windows give smoother but less responsive estimates.

diminish. More importantly, Figure 2 confirms the well documented empirical pattern of dramatic increase in the level of average correlation after 2000 and especially after 2007, which however has recently dropped from the peak of over 70% in September-October 2011 to levels around 30%-40% (still, however, above the historical average).[2]

Diversification Ratio

An alternative measure of diversification in a universe of assets is the Choueifaty and Coignard's (2008) Diversification Ratio (*DR*), which is defined as the ratio between the average volatility of portfolio constituents and the portfolio volatility:

$$DR = \frac{\sum_{j=1}^{N} w_j \sigma_j}{\sigma_P} \qquad (6)$$

Interestingly, the average volatility of portfolio constituents, which appears in the nominator, is effectively the maximum portfolio volatility for a given set of weights, as it can be easily deduced from equation (4) when $\rho_{i,j} = 1$ for every i and j:

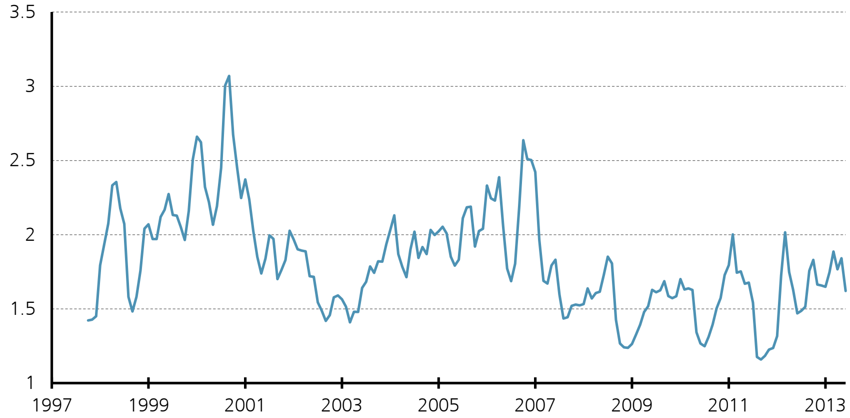
$$\sigma_{P,max}^2 = \sum_{j=1}^{N} w_j^2 \sigma_j^2 + 2 \sum_{j=1}^{N} \sum_{i>j}^{N} w_i w_j \sigma_i \sigma_j \cdot 1 = \left[\sum_{j=1}^{N} w_j \sigma_j \right]^2$$
$$\Rightarrow \sigma_{P,max} = \sum_{j=1}^{N} w_j \sigma_j \qquad (7)$$

Along these lines, the Diversification Ratio can be expressed as the ratio between the theoretically maximum volatility and the actual one:

$$DR = \frac{\sigma_{P,max}}{\sigma_P} \qquad (8)$$

Hence, a universe of perfectly correlated assets behaves effectively as a single asset and therefore minimises the *DR* to one. Figure 3 presents monthly estimates of the Diversification ratio calculated using a 3-month rolling window.

Figure 3: Diversification Ratio



Source: UBS Quantitative Research. Monthly estimates of the Diversification Ratio of the S&P500 universe over 3-month windows.

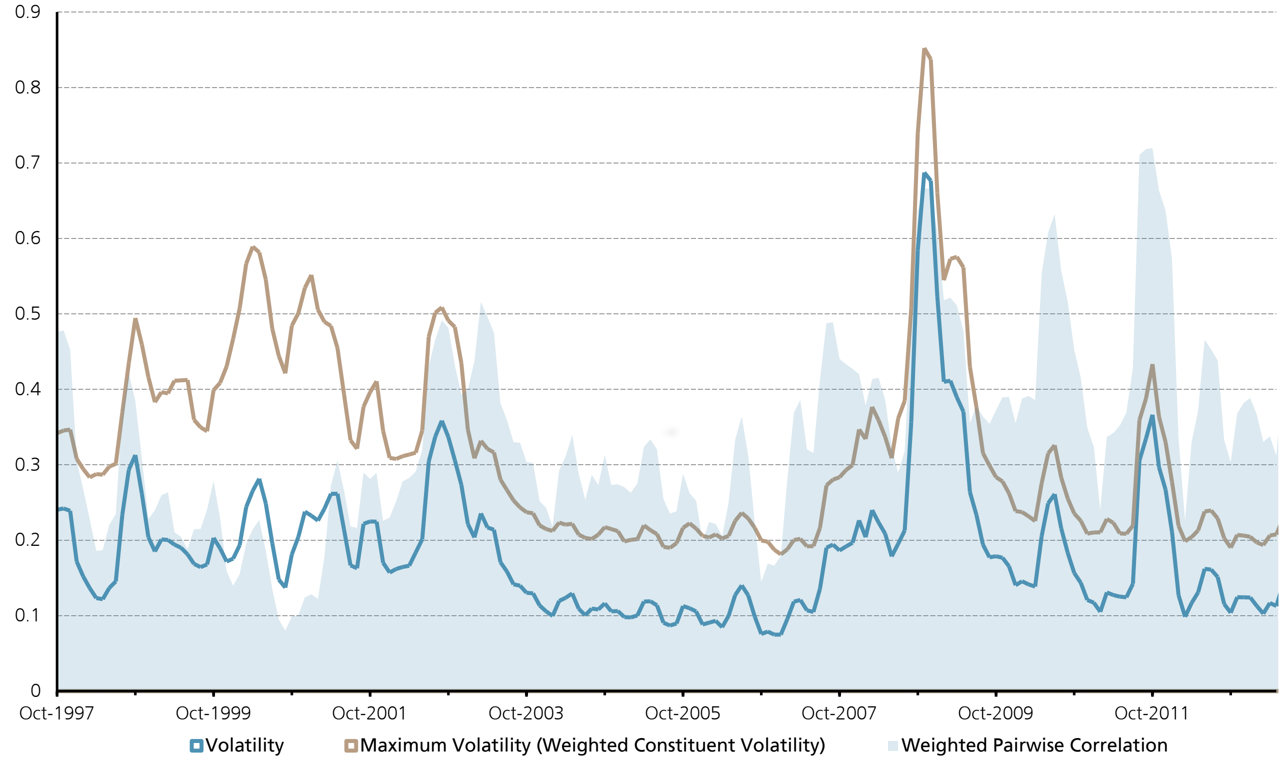
The evidence of recent increase in average pairwise correlation lends support to diversification benefits being currently close to the absolute minimum of 1.

[2] This empirical pattern of increasing correlations has already been identified in our Global Quantitative Research Monograph, *All Together Now* (19 August 2010). However, since the publication of that note and over the last 18 months, a very significant drop in the level of average correlation has taken place.

Pairwise Correlation and Portfolio Volatility

Portfolio volatility is a function of pairwise correlations (see equation (4)). It is plausible to argue that changes in portfolio volatility are contemporaneous to changes in pairwise correlations of the constituents. The true relationship however is more subtle, since portfolio volatility can still increase even when correlations remain constant, if constituents' volatilities increase. Figure 4 presents the 3-month S&P500 volatility along with its upper bound (by equation (7) the weighted volatility of the constituents) and the weighted correlation of its constituents.

