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Quantessentials

Conventional active returns can be misleading

Equities

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Take care: Investment decisions can be affected

Traditional measures of calculating active returns can produce illogical answers - something that could result in the wrong investment decision being made.

More than one approach to active returns

We examine five different approaches to calculating active returns. In general, each approach will yield a different multi-period result using the same data. Some approaches even yield results that are inconsistent with themselves.

Conventional metrics fail

We outline the minimum criteria that the active return metrics should satisfy. We find that three of the measures fail against at least one of these criteria.

Solution at hand

We present a solution that is well known, but not well used. We argue that the problems with other metrics are both material and can affect investment decisions, and so justify our slightly more complex solution. Applications of active returns include reporting, performance attribution and investment decisions, therefore avoidance of the problems or at the very least an understanding of the shortcomings of these metrics is paramount, in our view.

Introduction

Quick quiz

(1) What has been the active return of the FTSE 100 (in USD) relative to the S&P-500 since February 2009 (the low of the financial crisis)?

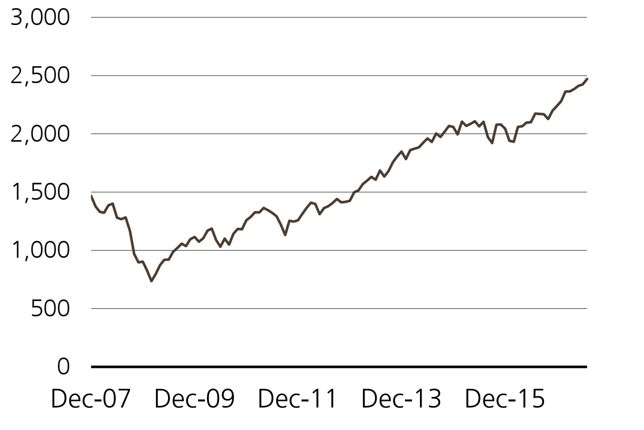
(a) –59% (b) –158% (c) –47% (d) –64% (e) –47%

(2) True or false: The FTSE 100 (in USD) has outperformed the S&P-500 over the period 31 December 2008 to 31 December 2012.

(a) True (b) False

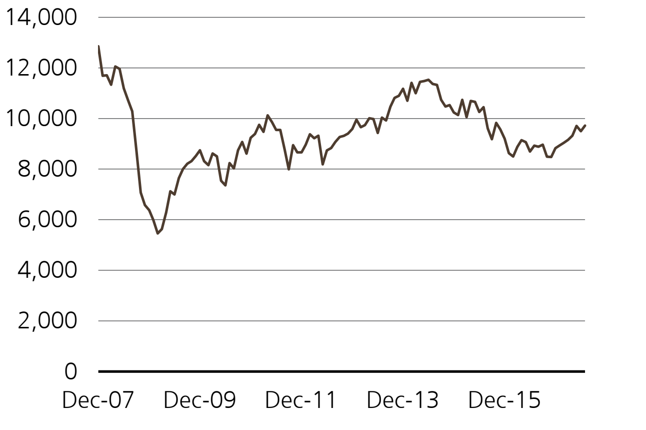
As a hint, we display the S&P 500 index in Figure 1 and the FTSE 100 index in Figure 2.

Figure 1: S&P 500 Index Values



Source: Datastream

Figure 2: FTSE 100 Index Values



Source: Datastream

The short answer to this quiz is that it depends on how you measure it.

'Active returns' is ambiguous

Many readers might imagine the definition of active returns is obvious and unambiguous – the difference between portfolio and benchmark returns. They will perhaps be surprised to learn that not only are there different definitions, but also that they can give rise to very different answers. In fact, using the same data to answer the quiz above, we obtain all of the answers depending on the approach used.

We give a brief definition of the five approaches here; we will give more detailed definitions later in the note.

(1) **Simple active**: The difference between the sum of portfolio returns and benchmark returns.

(2) **Index difference**: The difference between the compounded returns to the portfolio and the compounded returns to the benchmark.

(3) **Compounded active**: The difference between returns to the portfolio and returns to the benchmark, compounded (i.e. take the difference first).

(4) **Log returns**: The difference between portfolio log returns and benchmark log returns, summed.

(5) **Index ratio**: The ratio of the compounded returns to the portfolio and the compounded returns to the benchmark.

So what is the correct way? We advocate the use of log returns. This is neither a new technique nor is its validity questioned; it features in many econometrics texts. However, we believe practitioners often shy away from the use of log returns for three reasons.

(1) It is less intuitive and less conventional than other techniques;

(2) It is more work to aggregate log returns in a portfolio context; and

(3) It is widely argued that the results obtained using other techniques give rise to the same decisions.

Results are different and it matters

We do not disagree with the first two assertions, but believe that:

a. The results can be substantially different using different techniques;

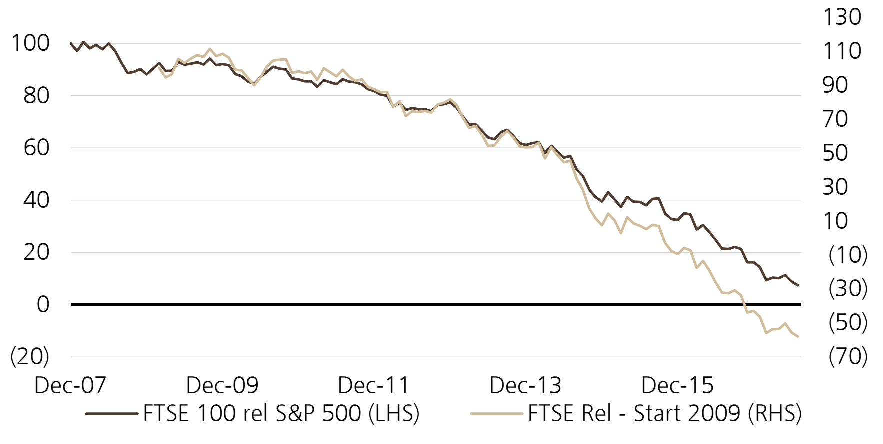
b. The results can lead to different conclusions and hence different investment decisions; and

c. The results can be inconsistent within a single methodology.

The aim of this report is to draw the reader's attention to the many problems with the commonly used methods of calculating active returns over multiple periods and to argue that log returns, while less common and more work, should be used for investment decision making. As a precursor to our discussion, let us consider a couple of inconsistencies within a single method of calculating active returns.

Using one method, we find that the active return of the FTSE 100 relative to the S&P-500 since Feb 2009 has been –158% or –92%, as shown in Figure 3[1]. While the same method is used, with the same data, the difference lies in whether we are considering the results in isolation, or in a wider context.

Figure 3: Active return of FTSE 100 relative to S&P starting at different times

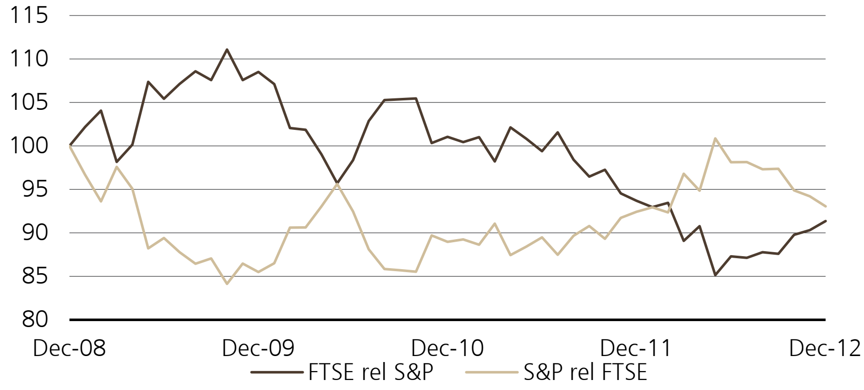


Source: UBS

Using another method, we find that both the FTSE 100 underperforms the S&P-500 and the S&P-500 underperforms the FTSE 100 over the period 31 December 2008 to 31 December 2012 (see Figure 4).

