Multi-Stage Stochastic Goal Programming Explained: Holistic Approach for Personalized Goal-Based Investing

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- Multi-stage stochastic goal programming (MSGP)
- Goal-based investing (GBI) examples

Section 1

INTRODUCTION: PERSONALIZED FINANCIAL PLANNING

Private Wealth Management

- Traditionally, private wealth management had been mainly focusing on **high-net-worth individuals**
  - Objective is to increase capital while maintaining a reasonable exposure to risk
  - Investment objectives are comparable to those of institutional investors
  - Objectives can be addressed by **Modern Portfolio Theory** (MPT)

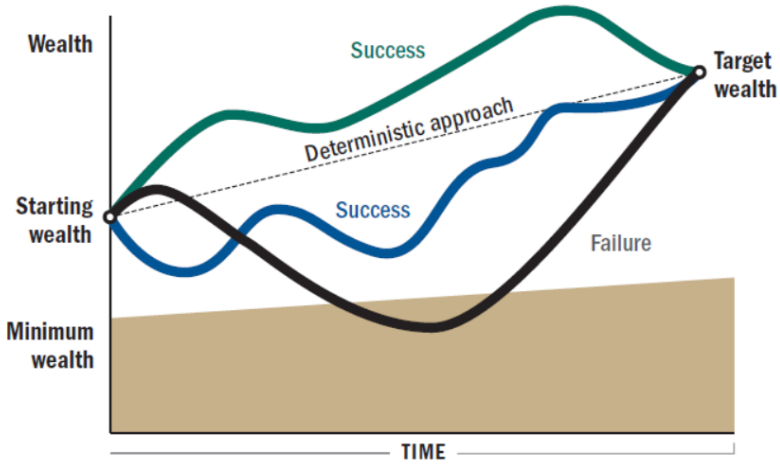
Automated Wealth Management

- **Automated wealth management** allows **average individuals**, who are not classified as high-net-worth individuals, to receive private wealth management services
  - Robo-advisors, online advisors, …

- There is surging demand for financial planning for average individuals due to
  - Growth of defined-contribution pension plans (DC plans)
  - Increase of life expectancy

Financial Planning for Average Individuals

- For average individuals,
  - Objective is **not simply high return**
  - **Spending goals (liabilities)** are very important
  - Investment is tightly constrained on individual requirements



(Chabbra, 2005)

- Major difference between high-net-worth individuals and average individuals:
  - Average individuals require **management in both assets and liabilities**

Personalized ALM

- However, **MPT** takes an ***asset-only* approach** (Fabozzi, Gupta, & Markowitz, 2002)
  - MPT does not incorporate consumption goals

- Therefore, wealth management for average individuals should be addressed within an **Asset-Liability Management (ALM)** framework (Mulvey, 1994)

  ➔ **Personalized ALM** (Consiglio, Cocco, & Zenios, 2007; Medova et al., 2008; Dempster et al., 2016; Martellini & Milhau, 2017)

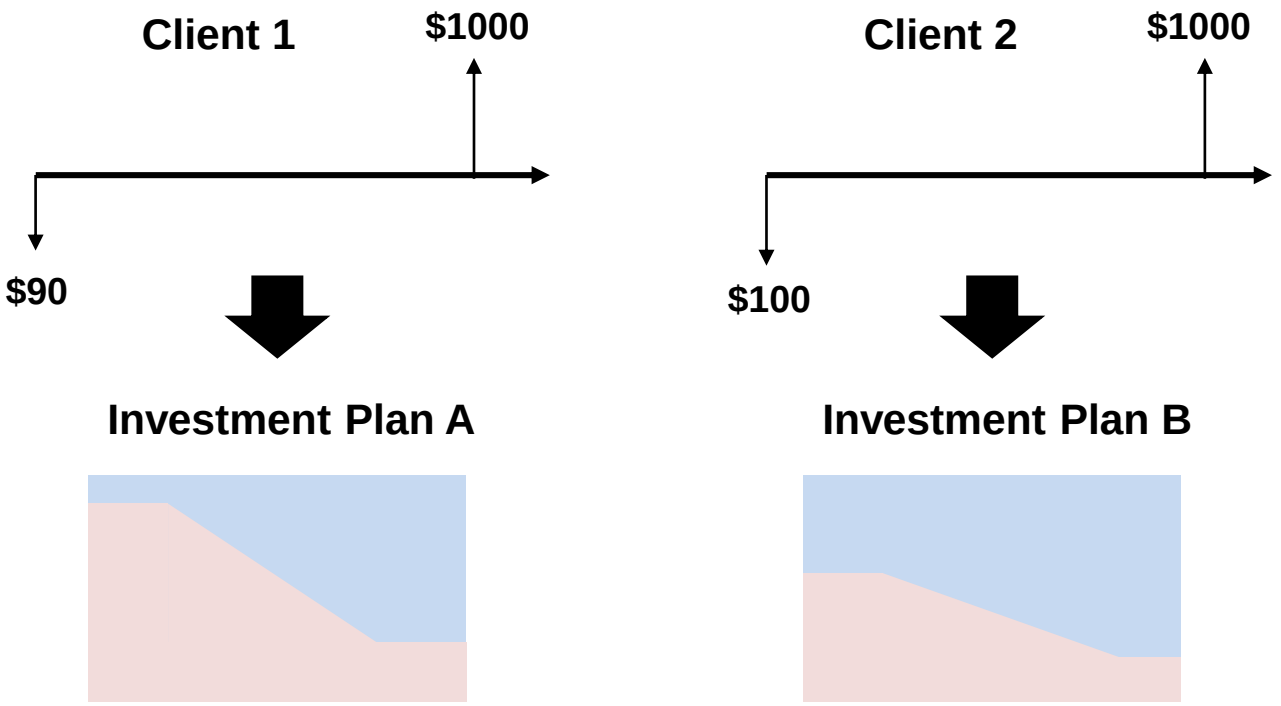
Goal-Based Investing

- In this regard, **Goal-Based Investing (GBI)** framework was proposed based on **Behavioral Portfolio Theory (BPT)**

- BPT states that (Shefrin and Statman, 2000)
  - individuals often have **multiple consumption goals** (or liabilities)
  - investors typically view these consumption goals with **different aspiration level or risk-averseness**

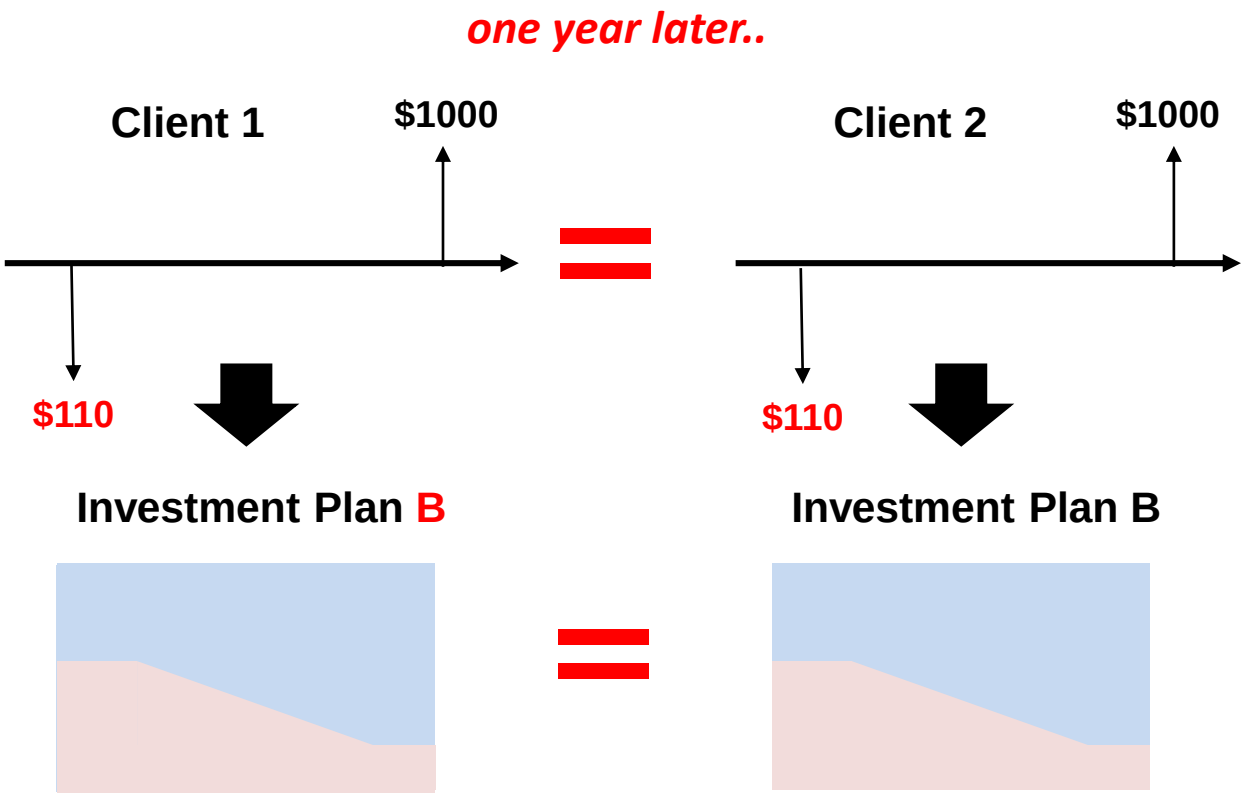
Recourse Decisions are Important

- A problem of deterministic model without recourse decisions



Recourse Decisions are Important

- Bellman's principle of optimality must hold



Principle of Optimality: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. (See Bellman, 1957, Chap. III.3.)