Figure 4: S&P500 3-month Volatility, Maximum Volatility and Weighted Pairwise Correlation



Source: UBS Quantitative Research. Monthly estimates (using 3-month windows of daily returns) for the S&P500 Volatility, its upper bound, calculated as the weighted volatility of the constituents and of the weighted pairwise correlation of the constituents. The volatility estimates are quoted in annual terms.

It is clear that the index volatility increases when the average volatility of constituents increases and of course the former is always lower than the latter due to the implied diversification benefit. The spread between the two measures is a function of the pairwise correlation. In periods of low average pairwise correlation, index volatility is significantly lower than its maximum upper bound, for instance during the period between 1999 and 2001 (from Figure 3, this is indeed the period when the *DR* reaches its maximum values). Conversely, when average pairwise correlation peaks, portfolio volatility approaches its maximum upper bound and both record local maxima. However, these local maxima do not have the same level; the three most recent correlation maxima at the end of 2008, mid-2010 and at the end of 2011 are of similar magnitude (approximately 60%-70%), but the respective volatility peaks cover a much broader range (25% to 70%).

Studying the dynamics of a tri-party relationship:

(1) index volatility

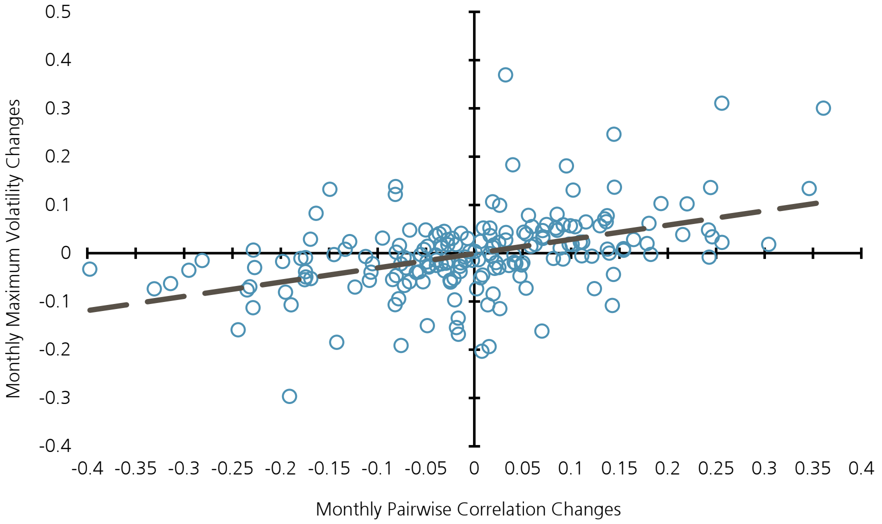
(2) maximum volatility

(3) average correlation

Apparently, even though portfolio volatility and pairwise correlation seem to co-move in the long-run, there are periods where changes in correlation are more extreme and decouple from changes in average volatility.

Finally, in order to uncover further aspects of this tri-party relationship (index volatility - maximum volatility - average correlation), Figure 5 presents a scatterplot of monthly changes of the maximum volatility and the average correlation. The positive correlation of the two variables is 44.5% (as reported later in Figure 9) and is easily deduced by the upward-sloping linear least-squares fit.

Figure 5: Monthly S&P500 Maximum Volatility vs. Pairwise Correlation Changes



Source: UBS Quantitative Research. Scatterplot between monthly changes in S&P500 theoretically maximum volatility and monthly changes in the weighted pairwise correlation of S&P500 constituents.

To sum up, the dynamics between index volatility, theoretically maximum volatility and pairwise average correlation are important determinants of the diversification benefits available in the stock market. The next step in our analysis aims to establish a relationship between equity index returns and the above variables.

Leverage Effects and Correlation Risk

What stats should an investor hedge against?

From an investment perspective, it is important to relate index/portfolio volatility and pairwise correlation to performance. As we will see later on in this research note, volatility-targeting strategies aim to timely adjust the leverage on an index in order to safeguard against high-volatility states. However, is it high-index-volatility states that an investor should hedge against? Or maybe high-average-volatility states (i.e. high-maximum-volatility states)? Or probably high-correlation states?

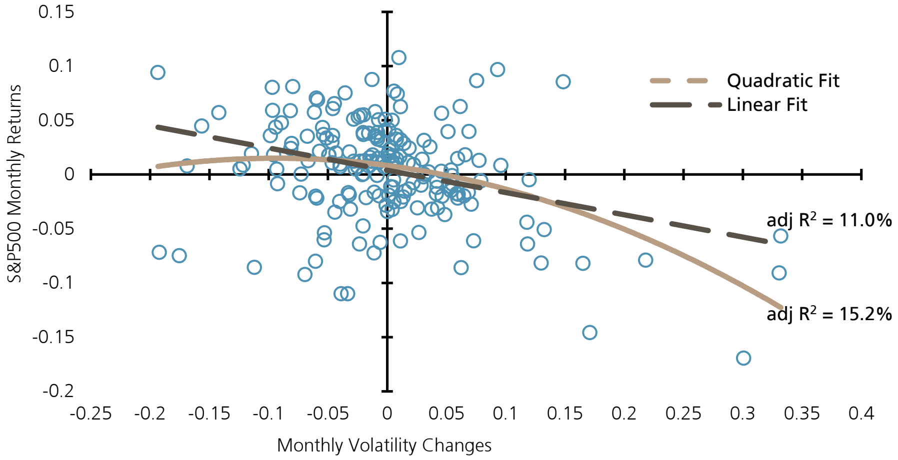
Index return-volatility relationship is not linear;

➔ it is concave

Figure 6 presents a scatterplot between monthly S&P500 returns and contemporaneous monthly changes in index volatility. The negative correlation between the two variables, widely known as the "leverage effect", is prevalent. Empirically, it is well-documented that high-volatility states are characterised by negative average returns and vice versa[3]. However, the relationship is not linear. A quadratic OLS fit significantly increases the adjusted R^2 to 15.2% (11.4% of the linear fit), giving rise to a concave relationship between returns and volatility. In other words, *when index volatility increases, stock market returns are more negative than what a linear relationship would predict*.

[3] We should be careful at this stage not to make any causality arguments. In fact, it can be theoretically argued that there exists a feedback effect between stock returns and volatility. When volatility increases, investors demand larger expected returns as compensation for the increased amount of risk and therefore prices fall for the expected returns to increase. On the other hand, when prices fall, the market value of the firms fall and, therefore, leverage, expressed as the ratio of debt-to-equity increases, hence aggregate risk, as measured by volatility, similarly increases. For this latter explanation, the negative correlation between stock market returns and volatility is known as the "*leverage effect*".

