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# Understanding volatility targeting strategies

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## ■ Why volatility targeting?

We analyse simple allocation strategies that consist of investing in a passive index and cash. The goal is to keep the volatility of the returns to the strategy at a constant target level. We argue that volatility targeting can be useful to control tracking errors, for risk budgeting and to achieve market neutrality. The resulting index lends itself naturally to being used as underlying for derivative securities.

## ■ The effect on the Sharpe ratio

Does volatility targeting affect the risk adjusted performance? We argue that in theory one should expect an improvement in the Sharpe Ratio, as the strategy is implicitly a market timing device.

## ■ Evidence from the backtest

In practice, for all the stock indices analysed in our backtest, the volatility targeting strategy improved on the Sharpe ratio of the passive index. This would also have been true for the Global Asset Allocation portfolio. In contrast, in the case of a long short value factor we found that volatility targeting would have got the timing remarkably wrong.

## ■ Implementing the strategy in practice

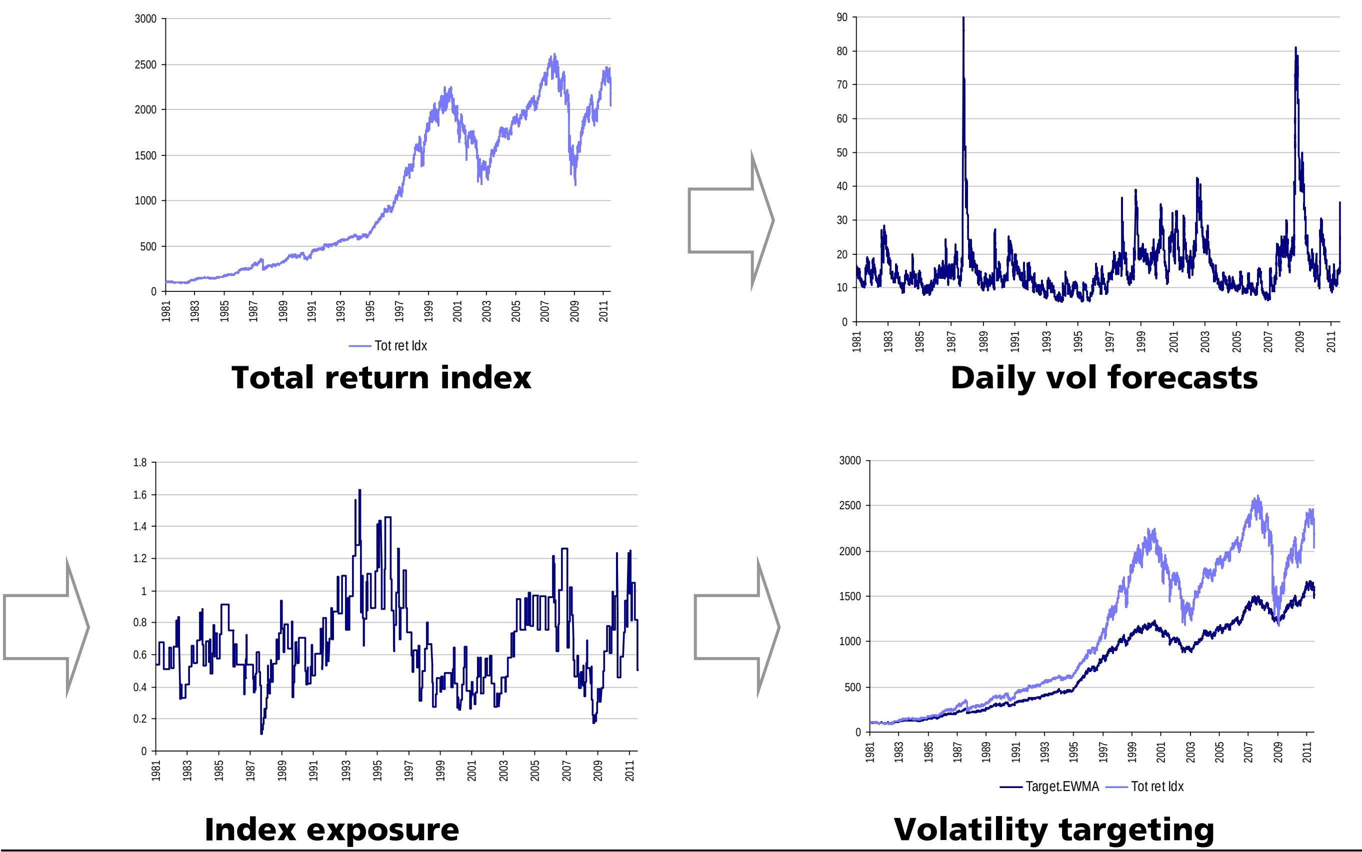
This report is an update of a document previously published on 14 September, 2011 and includes additional data on the effectiveness of targeting (page 7 onwards). We consider ways to curb turnover without sacrificing the improvement in performance. Among the volatility forecasts that we analysed, naïve estimators seem to do as good a job as model based ones.

# Introduction

We refer to a *volatility targeting strategy* (VTS) as a strategy that consists of changing allocation between cash and an index in order to keep the overall volatility stable at a target level. Chart 1 illustrates the concept with an example based on the S&P 500. At time *t* we generate a forecast of the volatility of the index for time *t+1*. We then adjust the exposure to the index itself so as to set the *ex ante* volatility of the strategy equal to the target. The index weight may be greater than one, resulting in a leveraged position. If the weight is between zero and one then we hold the remaining funds as cash.

Definition of volatility targeting strategy and example

Chart 1: Volatility targeting, the S&P 500 index




Source: UBS quant.

The motivation of our analysis is twofold.

First, it would be useful in a number of practical situations to have access to a strategy that provides exposure to a given factor with stable volatility. For example, the mandate of a fund manager is often described in terms of a target tracking error and we know that tracking errors are strongly impacted by market volatility. Risk budgeting across different strategies may be greatly simplified. In addition, it would be useful to hedge funds that aim to be market neutral. Another notable advantage of the VTS over a passive index is that it would be easier to write derivatives on it.

Useful to manage tracking error, budget risk, obtain market neutrality, design derivative securities

The second interesting feature of VTSs, which is the main focus of this report, is the risk adjusted return. It is natural to conjecture that if the expected return to the strategy is related to lagged volatility then volatility targeting can be used to enhance performance. Indeed, volatility targeting is implicitly a market timing strategy which takes more exposure to the index in times of low volatility and rotates into cash when volatility spikes.

It is also implicitly a market timing strategy...

Important examples of the relation between volatility and expected returns are:

...as there is empirical evidence that expected returns and volatility are related

1. The leverage effect and volatility feedback: expected returns to equity indices and stocks tend to be negatively impacted by upward movements in volatility.

2. The price momentum factor performs better in low volatility periods, as documented in our previous reports *Quant is not dead* (De Rossi, 7 December 2009) and *Momentum and volatility* (Jones, 24 March 2011).

To summarise, the main focus of our research is on alpha generation. By analysing both long only and long short portfolios, we show that in most cases the use of a VTS would have enhanced the Sharpe ratio of the passive index. We argue that the effect typically materialises over a relatively long period of time and it is most pronounced when volatility suddenly increases.

## Related work in academic literature

Several strands of academic literature are relevant to our analysis.

A few recent papers used an optimisation framework with time varying volatility: Fleming et al. (2001), Johannes, Polson, and Stroud (2011) and Basak and Chabakauri (2010). Some of the results provide the theoretical underpinning for the idea of choosing weights that are inversely proportional to volatility – it can be shown to be optimal under stochastic volatility in some special cases.

A brief survey of the related academic literature:

- Intertemporal asset allocation under time varying volatility

From the empirical point of view, the evidence on the feedback and leverage effect is key: Do expected returns depend on lagged volatility? A considerable amount of work has concentrated on the contemporaneous relation, starting with the ARCH in mean model of Engle et al. (1987). French, Schwert and Stambaugh (1987) and Campbell and Hentschel (1992) find evidence of a positive relation. In contrast, Glosten, Jagannathan and Runkle (1993) estimate a more general ARCH-M model and detect a negative intertemporal relation for the US stock market. Nelson (1991), Poon and Taylor (1992) and Koopman and Hol Uspensky (2002) find a weak negative relation, Assaf (2006) a positive one. Smith (2007) contains an extensive literature survey.

- Conflicting results about the relation between return and volatility

Bollerslev and Zhou (2006) try to make sense of the conflicting empirical evidence by using implied and realised volatility in their analysis. They show that the conclusion is unambiguous for *implied* volatility: Its contemporaneous correlation with return is positive. The contemporaneous relation between *realised* volatility and return depends instead on the structural parameters of the model.

Brandt and Kang (2010) made an interesting contribution to this vast literature. They used a stochastic volatility model in which both expected return and volatility are allowed to vary over time. This allows them to study the dynamic properties of the (conditional) Sharpe ratio, which appears to vary countercyclically.

A recent analysis of the leverage effect can be found in Carr and Wu (2010) who distinguish the three main components of the phenomenon and estimate their effects separately.

- Market neutrality and hedge fund returns

This paper is also related to the idea of variance neutrality of returns, i.e. for a strategy to be market neutral its volatility should not increase when market volatility does (Patton, 2009).