Section 2

FINANCIAL PLANNING

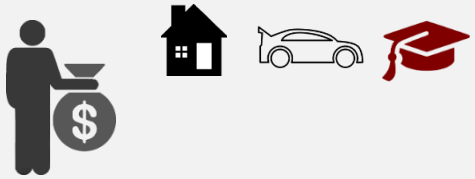
Investor Cases

- In order to discuss GBI, we consider four investor cases

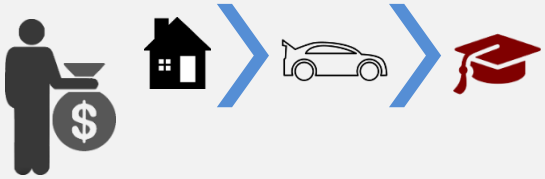
(A) No goal

(B) Single goal

(C) Multiple goals



(D) Multiple goals with priorities

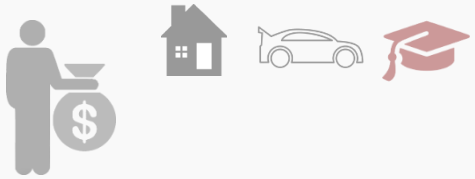


Case (A): No Goal

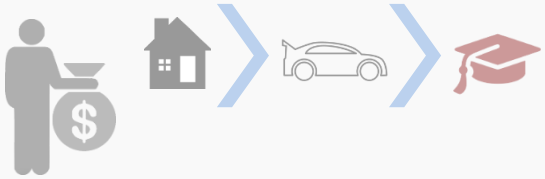
(A) No goal

(B) Single goal

(C) Multiple goals



(D) Multiple goals with priorities



Case (A): No Goal

- Conventional investor: No specific consumption goals

  - *Example*: conventional funds

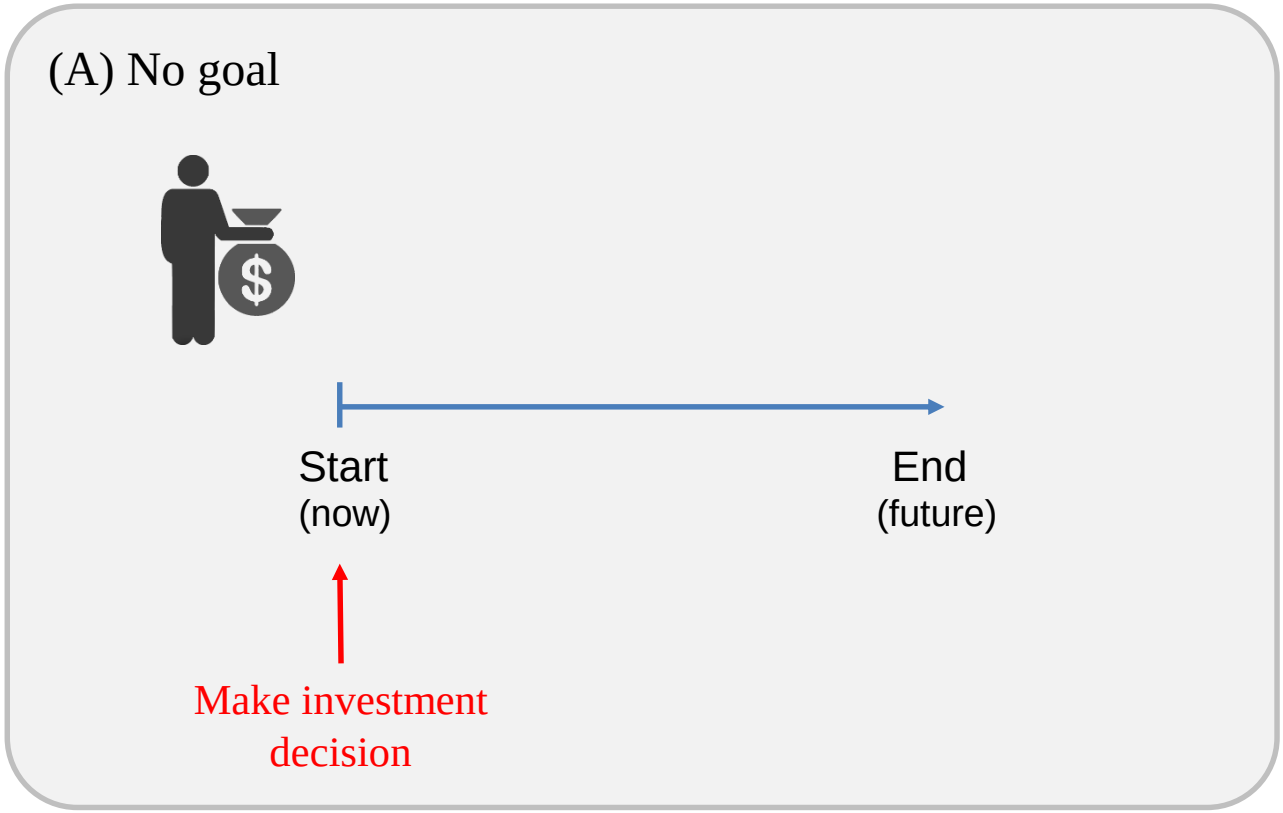
Case (A): No Goal

- Conventional investor: No specific consumption goals

  - Objective: **Maximize wealth**
    - Find efficient portfolio (portfolio with high return and low risk)
  - Model: **Markowitz mean-variance framework**