Figure 6: Monthly S&P500 Returns vs. Volatility Changes – "Leverage Effect"

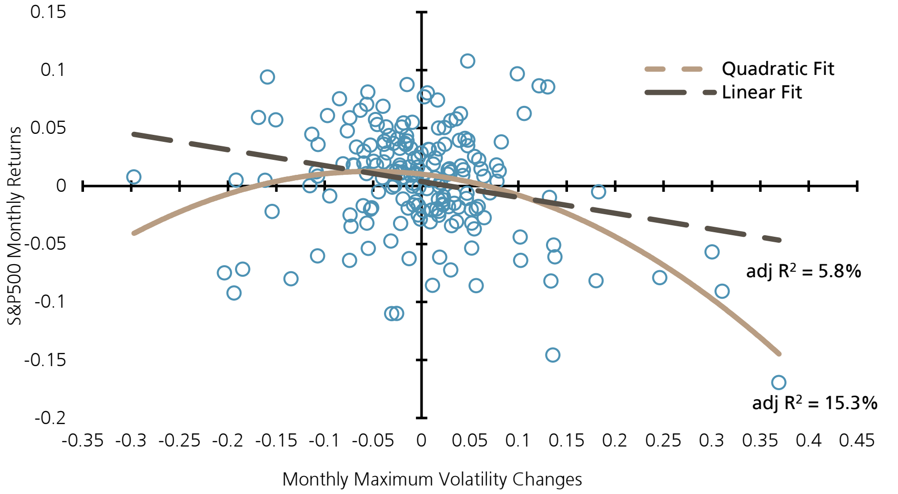


Source: UBS Quantitative Research. Scatterplot between monthly returns of the S&P500 and monthly changes in the S&P500 volatility.

The non-linearity effect is even stronger when we study the relationship between monthly S&P500 returns and monthly changes in the theoretically maximum volatility of the index as shown in Figure 7. The linear fit now has an adjusted R^2 of just 5.8% that is almost tripled to 15.32% for a quadratic fit.

The documented non-linearity second-order effects are most probably due to pairwise correlations that increase during bad times, hence causing an additional drop in asset prices. Academic literature justifies the existence of a correlation risk premium.[4] Buraschi, Kosowski and Trojani (2013) in their hedge fund study nicely describe the periods of increased average correlation as the times "*when there is no place to hide*"; in other words, when market volatility spikes, there is an additional price downward force coming from the increase in pairwise correlations that further hits an investor.

Figure 7: Monthly S&P500 Returns vs. Maximum Volatility Changes



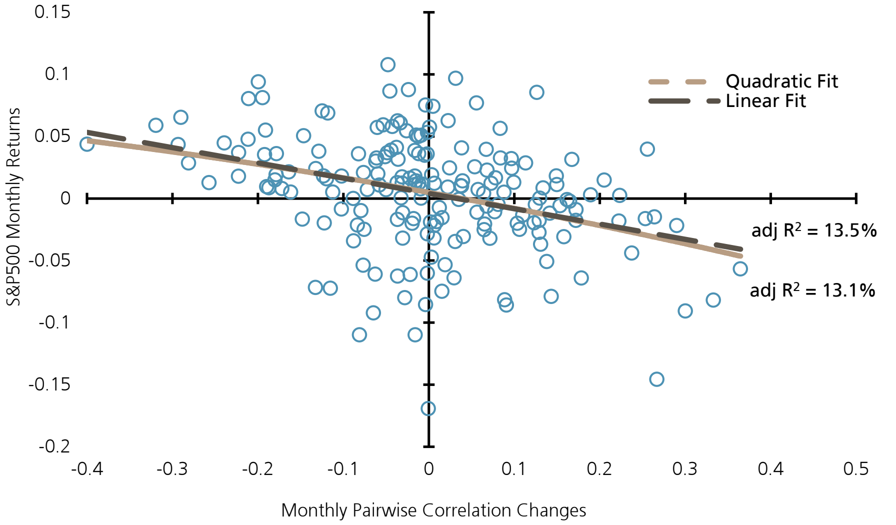
Source: Scatterplot between monthly returns of the S&P500 and monthly changes in the theoretically maximum volatility of the index (equal to the weighted sum of volatilities of the constituents).

Contrary to the non-linear relationship between market returns and volatility changes, the relationship between market returns and average correlation has no

[4] Indicatively, see Krishnan Petkova and Ritchken (2009), Driessen, Maenhout and Vilkov (2009, 2013), Buraschi, Trojani and Vedolin (2013) and references therein.

higher-order effects. Figure 8 shows that a quadratic term does not have explanatory power (in fact the adjusted R^2 drops). The correlation between monthly index returns and monthly changes in average correlation is -34.6% (see Figure 9) indicating larger degree of stock co-movement during down markets. It is exactly these states that a well-designed risk management scheme should address.

Figure 8: Monthly S&P500 Returns vs. Pairwise Correlation Changes



Source: UBS Quantitative Research. Scatterplot between monthly returns of the S&P500 and monthly changes in the weighted pairwise correlation of S&P500 constituents.

Figure 9: Correlation Matrix of Monthly Changes in 1-month Estimates

S&P500 Universe May 1997 – June 2013	**Returns**	**Index Volatility**	**Maximum Index Volatility**	**Average Pairwise Correlation**
Returns	1	-33.8%	-25.0%	-37.4%
Index Volatility		1	90.5%	68.6%
Maximum Index Volatility			1	44.5%
Average Pairwise Correlation				1

Source: UBS Quantitative Research. Correlation matrix between returns of the S&P500 and monthly changes in the index volatility, in the theoretically maximum volatility and in the weighted pairwise correlation of the index.

Summary of empirical patterns

Following the above documentation of empirical patterns, we next summarise our main findings that motivate the next sections of the research note:

- Portfolio volatility is always bounded above by the average constituent volatility.
- The distance between portfolio volatility and maximum portfolio volatility is a function of the weighted pairwise correlation.
- Correlation has been on average increasing after 2007, leading to diminishing diversification benefits.
- Increases in correlation tend to occur when volatilities increase. Hence, diversification becomes scarce, exactly when it is particularly needed.
- Portfolio returns are negatively correlated with portfolio volatility (or maximum volatility), but the relationship is not linear; it is concave. When volatility increases, portfolio loses more than what a linear relationship would predict.
- Portfolio returns are negatively correlated with average correlation and the relationship appears to be linear.

Risk Targeting, but Which Risk?

The essence of risk targeting lies in an appropriately designed adjustment for the amount of leverage employed on a particular asset or trading strategy. Given the empirical evidence of persistency of various risk measures, over- or under-weighting an asset/strategy can potentially safeguard against imminent drawdowns in the performance. Along these lines, we can illustratively represent such a risk-adjusted strategy (RAS) with the following equation:

$$r_{t,t+1}^{RAS} = \frac{\lambda}{RISK_t} r_{t,t+1} \tag{9}$$

where $RISK_t$ is an ex-ante estimate of some measure of risk of the asset/strategy available at time t and λ is a just a normalisation constant, so that the overall level of strategy volatility is within desirable levels. This strategy bears an implicit market timing mechanism as the leverage term $\frac{\lambda}{RISK_t}$ appropriately increases/decreases the exposure in low/high risk states and can therefore achieve a relatively stable risk profile. Obviously, the key research question is what measure to use as an estimate of "risk". In what follows, we suggest various choices that are both theoretically supported and empirically tested.