[1] Both active returns are calculated from December 2009, but one using the active return series starting in 2007, the other using the active return series from Dec 2009.

Figure 4: FTSE relative to S&P 500 and S&P 500 relative to FTSE



Source: UBS

Answering the quick quiz has uncovered some problems. The central issues that we consider in this report are:

- Does it matter if the results depend on past results?
- Does it matter if the results depend on whether daily or monthly returns are used?
- Does it matter if there are inconsistent results between a portfolio relative to a benchmark and a benchmark relative to a portfolio?
- Does it matter if one arrives at different results if one compares the portfolio to a benchmark, or compares both to a third portfolio?

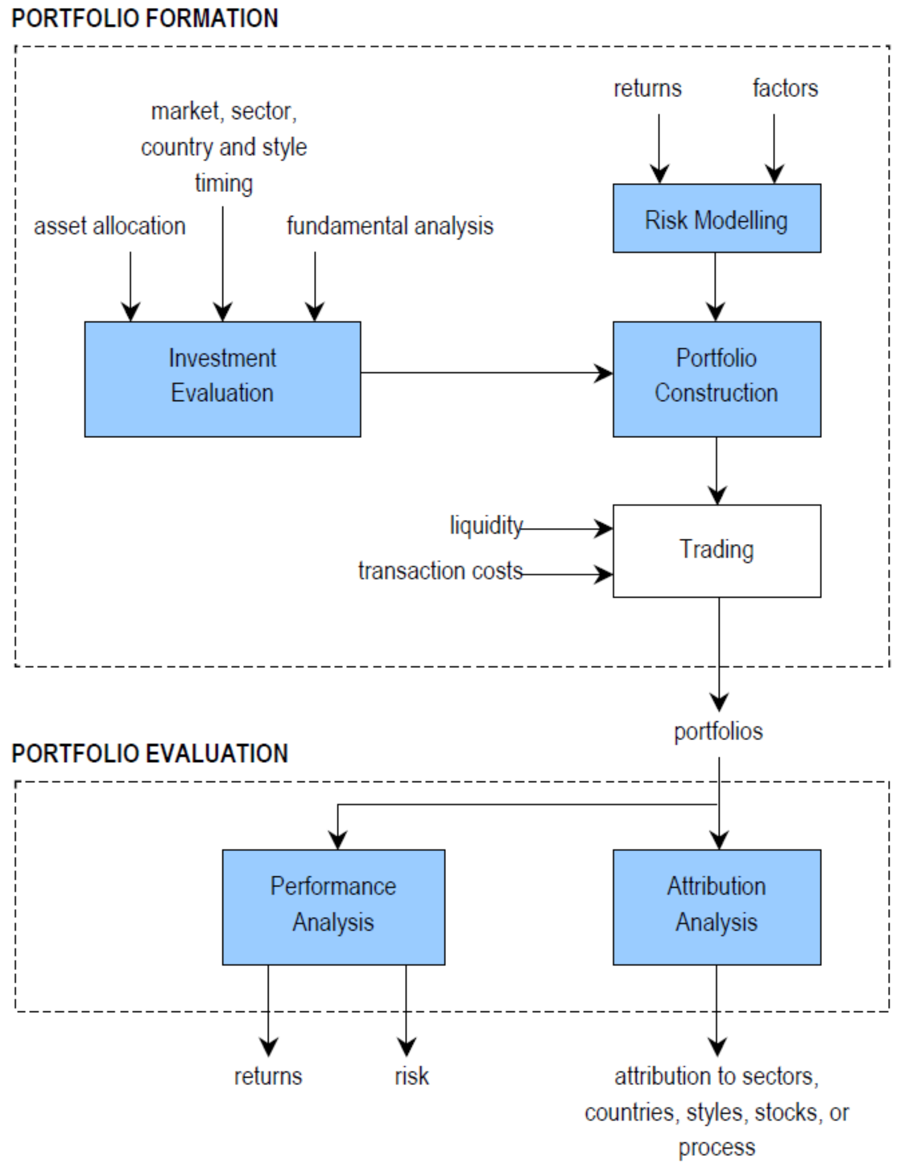
In all these cases, we find that it does matter, and that many of the standard approaches to calculating active returns fail in at least one of these ways. This can impact particularly on investment decisions, reporting, tracking error and performance attribution.

The big picture

A typical investment process may be stylistically depicted[2] as in Figure 5.

The investment process has a heavy reliance on active returns

Figure 5: Simple depiction of the investment process



Source: UBS

This report is not about the many tools that UBS has for helping with this process, such as the UBS Portfolio Analysis System but it is about active returns. Many people associate the use of active returns with portfolio evaluation, in particular, the reporting process.

Active returns are relevant to many different participants

However, active returns are used in nearly all parts of the process (save perhaps trading) and so can also affect portfolio formation. For this reason, this report should be relevant to anyone involved in evaluating investments, risk modelling,

[2] Clearly there are many more relationships than depicted here. For example, liquidity and transaction costs are usually inputs to the portfolio construction process as well, and there should be feedback from portfolio evaluation into portfolio formation.

portfolio construction, performance analysis or attribution analysis. Let us consider the impact of active return calculations with just two examples.

Illustration

Investment decisions can be affected by active returns

To buy or not to buy: Consider a portfolio manager who is approached by an analyst trying to convince the manager of buying into a stock, a sector or a country. The evidence that the analyst provides is based on a study of past relative performance in certain market conditions. It would be distressing to find another analyst come up with contradictory evidence based on the same data, over the same period and the same market conditions. It would be particularly unsettling if the differences in opinion came down to different active return numbers *using the same method of calculating active returns*. This can happen, for example:

- When one analyst uses daily data, and another uses monthly data; or
- When the active returns are calculated as part of a longer time series of active returns by one analyst and not the other.

The calculation of active returns can affect investment decisions, and consistency is very important.

Fair bonuses, an example: Harry, a CEO of a fund management organisation, is in the happy situation of allocating bonuses after a year in which the fund had performed very strongly. As the performance of the fund is measured in relative terms, so too are the staff. The analysts are assessed on their recommendations of stocks under their responsibility relative to their portion of the benchmark; portfolio managers are assessed on their value-added over and above that of the analysts. The asset allocation team must also be rewarded according to its contribution. Performance attribution needs to be performed and bonuses distributed. However, there is one stumbling block. The way Harry calculates relative returns produces results such as:

- Abi underperforms Boris, and yet Boris also underperforms Abi over the same period.
- Alfred outperforms Zac, Zac outperforms Betty and yet Alfred underperforms Betty.

Should Harry simply pay bonuses according to subjective criteria, or should he try to find a way of calculating relative performance without these problems?

Overview

In this work, we outline the desirable qualities of an active return metric; we give some alternatives and, by reference to a numeric example, see that they give rise to different values. We next give a replicating portfolio interpretation to the metrics. This allows us to discuss the problems with the different metrics.

We conclude with a look at different applications of the various metrics. In the appendices, we look at the materiality of our concerns and outline proofs of some of the claims made.

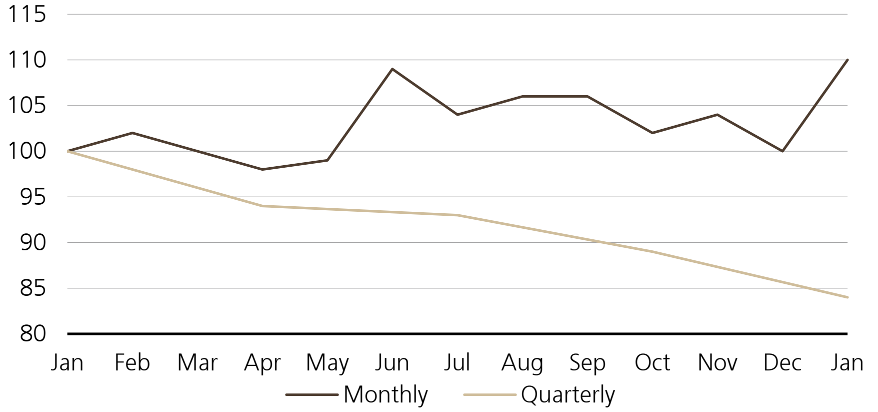
Some of this work is technical. As an aid, the next section provides a nontechnical summary with hand-waving arguments for some of our main findings.