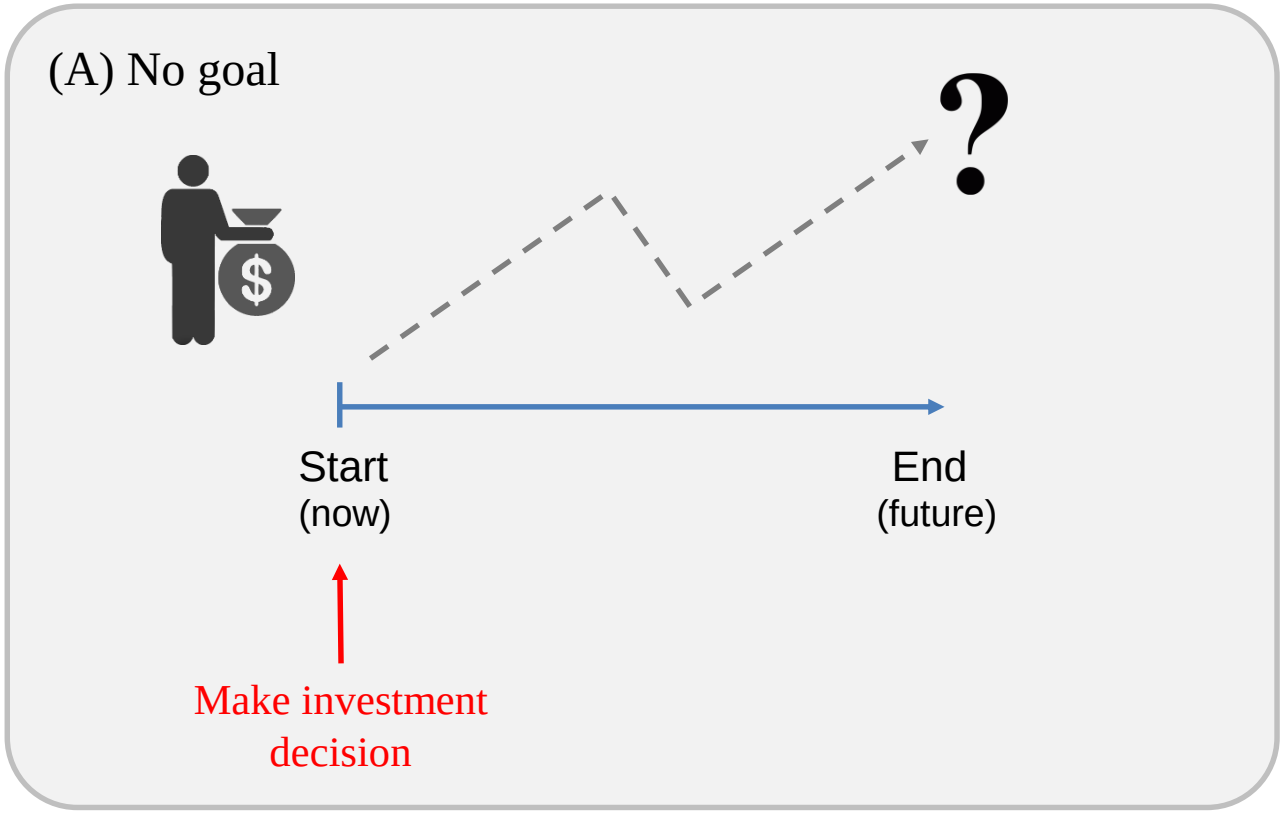
Case (A): No Goal

- Need to make decision investment decision now



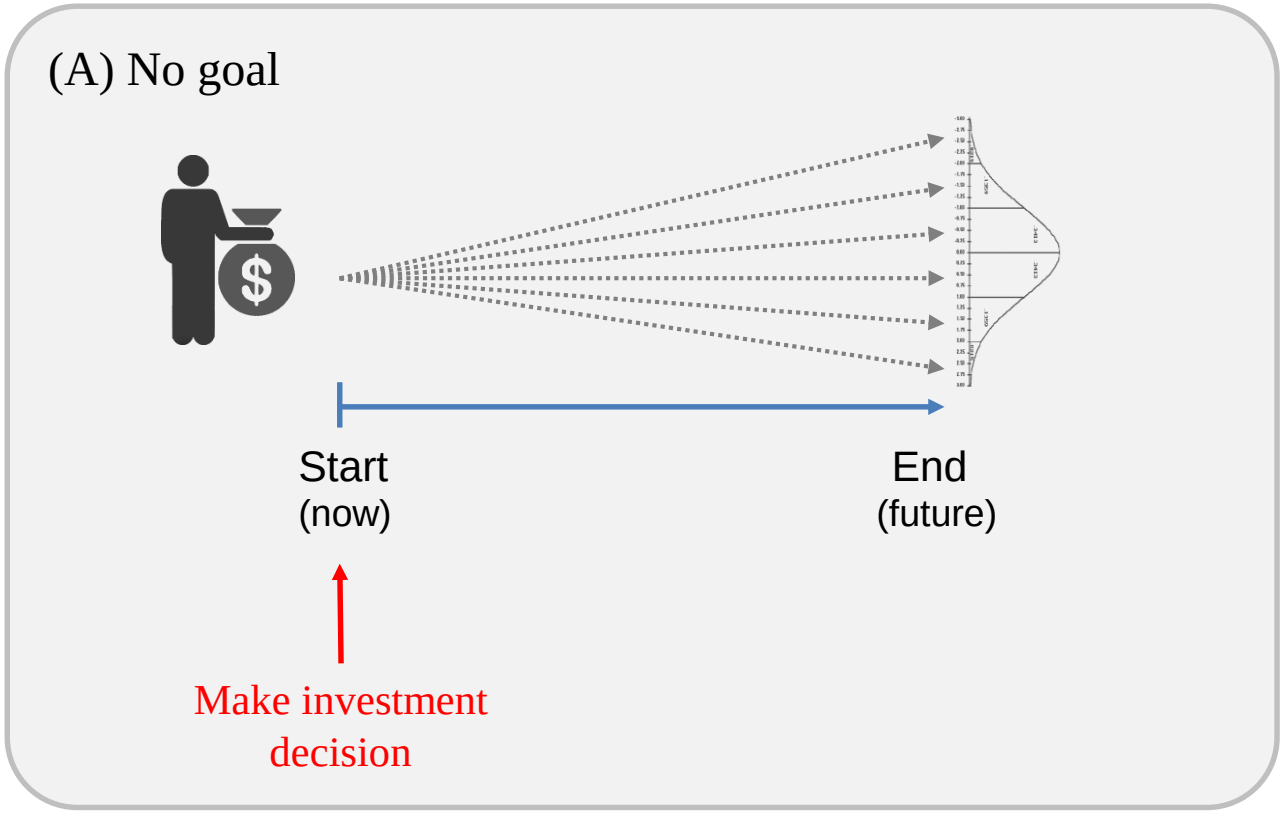
Case (A): No Goal

- But future is uncertain



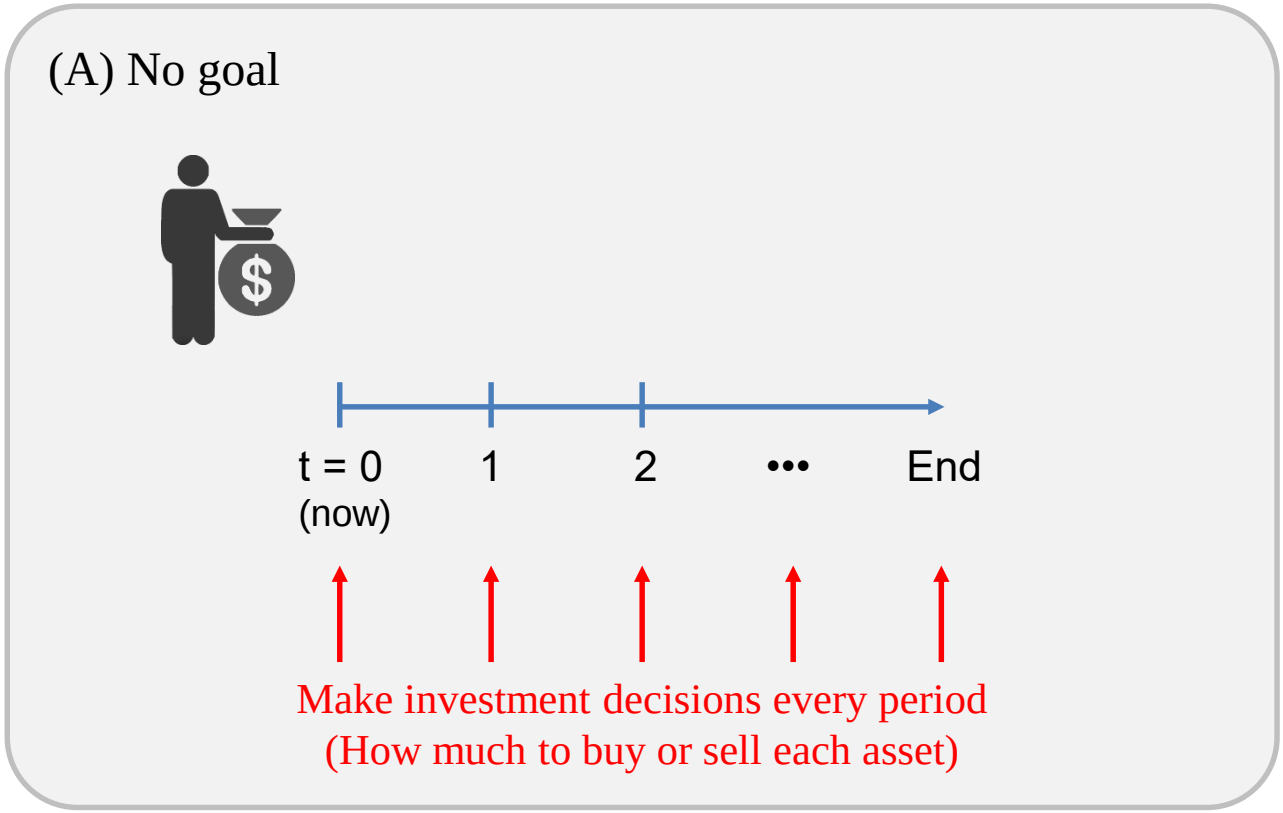
Case (A): No Goal

- Therefore, models include distribution of asset returns



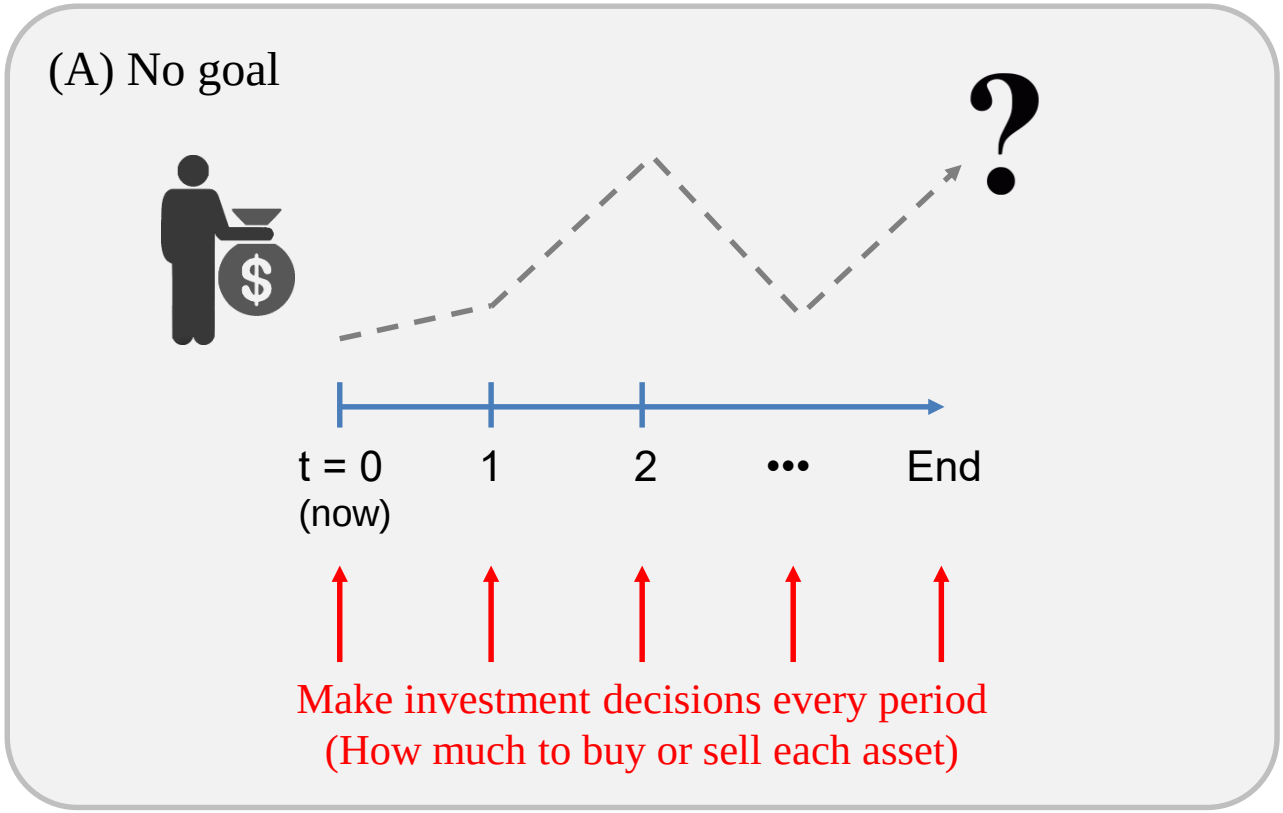
Case (A): No Goal

- Multi-stage models consider portfolio rebalancing



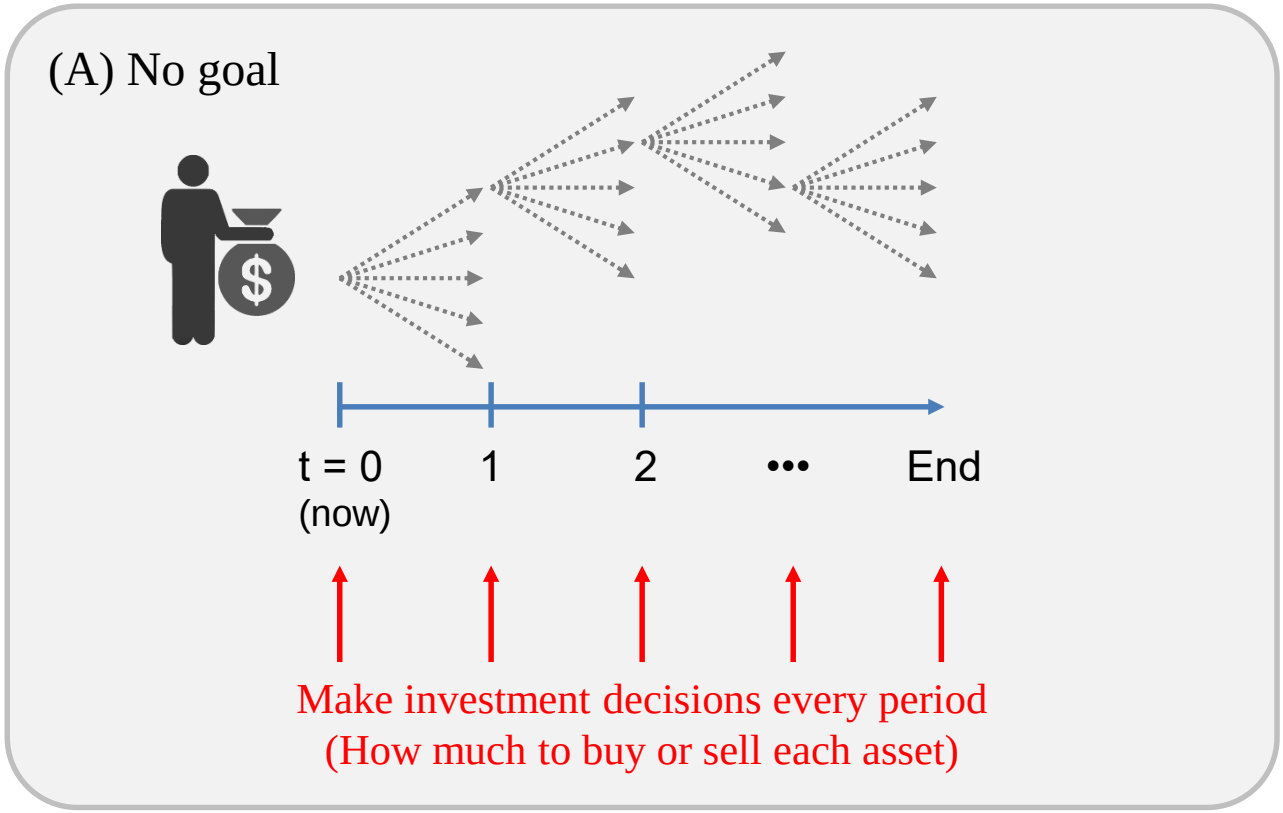
Case (A): No Goal