# Targeting volatility

## Measuring volatility

We use alternative estimators of volatility:

Volatility is not directly observed and must therefore be estimated. In our backtests we use both historical measures and model based ones.

- Historical

As for the historical volatility measure, we take the 90- and 180-day unweighted standard deviation of *price* returns. In addition, we consider an exponentially weighted moving average (EWMA) estimator with decay parameter 0.94.

- Moving average

- GARCH

Finally, two model-based volatility measures are used. The first one is the estimate from a GARCH(1,1) model. The equation for the return $y_t$ can be written

$$y_t = \mu + \sigma_t \varepsilon_t \quad \text{where}$$
$$\sigma_t^2 \equiv \omega + \alpha (y_t - \mu)^2 + \beta \sigma_{t-1}^2$$

where $\mu$, $\omega$, $\alpha$ and $\beta$ are constants.

- Stochastic volatility

We also consider a one factor stochastic volatility model, which can be written as

$$\begin{aligned} y_t &= \mu + \sigma_t \varepsilon_t \quad \text{where} \\ \sigma_t &\equiv \sigma^* e^{h_t/2}, \\ h_t &= \varphi h_{t-1} + \eta_t \end{aligned} \tag{1}$$

Here $y_t$ is the relevant market return, $\sigma_t$ is volatility, $\varepsilon_t$ is a random variable (i.i.d. standard normal) which represents *transitory* shocks to volatility. The process $h_t$ represents the fluctuations of the log variance $\log \sigma_t^2$ around its long term mean $\log \sigma^{*2}$. The innovation to $h_t$, which represents a *permanent* volatility shock, is i.i.d. and normally distributed: $\eta_t \sim N(0, q)$. $\sigma^*$, $\varphi$ and $q$ are constants. We assume $0 < \varphi < 1$ in order to impose stationarity.
The unconditional mean of *h* is zero, while it unconditional variance is given by

$$\sigma_h^2 \equiv Var(h_t) = \frac{q}{1-\varphi^2}.$$

## Choosing the weights

The weights are chosen so as to set the ex ante volatility equal to the target

In the theoretical derivation of the Appendix we use weights that are inversely proportional to the volatility:

$$w_t \equiv \frac{\bar{\sigma}}{\sigma_t}. \tag{2}$$

This ensures that the conditional volatility of $w_t y_t$ is constant over time. In practice, however, the weighting scheme (2) is infeasible for two reasons:

1. The volatility for time *t* is unknown at the beginning of the period.

2. It would result in an excessive amount of trading.

To address the first issue we can use a forecast based on the information available at time $t$-1:

$$w_t \equiv \frac{\bar{\sigma}}{\hat{\sigma}_{t|t-1}}.$$

For the historical volatility measures, the forecast is given by the estimate based on data as of time $t$-1. For the SV and GARCH models we can generate a one period ahead forecast which takes mean reversion into account.

We use volatility predictions one step ahead obtained from the models

To reduce turnover we considered rebalancing at a lower frequency, e.g. monthly. However, this solution may not be satisfactory because volatility typically spikes when market conditions deteriorate. If the spike occurs at the beginning of the calendar month, the strategy may turn out to be very ineffective.

Less frequent rebalancing reduces turnover but it is crucial to react promptly to volatility spikes

A better alternative consists of defining two trigger points, one above and one below the target. Each time the estimated volatility crosses the threshold, we rebalance so as to bring the volatility of the strategy in line with the target. However, if the estimated volatility keeps fluctuating between the lower and upper thresholds, we do not change the allocation.

This can be achieved by rebalancing only when volatility ends up outside a given range

# Empirical results

## Data and setup

Data sources

Daily data on closing prices and dividends between 1/1/1980 and 8/8/2011 are taken from Bloomberg. As for the risk free rate, we used short term generic government bill yields where available (US and Germany), money market rates otherwise. In particular, we used a generic 3 month Bloomberg government yield for the US. For Germany, a 6 month government yield published by the Bundesbank (WZ9807) until 24/5/1993, a Bloomberg government yield thereafter. For the other countries we used short term interbank rates provided by the OECD until 1986 and then 3 month Libor.

European style indices are part of the UBS quant style database, described in detail in our publications such as *Style Watch*.

Returns to the Global Asset Allocation (GAA) portfolio are calculated by the Asset Allocation team.

Choosing a target

As we noted above, daily rebalancing is an unrealistic assumption. While we do analyse a version of the backtests with daily rebalancing, we also consider a target range for each index volatility. The target is 10% for all our stock indices (this does not affect the SR). As for the volatility range, we target an arbitrary 8% - 12% range and rebalance only when the (predicted) volatility falls outside of it.

Recursive estimation of the models

Estimates of model based volatility are obtained as follows. For the one factor stochastic volatility model of Harvey et al. (1994) we use their quasi maximum likelihood procedure based on the extended Kalman filter. One day ahead forecasts are obtained from the filter at the optimal value.
Each GARCH(1,1) model is estimated by maximum likelihood in R using the fGARCH package.
All the models are re-estimated at intervals of five trading days.

We lag the available information by one day to obtain robust results

To allow for a certain lag between the arrival of price information and portfolio rebalancing, we always use the model forecast based on closing prices from day $t$-1. To summarise, at the end of day $t$ we compute the forecast based on the time $t$-1 price and rebalance. The resulting index weight is used between closing time of day $t$ and closing time of day $t$+1.

We think it is worth pointing out that SRs are computed from the monthly *total return* index of each strategy, while volatility models are fitted to the (daily) *price* index.

## Does volatility targeting work?

In this section we analyse the returns to our simulated VTSs in order to assess whether they achieve the desired constant level of volatility.

We plot the rolling 100-day volatility of returns...

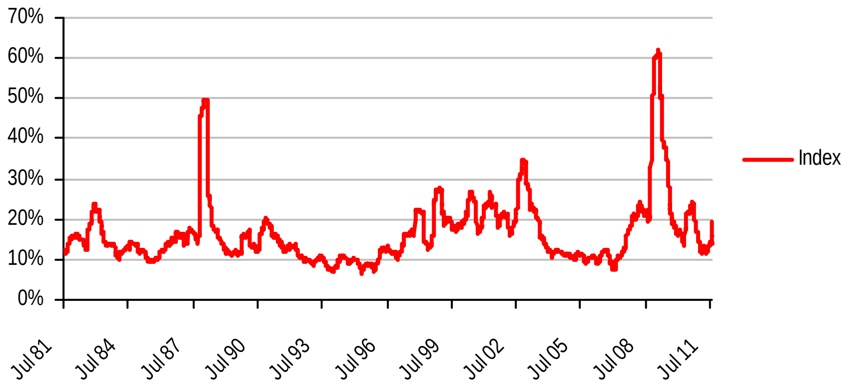
From a statistical point of view, there is no obvious way to check how stable volatility is in our backtest. We choose to calculate the annualised standard deviation of returns over rolling windows of 100 days (Chart 2 through Chart 6). If the return series had constant volatility then we would expect its rolling standard deviation to be stable over time. To give an idea, if returns were independent and normally distributed with mean zero and volatility 10%, then 99% of the time the standard deviation should lie between 8.2% and 11.8%.

...although this simple measure introduces an obvious bias

It is worth stressing, however, that this particular measure of volatility has obvious limitations. A single outlier (e.g. a daily return of -10%) will likely cause rolling volatility to remain high for 100 days after the extreme event has occurred, even if the strategy has adjusted in the meantime.[1]

Chart 2 shows the rolling 100-day volatility calculated for the index. A familiar pattern emerges, with periods of high volatility in 1999-2003 and 2007-2009. The spike corresponding to the stock market crash of 1987 is also evident in the chart.

Chart 2: Rolling 100-day volatility, S&P 500




Source: Bloomberg, UBS quant.

Volatility targeting is effective in maintaining volatility at a stable level

Chart 3 displays the rolling volatility of the VTSs based on daily rebalancing. If we compare the overall patterns of Chart 2 and Chart 3 we can immediately see that the cyclical fluctuations in volatility have been completely eliminated: Each of the series in Chart 3 hovers around the target level of 10% most of the time. Some large deviations from the target can be observed in 1987 and around the major spikes that characterise the volatility of the index. However, the deviations from the target appear to be short lived and are likely to be exaggerated by two factors:

1. The use of rolling volatility which introduces spurious persistence in the estimate, as argued above

2. The fact that we use data lagged by one day when simulating the strategies.

Stochastic volatility appears to be less effective than GARCH or EWMA...

By comparing the alternative model based approaches we can conclude that SV tends to do a slightly worse job when volatility peaks. This result is probably due to the structure of the model, which, compared to GARCH and EWMA,

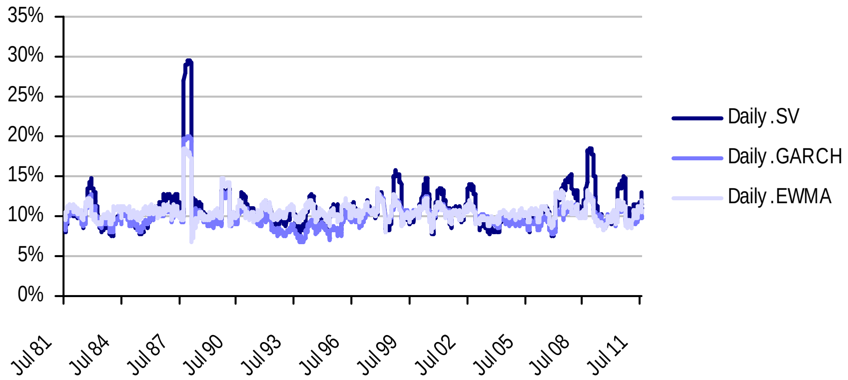
[1] A better solution would be to use intraday data on index returns in order to calculate the realised volatility of the strategy. Unfortunately, the data is only available for a small portion of our sample period.

tends to result in smoother changes in forecast over time. We return to this topic when we estimate turnover.