Non-technical summary

Perhaps the simplest way of calculating multi-period active returns is to take the sum of portfolio returns less benchmark returns. However, this does not take into account the compounding nature of returns. As such, one can arrive at different results whether one chooses daily, monthly or quarterly returns to calculate it (see Figure 6).

Results inconsistent for different data frequencies

Figure 6: Illustrative example of active returns using monthly and quarterly returns

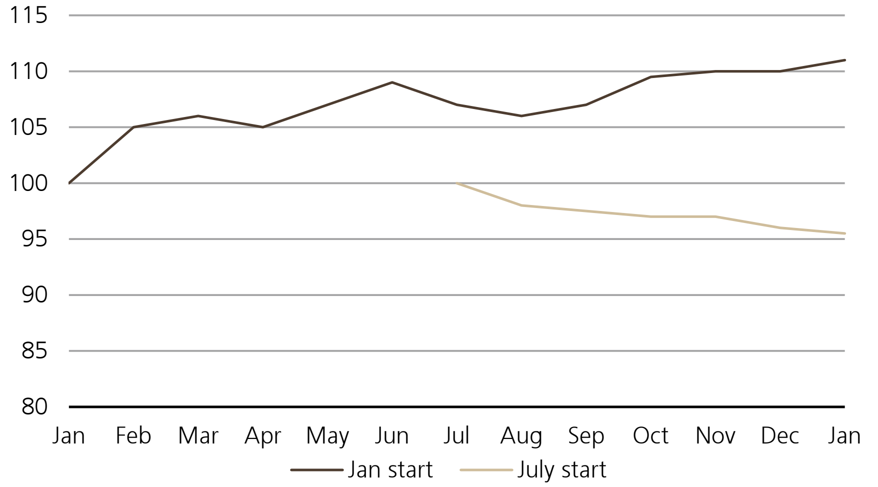


Source: UBS

In order to take into account the compounding nature of returns, some practitioners adopt a different method for calculating active returns. One method is to take the difference between the compounded portfolio returns and the compounded benchmark returns. Unfortunately, prior returns can affect the results: there can be a mirage of outperformance during a period of underperformance.

Results are inconsistent for different starting periods

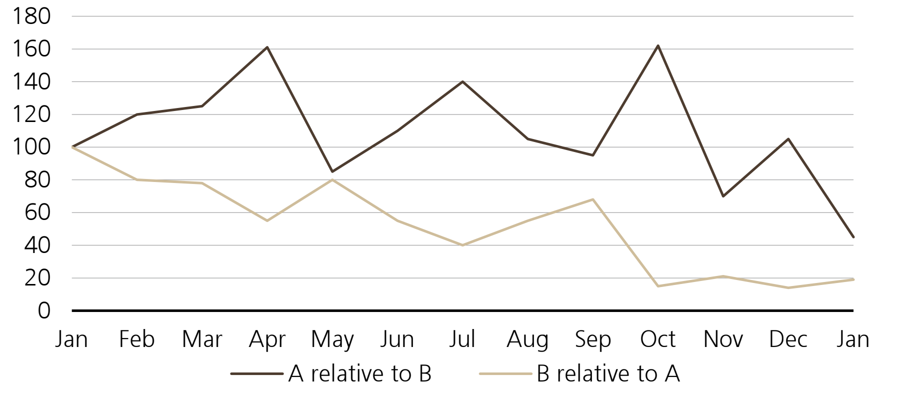
Figure 7: Illustrative example of different starting times



Source: UBS

An alternative approach to the compounding problem that is often adopted is to compound the differences between the portfolio returns and the benchmark returns. However, this approach suffers from several problems, most disturbing of which is that one can have situations in which a portfolio underperforms the benchmark and the benchmark underperforms the portfolio at the same time. For example, Figure 8 shows A underperforming B by around 55% and B underperforming A by around 85%.

Figure 8: Illustrative example showing A relative to B and B relative to A



Source: UBS

Results are inconsistent depending on what is regarded as the benchmark

The approach we would advocate uses log returns. While the numbers are harder to interpret over a single period, the numbers coming from all approaches over multiple periods do not have a straightforward interpretation because of these problems. In any case, using log returns to calculate the relative performance does not result in the problems illustrated and so always gives an outcome: outperformance means outperformance.

In this document we discuss these issues in more depth as well as giving some other results. For the non-technical reader, the materiality of the problems and the appendix can be skipped over without any great loss in understanding. We do, however, recommend reading the other sections to obtain a fuller picture. Definitions of the criteria and methods follow in the next two sections, and then we provide a long/short replicating portfolio interpretation. The following section attempts to describe the mechanics behind the problems with some of the methods for calculating active returns. Finally, the applications and conclusions section gives some applied perspective to the problem.

Criteria for active returns

For a portfolio, A, and a benchmark, B, it is desirable that an active return measure induce the relations:

- 'Underperform': A relative to B < 0
- 'Perform the same as': A relative to B = 0
- 'Outperform': A relative to B > 0,

with the following qualities, at least.

(1) **Path independent**: The induced relations do not depend on prior or subsequent returns.

(2) **Sampling frequency invariance**: The induced relations do not depend on the frequency of returns used to calculate them.

(3) **Equivalence**: 'Perform the same as' is an equivalence relation.

  a. **Reflexive**: A 'performs the same as' A.

  b. **Symmetric**: A 'performs the same as' B if, and only if, B 'performs the same as' A.

  c. **Transitive**: A 'performs the same as' Z and Z 'performs the same as' B implies that A 'performs the same as' B.

(4) **Strict inequality**: Both 'underperforms' and 'outperforms' are strict inequality relations.

  a. **Non-reflexive**: It is not the case that A 'underperforms' A, or that A 'outperforms' A.

  b. **Asymmetric**: A 'underperforms' B implies that it is not the case that B 'underperforms' A. Similarly, A 'outperforms' B implies that it is not the case that B 'outperforms' A.

  c. **Transitive** A 'underperforms' Z and Z 'underperforms' B implies that A 'underperforms B. Similarly, A 'outperforms' Z and Z 'outperforms' B implies that A 'outperforms' B.

This may seem both technical and yet obvious, but unfortunately not all metrics satisfy these minimum criteria. Intuitively, we would like our measure to be able to tell us when our portfolio was outperforming and when it was underperforming some benchmark, and we want the meaning of this to be unambiguous

Alternatives for active returns

We list the main alternatives for calculating active returns used by practitioners. Over the period t_1 to t_2, we use the notation: $r_A^{t_1,t_2}$ for the return to the portfolio, $r_B^{t_1,t_2}$ for the return to the benchmark.