- But since future is uncertain…



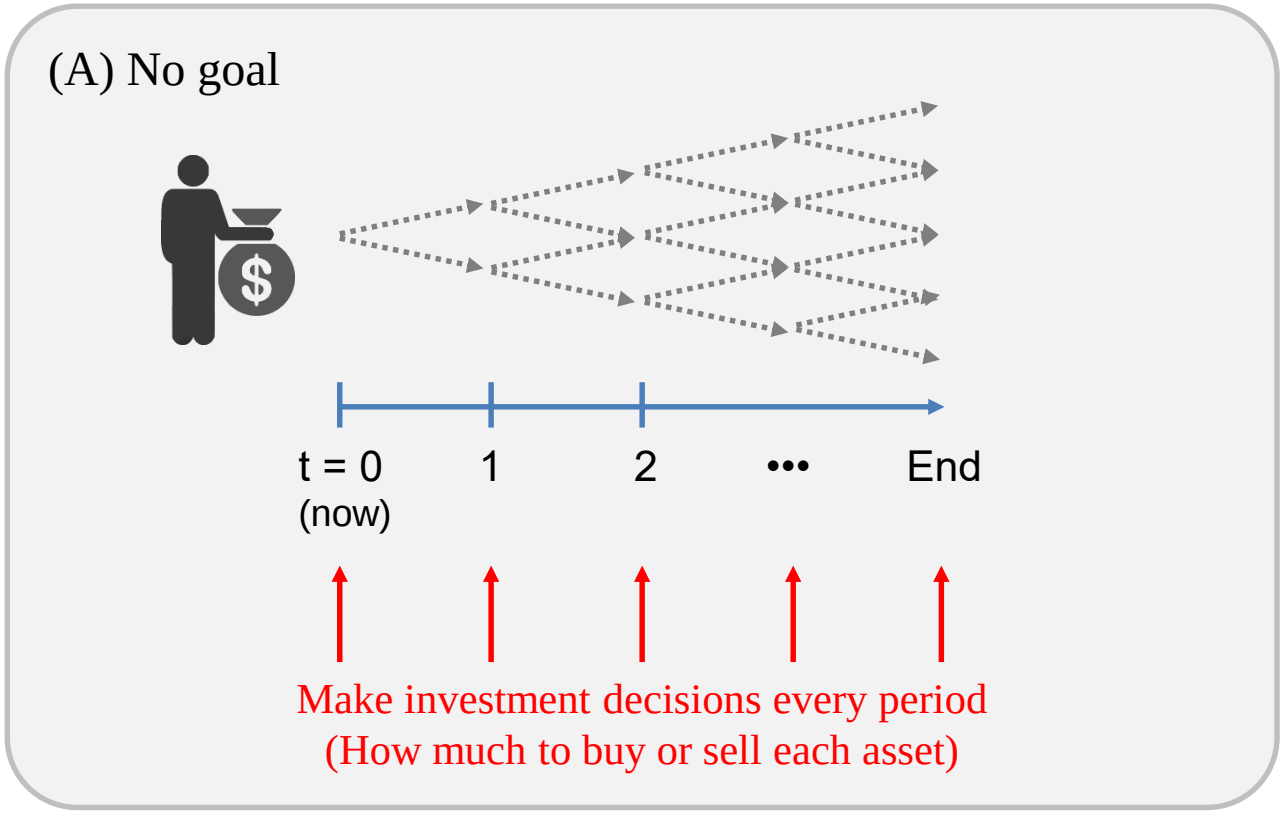
Case (A): No Goal

- Consider many scenarios of future events



Case (A): No Goal

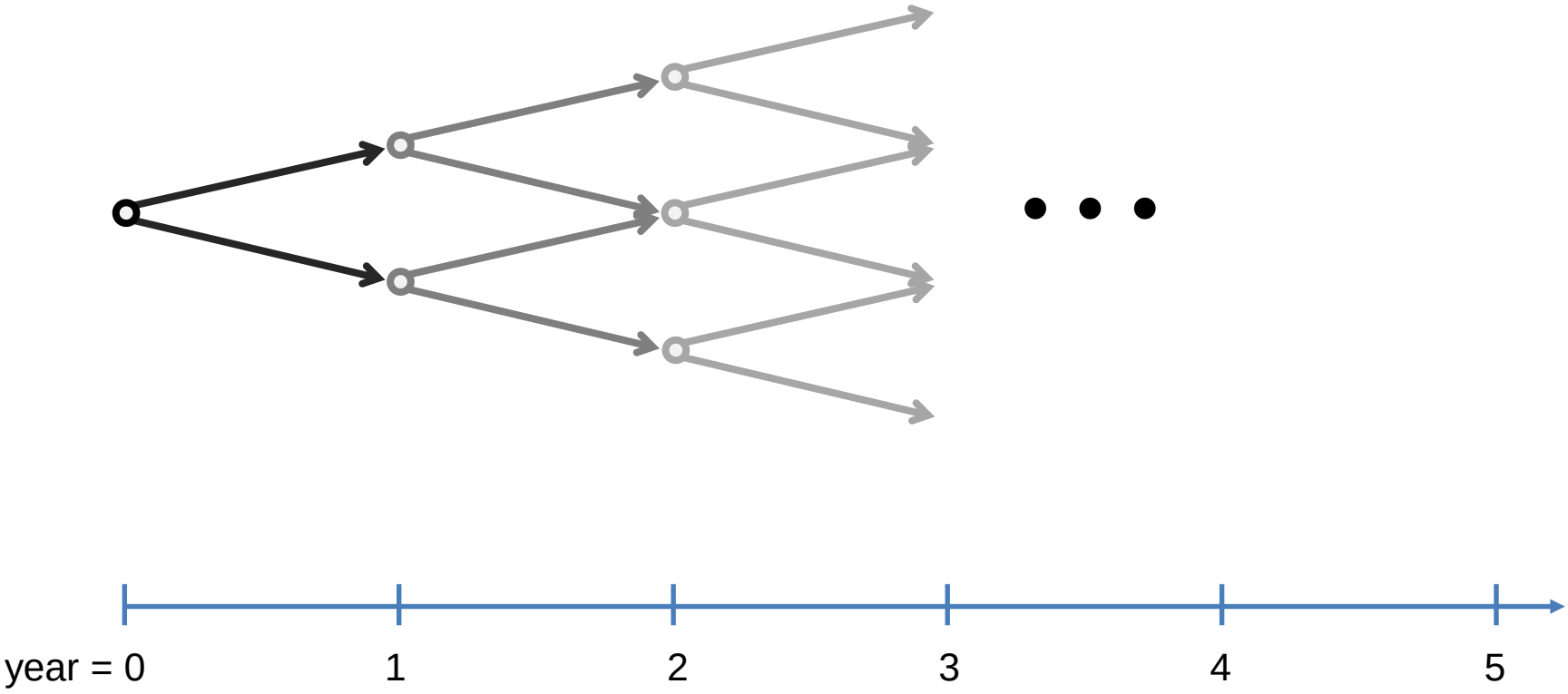
- Use scenarios (scenario trees) to model future asset returns



  - Multi-stage stochastic programming (Consigli & Dempster, 1998; Ziemba & Mulvey, 1998)

Case (A): No Goal

- Scenario trees

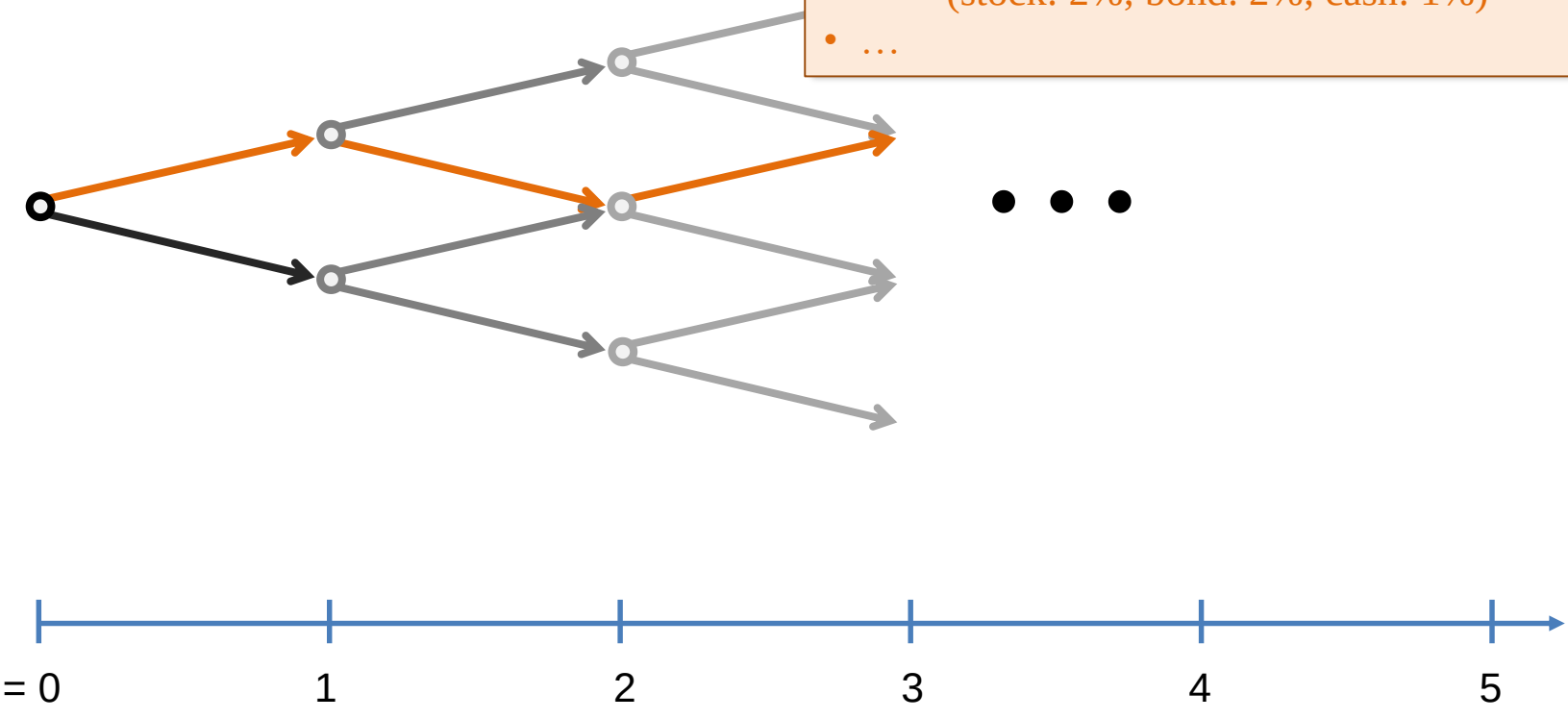


  - Multi-stage stochastic programming (Consigli & Dempster, 1998; Ziemba & Mulvey, 1998)