Constant-volatility strategy

Volatility is obviously the first and most natural candidate. In our Global Quantitative Research Monograph, *Understanding Volatility Targeting* (4 October 2011), we construct volatility-targeting strategies[5], also known as constant-volatility (*CVOL*) strategies, by scaling the position with the ratio of a target volatility level and an estimate of its realised volatility[6]:

$$r_{t,t+1}^{CVOL} = \frac{\sigma_{target}}{\sigma_{P,t}} r_{t,t+1} \tag{10}$$

As long as the ex-post realised volatility of the portfolio remains close to the ex-ante volatility estimate $\sigma_{P,t}$ (which is not an unrealistic assumption, given the large persistence of the volatility process), the *CVOL* achieves, on average, an ex-post constant volatility equal to the chosen level of target volatility (see Appendix A).

Identifying the components of constant-volatility strategies:

➔ It's merely covariances

The index/portfolio volatility consists of two main components, namely the constituent volatilities and the pairwise covariances. Following from equation (3), we can illustratively write:

$$\sigma_{P,t}^2 = \underbrace{\sum_{j=1}^{N} w_j^2 \sigma_{j,t}^2}_{VARS_t} + \underbrace{2\sum_{j=1}^{N}\sum_{i>j}^{N} w_i w_j \sigma_{i,j,t}}_{COVS_t} \tag{11}$$

Hence, the leverage factor that is proportional to the reciprocal of index volatility implies an adjustment with respect to constituent variances and covariances:

$$\frac{1}{\sigma_{P,t}} = \frac{1}{\sqrt{VARS_t + COVS_t}} \tag{12}$$

5 See Barroso and Santa-Clara (2013) and Daniel and Moskowitz (2013) for other applications of such strategies.

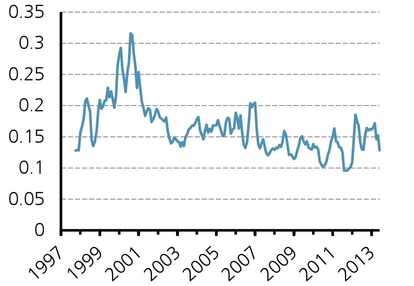
6 More recently, in our Quant Keys issue, *Beyond Volatility Targeting* (18 June 2012), we generalise the concept of targeting a constant level of volatility by constructing strategies that have a weight on the index proportional to a general power γ of the ex-ante volatility and find that the Sharpe ratio of the strategy is maximised when dividing by a factor that is closer to the variance (i.e. $\gamma = 2$) instead of the volatility ($\gamma = 1$).

Measuring the relative effects of variances and covariances is critical in our deeper understanding of *CVOL* strategies. Figure 10 presents the ratio $\sqrt{VARS_t}/\sigma_{P,t}$ for S&P500. Clearly, constituent variances account for a smaller part of portfolio volatility, or in other words *CVOL* strategies effectively control for covariances between portfolio constituents. In fact, using the Taylor expansion of the function $1/\sqrt{x+y}$ (see Appendix B), we can deduce the following first-order approximation:

$$\frac{1}{\sigma_{P,t}} = \frac{1}{\sqrt{VARS_t + COVS_t}} \approx \frac{1}{\sqrt{COVS_t}}, \quad \text{if} \quad VARS_t < COVS_t \tag{13}$$

We can therefore conjecture that the *CVOL* strategy is almost the same as adjusting for pairwise covariances/correlations (we name this strategy *COVS*), under the assumption that constituent variances are considerably smaller in value than the pairwise covariances. We empirically test this in the following section.

Figure 10: Ratio $\frac{\sqrt{VARS_t}}{\sigma_{P,t}}$ of S&P500



Source: UBS Quantitative Research. Proportion of S&P500 Volatility due to Constituent Volatilities, estimated using 3-month windows.

Adjusting by the index volatility, or equivalently, by the weighted sum of covariances is, in effect, a risk-adjustment that is highly dependent on the contemporaneous level of pairwise correlation. However, when correlation unexpectedly increases during the holding period, the portfolio would be under-hedged. Given that such increases are often accompanied by negative market returns (see Figure 8), the *CVOL* and *COVS* strategies are likely to suffer excessive losses due to the under-hedging argument.

Suggestion: Scale by the theoretically maximum volatility (when all correlations are one)

In an effort to account for the "worst-case" scenario of correlations peaking unexpectedly to one over the holding period, we argue that a more plausible risk adjustment is to scale the index holding by the theoretically maximum volatility, $\sigma_{P,t,max}$ (we name this strategy *MVOL*), that is derived as the weighted sum of constituent volatilities as shown in equation (7). It is well understood that such an adjustment would definitely lead to over-hedging, but is theoretically expected to safeguard against unexpected changes in the correlation dynamics of the portfolio. On the contrary, such an adjustment is not expected to have great effects when pairwise correlations stay the same but individual volatilities increase; this however is a more general limitation for all strategies studied in this note.

Suggestion: Scale by the downside volatility

Finally, the last strategy that we aim to empirically test controls for the so-called *downside* volatility that is defined as the average of squared daily *negative* returns over a certain reference period, usually a month, a quarter etc.:

$$\sigma_{P,t,down} = \frac{\sum_{i=1}^{M} r_i^2 I_{[r_i<0]}}{\sum_{i=1}^{M} I_{[r_i<0]}} \tag{14}$$

where M denotes the number of daily portfolio returns, r_i, over the reference period. We name this strategy *DVOL* for obvious reasons. Downside volatility is numerically -in general- close to the overall volatility, except for periods when the return distribution becomes largely negatively skewed. Hence, the argument that downside volatility can more appropriately capture the dynamics of left-tail risk is plausible.

Strategy Performance Evaluation

Following from the above section, Figure 11 lists the handful of risk-targeting strategies, whose performance we next evaluate.

Figure 11: Risk-Adjusted Strategies

Strategy	Description	Candidates for $RISK_t$
INDEX	Unweighted	1
CVOL	Volatility	$\sigma_{P,t}$
COVS	Weighted Covariances (Correlations)	$\sqrt{2\sum_{j=1}^{N}\sum_{i>j}^{N} w_i w_j \sigma_{i,t}\sigma_{j,t}\rho_{i,j,t}}$
DVOL	Downside Volatility	$\sigma_{P,t,down}$
MVOL	Maximum Volatility (Weighted Volatilities)	$\sigma_{P,t,max}$ $\left(\text{i.e. } \sum_{j=1}^{N} w_j \sigma_{j,t}\right)$

Source: UBS Quantitative Research

S&P500 Universe

Choosing the constant λ

Using data on the S&P500 universe for the period between May 1996 and June 2013, we evaluate the performance of the risk-adjusted strategies of equation (9) where the leverage factor $\frac{\lambda}{RISK_t}$ is defined for all the different risk measures listed in Figure 11. As already mentioned, the constant λ is effectively chosen in order to keep the ex-post volatility of the strategy within desirable levels. For the CVOL strategy, λ is trivially, by equation (10), the level of target volatility. We choose this to be 20%. Since *COVS* and *DVOL* values are expected to be numerically close to the total volatility, we use the same value of 20% for the respective strategies.