(5) **Simple active**: The difference between returns to the portfolio and returns to the benchmark, summed, or, equivalently, the difference between the sum of portfolio returns and benchmark returns.

$$\sum_r \left(r_A^{t,t+1} - r_B^{t,t+1}\right) = \sum_t r_A^{t,t+1} - \sum_t r_B^{t,t+1}$$

(6) **Index difference**: The difference between the compounded returns to the portfolio and the compounded returns to the benchmark.

$$\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_B^{t,t+1}\right)$$

(7) **Compounded active**: The difference between returns to the portfolio and returns to the benchmark, compounded.

$$\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right) - 1$$

(8) **Log returns**: The difference between portfolio log returns and benchmark log returns, summed.

$$\sum_t \left(ln\left(1 + r_A^{t,t+1}\right) - ln\left(1 + r_B^{t,t+1}\right)\right)$$

(9) **Index ratio**: The ratio of the compounded returns to the portfolio and the compounded returns to the benchmark.

$$\frac{\prod_t \left(1 + r_A^{t,t+1}\right)}{\prod_t \left(1 + r_B^{t,t+1}\right)} - 1$$

The simplest of these is the simple active returns metric, but as it does not take into account the compounding nature of returns practitioners tend to opt for either the index difference or compounded active returns metrics. Many quantitative analysts would agree that single asset returns should be modelled in log-space. However, when it comes to portfolios, the lack of additivity of the portfolio's constituent returns combined with the fact that

$$\ln(1 + r) \approx r \text{ when } r \approx 0$$

means that practitioners do not usually work with log returns. The index ratio that we would prescribe for those who would shy from the log returns metric can be seen to yield the same results, as there is a one-to-one mapping between the two metrics. In fact, the log returns metric can be written in many useful forms:

$$\sum_t \left(ln\left(1+r_A^{t,t+1}\right) - ln\left(1+r_B^{t,t+1}\right)\right) = \sum_t ln\left(1+r_A^{t,t+1}\right) - \sum_t ln\left(1+r_B^{t,t+1}\right)$$

$$= ln\left(\frac{\prod_t\left(1+r_A^{t,t+1}\right)}{\prod_t\left(1+r_B^{t,t+1}\right)}\right)$$

$$= \ln\left(\prod_t\left(1+r_A^{t,t+1}\right)\right) - ln\left(\prod_t\left(1+r_B^{t,t+1}\right)\right)$$

Problems with metrics 1 to 3; but metrics 4 and 5 are less conventional

As we will soon see, metrics 1 to 3, above, do not satisfy our criteria for an active return metric. The main problems with the 4th and 5th metrics are that they are not so well recognised and are somewhat incomprehensible to non-practitioners. Hence, we would argue that practitioners should use either the 4th or 5th metrics in the decision making process, resorting to one of the other metrics (perhaps the 3rd) for reporting, fully aware of the weaknesses of that metric.

An example

Let us just consider a two-period example, in which a portfolio, A, returns 55% and then 49%, and the benchmark, B, returns 50% and then 50%. For future reference we calculate the alternate active return metrics in Figure 9.

Figure 9: Active return calculations using five alternative metrics

Returns (%)	0 →1	1 →2	0 →2
Portfolio A	55.00	49.00	130.95
Benchmark B	50.00	50.00	125.00
Simple active	5.00	-1.00	4.00
Index difference	5.00	-1.00	5.95
Compound active	5.00	-1.00	3.95
Log returns	3.28	-0.67	2.61
Index ratio	3.33	-0.67	2.64

Source: UBS

As we can see from Figure 9, each of the five metrics gives us a different answer for the active returns. But how do we know which is correct? We will soon discuss the shortcomings of the first 3 of these metrics, but first we believe that it is of interest to describe the different approaches in terms of replicating portfolios.

Long/short interpretation

We give a long/short interpretation to four of these metrics by considering an investment of $100. For our purposes, in these theoretical replications, cash earns 0% return and there are no transaction costs or other market frictions. As index difference is the most straightforward metric to replicate, we start there.

Long / short interpretation: start with $100

Index difference replication: Buy $100 worth of portfolio A, (short) sell $100 worth of benchmark B and keep $100 worth of cash. We hold the position for the entire analysis period, observing changes to our replication portfolio value to generate our return series.

Do not rebalance exposures

Simple active replication: Buy $100 worth of portfolio A, (short) sell $100 worth of benchmark B and keep $100 worth of cash. At the end of each period, we rebalance the portfolio such that we are long $100 of portfolio A, short $100 of benchmark B and keep any surplus as cash. One might think of this strategy as the same as the index difference replication but with frequent rebalancing to ensure that the exposure to the two assets is maintained at $100.

Rebalance exposures to $100

Compounded active replication: Buy $100 worth of portfolio A, (short) sell $100 worth of benchmark B and keep $100 worth of cash. At the end of each period, we value the replicating portfolio as $PV. We then rebalance the portfolio such that we are long $PV of portfolio A, short $PV of benchmark B and keep $PV worth of cash. One might think of this strategy as the same as the index difference replication but with frequent rebalancing to ensure that the exposure to the two assets is maintained at the portfolio value.

Rebalance exposures to portfolio value

Index ratio replication: Buy $\gamma_0 \times$ \$100 worth of portfolio A, (short) sell $\gamma_0 \times$ \$100 worth of benchmark B and keep $100 worth of cash. At the end of each period, we value the replicating portfolio as $PV. Rebalance the portfolio at the end of each period such that we are long $\gamma_t \times$\$PV of portfolio A, short $\gamma_t \times$\$PV of benchmark B and have keep $PV worth of cash. Again we are rebalancing frequently, but this time with respect to γ_t. As we will see, γ_t is not known at the time of rebalancing, hence it is not practically possible to implement this strategy.

Initially invest $\gamma_0 \times$ \$100; rebalance exposures to $\gamma_t \times$\$PV

Log returns replication: Log returns replication might have an interpretation similar to that of the index ratio replication, where rebalancing is performed in continuous time, but we choose not to examine that here.

Continuous rebalancing?

Illustration

Let us return to our two-period example. Here we write Time 1^- for the end of the first period prior to rebalancing and Time 1^+ for the end of the first period after rebalancing. No rebalancing is done for the index difference replicating portfolio example, as seen in Figure 10.

Figure 10: Index difference replicating portfolio example

Do not rebalance exposures

	Time 0	Time 1^-	Time 1^+	Time 2
A	$100.00	$155.00	$155.00	$230.95
B	-$100.00	-$150.00	-$150.00	-$225.00
C	$100.00	$100.00	$100.00	$100.00
Total	$100.00	$105.00	$105.00	$105.95

Source: UBS

For the simple active replicating portfolio, the exposure to the portfolio A is rebalanced to long \$100, and the exposure to benchmark B is rebalanced to short \$100, after period 1, as shown in Figure 11.

Figure 11: Simple active replicating portfolio example

Rebalance exposures to \$100

	Time 0	Time 1⁻	Time 1⁺	Time 2
A	\$100.00	\$155.00	\$100.00	\$149.00
B	-\$100.00	-\$150.00	-\$100.00	-\$150.00
C	\$100.00	\$100.00	\$105.00	\$105.00
Total	\$100.00	\$105.00	\$105.00	\$104.00

Source: UBS

After period 1, the value of the compounded active replicating portfolio is \$105. Hence, the exposure to the portfolio A is rebalanced to \$105, and the exposure to benchmark B is rebalanced to short \$105, after period 1, as shown in Figure 12.

Figure 12: Compound active replicating portfolio example

Rebalance exposures to portfolio value (\$105)

	Time 0	Time 1⁻	Time 1⁺	Time 2
A	\$100.00	\$155.00	\$105.00	\$156.45
B	-\$100.00	-\$150.00	-\$105.00	-\$157.50
C	\$100.00	\$100.00	\$105.00	\$105.00
Total	\$100.00	\$105.00	\$105.00	\$103.95

Source: UBS

Finally, for the index ratio replicating portfolio, the exposure to the portfolio A is initially \$66.67, and the exposure to benchmark B is initially to short \$66.67. After period 1, they are rebalanced to long \$68.89 and short \$68.89, as shown in Figure 13.