Case (A): No Goal

- *Year 1:*
  (stock: 8%; bond: 2%; cash: 1%)
- *Year 2:*
  (stock: −1%; bond: 3%; cash: 1%)
- *Year 3:*
  (stock: 2%; bond: 2%; cash: 1%)
- …

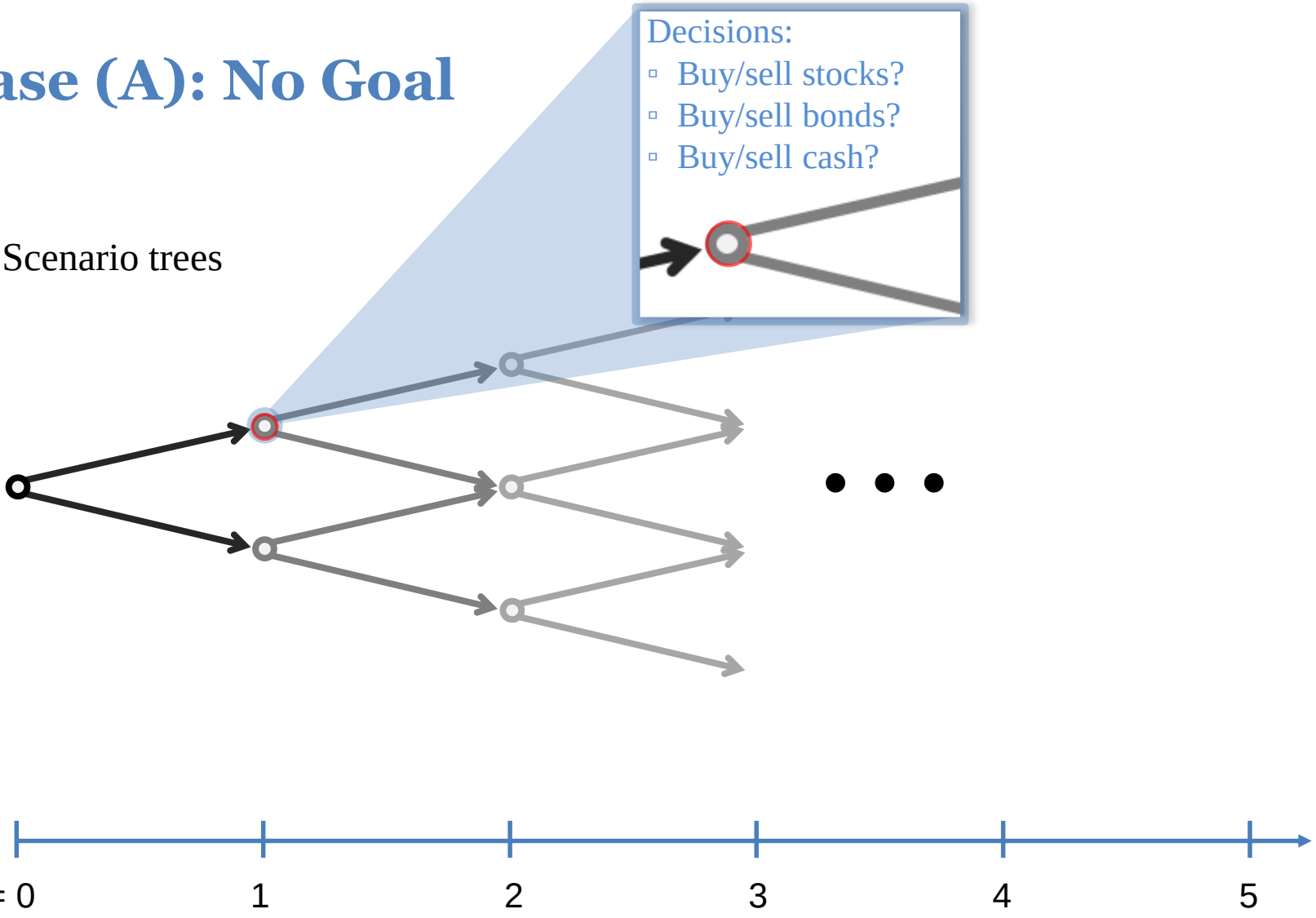
- Scenario trees



- Multi-stage stochastic programming (Consigli & Dempster, 1998; Ziemba & Mulvey, 1998)

Case (A): No Goal

- Scenario trees



- Multi-stage stochastic programming (Consigli & Dempster, 1998; Ziemba & Mulvey, 1998)

Case (A): No Goal

- Scenario trees

From a modeling perspective,

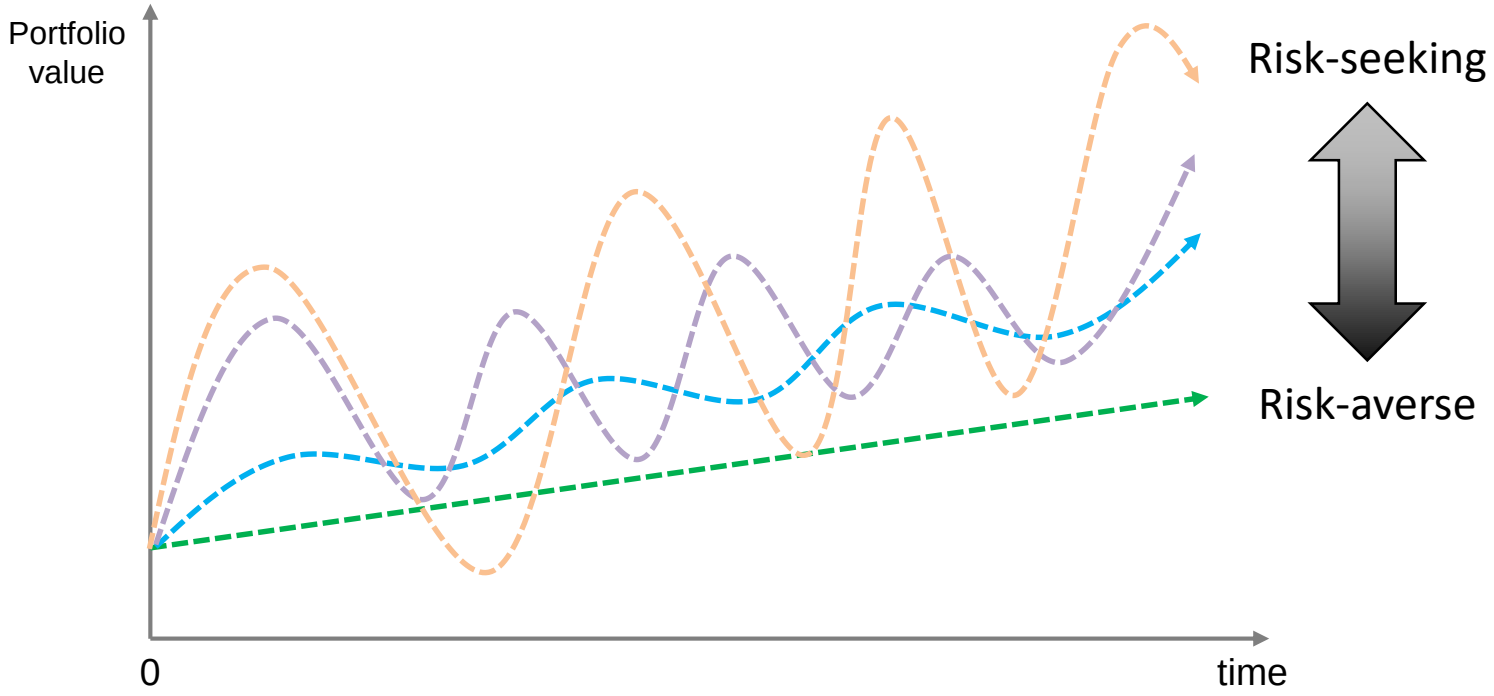
Multi-stage stochastic programming

is important

- Multi-stage stochastic programming (Consigli & Dempster, 1998; Ziemba & Mulvey, 1998)