...while the purely historical volatility measure come last in the ranking

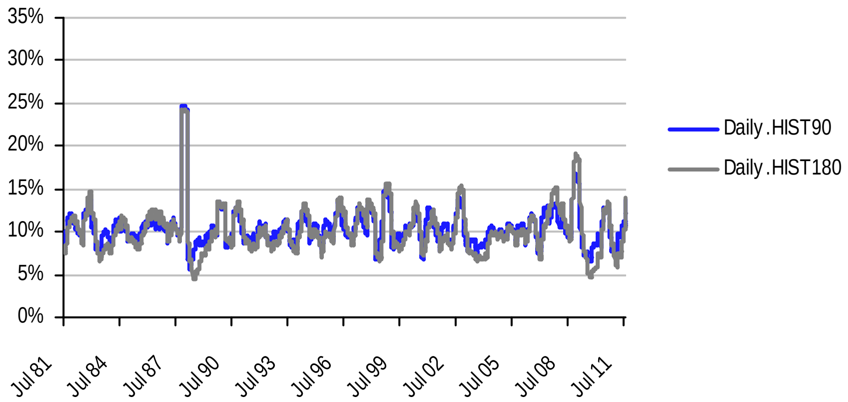
Chart 4 shows that the strategies based on historical volatility measures, again assuming daily rebalancing, would have been less effective in curbing volatility. The fluctuations are clearly wider than in Chart 2. It is also noticeable that, while the model based approaches rarely undershoot the target, the historical measures have a more symmetric pattern of errors, resulting in a rolling volatility as low as 5% after the two worst shocks.

Chart 3: Rolling 100-day volatility, daily rebalancing (model based)




Source: Bloomberg, UBS quant.

Chart 4: Rolling 100-day volatility, daily rebalancing (historical)

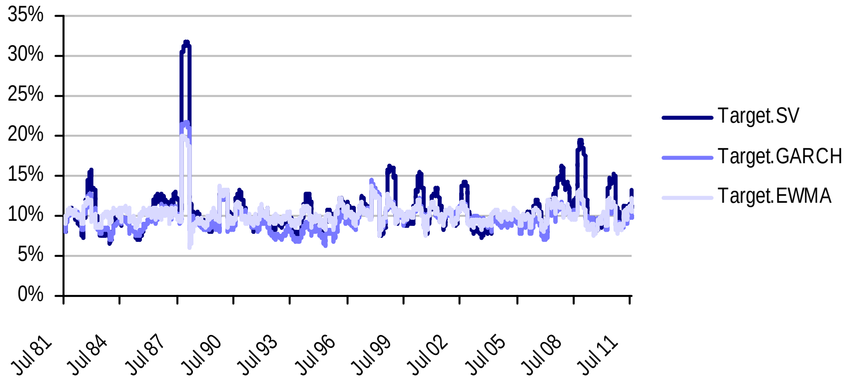



Source: Bloomberg, UBS quant.

Targeting a range does as good a job as daily rebalancing

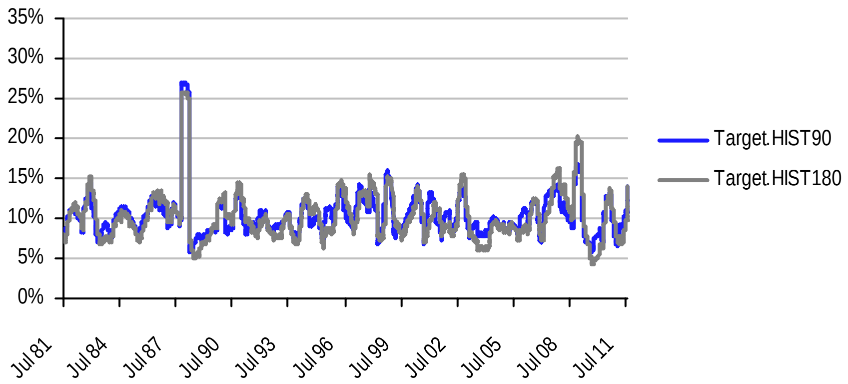
Finally, Chart 5 and Chart 6 suggest that the same level of effectiveness can be achieved by targeting a range of values around 10% instead of rebalancing daily. No significance deterioration can be detected in the ability to control volatility but, as we shall argue in the next section, the impact on transaction costs is likely to be sizeable.

Chart 5: Rolling 100-day volatility, range targeting (model based)




Source: Bloomberg, UBS quant.

Chart 6: Rolling 100-day volatility, range targeting (historical)




Source: Bloomberg, UBS quant.

## The leverage effect

Definition

The leverage effect can be defined as the asymmetric response of current volatility to lagged negative and positive returns. It is a well established empirical fact that sharp increases in volatility tend to be associated with negative returns. At the single firm level, this phenomenon is interpreted as the effect of a change in the company's leverage: A sharp negative return lowers the market value of the equity and thus increases leverage, thereby amplifying the future fluctuations in price.

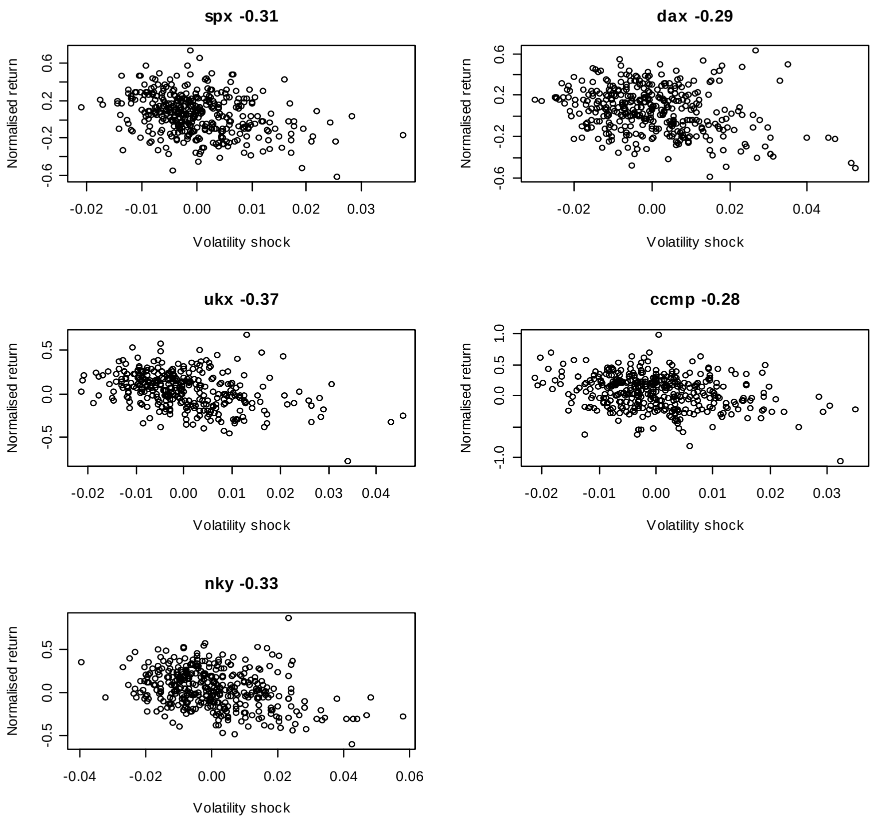
We estimate the leverage effect from our data

We can gauge the leverage effect by looking at the correlation between innovations to the volatility process in the SV model and the normalised return. Here the estimated time series of volatility is taken from the smoother in the SV model estimated from the full sample.

The effect is clearly present in our stock indices (Chart 7). The negative correlation is consistent over time – the estimate is negative over each subperiod of 10 years and for each of the indices considered here.[2]

Strong evidence in stock indices...

Chart 7: Leverage effect, equity indices



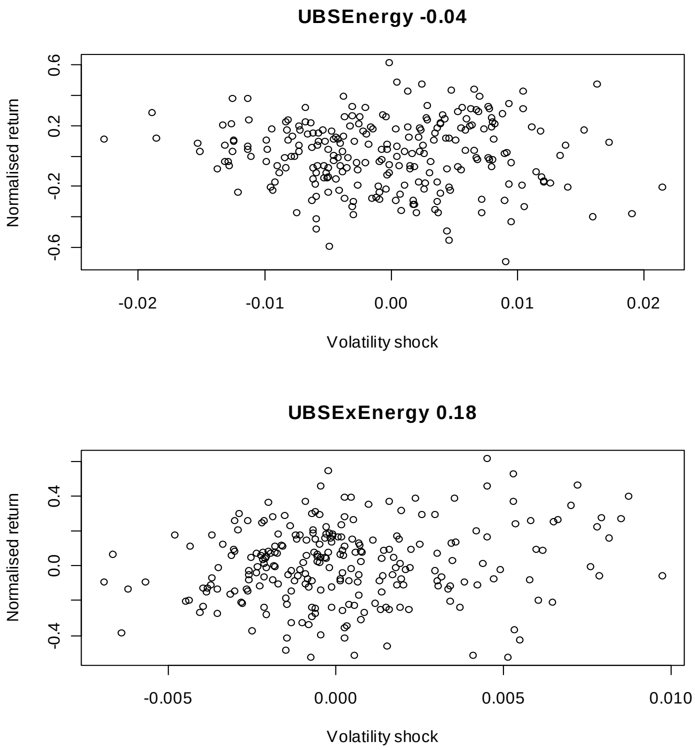

Source: UBS quant. Plot of the cumulated innovations to the volatility process against the normalised return. Monthly data. Correlation coefficients are given next to the titles. “CCMP” stands for Nasdaq, “UKX” for the FTSE 100 index and “NKY” for the Nikkei.