Figure 13: Index ratio replicating portfolio example

Start with \$66.67 exposures; rebalance exposures to \$68.89

	Time 0	Time 1⁻	Time 1⁺	Time 2
A	\$66.67	\$103.33	\$68.89	\$102.64
B	-\$66.67	-\$100.00	-\$68.89	-\$103.33
C	\$100.00	\$100.00	\$103.33	\$103.33
Total	\$100.00	\$103.33	\$103.33	\$102.64

Source: UBS

Why are such odd amounts chosen for the replicating portfolio in the case of the index ratio metric? The return over all periods should be equal to:

$$\frac{\prod_t \left(1 + r_A^{t,t+1}\right)}{\prod_t \left(1 + r_B^{t,t+1}\right)} - 1 = \prod_t \left(\frac{1 + r_A^{t,t+1}}{1 + r_B^{t,t+1}}\right) - 1 = \prod_t \left(1 + \frac{r_A^{t,t+1} - r_B^{t,t+1}}{1 + r_B^{t,t+1}}\right) - 1$$

In other words, it is the same as the compounding of the one-period returns:

$$\frac{r_A^{t,t+1} - r_B^{t,t+1}}{1 + r_B^{t,t+1}}$$

This means that, as long as we rebalance the portfolio to the portfolio value multiplied by $\gamma_t = \frac{1}{1+r_B^{t,t+1}}$ we will be able to replicate the index ratio metric.

Rebalance to replicate index ratio returns

Unfortunately, as γ_t depends on the benchmark returns over the next period, it is not known at time t, so it is not possible to construct a portfolio that replicates the index ratio metric in practice.

Not practical (requires knowledge of the future)

We see that in each of these cases, the value of the replicating portfolios through time lead to the same returns as the metrics they replicate. But what are the problems that face these metrics?

Problems with the active return metrics

Index difference

Index difference depends on earlier returns

From the replicating portfolio analogy of Figure 10, we see that from Time 1 to Time 2 the index difference metric increases in our example from $105.00 to $105.95. This is despite the fact that the portfolio underperforms the benchmark over this period. Why is this a problem? Suppose that we only started investing in this strategy after Time 1. Then our active return would be –1%, even though the two-period analysis suggests that we would make $0.95 on our $105 = +0.90%. Hence the results that we obtain for the period 1→2 using the index difference metric depend on the results that we obtain for the period 0→1: The index difference active returns are not path independent.

What is the cause of the problem? Path dependence is because the performance over 1→2 is measured on exposures of $155 and –$150, rather than matched exposures such as $100 and -$100.

Simple active

Simple active returns depend on sampling frequency

Simple active returns are path independent (see the appendix). However, in Figure 9 we notice another problem. The active return that we calculate using one-period returns give us a 4% active return over the two periods. However, if we use the two-period return numbers for portfolio A and benchmark B to derive the returns using the simple active return metric, we get a return of 5.95%. In general, *simple active returns are not sampling frequency invariant.*

What is the cause of the problem? Ironically, as we have changed the exposures to circumvent the problem of path dependence, we have introduced this problem. The exposures of $100 and –$100 at Time 1^+ disregard the performance over the period 0→1 and so active returns are unable to compound appropriately over different sampling frequencies.

Compounded active

Compounded active is again path independent and not sampling frequency invariant. This follows for exactly the same reasons as for the simple active returns. However, there is perhaps a more profound problem. Suppose that we now look at the active return of the benchmark against the portfolio. We turn again to our replicating portfolio analogy to see what is happening.

Figure 14: Compounded active replicating portfolio example

	Time 0	Time 1^-	Time 1^+	Time 2
B	$100.00	$150.00	$95.00	$142.50
A	-$100.00	-$155.00	-$95.00	-$141.55
C	$100.00	$100.00	$95.00	$95.00
Total	$100.00	$95.00	$95.00	$95.95

Source: UBS

As the roles of the benchmark and the portfolio have been reversed, this time when we rebalance, we rebalance to $95 (*cf* $105 in Figure 12). This means that ultimately we get an active return of the benchmark over the portfolio of –4.05% compared with +3.95% for the portfolio relative to the benchmark. The fact that these do not sum to zero, and that the product of one plus these numbers do not multiply to one, should, we believe, flag a potential problem. The problems are best illustrated if we change the numbers in our example.

Suppose now that A returns 0% then 0% and that B returns 20% then –20%. Then in Figure 15 we see that A underperforms B and B underperforms A according to this metric. Hence, compounded active does not induce an asymmetric underperformance relation.

Figure 15: Compounded active example with A: 0%, 0% and B: 20%, -20%

	Time 0	Time 1-	Time 1+	Time 2
A	$100	$100	$80	$80
B	-$100	-$120	-$80	-$64
C	$100	$100	$80	$80
Total	$100	$80	$80	$96

	Time 0	Time 1-	Time 1+	Time 2
B	$100	$120	$120	$96
A	-$100	-$100	-$120	-$120
C	$100	$100	$120	$120
Total	$100	$120	$120	$96

Source: UBS

Suppose now that A returns 0% then 0% and that B returns 25% then –20%. Then in A underperforms B even when nothing happens

Figure 16 we see that although A and B both have the same price as when they started, that the portfolio could yield an active return of –10% depending on which is relative to which. This is another illustration that this metric is not sample frequency invariant, but also that compounded active does not induce a symmetric 'performs the same as' relation. Note that in the previous example from Figure 15, A has a two-period return of 0% and B has a return of –4%, so A relative to B is negative, even though A outperformed B over the two periods, also illustrating sample frequency dependence.

A underperforms B even when nothing happens

Figure 16: Compounded active example with A: 0%, 0% and B: 25%, -20%

	Time 0	Time 1^-	Time 1^+	Time 2
A	$100.00	$100.00	$75.00	$75.00
B	-$100.00	-$125.00	-$75.00	-$60.00
C	$100.00	$100.00	$75.00	$75.00
Total	$100.00	$75.00	$75.00	$90.00

Source: UBS

Finally, let us consider three portfolios: A, Z, B with returns (0%, 4%), (25%, -24%) and (0%, 0%), respectively. Then Figure 17 shows us that this metric would give a return of A relative to Z of –4% and Z relative to B of –5%. In practice this might be enough for us to dismiss both strategies, even though this metric would give a return of 4% for A relative to B. Hence, A underperforms Z and Z underperforms B but A outperforms B – *compounded active does not induce a transitive relation.*

Not transitive; A underperforms Z and Z underperforms B; but A outperforms B

Figure 17: Compounded active with A: 0%, 4%; Z: 25%, -24% and B: 0%, 0%

	Time 0	Time 1-	Time 1+	Time 2
A	$100	$100	$75	$78
Z	-$100	-$125	-$75	-$57
C	$100	$100	$75	$75
Total	$100	$75	$75	$96

	Time 0	Time 1-	Time 1+	Time 2
Z	$100	$125	$125	$95
B	-$100	-$100	-$125	-$125
C	$100	$100	$125	$125
Total	$100	$125	$125	$95

Source: UBS

Again the cause of the problems is the rebalancing. In Figure 12 and Figure 14 we see rebalancing to $105 and $95 respectively is inconsistent and in Figure 15 rebalancing to $80 and $120 is also inconsistent. This leads to the conclusion that underperformance can be asymmetric. Similarly, the inconsistency of rebalancing to $75 and $125 in Figure 17 induces a lack of transitivity in performance relations.

Log returns and the index ratio

No problems; but less conventional

Against the criteria to which we believe the metrics should adhere, log returns and index ratio metrics have no problems. However, because these metrics are less conventional, we understand that the other metrics will continue to be used for reporting. We hope that only the log returns and index ratio metrics will be used for decision making.

Summary of problems with active return metrics

Figure 18 lists the metrics against the criteria that we listed at the outset. Proofs for the satisfaction of the criteria are given in Appendix II.