However, the theoretically maximum volatility is by definition greater than the portfolio's realised volatility (see Figure 4). The Diversification Ratio, defined as their ratio as shown in equation (8), can give an indication of how larger the maximum volatility is (see Figure 3). The average value of *DR* over the entire sample period is 1.82. Along these lines, and in order to keep things simple, we propose as a rule of thumb to choose a conservative $\lambda_{MVOL} = 1.5 \cdot \lambda_{CVOL}$ (effectively choosing a target for the *maximum* volatility that is 50% larger than the target level of a *CVOL* strategy). Such an adjustment will hopefully result in the *MVOL* strategy having similar ex-post volatility to the rest of the risk-adjusted strategies.

In what follows, we assume monthly rebalancing and make use of 1-month, 3-month and 6-month windows for the estimation of volatilities and correlations. It is expected that smaller estimation windows result in estimates that are more responsive to abrupt changes, hence delivering larger risk-adjusted returns. Our results do confirm this expectation.

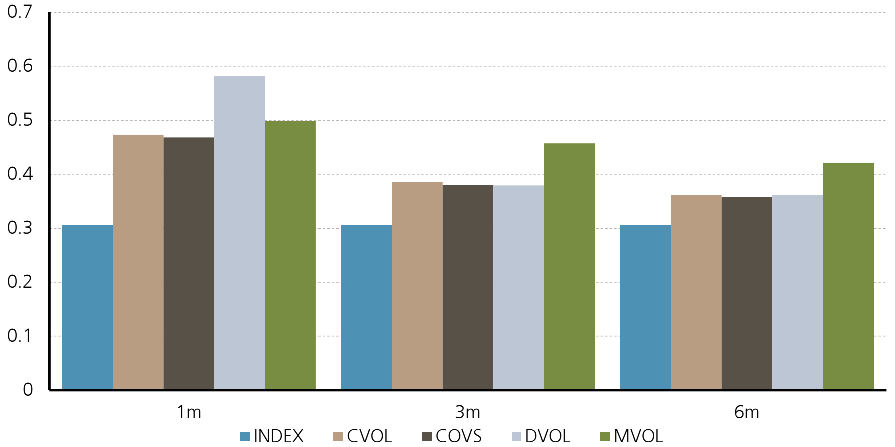
Main findings

Figure 12 presents annualised risk-adjusted returns (defined as average return/volatility) for the strategies that we test. The unweighted index exhibits a Sharpe ratio of 0.31 over the sample period. The main key takeaways are summarised below:

- Without *any* exceptions, *all* risk-targeting strategies deliver superior performance compared to the unweighted strategy.

- The effects are stronger for 1-month volatility/correlation estimates.
- As expected, *CVOL* and *COVS* strategies are almost identical. In other words, adjusting by portfolio volatility is almost as effective as adjusting for the sum of weighted covariances.
- With the exception of the 1-month results, *DVOL* strategy exhibits similar Sharpe ratios to *CVOL* and *COVS* strategies. There indeed seems to exists an improvement, in comparison to *CVOL*, for the strategy that uses 1-month estimates, but this (as we see later on) is not stable across universes and comes at the expense of increased turnover.
- Unanimously, the strategy that adjusts for the theoretically maximum portfolio volatility, *MVOL*, exhibits consistently larger Sharpe ratio compared to the unweighted index and also compared to the widely used *CVOL* strategy.

Figure 12: Annualised Sharpe Ratios of S&P500 Risk-Adjusted Strategies



Source: UBS Quantitative Research. Annualised Sharpe ratios (measured as the ratio of mean return over volatility) for various risk-adjusted strategies applied on S&P500 using 1-month, 3-month and 6-month windows for the estimation of volatilities and correlations.

Further performance statistics for strategies with a 3-month estimation window are presented in Figure 13. The ex-post volatility levels of all strategies are fairly similar, which is an indication of good choice of the normalisation constant λ. The most impressive statistic, other than the Sharpe ratio that was already commented upon, is the annualised turnover estimate. *CVOL* and *COVS* strategies exhibit a turnover of about 68%, *DVOL* strategy exhibits the largest turnover of 85%, but the *MVOL* strategy, after exhibiting the largest Sharpe ratio, effectively halves the turnover of the *CVOL* strategy, delivering even greater after transaction costs performance.

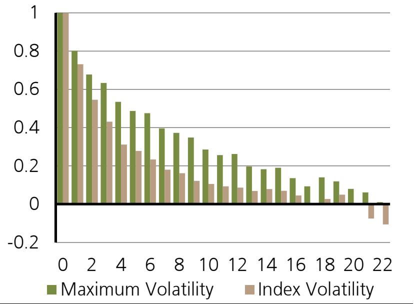
Figure 13: S&P500 Risk-Adjusted Strategies, using 3-month estimation window

	INDEX	CVOL	COVS	DVOL	MVOL
Risk control λ	N/A	20%	20%	20%	30%
Mean	4.90%	6.45%	6.47%	6.24%	6.50%
Volatility	16.01%	16.76%	17.04%	16.46%	14.22%
Sharpe Ratio	0.31	0.39	0.38	0.38	**0.46**
Two-way Turnover	0.00	0.68	0.69	0.85	**0.38**
AC(1) of 1-month $RISK_t$	1.00	0.73	0.69	0.69	0.80

Source: UBS Quantitative Research. Annualised performance statistics for various risk-adjusted strategies applied on S&P500 using a 3-month window for the estimation of volatilities and correlations. AC(1) denotes the 1st order serial correlation of the different measure of risk used for each different strategy.

What is really the reason for the *MVOL* strategy to exhibit by far the lowest turnover? Since all risk-adjusted strategies adjust the leverage to the index based on an estimate of risk, denoted as $RISK_t$, the strategy with the most persistent measure of risk would effectively limit the amount of rebalancing, hence the turnover. The last row of Figure 13 reports the 1st order of serial correlation for each of the risk variables (estimated over 1-month horizons to avoid econometric issues due to overlap) of the risk-adjusted strategies that we test. It is obvious that the maximum volatility is the most persistent measure of risk with a serial correlation of 80% against 73% for the monthly index volatility. Moreover, Figure 14 presents the autocorrelation function for both measures and is easily deduced that maximum volatility, given its persistence, decays at a much slower pace compared to the index volatility (half-life of 6.2 months versus 3.7 months assuming exponential decay). This is exactly the reason why the turnover of *MVOL* constitutes a great improvement to the rest of the strategies.