The correlation, however, is only mildly negative for the UBS DJ Energy index and positive for the UBS DJ Ex Energy commodity index (Chart 8).

...but not in commodity indices

[2] The full results are available on request.

Chart 8: Leverage effect, commodity indices




Source: UBS quant. Plot of the cumulated innovations to the volatility process against the normalised return. Monthly data. Correlations coefficients are given next to the titles.

## Equity indices

Charts 9 through 13 illustrate the main result of our backtest. The red bar represents the SR of the passive index (including dividends). The dark blue bars show the gross SR which would have resulted from following the VTS with daily rebalancing. Finally, the light blue bars refer to a strategy where we target a volatility range with thresholds 8- and 12%. We display one pair of columns for each of the volatility measures.

Nearly all VTS would have outperformed the passive index

The most notable result is the outperformance of VTSs. With no exceptions, across all stock indices and estimation methods, by targeting volatility we would have increased the SR over the whole period. The increase for the S&P 500 is roughly in line with the theoretical value estimated in the Appendix.

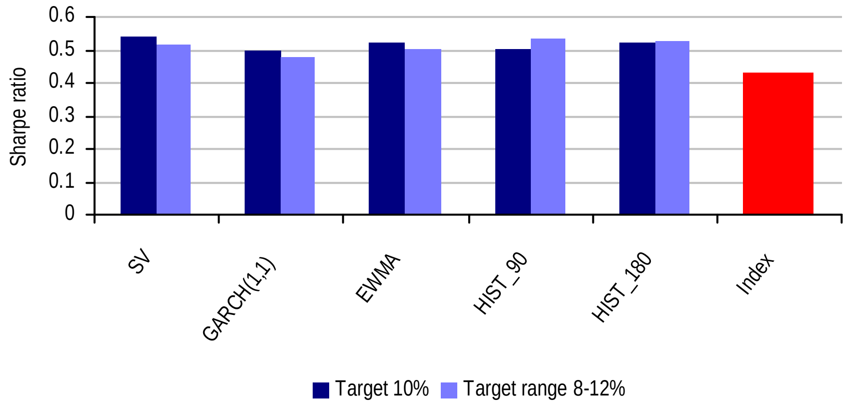
The choice of volatility does not seem to matter...

A surprising conclusion can be drawn by comparing the SRs across different volatility measures in each case. There appears to be no pattern from the charts and, in particular, we found no evidence that a model based measure outperforms a simple historical one.

...nor does the choice of rebalancing scheme

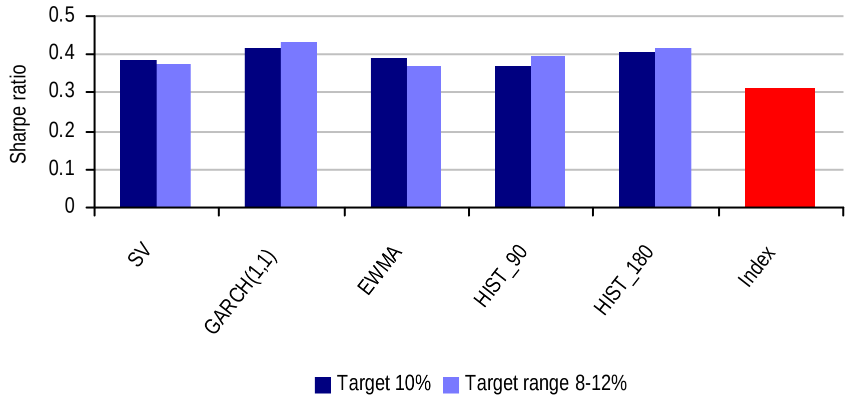
Similarly, it appears that daily rebalancing does not systematically outperform the more realistic range targeting rule. Again no clear pattern emerges. This result is particularly encouraging because it suggests that the large amount of turnover implicit in the strategy based on daily rebalancing is not necessary to obtain the improvement in SR.

## Chart 9: Performance, S&P 500



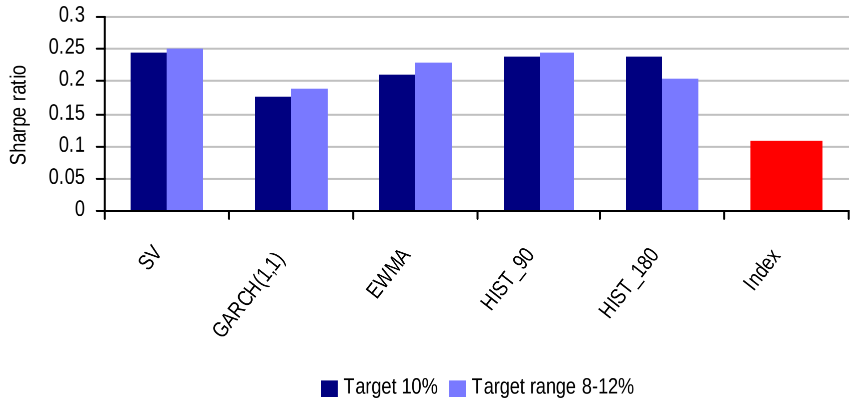

Source: UBS quant.

## Chart 10: Performance, DAX



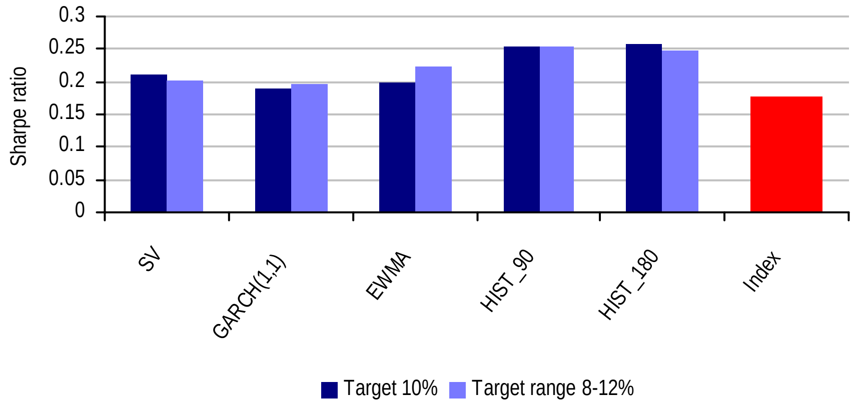

Source: UBS quant.

## Chart 11: Performance, NIKKEI



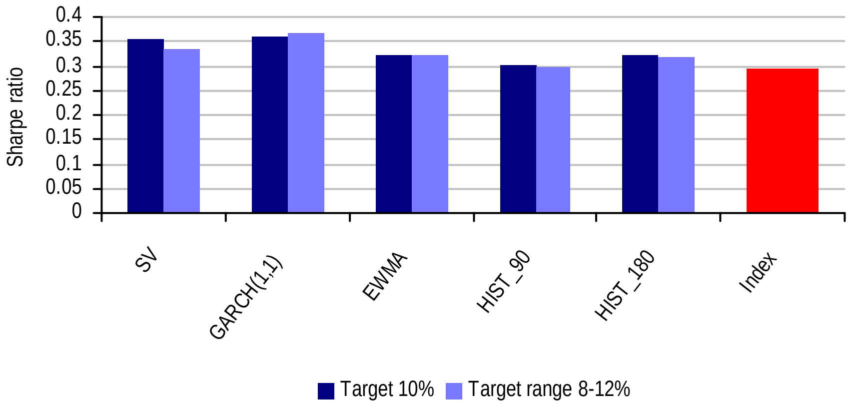

Source: UBS quant.

## Chart 12: Performance, FTSE 100




Source: UBS quant.

Chart 13: Performance, Nasdaq




Source: UBS quant.

Chart 14 illustrates how the difference in SR accrues over time. The last point in the chart corresponds to the SRs of the SV strategy (light blue 'SV' bar in Chart 9) and the index (red bar in Chart 9). The lines are obtained by recalculating the SR recursively, starting with the 1980s as the smallest sample and progressively extending the window.

VTSs would have underperformed in the 1980s...

The 1980s would not have been a good decade for volatility targeting – a result that seems to confirm our theoretical contention that in times of strong upward trending prices it is not a good idea. However, over the period 1990-2011 the SR of our VTS gradually caught up with the index. In particular, it is during periods of high volatility and sharp market falls that the strategy does well compared to the market: The tech bubble and the credit crisis.