Figure 18: Active return metrics and their criteria

		Index difference	Simple active	Compound active	Log returns	Index ratio
Path independent		×	✓	✓	✓	✓
Sampling frequency invariance		✓	×	×	✓	✓
"Performs the same as"	Reflexivity	✓	✓	✓	✓	✓
	Symmetry	✓	✓	×	✓	✓
	Transitivity	✓	✓	×	✓	✓
"Underperforms"	Non-reflexivity	✓	✓	✓	✓	✓
or	Asymmetry	✓	✓	×	✓	✓
"Outperforms"	Transitivity	✓	✓	×	✓	✓

Source: UBS

Materiality of problems

As we show in Appendix I these differences can be material:

- Errors associated with path dependence can be 200% of the value being measured!
- Errors associated with sampling frequency invariance are typically 20% to 30% over a 10-year study.
- Errors associated with symmetry are often small, but disturbing: for this kind of error, even if small, can dramatically undermine confidence in the analysis.
- Errors associated with transitivity are typically around 2–3% pa.

For all these kinds of errors, problems of this magnitude can lead to incorrect conclusions and undermine the value of other analysis.

Applications and conclusions

Reporting

As the fund owners do often not accept the use of log returns for reporting, it is necessary to find a different way of reporting the result. As all metrics described in this report can give rise to different results, the choice of metric can greatly affect the results that the fund owners see. By selectively choosing the parameters such as the sampling frequency, fund managers may give a false impression of their performance. While we would not advocate this practice, managers should be aware of the degree to which their results can vary according to the way the results are reported. Furthermore, an understanding of the limitations of different metrics can be used to defend investment decisions.

Performance attribution / style analysis

Performance attribution is one area in which active return calculations are used, and some of the problems that are described in this report come to the fore. In particular, performance attribution and style analysis is about disaggregating historical performance and apportioning it to different factors, normally relative to a benchmark.

In some simple cases, attribution may be calculated by classifying the assets in the portfolio according to the factors in question. This often results in factor portfolios with few assets. As small portfolios tend to have higher volatility, any active return results can be more prone to methodological problems, depending on the approach adopted. In this case, path independence, sampling frequency invariance and asymmetry of returns are all important considerations.

In the more general case, the analysis is calculated by partitioning the returns for each asset for attribution to each factor. This occurs when the factors are not mutually exclusive in terms of the asset exposures. In this case, the partitioning may be calculated sequentially or simultaneously. In the simultaneous calculation case, once the results are known, it is possible to rewrite the partitioning problem in terms of sequential analysis. So in either case, we may split the attribution problem into two parts for each factor:

(1) Determine the attribution for that factor and

(2) Residualise for attribution against other factors.

The residualisation process has much in common with the calculation of active returns. This means that many of the concerns we have raised with the calculation methodologies map onto the performance attribution problem. In particular, note that the importance of a metric being transitive if it is being used in performance attribution for multiple factors. Again, the volatility of the factors can cause problems in performance attribution.

Investment decisions

Where outside parties are not involved, we would strongly advocate the use of log returns (or equivalently, the index ratio method) for calculating multi-period active returns. These methods are path independent, so are appropriate for making decisions concerning factor timing. These methods are not dependent on the sampling frequency, so will give consistent results in this respect. These methods have asymmetry, so that the user can be assured that shorting a strategy that produces negative returns would have yielded positive returns. Finally, these methods are transitive, so the combination of strategies will be consistent with the parts. In short, the use of other metrics can lead to incorrect conclusions; these metrics, however, are not subject to these problems.

Quiz answers

1. (d) The FTSE has had an active return of –63% relative to the S&P-500 since February 2009, using the log returns metric.

2. False. It is not the case that the FTSE 100 outperformed the S&P-500 over the period 31 December 2008 to 31 December 2012. In fact, it underperformed by around 4.9% over this period, using the log returns metric.

Footnote

It is hoped that this report will allow the portfolio manager who was approached by analysts with contradictory results to be able to resolve this issue, and that Harry, our fund manager CEO from our initial illustration, will allocate bonuses fairly.

Appendix I: Are these concerns material?

As violation of these criteria can lead to incorrect conclusions and therefore incorrect investment decisions, then the short answer is "yes". However, in this section we strive to quantify the degree of materiality.

Path dependence

If we break up the **index difference** metric into two disjoint but arbitrary periods, $[t_1, t_2)$ and $[t_2, t_3)$, then we can see the effect of the first period on the second period's values. Writing r_Δ for the active return of A against B (using the index difference metric), and using the shorthand for each index,

$$A_i = \prod_{t=t_i}^{t_{i+1}-1} \left(1 + r_A^{t,t+1}\right)$$

$$B_i = \prod_{t=t_i}^{t_{i+1}-1} \left(1 + r_B^{t,t+1}\right)$$

$$\Delta_i = 1 + \prod_{t=t_i}^{t_{i+1}-1} \left(1 + r_A^{t,t+1}\right) - \prod_{t=t_i}^{t_{i+1}-1} \left(1 + r_B^{t,t+1}\right)$$

Then we can write the one- and two-period version of the index difference metric, writing ε for the error due to path dependence:

$$\Delta_0 = 1 + A_0 - B_0$$
$$\Delta_1 = 1 + A_1 - B_1$$
$$\Delta_0\Delta_1 = 1 + A_0A_1 - B_0B_1 + \varepsilon$$

Solving for ε and re-arranging:

$$\varepsilon = (1 - B_0)(A_1 - B_1) + (A_0 - B_0)(1 + B_1)$$

So we see that the error comes in two parts: (a) there is an error in future index differences multiplied by past differences of the benchmark index from 1, and (b) there is an error due to past index differences multiplied by future benchmark index differences from 1.

An error due to path dependence can be twice the value being measured

In the case of a 10-year study, typical benchmark indices might well have tripled, so the first term might contribute around minus twice the difference between the portfolio and benchmark indices to the error. In other words, this could contribute an error of around 200% of the value that we are trying to measure. The second term contributes an error regardless of the future active returns. In the case of a 10-year study where the portfolio has returned only 2% pa over a typical benchmark, about 60% of any future benchmark moves will be contributed to the error. If the portfolio has greater deviation from the benchmark, then the error will be even more significant.

The index difference method of calculating active returns is path dependent and errors associated with this over long periods are typically very material.

Sampling frequency dependence

Simple active returns are dependent on the sampling frequency. For example, consider the results that we get for a period using high frequency data:

$$\sum_t r_A^{t,t+1} - \sum_t r_B^{t,t+1}$$

With the results that we get using single period data

$$\prod_t (1 + r_A^{t,t+1}) - \prod_t (1 + r_B^{t,t+1})$$

We can express the error as:

$$\varepsilon = \sum_t r_A^{t,t+1} - \sum_t r_B^{t,t+1} - \prod_t (1 + r_A^{t,t+1}) + \prod_t (1 + r_B^{t,t+1})$$

$$= \sum_t r_A^{t,t+1} - \sum_t r_B^{t,t+1}$$

$$- \left(1 + \sum_t r_A^{t,t+1} + \sum_{t_1 \neq t_2} r_A^{t_1,t_1+1} r_A^{t_2,t_2+1} + O\left(\sum_{t_1 \neq t_2, t_2 \neq t_3} r_A^{t_1,t_1+1} r_A^{t_2,t_2+1} r_A^{t_3,t_3+1}\right)\right)$$

$$+ \left(1 + \sum_t r_B^{t,t+1} + \sum_{t_1 \neq t_2} r_B^{t_1,t_1+1} r_B^{t_2,t_2+1} + O\left(\sum_{t_1 \neq t_2, t_2 \neq t_3} r_B^{t_1,t_1+1} r_B^{t_2,t_2+1} r_B^{t_3,t_3+1}\right)\right)$$

$$= - \sum_{t_1 \neq t_2} r_A^{t_1,t_1+1} r_A^{t_2,t_2+1} + \sum_{t_1 \neq t_2} r_B^{t_1,t_1+1} r_B^{t_2,t_2+1} + Higher\ order\ terms$$

Hence, the error depends on the volatility of the portfolio and the volatility of the benchmark as well as higher order terms. For well-diversified portfolios, such as typical benchmarks, calculating the results using daily returns would lead to an error of around 7% over 10 years compared with the calculation using annual returns, and the errors would tend to cancel between the portfolio and the benchmark. However, if the portfolio has a high volatility, say of 30% pa, then the active returns calculated using the simple active metric might be different by around 40% using daily versus annual data, over a 10-year study. This could be very material, except to say that highly volatile processes would probably be penalised elsewhere in the investment research process.