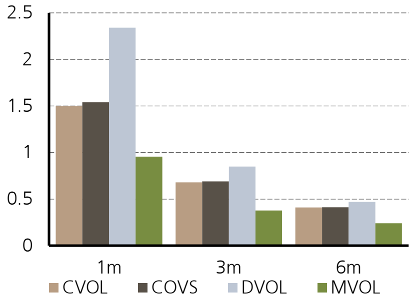
Figure 14: Autocorrelation of Risk



Source: UBS Quantitative Research. Autocorrelation plot for the S&P500 1-month volatility and the respective theoretically maximum volatility, estimated as the weighted sum of volatilities of the constituents.

Besides, it is expected that a larger estimation window for volatilities and correlations will result in lower turnover costs due to the slower-moving risk estimates. This is depicted in Figure 15. Notice, that the length of the estimation window does not affect our main finding whatsoever; the turnover of *MVOL* strategy is always almost half that of *CVOL* strategy. It must however be stressed that the overall reduction of turnover via using larger estimation windows should be balanced with the relative drop in the performance of the strategies, as the estimated risk measures become less responsive (see the drop in the Sharpe ratios in Figure 12).

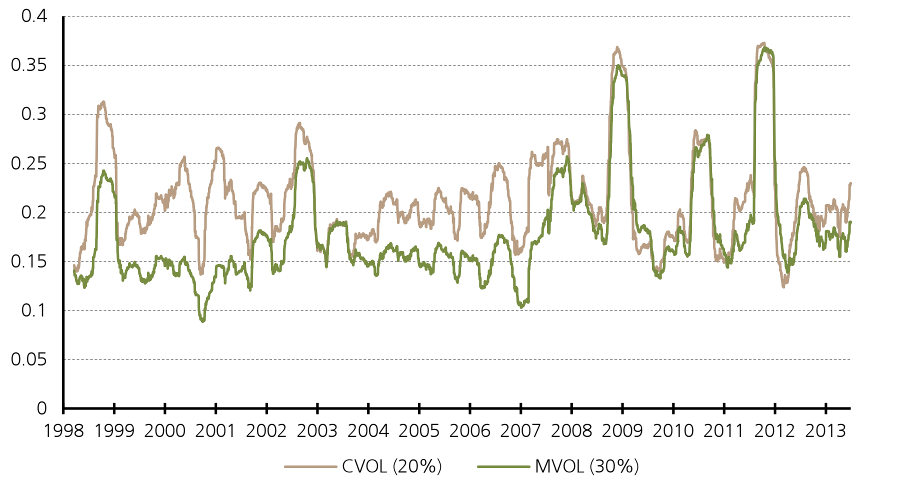
Figure 15: Turnover Estimates



Source: UBS Quantitative Research. Annualised two-way turnover estimates for various risk-adjusted strategies applied on S&P500 using 1-month, 3-month and 6-month windows for the estimation of volatilities and correlations.

As a final note at this stage, Figure 16 presents the running volatility of the *CVOL* and *MVOL* strategies estimated over a 100-day window, in order to gauge how effective the risk-adjustment has been for the two schemes of interest. As a reminder, the *CVOL* strategy is using a risk control value $\lambda_{CVOL} = 20\%$, whereas the *MVOL* strategy uses a 50% larger value, $\lambda_{MVOL} = 30\%$.

Figure 16: 100-day Running Volatility of *MVOL* and *CVOL* Strategies



Source: UBS Quantitative Research. Time-series of 100-day running volatility for the risk-adjusted strategies controlling for S&P500 volatility (*CVOL*) and the theoretically maximum volatility of the index (*DVOL*)..

By and large, the volatility profile of the risk-adjusted strategies exhibits a fluctuating, mean-reverting behaviour around the 20% level. In theory, the *CVOL* strategy should, by construction, result in a flat 20% volatility profile. However, this would imply continuous rebalancing, instead of the end-of-month rebalancing scheme that we employ[7]. Moreover, higher-frequency rebalancing would also dramatically increase the turnover of the strategies. If anything, Figure 16 does confirm that the risk-adjusted strategies can satisfactorily manage to keep the volatility within reasonable and acceptable bounds. Interestingly, the *MVOL* strategy, even though not specifically designed to achieve a *constant* level of volatility (controlling for the theoretically maximum volatility is not directly equivalent to targeting a certain level of volatility), it ex-post manages to do so.

An In-Sample Exercise

Adjust all strategies, so to have an ex-post volatility of 16%

Even though the risk-adjusted returns presented in the section above are not dependent on the constant λ, the mean return, volatility and other risk measures are dependent on the amount of leverage employed. In order to provide the fairest comparison between the available risk-adjustment schemes, we appropriately adjust the constant λ for each and every strategy, so that all strategies have an ex-post volatility equal to 16%, which coincides with the volatility of the unadjusted index. Figure 17 presents a number of statistics for these strategies including the maximum drawdown and the Calmar ratio, which is defined as the ratio between the mean return and the maximum drawdown.

Figure 17: S&P500 Risk-Adjusted Strategies, with 16% ex-post Volatility

	INDEX	CVOL	COVS	DVOL	MVOL
Risk control λ	N/A	19.1%	18.8%	19.5%	33.8%
Mean	4.90%	6.16%	6.08%	6.07%	7.33%
Volatility	16.01%	16.01%	16.01%	16.01%	16.02%
Skewness	-0.64	-0.71	-0.71	-0.66	**-0.57**
Kurtosis	3.90	3.81	3.82	3.63	**3.35**
Sharpe Ratio	0.31	0.39	0.38	0.38	**0.46**
Maximum Drawdown	52.56%	46.82%	47.22%	49.79%	**42.32%**
Calmar Ratio	0.07	0.11	0.10	0.10	**0.15**
Two-way Turnover	0.00	0.68	0.69	0.85	**0.38**

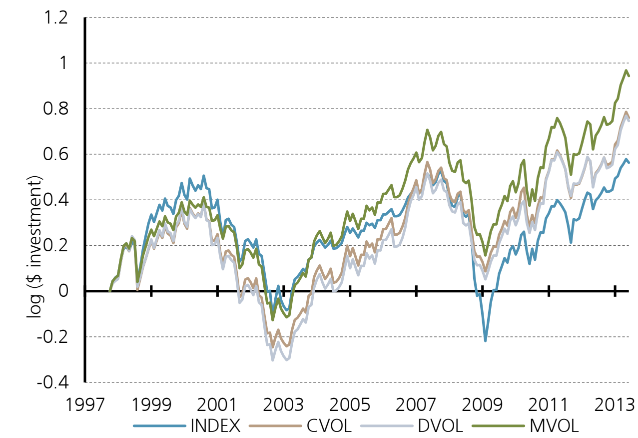
Source: UBS Quantitative Research. Annualised performance statistics for various risk-adjusted strategies applied on S&P500 that are appropriately adjusted in order to exhibit an ex-post volatility of 16%.