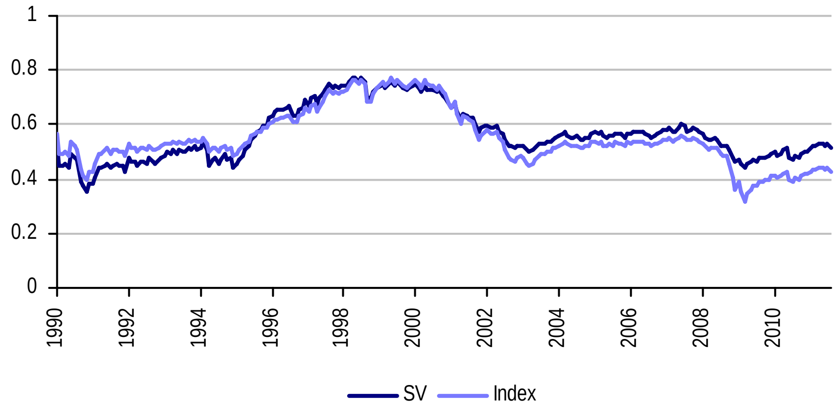
...and outperformed significantly in periods of high volatility...

...and between 1995 and 1996

It is also interesting to note that the VTS outperforms the index between 1995 and 1996. This happened as the stock market rally gathered pace in early 1995 and volatility plummeted. In these circumstances the timing of our strategy was quite effective: It increased the exposure to the index exactly as the strongest phase of the rally was about to begin.

We now examine in more detail the evolution of index weights over time for the S&P 500 index (Chart 15 and Chart 16). As we would expect, index exposure is particularly low when volatility spikes, i.e. in 1987, during the tech bubble and during the credit crisis. Given the relatively low target value (10%) our strategy rarely borrows money to fund increased leverage: The weight is above one in 1995-1996 and in 2005-2006.

Chart 14: Recursive Sharpe ratios, S&P 500

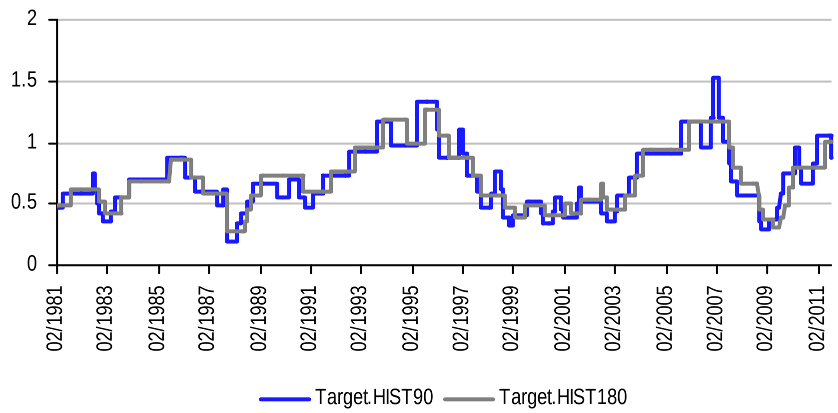



Source: UBS quant. The volatility targeting strategy is implemented by targeting a range between 8 and 10%.

GARCH and EWMA generate a higher turnover

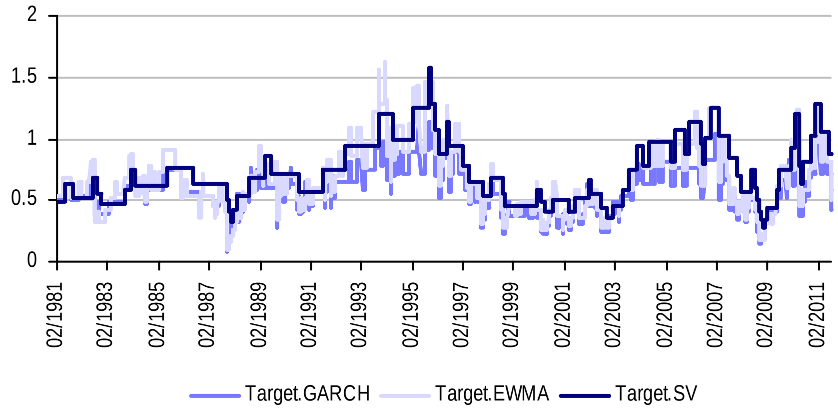
Among the model based estimators (Chart 16) SV appears to result in a smoother pattern over time. This can be explained by the structure of the model: SV distinguishes between short term shocks and long term ones. As a result, when a large negative return is observed it does not necessarily translate into a permanent increase in volatility. In GARCH and EWMA, in contrast, large negative returns tend to drive volatility up over a long period.

Chart 15: Index weight in the volatility targeting strategies, S&P 500




Source: UBS quant.

Chart 16: Index weight in the volatility targeting strategies, S&P 500




Source: UBS quant.

Targeting a range instead of a fixed level has a significant impact on turnover

How significant is the reduction in turnover obtained by targeting a range of volatility levels? Table 1 displays the turnover figures for each of the S&P 500 strategies, both for the full sample and for alternative subsamples. The amount of trading is significantly lower when the strategy targets a range of values instead of rebalancing continuously. In addition, some marked differences across the volatility estimators emerge. GARCH and EWMA consistently display the highest turnover values. SV is in the middle and the historical measures result in the lowest amount of trading.

Table 1: Average annual turnover, S&P 500

| | Daily rebalancing | | | | | Targeting 8-12% range | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | SV | GARCH | EWMA | HIST90 | HIST180 | SV | GARCH | EWMA | HIST90 | HIST180 |
| All months | 3.24 | 4.86 | 6.03 | 1.68 | 1.01 | 0.44 | 1.15 | 1.50 | 0.40 | 0.23 |
| 2000-2011 | 3.97 | 5.34 | 5.94 | 1.81 | 1.14 | 0.65 | 1.29 | 1.54 | 0.48 | 0.26 |
| 1990s | 3.44 | 5.46 | 7.08 | 1.80 | 1.02 | 0.36 | 1.29 | 1.66 | 0.39 | 0.22 |

Source: UBS quant. Turnover is measured as the average absolute change in the index weight, annualised from daily data by multiplying by a factor of 252.

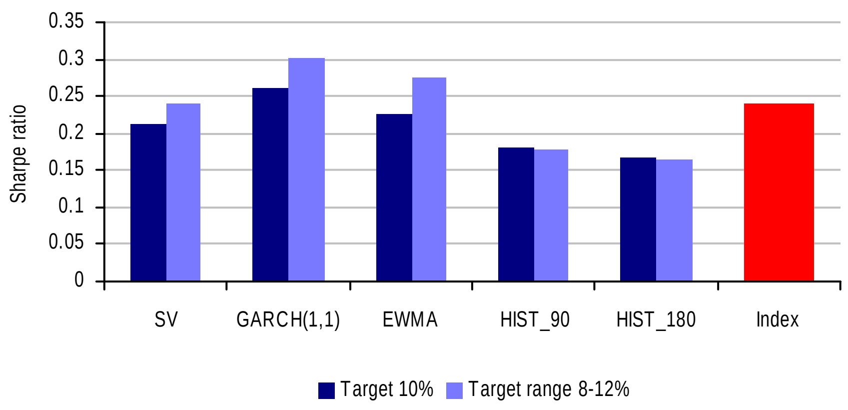
## Long short style portfolios

We tested the volatility targeting methodology on two long short style factors. Each portfolio is obtained by simulating a strategy that takes a long position in the top third of names ranked by the factor and a short position in the overall market.

The results for price momentum are mixed (Chart 17). The model based estimators would have performed roughly in line with the index, while the historical measures of volatility display lower SRs.

No clear evidence of outperformance for momentum…

Chart 17: Performance, High price momentum

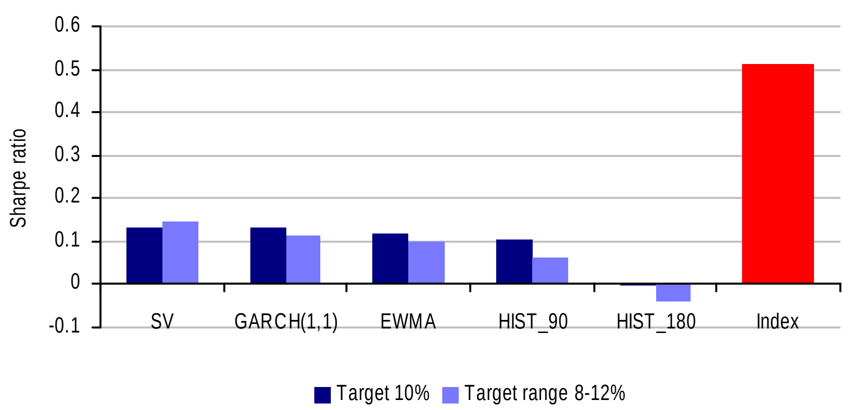



Source: UBS quant. The universe is DJ Europe.

The results for our composite value factor paint a familiar picture: As we argued in *Quant is not dead*, adjusting the amount of leverage dynamically has a detrimental effect on the performance (Chart 18). The result is consistent across all volatility estimators and for both versions of the strategy.