Case (A): No Goal

- Optimal portfolio is chosen depending on risk appetite

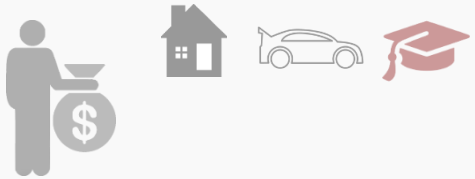


Case (B): Single Goal

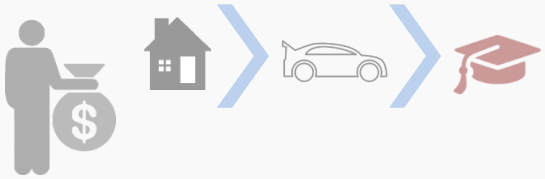
(A) No goal

(B) Single goal

(C) Multiple goals



(D) Multiple goals with priorities



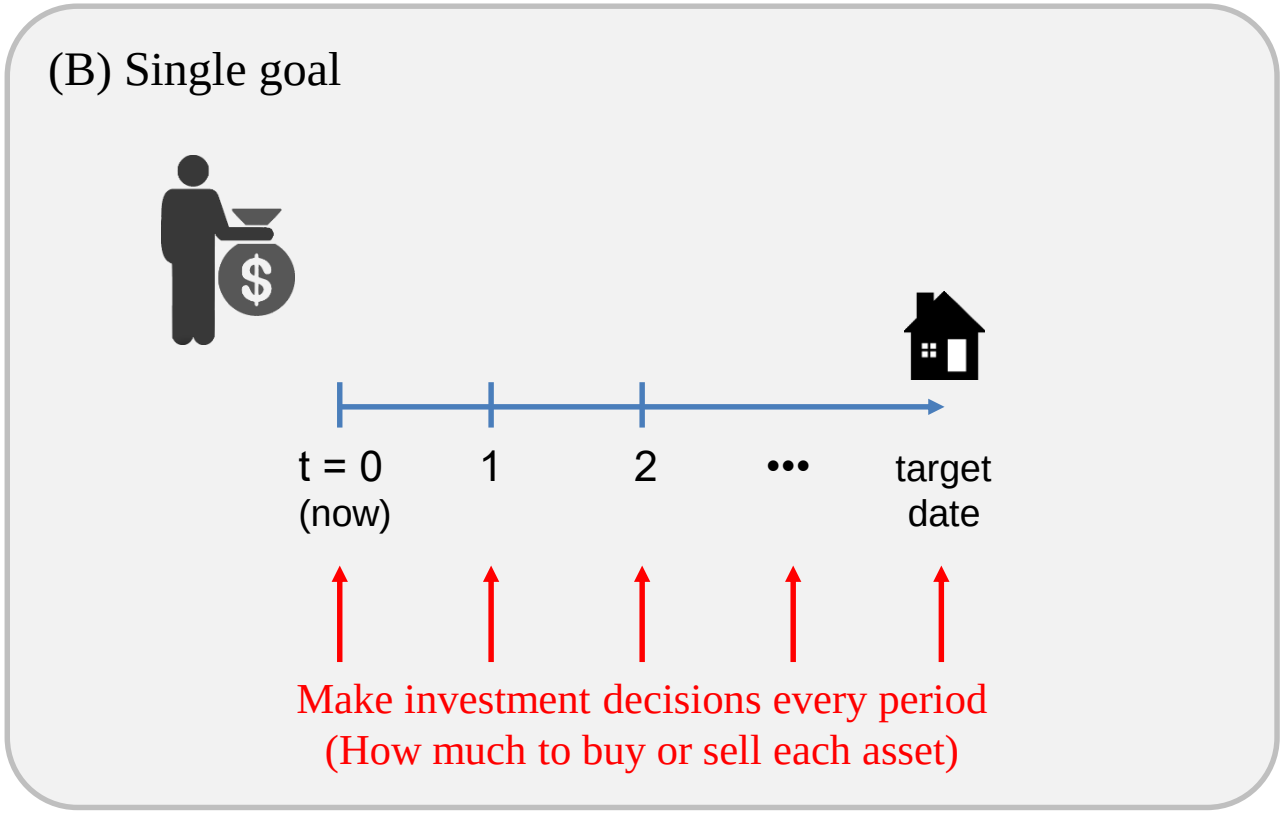
Case (B): Single Goal

- Investor with one consumption goal

  - Objective is to **maximize goal achievement** (*not* simply maximize wealth)
  - *Example*: target-date funds (TDFs), goal-based investing