By any measure, *MVOL* dominates

Our purely out-of-sample results find strong support from this simple in-sample tweak. Not only does the *MVOL* strategy dominate the long-only passive index investing and the *CVOL* strategy having the largest Sharpe ratio with the lowest turnover, but also exhibits the lowest in absolute value negative skewness (surprisingly, all other risk-adjustment schemes -*CVOL, COVS, DVOL*- result in more negatively skewed distributions than the unadjusted strategy) and the lowest kurtosis. Naturally, *CVOL* exhibits the most conservative drawdown that, in turn, more than doubles the Calmar ratio of the unadjusted strategy. For completeness, Figure 18 and Figure 19 present the cumulative returns and the running drawdowns of the risk-adjustment strategies (we exclude *COVS* as it almost always coincides with *CVOL*).

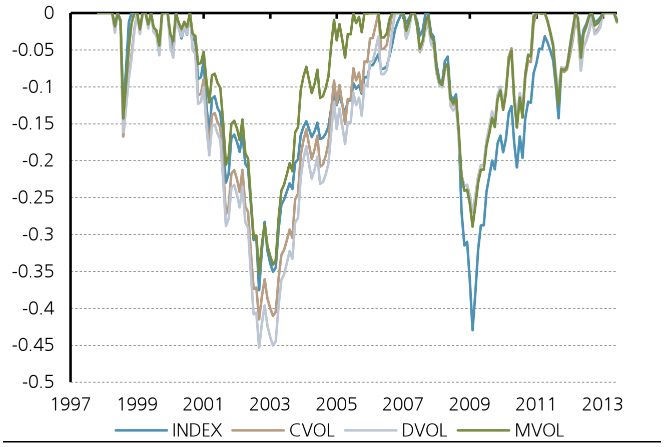
[7] In our Global Quantitative Research Monograph, *Understanding Volatility Targeting* (4 October 2011), we employ daily rebalancing and show that the running volatility of a constant-volatility strategy can track more closely the target volatility level. However, even the daily rebalancing cannot safeguard against some infrequent and pronounced short-term spikes in volatility, while at the same time the turnover of the strategy is dramatically increased.

Figure 18: Cumulative Returns



Source: UBS Quantitative Research. Cumulative returns for various risk-adjusted strategies applied on S&P500 that are appropriately adjusted in order to exhibit an ex-post volatility of 16%.

Figure 19: Drawdowns



Source: UBS Quantitative Research. Running drawdown estimates for various risk-adjusted strategies applied on S&P500 that are appropriately adjusted in order to exhibit an ex-post volatility of 16%.

As a final round of empirical evidence, Figure 20 presents the time-series of the leverage factor $\frac{\lambda}{RISK_t}$ for the *CVOL* and *MVOL* strategies in order to further illustrate where the benefit of reduced turnover stems from. Evidently, the fact that portfolio volatility is less persistent than the theoretically maximum volatility leads to more "volatile" weights, hence significantly larger turnover.

By all measures, *MVOL* dominates.

Figure 20: Leverage Factor (Index Weight) for *CVOL* and *MVOL* Strategies

Source:

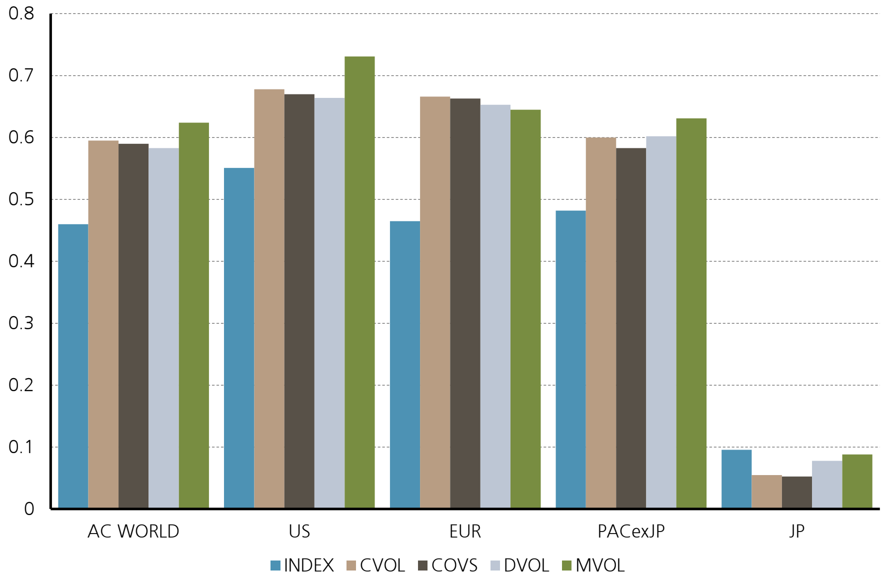
Empirical Evidence from Other Regions

Following the evidence from the S&P500 universe, we now turn to other broader universes and regions in order to corroborate our findings. We collect data for all countries covered by MSCI over the period between January 1996 and June 2013. We then form risk-adjusted strategies using all firms that are members of the constituent countries of MSCI AC World (45 countries), MSCI USA, MSCI Europe

(16 countries), MSCI Pacific excluding Japan[8] (4 countries) and MSCI Japan. All these indices are value-weighted and the returns are calculated in USD.

Similar to the analysis on the S&P500 universe, regarding the constant λ, we choose the value of 20% for all risk-adjusted strategies apart from *MVOL*, for which we use the value 30%. Figure 21 presents annualised Sharpe ratios for all regions. All volatility/correlation estimates have been calculated using 3-month windows.[9] With the exception of Europe (where all risk-adjusted strategies deliver very similar Sharpe ratios), the *MVOL* strategy continues to exhibit the best ex-post risk-adjusted returns.

Figure 21: Annualised Sharpe Ratios of MSCI Regions' Risk-Adjusted Strategies



Source: Annualised Sharpe ratios (measured as the ratio of mean return over volatility) for various risk-adjusted strategies applied on various MSCI universes using a 3-month window for the estimation of volatilities and correlations.