…and major underperformance when volatility targeting is used for a value index

Chart 18: Performance, Composite value



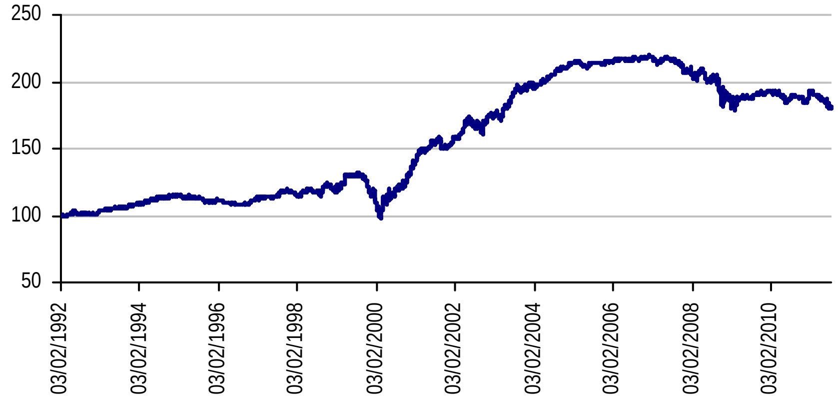

Source: UBS quant.

Why does the performance of value deteriorate so markedly when we implement volatility targeting? We can try to answer this question by inspecting Chart 19 and Chart 20. The former depicts the total return index, which displays three clear turning points: The moment when the tech bubble burst (end 1999), the start of the ensuing *dash to trash* (March 2000) and the end of the value rally (March 2005). As can be seen from Chart 20, the VTS would have performed well relative to the index at the end of 1999 (the two lines caught up at the trough despite the large gap that accumulated in the early 1990s). This is not surprising as we would expect the VTS to be very cautious in a period of high

The timing of the strategy would have been remarkably wrong volatility. However, volatility remained high over the following months and therefore the VTS missed the sudden turn in March 2000, causing the gap in SR to widen.

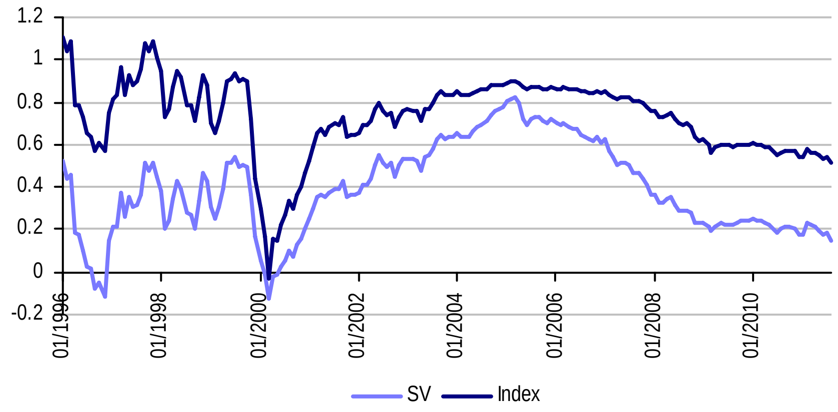
Something similar happened in the period 2005 – H1 2007, when volatility remained low and performance flattened considerably. Once again timing worked against the VTS.

Chart 19: Daily performance of the long short value factor




Source: UBS quant. The signal is built by simulating a long position in the top third of stocks ranked by composite values and a short position in the market.

Chart 20: Recursive Sharpe ratios, Value




Source: UBS quant.

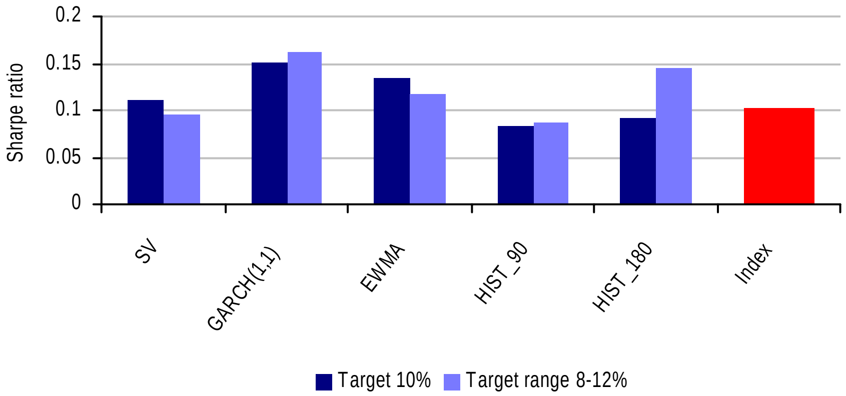
## Commodity indices

Volatility targeting would have worked in some cases for an index of energy commodities...

Another category of indices which we have considered is the family of DJ UBS commodity indices. In particular, we have run our analysis by targeting a volatility level of 10% on the DJ UBS Energy and Ex Energy indices.

The SRs of the various implementations of VTS for the Energy index are displayed in Chart 21. Most of the strategies would have obtained gross returns in line with the index or slightly outperformed. GARCH would have performed particularly well in this case.

Chart 21: Performance of the DJ UBS Energy commodity index




Source: Bloomberg, UBS quant.

...but for the non energy one

All the SRs for the VTSs based on the Ex Energy index would have been negative. The index itself displayed a negative average return over the sample period. By targeting volatility one would have obtained slightly lower gross returns with a lower volatility.

Table 2: Performance, DJ UBS Ex Energy commodity index

| | SV | GARCH | EWMA | HIST90 | HIST180 | Index |
|---|---|---|---|---|---|---|
| Mean | -1.26% | -0.96% | -1.25% | -1.14% | -1.27% | -0.08% |
| Standard dev | 10.68% | 10.53% | 10.62% | 10.65% | 10.95% | 13.48% |
| Sharpe ratio | -11.78% | -9.14% | -11.82% | -10.69% | -11.64% | -0.56% |

Source: UBS quant, Bloomberg.

One possible explanation for the disappointing performance of volatility targeting in this particular case is the lack of a leverage effect in the non energy commodity index. From a theoretical standpoint, it is conceivable that such commodities display countercyclical performance. The recent dynamics of the price of precious metals suggests that non energy commodities may be likely to outperform in periods of high volatility.

Our interpretation: It tends to work when returns are procyclical

## Cross-Asset Constant Volatility

One of the arguments for having a diversified portfolio is to reduce volatility. Assuming that asset returns are not correlated diversification succeeds in this regard. However during the Credit Crisis and the current sovereign debt crisis we have seen that systemic shocks to the financial system have affected all assets. In terms of daily returns this has resulted in a rise in correlation between assets. However, as we have shown previously, increasing correlation does not rule out diversification benefits. During a crisis risky assets become more positively correlated with one another, and "safe" assets become more positively correlated with one another. Risky and "safe" assets become more negatively correlated with one another. Consequently a portfolio that contains both "safe" and risky assets benefits by having reduced volatility even during a crisis.

Correlation among assets increases during a crisis...

...hence targeting a *low* level of volatility helps when markets fall

Even though a portfolio is diversified it can still show large variations in volatility. In the following comparisons we will use the Global Asset Allocation model portfolio benchmark because it is diversified across assets and regions. The benchmark is roughly market weighted and allocations are static over time. In order to keep volatility constant we re-balance a portfolio containing cash and the benchmark daily. Re-balancing is performed in order to keep volatility constant at some target level over a three month window. The cash weight is increased when benchmark volatility is above the target and reduced when volatility is below the target.

The Global Asset Allocation model portfolio

Volatility is widely used as a risk measure, but is not necessarily the best measure. Few investors would consider a daily rise of +5% to be equally risky as a fall of -5% in the value of their portfolio. And yet volatility would consider both returns as equivalent. Some have suggested that using a lower partial moment as a risk measure might be more in tune with the utility assigned to returns by investors. Downside deviation is the second order lower partial moment, where we specify some minimum acceptable return *m*, and

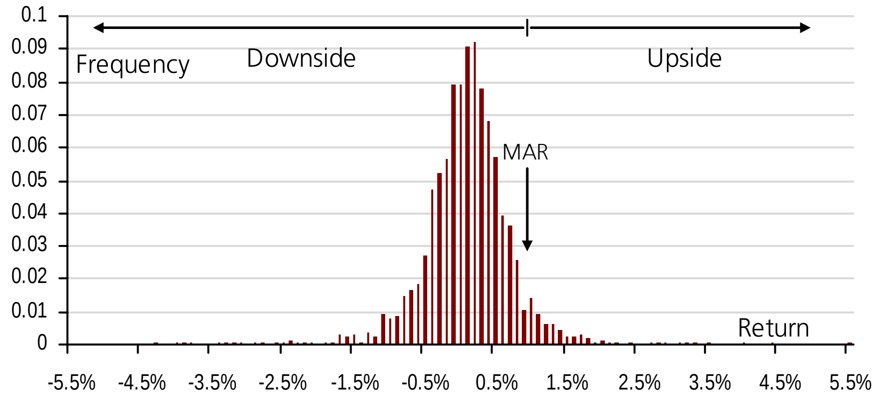
One of the limitations of volatility as a measure of risk is that it does not distinguish between the downside and the upside

$$DD = \sqrt{\frac{\sum_{i=1}^{n} I_i (r_i - m)^2}{n}}$$

The indicator function $I_i$ is 1 if the return is less than the minimum acceptable return and 0 otherwise because we only choose to count returns below the MAR as risk. Any returns above the MAR would be upside. This measure will differ most from standard deviation for return distributions that have a heavy upside tail. This is because the upside tail would count towards risk using standard deviation but would be ignored by downside deviation. Downside deviation is illustrated in Chart 22 using a return histogram for our Global Asset Allocation model portfolio.