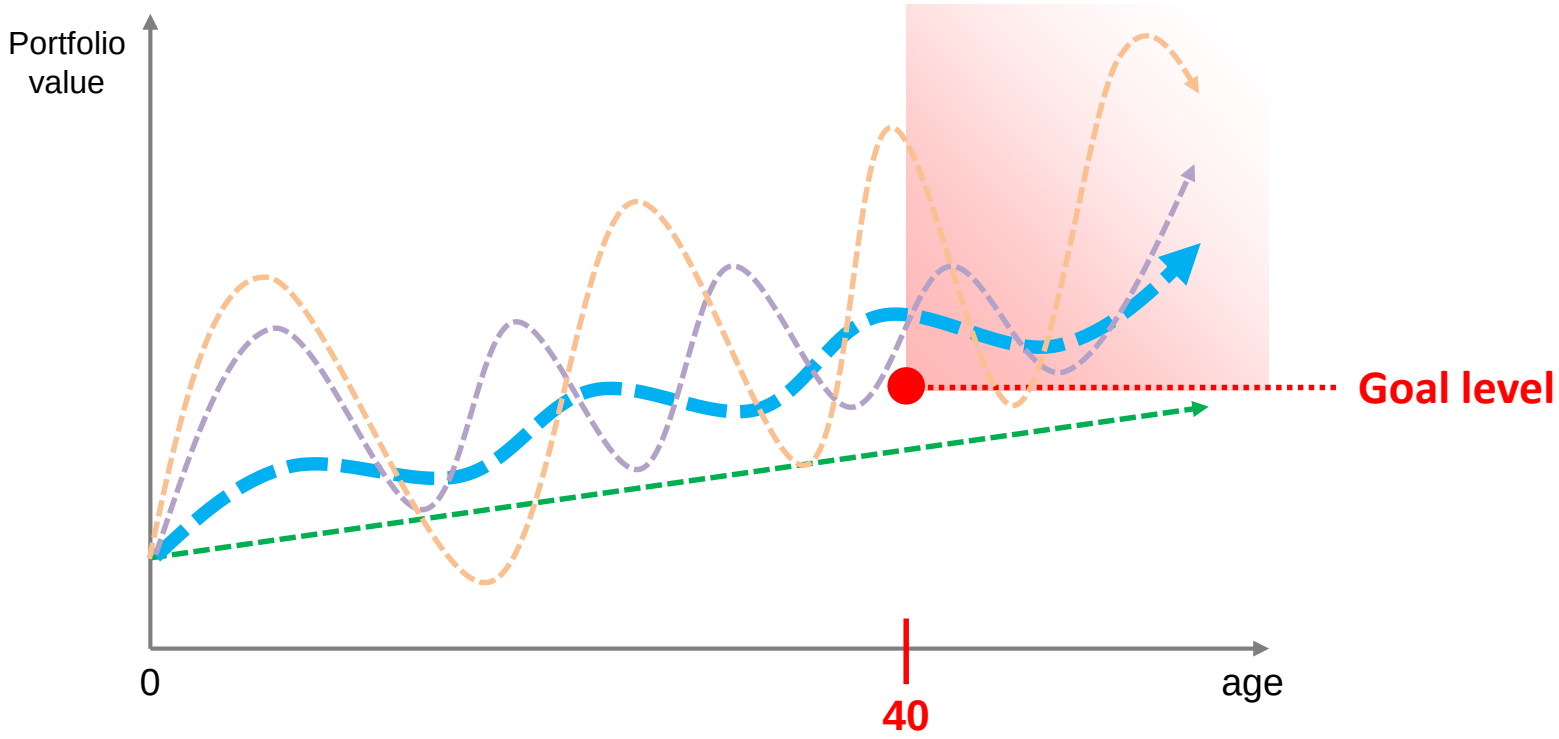
Case (B): Single Goal

- Multi-stage model with a consumption goal



Case (B): Single Goal

- Higher expected return is not desired if goal achievement is less probable



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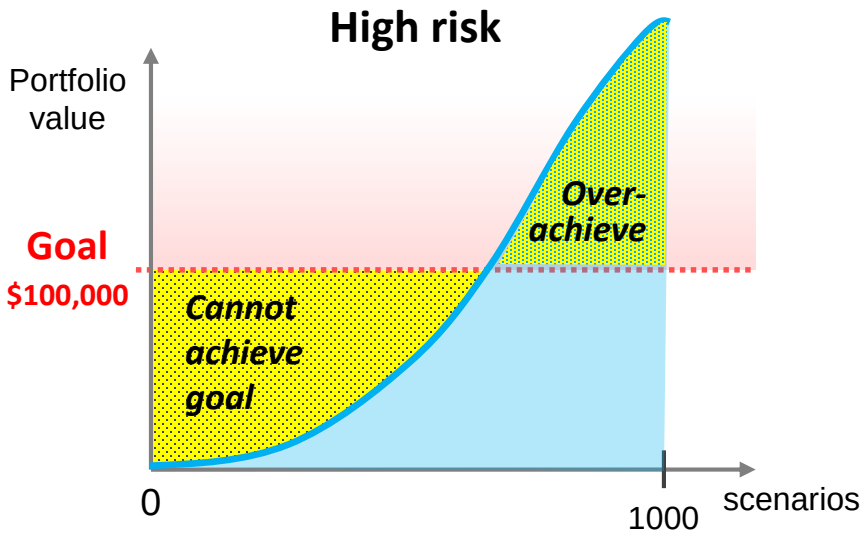
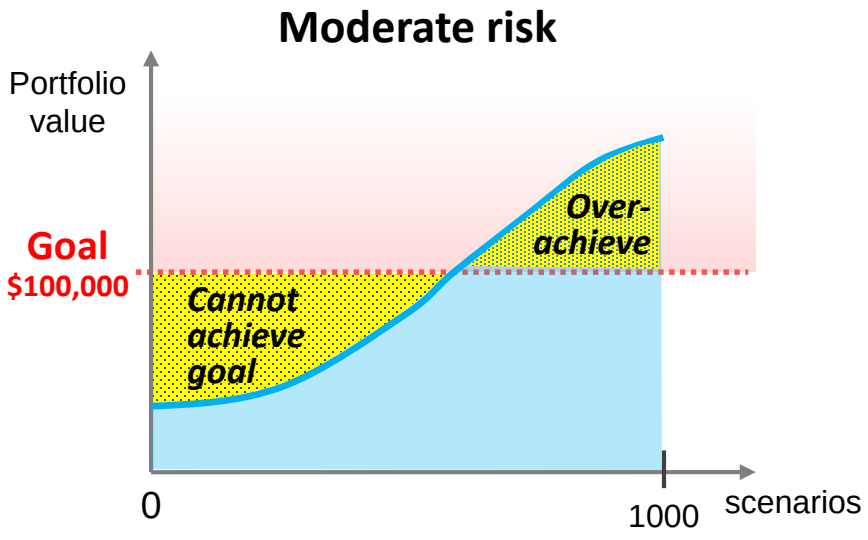
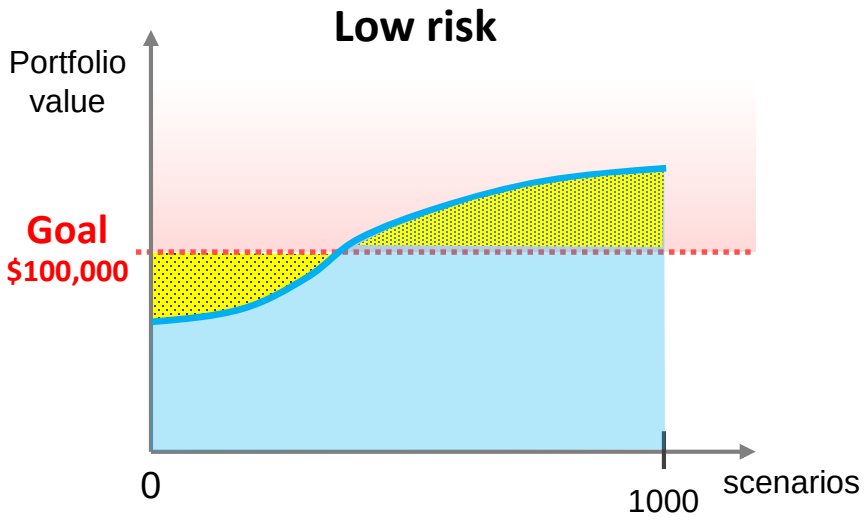
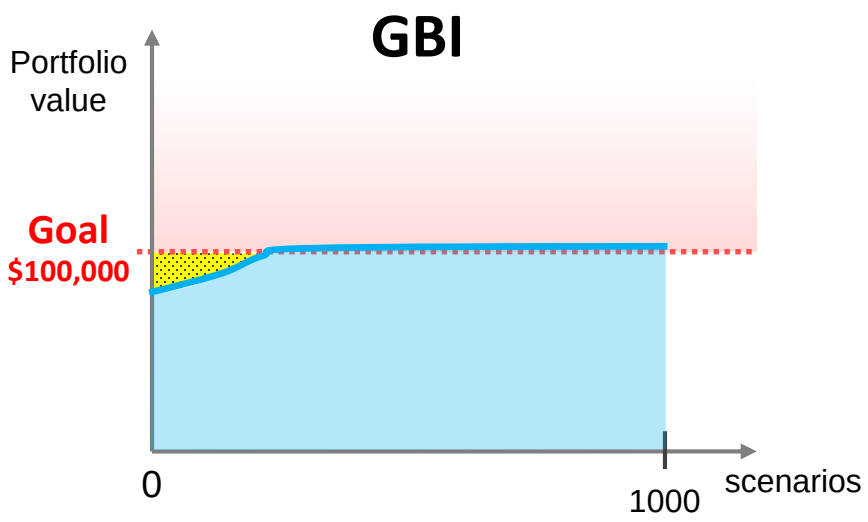
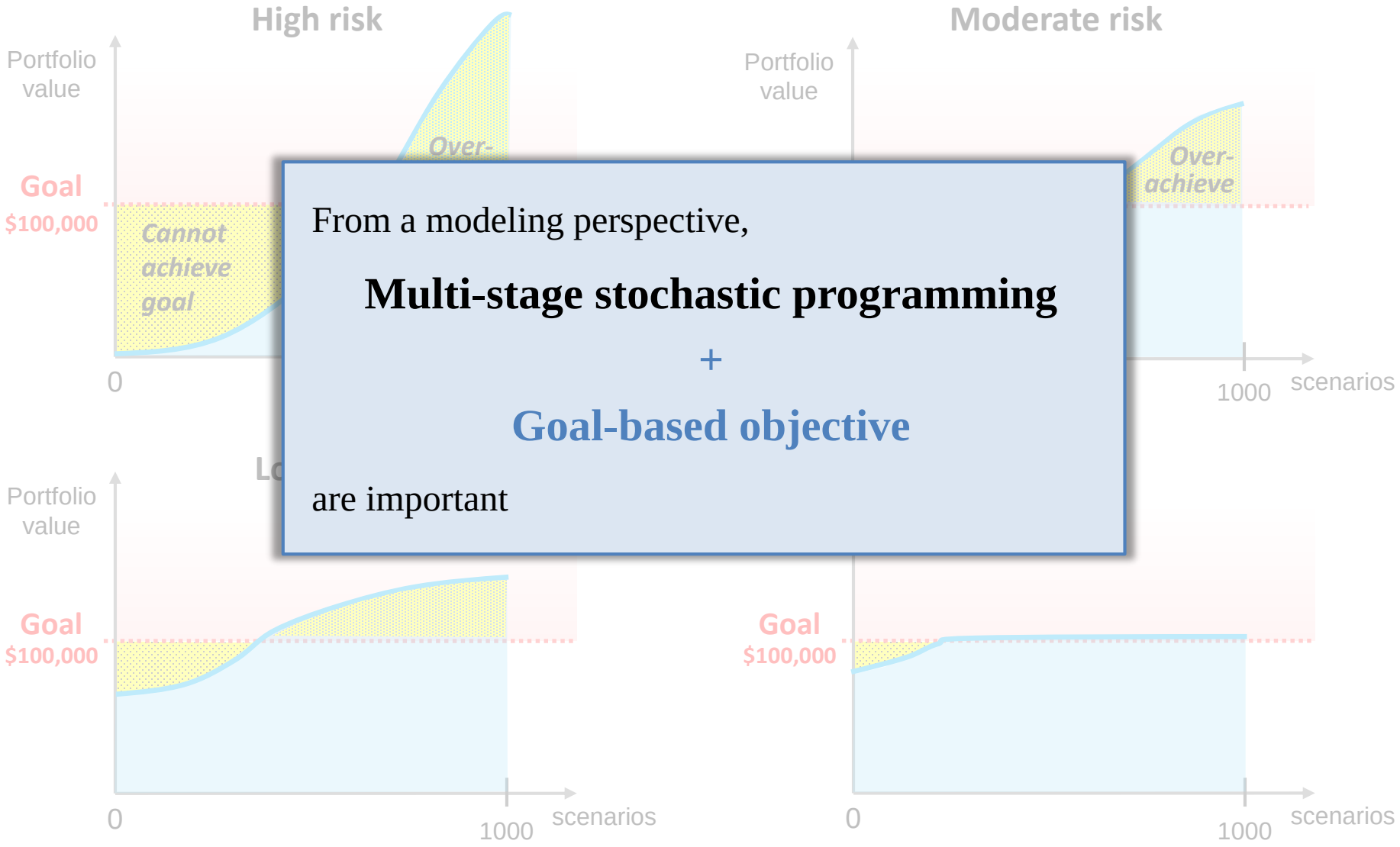








- Higher expected return is not desired if goal achievement is less probable



From a modeling perspective,

Multi-stage stochastic programming

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Goal-based objective

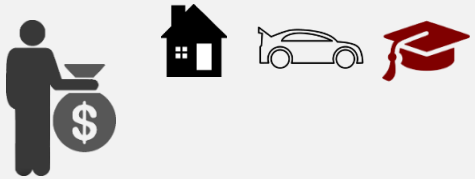
are important

Case (C): Multiple Goals

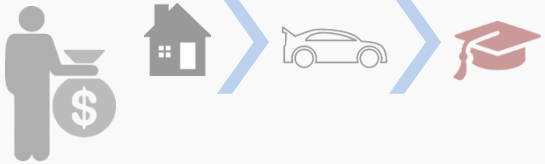
(A) No goal

(B) Single goal

(C) Multiple goals

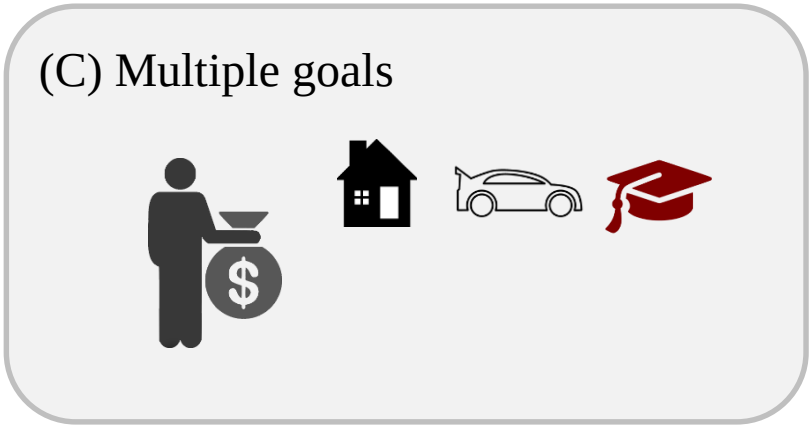


(D) Multiple goals with priorities



Case (C): Multiple Goals

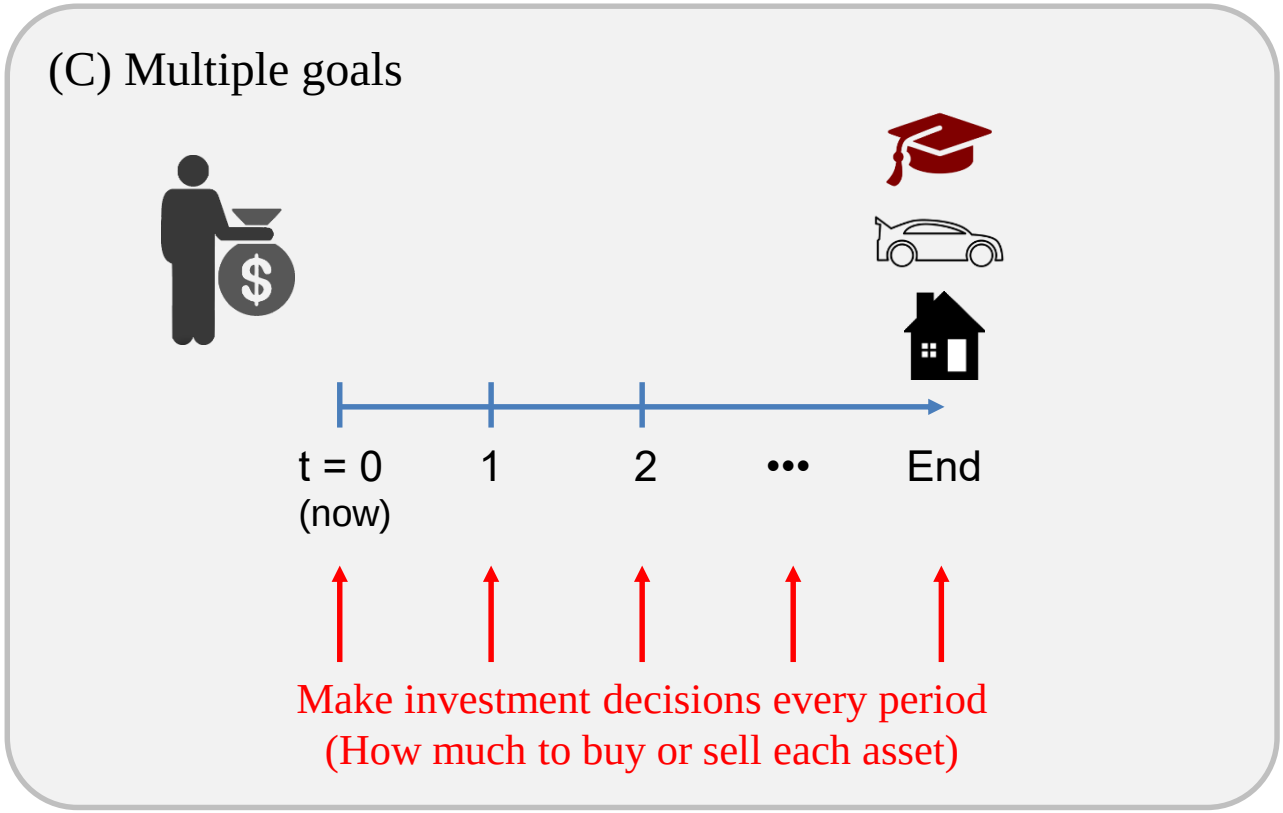
- Investor often have more than one consumption goals