The error is typically around 7% over 10 years

The **compounded active** metric is also dependent on the sampling frequency. For example, consider the results that we get for a period using high frequency data:

$$\prod_t (1 + r_A^{t,t+1} - r_B^{t,t+1}) - 1$$

with the results that we get using single period data:

$$\prod_t (1 + r_A^{t,t+1}) - \prod_t (1 + r_B^{t,t+1})$$

We can express the error as:

$$
\begin{aligned}
\varepsilon = &\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right) - 1 - \prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_B^{t,t+1}\right) \\
&= \Bigg(1 + \sum_t \left(r_A^{t,t+1} - r_B^{t,t+1}\right) \\
&- \sum_{t_1 \neq t_2} \left(r_A^{t_1,t_1+1} - r_B^{t_1,t_1+1}\right)\left(r_A^{t_2,t_2+1} - r_B^{t_2,t_2+1}\right)\Bigg) - 1 \\
&- \left(1 + \sum_t r_A^{t,t+1} + \sum_{t_1 \neq t_2} r_A^{t_1,t_1+1} r_A^{t_2,t_2+1}\right) \\
&+ \left(1 + \sum_t r_B^{t,t+1} + \sum_{t_1 \neq t_2} r_B^{t_1,t_1+1} r_B^{t_2,t_2+1}\right) + Higher\ order\ terms \\
\approx &- \sum_{t_1 \neq t_2} \left(r_B^{t_1,t_1+1}\left(r_A^{t_2,t_2+1} - r_B^{t_2,t_2+1}\right) + \left(r_A^{t_1,t_1+1} - r_B^{t_1,t_1+1}\right) r_A^{t_2,t_2+1}\right)
\end{aligned}
$$

The errors are often 20-30% over 10 years

So for the compounded active metric, the difference between results based on high- and low-frequency data is related to the covariance between the benchmark returns and the returns of the portfolio less the benchmark. The errors due to the compounded active metric are of a similar magnitude to those using the simple active metric. As the error depends on the covariance, the results will be considerably different depending on the relationship between the portfolio and the benchmark.

Simple active and compounded active metrics for calculating active returns depend on the sampling frequency of the data used, and we find that the discrepancies are material.

Non-asymmetry

The **compounded active** metric is not asymmetric. This means that, under this measure, if the portfolio's return relative to the benchmark is negative, then the benchmark's return relative to the portfolio is not necessarily positive.

Mathematically, we would like it if:

$$
\begin{aligned}
\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right) - 1 > 0 &\Leftrightarrow \prod_t \left(1 + r_B^{t,t+1} - r_A^{t,t+1}\right) - 1 < 0 \\
\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right) > 1 &\Leftrightarrow \prod_t \left(1 + r_B^{t,t+1} - r_A^{t,t+1}\right) < 1
\end{aligned}
$$

Furthermore, it would be nice if the 'outperformance' signal were as strong as the 'underperformance' signal. Incorporating an error term, we can write our desired equality:

$$
\begin{aligned}
1 + \varepsilon &= \left(\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right)\right)\left(\prod_t \left(1 + r_B^{t,t+1} - r_A^{t,t+1}\right)\right) \\
&= \prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right)\left(1 + r_B^{t,t+1} - r_A^{t,t+1}\right) \\
&= \prod_t \left(1 + \left(r_A^{t,t+1} - r_B^{t,t+1}\right)^2\right)
\end{aligned}
$$

For a portfolio with a tracking error of 3% pa, we see that the error term would be around 0.9% for a 10-year study. While this is probably not material, a tracking error of 10% pa gives rise to an error term of almost 10% over the 10 years. This may still appear to be immaterial, but the result that A underperforms B and that B underperforms A over the same period is disturbing.

Errors are normally small but disturbing

Appearance of this problem, no matter how small, can compromise the acceptance of the research applied. Furthermore, later in this report we discuss applications like performance attribution in which the volatility can grow.

Transitivity

The **compounded active** metric does not exhibit transitivity. Ideally, we would like the product of active returns of A relative to Z and of Z relative to B to be equal to the active returns of A relative to B. Again using an error term, we can assess the degree to which this is inaccurate.

$$\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right) + \varepsilon = \left(\prod_t \left(1 + r_A^{t,t+1} - r_Z^{t,t+1}\right)\right)\left(\prod_t \left(1 + r_Z^{t,t+1} - r_B^{t,t+1}\right)\right)$$
$$= \prod_t \left(1 + r_A^{t,t+1} - r_Z^{t,t+1}\right)\left(1 + r_Z^{t,t+1} - r_B^{t,t+1}\right)$$

Introducing a new error variable on a per-period basis,

$$\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1} + \varsigma^t\right) = \prod_t \left(1 + r_A^{t,t+1} - r_Z^{t,t+1}\right)\left(1 + r_Z^{t,t+1} - r_B^{t,t+1}\right)$$
$$\varsigma^t = r_A^{t,t+1} r_Z^{t,t+1} - r_Z^{t,t+1} r_Z^{t,t+1} - r_A^{t,t+1} r_B^{t,t+1} + r_Z^{t,t+1} r_B^{t,t+1}$$

In assessing the materiality of this, if we assume that Z is uncorrelated (worse if negatively correlated) with A and with B, then the per-period error will be related to the variance of Z and the covariance of A and B. This could be around 2-3% pa for fairly well diversified portfolios but around 20% pa for portfolios with a risk of around 30% pa. The per-period error compounds, but it also magnified by the size of the relative return calculation. Hence, having a strong result for A relative to B means that the errors associated with the result are also going to be large.

Errors are often around 2-3% pa

The compounded active metric does not induce transitive relations, and we find that this can be fairly material.

Appendix II: Outline of proofs

Index difference

Sampling frequency invariant

For all low frequency data, consider a partitioning (perhaps unevenly) into two parts. For X = the portfolio, A, and the benchmark, B, write the returns for the low frequency data as $R_X^{t_1,t_2}$ and the returns for high frequency data as $r_X^{t_1,t_2}$, such that

$$1 + R_X^{t_1,t_3} = \left(1 + r_X^{t_1,t_2}\right)\left(1 + r_X^{t_2,t_3}\right)$$

Then

$$\begin{aligned}
&\prod_t \left(1 + R_A^{t,t+1}\right) - \prod_t \left(1 + R_B^{t,t+1}\right) \\
&= \prod_t \left(1 + r_A^{t,t\cdot}\right)\left(1 + r_A^{t\cdot,t+1}\right) - \prod_t \left(1 + r_B^{t,t\cdot}\right)\left(1 + r_B^{t\cdot,t+1}\right) \\
&= \prod_s \left(1 + r_A^{s,s+1}\right) - \prod_s \left(1 + r_B^{s,s+1}\right)
\end{aligned}$$

The general case may be derived through induction on the proof above over the partitioning adopted.