Downside deviation is a better alternative...

Chart 22: Upside and downside relative to the minimum acceptable return (MAR)




Source: Bloomberg. UBS

Extending the idea of constant volatility slightly we can think in terms of constant risk. Any risk measure could be held constant by adjusting allocation to cash, which has a volatility of almost zero, and a portfolio of risky assets. Allowing negative cash allocation means that it is possible to boost volatility by taking a leveraged position in the risky portfolio.

...which can also be used to target a constant risk level in our dynamic asset allocation

Here we compare a constant volatility approach using both standard deviation and downside deviation as risk measures. The portfolio we use is our Global Asset Allocation (GAA) portfolio using daily returns between 2002 and the present day. We measure volatility over a window of three months of daily returns and solve for the cash and risky portfolio allocation that matches the desired risk. The portfolio is re-balanced on a daily basis. As we show above the rebalancing frequency does not greatly affect the theoretical results and in practice rebalancing would occur much less frequently. One difficulty with comparing volatility and downside deviation risk measures is that downside deviation requires a minimum acceptable return. In this comparison we have used a minimum acceptable return of zero.

We apply the methodology to the GAA portfolio

In Chart 23 we compare cumulative returns since 2002 using the raw GAA benchmark portfolio and a selection of volatility targeted portfolios. The volatility of the benchmark portfolio was 10.7% so, as we would expect, a volatility target of 10% gave a return that was comparable with the benchmark. Volatility targets below that of the benchmark portfolio reduced return but increased risk-adjusted return. To put this another way, the cost of reduced return was more than compensated by the benefit of decreased risk.

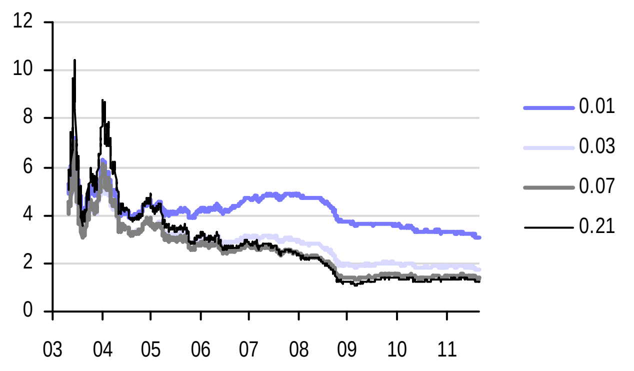
Volatility targeting boosts the Sharpe ratio

Chart 24 shows a comparison between risk and return using standard deviation and downside deviation as risk measures. In order to make the two lines comparable we also calculated the standard deviation of returns for the constant downside deviation strategy returns. The results show that the two risk measures produce comparable returns for this benchmark, with downside deviation outperforming standard deviation very slightly for the same level of risk. This would not be the case if instead the portfolio had exhibited strongly positive

The two alternative risk measures give similar results...

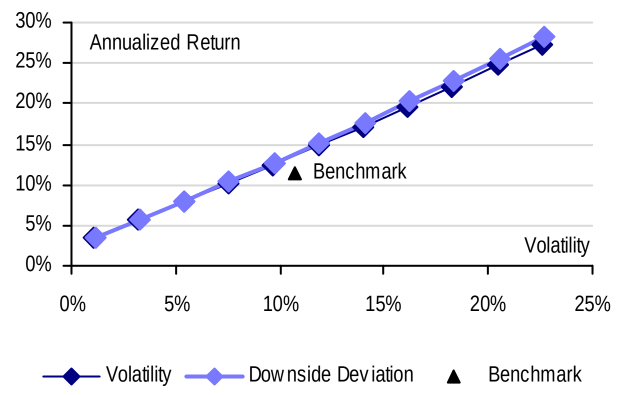
...arguably because the returns are not markedly asymmetric skew but the benchmark portfolio returns were almost symmetric, as shown in Chart 22.

Chart 23: Cumulative Sharpe ratio varying the volatility target




Source: Bloomberg, UBS

Chart 24: Volatility and return varying the risk target

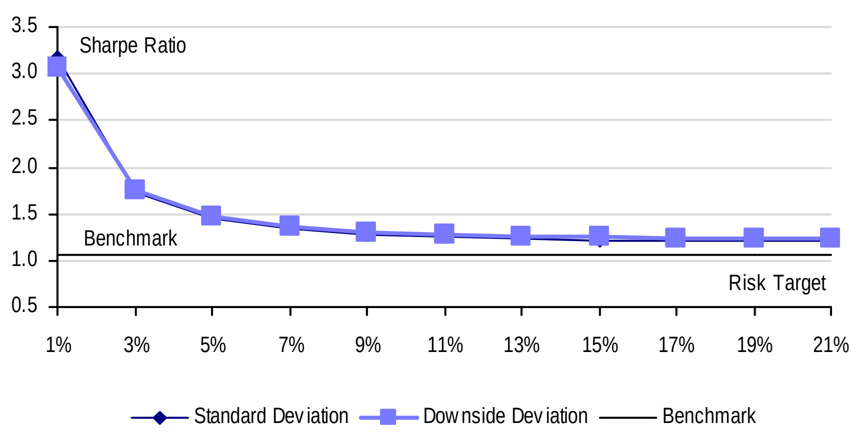



Source: UBS

The strategy works particularly well for relatively low volatility targets

These results show that a combination of a diversified portfolio with a constant volatility strategy offers great flexibility to investors. The strategy makes it possible to dial up or dial down risk and return. As we show in Chart 25 the risk-adjusted return of the strategy is higher than the benchmark portfolio (benchmark return 11.4%, volatility 10.7%, SR 1.1) for all values of the risk target. Lower risk targets gave better risk-adjusted return, and there was little difference between the two risk measures on a risk-adjusted basis.

Chart 25: Sharpe ratio varying the risk target




Source: Bloomberg, UBS

# Conclusion

Does volatility targeting improve on the Sharpe ratio of an index? This report has attempted to answer the question both from a theoretical and from an empirical point of view.

The main question: Can we improve performance through volatility targeting?

We show that in a stylised model that allows for time varying volatility the outcome depends on the Sharpe ratio of the passive index and the volatility of its volatility. For realistic parameter values, however, the model implies that a VTS should always have a higher Sharpe ratio compared to the passive index. The more volatile volatility is, the more significant the improvement in performance.

The theoretical answer: Yes…

We then considered an extension of the model in which volatility-in-mean effects can arise. The conclusion is that, for realistic parameter values, a negative relation between expected return and volatility causes the difference between the Sharpe ratios to increase further in favour of the VTS.

… particularly if return and volatility are negatively related

Intuitively, this can be seen as a timing effect. The VTS takes more exposure to the index in times of low volatility and rotates into cash when volatility spikes.

Our empirical results lend some support to the theoretical analysis. We found that, over a thirty year period, volatility targeting would have improved the Sharpe ratios of stock indices and cyclical commodity indices.

The empirical evidence: Volatility targeting typically enhances risk adjusted return...

However, we also found that for a long short value portfolio the opposite would have been true. The explanation given in our analysis is centred on the relation between volatility and average return of the factor around turning points. A VTS would have missed most of the gains during the critical *dash to trash* phase because it is typically characterised by high volatility.

...although there are exceptions, e.g. a long short value factor

As a further empirical application, we backtested the methodology on the Global Asset Allocation portfolio over the period 2002-2011. Once again, the Sharpe ratio is enhanced by targeting constant volatility. Moreover, we considered an alternative measure of risk as the target: Downside deviation. The improvement obtained by targeting a better measure of risk appears to be modest, arguably because the historical returns to the GAA portfolio do not display significant asymmetry.

Our Global Asset Allocation portfolio also produced encouraging results

Finally, some of our empirical results can give useful guidance in implementing the strategy. We show that targeting a range, as opposed to a specific level of volatility, produces very similar levels of performance while reducing significantly the turnover. We also found that, somewhat surprisingly, the differences among alternative volatility estimators in terms of performance are very modest.

The turnover can be limited by targeting a range of volatility values

# Appendix A: The Sharpe ratio under stochastic volatility

This section explores the following statistical question: Assuming that the world is described by the SV model (1), what would the Sharpe ratio (SR) of a VTS be?

We derive the theoretical SR for a VTS and the passive index in a simple SV model

## The main intuition

The SR depends on unknown (and in this case time-varying) quantities, mean and variance, which need to be estimated from the returns of the strategy. Lo (2002) described a set of conditions under which the estimated SR converges, in large samples, to the population value. The crucial condition is that returns are not serially correlated.

The return to a VTS

We assume that the risk free interest rate is constant. Hence the excess return on the index is simply $y_t - r$, while the excess return on the VTS is $w_t y_t + (1-w_t) r - r = w_t (y_t - r)$. This can in turn be written, by plugging (1) into (2), as

$$w_t (y_t - r) = \bar{\sigma}\left(\frac{\mu - r}{\sigma_t} + \varepsilon_t\right)$$

i.e. the target volatility multiplied by the sum of the one-period conditional SR plus a standard normal disturbance.