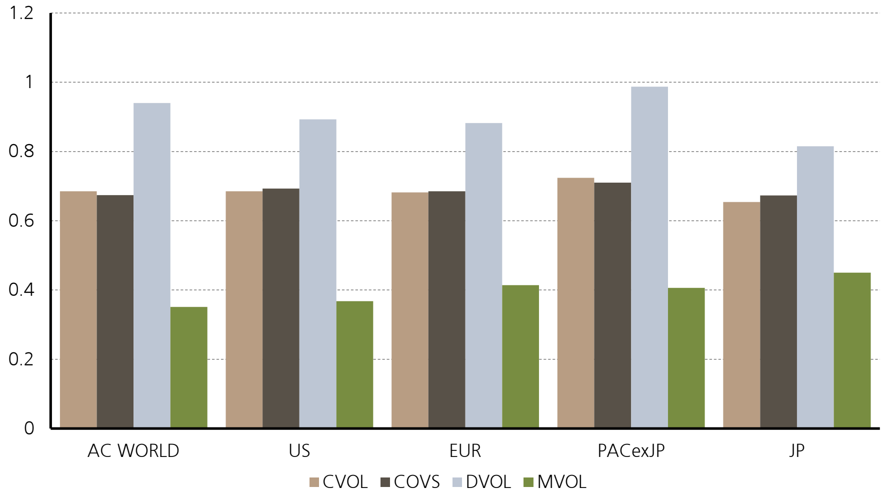
***MVOL* almost halves the turnover of the constant-volatility strategy**

Besides, the analysis of the S&P500 universe showed impressive improvement in the turnover of the risk-adjusted strategy when scaled by the theoretically maximum volatility. Figure 22 presents annualised two-way turnover estimates for all MSCI regions of interest. Once again, the empirical evidence is indisputable. Irrespective of the universe, the *MVOL* strategies reduce the turnover of the *CVOL* strategies by about 40-50%, whereas the *DVOL* strategies imply by far the largest amount of rebalancing.

[8] MSCI only publishes the "MSCI Pacific index" that includes Japan (along with the remaining four developed market countries Australia, Honk Kong, New Zealand and Singapore). We decide to exclude Japan from this universe and study it separately.

[9] Our results remain qualitatively the same irrespective of the window of estimation of these quantities. Similar to the S&P500 analysis, using 1-month, 3-month or 6-month volatility/correlation estimates does not change the relative ranking of the strategies. The figures are available upon request.

Figure 22: Annualised Turnover of MSCI Regions' Risk-Adjusted Strategies



Source: UBS Quantitative Research. Annualised two-way turnover estimates for various risk-adjusted strategies applied on various MSCI universes using a 3-month window for the estimation of volatilities and correlations.

Conclusion

Volatility-targeting strategies have been successful in timing the market…

Volatility-targeting strategies have been shown to empirically deliver better risk-adjusted returns, because they reduce the exposure to an asset or trading strategy when volatility peaks and therefore constitute an appropriate market-timing mechanism that provides a hedge against imminent drawdowns.

In fact, these strategies would in theory achieve a constant volatility profile if realised volatility over the holding period was equal to the ex-ante estimate of historical volatility that is used to weight the portfolio. However, in practice, volatility is time-varying and it is on average increasing during down-markets. The correlation between volatility and stock market returns is apparently negative, but the relationship between the two variables is not linear. In fact, it's concave, meaning that when volatility spikes, market returns are more negative than what a linear relationship would predict.

…but what happens when pairwise correlations increase?

Who takes the blame for this non-linearity? It is the pairwise correlations of the portfolio constituents, which tend to increase during high-volatility down-market states. A constant-volatility strategy is apparently very sensitive to an unexpected increase in pairwise correlations and, even though "hedged", it appears to be under-hedged in such states.

We control for the "worst-case" scenario; correlations increasing to one

For that reason, we suggest an improved risk-adjusted strategy that scales the exposure by the theoretically maximum volatility of the portfolio, which is calculated as the weighted sum of the constituent volatilities. This measure of risk assumes that all pairwise correlations are equal to one, hence accounting for the "worst case" scenario of correlations unexpectedly spiking.

The strategy improves risk-adjusted returns, with almost half the turnover!

Our empirical results show that this strategy, not only delivers similar, if not better, risk-adjusted returns compared to the constant-volatility strategy, but most importantly achieves so with almost half the turnover, hence exhibiting significantly greater net-of-transaction-costs performance. The results are pervasive across various regions and equity universes that we test over a relatively long period of 17 years.

APPENDIX

A: Conditional Variance of *CVOL*

The conditional variance of the *CVOL* strategy is:

$$var_t\left(r_{t,t+1}^{CVOL}\right) = var_t\left(\frac{\sigma_{target}}{\sigma_t} r_{t,t+1}\right) = \left(\frac{\sigma_{target}}{\sigma_t}\right)^2 var_t\left(r_{t,t+1}\right)$$

As long as the realised volatility of the index remains close to the ex-ante volatility estimate σ_t (which is not an unrealistic assumption, given the large persistence of the volatility process), the *CVOL* strategy achieves on average an ex-post constant volatility equal to the chosen level of target volatility σ_{target}.

B: Taylor Expanding $\mathbf{1}/\sqrt{\boldsymbol{x}+\boldsymbol{y}}$

The Taylor expansion of the function $f(x,y) = 1/\sqrt{x+y}$ around $x = 0$ is given by:

$$f(x,y) = \frac{1}{\sqrt{x+y}} = \frac{1}{\sqrt{y}} - \frac{1}{2\sqrt{y}} \cdot \frac{x}{y} + \frac{3}{8\sqrt{y}} \cdot \left(\frac{x}{y}\right)^2 - \frac{5}{16\sqrt{y}} \cdot \left(\frac{x}{y}\right)^3 + O(x^4)$$

and converges when $|x| < |y|$.

References

Barroso, P. & Santa-Clara, P. (2013) Momentum Has Its Moments, Available at SSRN: http://ssrn.com/abstract=2041429

Buraschi, A, Kosowski, R. & Trojani, F. (2013) When There is No Place to Hide - Correlation Risk and the Cross-Section of Hedge Fund Returns, Available at SSRN: http://ssrn.com/abstract=1639984

Buraschi, A., Trojani, F., & Vedolin, A. (2013). When Uncertainty Blows in the Orchard: Comovement and Equilibrium Volatility Risk Premia. *Journal of Finance*, forthcoming.

Choueifaty, Y., & Coignard, Y. (2008). Toward maximum diversification. *The Journal of Portfolio Management, 35*(1), 40-51.

Daniel, K. & Moskowitz T. (2013), Momentum Crashes, Available at: http://www.columbia.edu/~kd2371/papers/unpublished/mom_crash.pdf

Driessen, J., Maenhout, P. J., & Vilkov, G. (2009). The Price of Correlation Risk: Evidence from Equity Options. *Journal of Finance*, 64(3), 1377-1406.

Driessen, J., Maenhout, P. J., & Vilkov, G. (2013). Option-Implied Correlations and the Price of Correlation Risk. Available at SSRN: http://ssrn.com/abstract=2166829

Krishnan, C.N.V., Petkova, R., & Ritchken, P. (2009). Correlation Risk. *Journal of Empirical Finance*, 16(3), 353-367.

Longin, F., & Solnik, B. (2001). Extreme Correlation of International Equity Markets. *Journal of Finance*, 56(2), 649-676.

Moskowitz, T. J. (2003). An Analysis of Covariance Risk and Pricing Anomalies. *Review of Financial Studies*, 16(2), 417-457.

Pollet, J.M., & Wilson, M. (2010). Average Correlation and Stock Market Returns. *Journal of Financial Economics*, 96(3), 364-380.

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Sell	FSR is > 6% below the MRA.	10%	21%
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Source: UBS. Rating allocations are as of 30 June 2013.
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