Induces equivalence of ‘performs the same as’

‘Performs the same as’ is an equivalence relation as it is reflexive, symmetric and transitive:

$$\begin{aligned}
&\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_A^{t,t+1}\right) = 0 \\
&\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_B^{t,t+1}\right) = 0 \Rightarrow \prod_t \left(1 + r_B^{t,t+1}\right) - \prod_t \left(1 + r_A^{t,t+1}\right) = 0 \\
&\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_Z^{t,t+1}\right) = 0 \; and \prod_t \left(1 + r_Z^{t,t+1}\right) - \prod_t \left(1 + r_B^{t,t+1}\right) = 0 \Rightarrow \\
&\prod_t \left(1 + r_A^{t,t+1}\right) + \left(-\prod_t \left(1 + r_Z^{t,t+1}\right) + \prod_t \left(1 + r_Z^{t,t+1}\right)\right) - \prod_t \left(1 + r_B^{t,t+1}\right) = 0
\end{aligned}$$

Induces strict inequality of "underperformance" and "outperformance"

Outperformance is a strict inequality as it is non-reflexive, asymmetric and transitive:

$$\begin{aligned}
&\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_A^{t,t+1}\right) = 0 \\
&\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_B^{t,t+1}\right) > 0 \Rightarrow \prod_t \left(1 + r_B^{t,t+1}\right) - \prod_t \left(1 + r_A^{t,t+1}\right) < 0 \\
&\prod_t \left(1 + r_A^{t,t+1}\right) - \prod_t \left(1 + r_Z^{t,t+1}\right) > 0 \; and \prod_t \left(1 + r_Z^{t,t+1}\right) - \prod_t \left(1 + r_B^{t,t+1}\right) > 0 \Rightarrow \\
&\prod_t \left(1 + r_A^{t,t+1}\right) + \left(-\prod_t \left(1 + r_Z^{t,t+1}\right) + \prod_t \left(1 + r_Z^{t,t+1}\right)\right) - \prod_t \left(1 + r_B^{t,t+1}\right) > 0
\end{aligned}$$

Underperformance is a strict inequality relation for the similar reasons.

Simple active

Path independent

As summation and subtraction are associative, path independence follows.

Induces equivalence of "performs the same as"

"Performs the same as" is an equivalence relation as it is reflexive, symmetric and transitive:

$$\sum_t r_A^{t,t+1} - \sum_t r_A^{t,t+1} = 0$$
$$\sum_t r_A^{t,t+1} - \sum_t r_B^{t,t+1} = 0 \Rightarrow \sum_t r_B^{t,t+1} - \sum_t r_A^{t,t+1} = 0$$
$$\sum_t r_A^{t,t+1} - \sum_t r_Z^{t,t+1} = 0 \; and \; \sum_t r_Z^{t,t+1} - \sum_t r_B^{t,t+1} = 0 \Rightarrow$$
$$\sum_t r_A^{t,t+1} + \left(-\sum_t r_Z^{t,t+1} + \sum_t r_Z^{t,t+1}\right) - \sum_t r_B^{t,t+1} = 0$$

Induces strict inequality of "underperformance" and "outperformance"

Outperformance is a strict inequality as it is non-reflexive, asymmetric and transitive:

$$\sum_t r_A^{t,t+1} - \sum_t r_A^{t,t+1} = 0$$
$$\sum_t r_A^{t,t+1} - \sum_t r_B^{t,t+1} > 0 \Rightarrow \sum_t r_B^{t,t+1} - \sum_t r_A^{t,t+1} < 0$$
$$\sum_t r_A^{t,t+1} - \sum_t r_Z^{t,t+1} > 0 \; and \; \sum_t r_Z^{t,t+1} - \sum_t r_B^{t,t+1} > 0 \Rightarrow$$
$$\sum_t r_A^{t,t+1} + \left(-\sum_t r_Z^{t,t+1} + \sum_t r_Z^{t,t+1}\right) - \sum_t r_B^{t,t+1} > 0$$

For similar reasons, underperformance is a strict inequality.

Compounded active

Path independent

As multiplication is associative, $\prod_t \left(1 + r_A^{t,t+1} - r_B^{t,t+1}\right)$ is path independent.

Log return

Path independent

As summation and subtraction are associative, path independence follows.

Sampling frequency invariant

For all low frequency data, consider a partitioning (perhaps unevenly) into two parts. For X = the portfolio, A, and the benchmark, B, write the returns for the low frequency data as $R_X^{t_1,t_2}$ and the returns for high frequency data as $r_X^{t_1,t_2}$, such that

$$1 + R_X^{t_1,t_3} = \left(1 + r_X^{t_1,t_2}\right)\left(1 + r_X^{t_2,t_3}\right)$$

and so

$$ln\left(1 + R_X^{t_1,t_3}\right) = ln\left(1 + r_X^{t_1,t_2}\right) + ln\left(1 + r_X^{t_2,t_3}\right)$$

Then

$$\sum_t \ln\left(1 + R_A^{t,t+1}\right) - \sum_t \ln\left(1 + R_B^{t,t+1}\right)$$

$$= \sum_t \left(\ln\left(1 + r_A^{t,t\cdot}\right) + \ln\left(1 + r_A^{t\cdot,t+1}\right)\right) - \sum_t \left(\ln\left(1 + r_B^{t,t\cdot}\right) + \ln\left(1 + r_B^{t\cdot,t+1}\right)\right)$$
$$= \sum_s \ln\left(1 + r_A^{s,s+1}\right) - \sum_s \ln\left(1 + r_B^{s,s+1}\right)$$

The general case may be derived through induction on the proof above, over the partitioning adopted.

Induces equivalence of 'performs the same as'

'Performs the same as' is an equivalence relation as it is reflexive, symmetric and transitive:

$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) - \sum_t \ln\left(1 + r_A^{t,t+1}\right) = 0$$
$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) - \sum_t \ln\left(1 + r_B^{t,t+1}\right) = 0 \Rightarrow \sum_t \ln\left(1 + r_B^{t,t+1}\right) - \sum_t \ln\left(1 + r_A^{t,t+1}\right) = 0$$
$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) - \sum_t \ln\left(1 + r_Z^{t,t+1}\right) = 0 \; and \sum_t \ln\left(1 + r_Z^{t,t+1}\right) - \sum_t \ln\left(1 + r_B^{t,t+1}\right) = 0 \Rightarrow$$
$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) + \left(-\sum_t \ln\left(1 + r_Z^{t,t+1}\right) + \sum_t \ln\left(1 + r_Z^{t,t+1}\right)\right) - \sum_t \ln\left(1 + r_B^{t,t+1}\right) = 0$$

Induces strict inequality of 'underperformance' and 'outperformance'

Outperformance is a strict inequality as it is non-reflexive, asymmetric and transitive:

$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) - \sum_t \ln\left(1 + r_A^{t,t+1}\right) = 0$$
$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) - \sum_t \ln\left(1 + r_B^{t,t+1}\right) > 0 \Rightarrow \sum_t \ln\left(1 + r_B^{t,t+1}\right) - \sum_t \ln\left(1 + r_A^{t,t+1}\right) < 0$$
$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) - \sum_t \ln\left(1 + r_Z^{t,t+1}\right) > 0 \; and \sum_t \ln\left(1 + r_Z^{t,t+1}\right) - \sum_t \ln\left(1 + r_B^{t,t+1}\right) > 0 \Rightarrow$$
$$\sum_t \ln\left(1 + r_A^{t,t+1}\right) + \left(-\sum_t \ln\left(1 + r_Z^{t,t+1}\right) + \sum_t \ln\left(1 + r_Z^{t,t+1}\right)\right) - \sum_t \ln\left(1 + r_B^{t,t+1}\right) > 0$$

Index ratio

Path independent

See log return.

Sampling frequency invariant

See log return.

Induces equivalence of "performs the same as"

See log return.

Induces strict inequality of "underperformance" and "outperformance"

See log return.

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Buy	FSR is > 6% above the MRA.	45%	28%
Neutral	FSR is between -6% and 6% of the MRA.	38%	27%
Sell	FSR is > 6% below the MRA.	17%	11%
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Buy	Stock price expected to rise within three months from the time the rating was assigned because of a specific catalyst or event.	<1%	<1%
Sell	Stock price expected to fall within three months from the time the rating was assigned because of a specific catalyst or event.	<1%	<1%

Source: UBS. Rating allocations are as of 30 June 2017.
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