We have thus separated out the two sources of uncertainty: volatility dynamics ($\sigma_t$) and unpredictable component of return ($\varepsilon$). The dynamics of volatility causes weights to fluctuate over time. Intuitively, if the trend $\mu - r$ is relatively steep (compared to the variability of the innovation $\bar{\sigma}\varepsilon_t$) then the first term will be the main driver of returns.

In practice, it is easy to show that for reasonable parameter values the first term in the expression makes a negligible contribution to its variance. If the annualised SR varies between, say, 0 and 3, then the one day SR varies between 0 and 3/√252=0.19. It is easy to see that even in this extreme example the variability of the normal disturbance $\varepsilon$ would predominate.

Approximate expression for the SR

Suppose therefore that we can ignore, at a first approximation, the contribution of the SR term to the variance of $w_t (y_t - r)$. It is straightforward to show that, using this approximation and the properties of the lognormal distribution, the SR of the VTS is equal to

$$E\left(\frac{\mu - r}{\sigma_t}\right) = \frac{\mu - r}{\sigma^*} e^{\sigma_h^2/8}.$$

In contrast, the SR of the index is

$$\frac{E(y_t)}{\sqrt{Var(y_t)}} = \frac{\mu - r}{\sigma^* e^{\sigma_h^2/4}}.$$

A VTS should outperform the index

We can therefore expect the SR of a VTS to improve on the index by a factor of approximately $e^{3\sigma_h^2/8}$. In our estimated daily SV model for the S&P 500, this is roughly equal to 25%.

## The derivation in more detail

The expectation of the excess return to the index is $\mu - r \equiv \mu^*$.

To compute the moments of the VTS we need to set a target volatility level in (2), $\bar{\sigma}$. A natural choice is $\bar{\sigma} = \sigma^* \exp\left(-\frac{\sigma_h^2}{8}\right)$ which implies that the expected weight on the index, $E(w_t)$, is equal to one. In other words, we are on average holding the index.

This value of $\bar{\sigma}$ yields the same expected return for passive index and VTS:

$$E\left(w_t\left(y_t - r\right)\right) = \mu^*.$$

What about the volatilities? The unconditional variance of the index is

$$Var\left(y_t\right) = \sigma^{*2} \exp\left(\frac{\sigma_h^2}{2}\right). \tag{A1}$$

The expression represents two sources of uncertainty: The unpredictable component of returns ($\sigma^{*2}$) and the uncertainty surrounding future volatility ($e^{\sigma_h^2/2}$). The latter amplifies the former. If volatility were constant then the variance would boil down to $\sigma^{*2}$.

For the VTS we get

$$Var\left(w_t\left(y_t - r\right)\right) = \sigma^{*2} \exp\left(-\frac{\sigma_h^2}{4}\right) + \left(\exp\left(\frac{\sigma_h^2}{4}\right) - 1\right)\mu^{*2}.$$

The first term in the variance is smaller than the variance of the index (A1) because it reflects the fact that by targeting volatility we eliminate the effect of its fluctuations. The second term arises because by targeting volatility we introduce uncertainty on the future weights of the portfolio.

In order to compare the SRs of the two strategies we can consider the difference between the variances:

$$Var\left(y_t\right) - Var\left(w_t\left(y_t - r\right)\right) = \sigma^{*2}\left[\exp\left(\frac{\sigma_h^2}{2}\right) - \exp\left(-\frac{\sigma_h^2}{4}\right)\right] - \left[\exp\left(\frac{\sigma_h^2}{4}\right) - 1\right]\mu^{*2}$$

If the difference is positive then the SR of the VTS is higher compared to the SR of the index (it would have the same expected excess return and lower volatility). Both expressions in square brackets are strictly positive, unless volatility is constant. It is easy to show that

$$\left[\exp\left(\frac{\sigma_h^2}{2}\right) - \exp\left(-\frac{\sigma_h^2}{4}\right)\right] > \left[\exp\left(\frac{\sigma_h^2}{4}\right) - 1\right].$$

In addition, for a typical stock index the SR, which can be expressed as $\mu^* e^{\sigma_h^2/4} / \sigma^*$, is well below one. Given the model that we estimated for daily data on the S&P 500 index, $e^{\sigma_h^2/2} \approx 4/3$. As a result $\sigma^{*2} > \mu^{*2}$ for typical parameter values, which implies that the SR of the VTS is typically higher.

We can estimate the size of the improvement by using the estimated S&P 500 daily model. Plugging in the values for the mean and variance parameters, we obtain an increase in the SR of roughly 24% from 0.4 to 0.496. It is crucial to note that the autocorrelation of daily returns is very modest for all the VTSs. If this were not the case then, as argued by Lo (2002), scaling up from the daily SR would result in biased estimates.

In the appendix we show that if the expected return depends negatively on volatility then the improvement in SR, for typical parameter values, is even greater. Koopman and Hol Uspensky (2002) allowed for a variance in mean term and found a mild negative relationship for the stock indices they considered.

# Appendix B: Stochastic volatility in mean and the Sharpe ratio

In this section we show that, for realistic parameter values, the leverage effect implies that a VTS should outperform the passive index. The stronger the relation between volatility and expected return, the more significant the outperformance should be. This result formalises the intuition that VTSs work as market timing strategies because returns tend to be lower when volatility is high.

We generalise the SV model (1) by introducing, along the lines of Koopman and Hol Uspensky (2002), a variance in mean term:

$$\begin{aligned}
&y_t = \mu_t + \sigma_t \varepsilon_t \quad \text{where} \\
&\sigma_t \equiv \sigma^* e^{h_t/2}, \\
&h_t = \varphi h_{t-1} + \eta_t, \\
&\mu_t = \mu + d\sigma_t^2
\end{aligned}$$

The coefficient $d$ represents a contemporaneous relation between variance and expected return. We can now repeat for this more general model the calculations carried out in Appendix A. The first point to notice is that, unless $d$=0, the expected values of the index and the VTS are no longer equal:

$$E\left(w_t\left(y_t - r\right)\right) = \mu^* + d\sigma^{*2}$$

while

$$E\left(y_t - r\right) = \mu^* + d\sigma^{*2} e^{\sigma_h^2/2}.$$

Similarly, for the variances we get:

$$Var\left(y_t\right) = \sigma^{*2} e^{\sigma_h^2/2}\left[d\sigma^{*2}\left(e^{3\sigma_h^2/2} - e^{\sigma_h^2/2}\right) + 1\right]$$

and

$$Var\left(w_t\left(y_t - r\right)\right) = \sigma^{*2} e^{-\sigma_h^2/4}\left[\left(d^2\sigma^{*2} + \frac{\mu^*}{\sigma^{*2}}\right)V_1 + d\mu^* V_2 + 1\right]$$

$$V_1 = e^{\sigma_h^2/2} - e^{\sigma_h^2/4}$$

$$V_2 = 2\left(1 - e^{\sigma_h^2/4}\right)$$

We know from the previous analysis that, for realistic parameter values, when $d$=0 the SR of a VTS is higher than the SR of the passive index. What happens if $d$<0? We compute the partial derivatives below:

$$\frac{\partial}{\partial d} E\left(y_t - r\right) = \sigma^{*2} e^{\sigma_h^2/2}$$

$$\frac{\partial}{\partial d}E\left(w_t\left(y_t-r\right)\right)=\sigma^{*2}$$

This implies that, as *d* becomes more and more negative, the expected return will fall less for the VTS. Let us now turn our attention to the variance.

$$\frac{\partial}{\partial d}Var\left(y_t\right)=2\sigma^{*4}\left(e^{2\sigma_h^2}-e^{\sigma_h^2}\right)d$$

$$\frac{\partial}{\partial d}Var\left(w_t\left(y_t-r\right)\right)=2\sigma^{*4}\left(e^{\sigma_h^2/4}-1\right)d-2\mu^{*}\left(1-e^{-\sigma_h^2/4}\right) \tag{A2}$$

The last term in (A2) is negligible for typical parameter values, following the argument discussed in the previous appendix. If we ignore it then it is easy to show that both derivatives are negative when $d<0$ but

$$\frac{\partial}{\partial d}Var\left(y_t-r\right)<\frac{\partial}{\partial d}Var\left(w_t\left(y_t-r\right)\right)$$

As a result, as *d* becomes more and more negative both variances increase but the variance of a VTS increases less. This, together with the result we found for the expected return, implies that the SR of the VTS will decrease less than the SR of the index and therefore the difference between the two will widen as *d* becomes more and more negative.

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|---|---|---|---|
| **Buy** | **Buy** | 59% | 35% |
| **Neutral** | **Hold/Neutral** | 35% | 33% |
| **Sell** | **Sell** | 6% | 14% |
| **UBS Short-Term Rating** | **Rating Category** | **Coverage[3]** | **IB Services[4]** |
| **Buy** | **Buy** | less than 1% | 0% |
| **Sell** | **Sell** | less than 1% | 20% |

1:Percentage of companies under coverage globally within the 12-month rating category.
2:Percentage of companies within the 12-month rating category for which investment banking (IB) services were provided within the past 12 months.
3:Percentage of companies under coverage globally within the Short-Term rating category.
4:Percentage of companies within the Short-Term rating category for which investment banking (IB) services were provided within the past 12 months.

Source: UBS. Rating allocations are as of 30 September 2011.

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|---|---|
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| **Neutral** | FSR is between -6% and 6% of the MRA. |
| **Sell** | FSR is > 6% below the MRA. |
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