  - *Example*: advanced goal-based investing

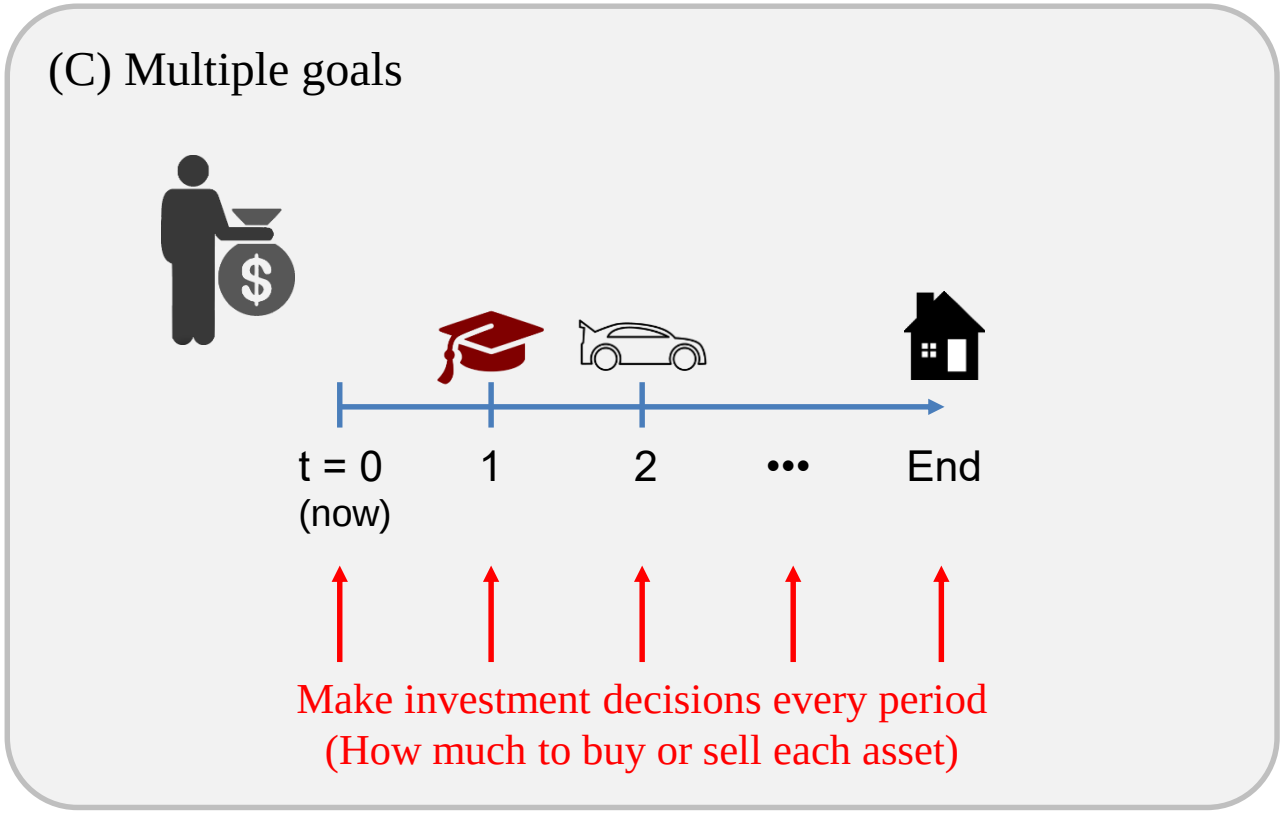
Case (C): Multiple Goals

- Multi-stage model

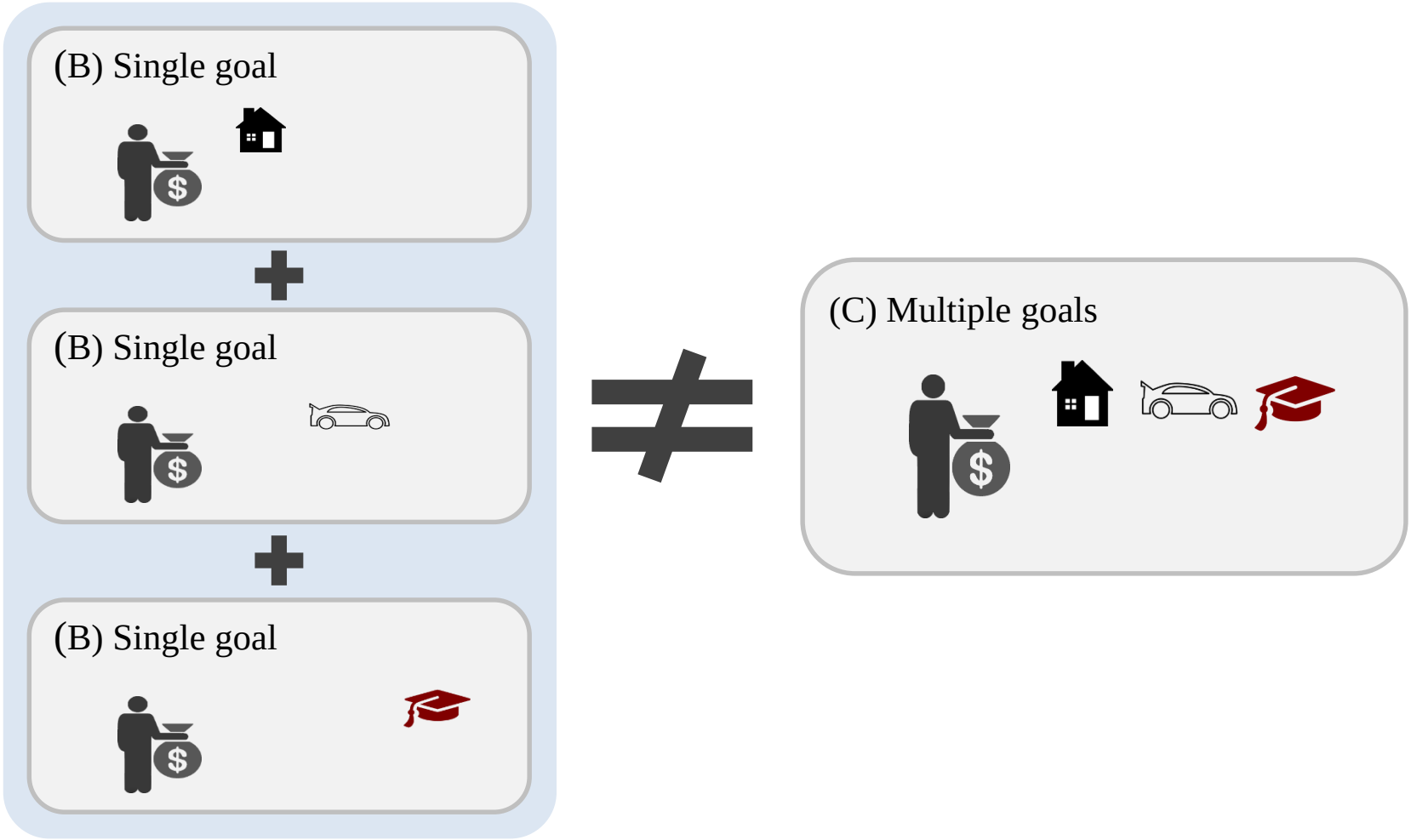


Case (C): Multiple Goals

- Multi-stage model



Case (C): Multiple Goals



Case (C): Multiple Goals

- Thus, cannot consider each goal separately

  → Need a **holistic approach** (Fowler & de Vassal, 2006)

- Find optimal decision that maximizes total achievement probabilities of all goals

Case (C): Multiple Goals

- Thus, cannot consider each goal separately

  → Need a

- Find optima of all goals

From a modeling perspective,

Multi-stage stochastic programming

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Goal-based objective

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Multi-objective (multi-goals)

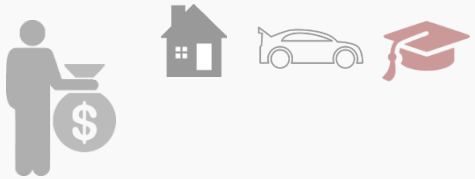
are important

Case (D): Multiple Goals with Priorities

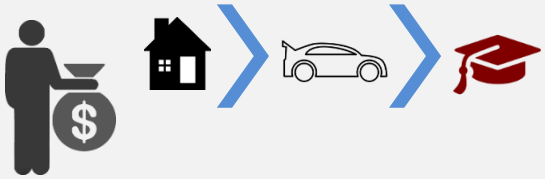
(A) No goal

(B) Single goal

(C) Multiple goals

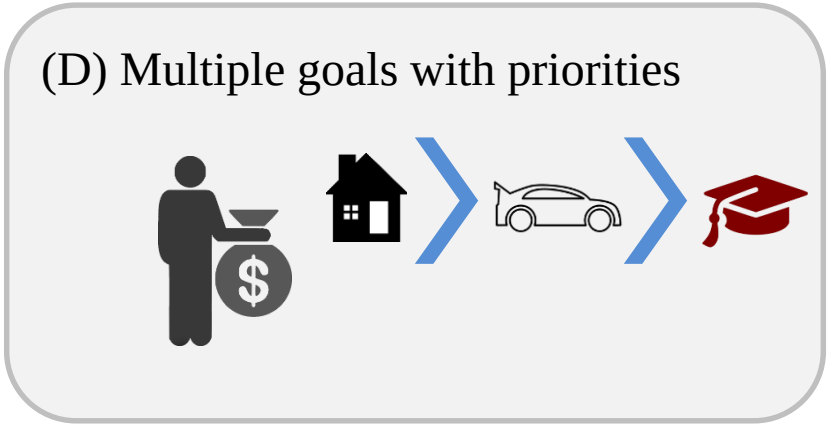


(D) Multiple goals with priorities

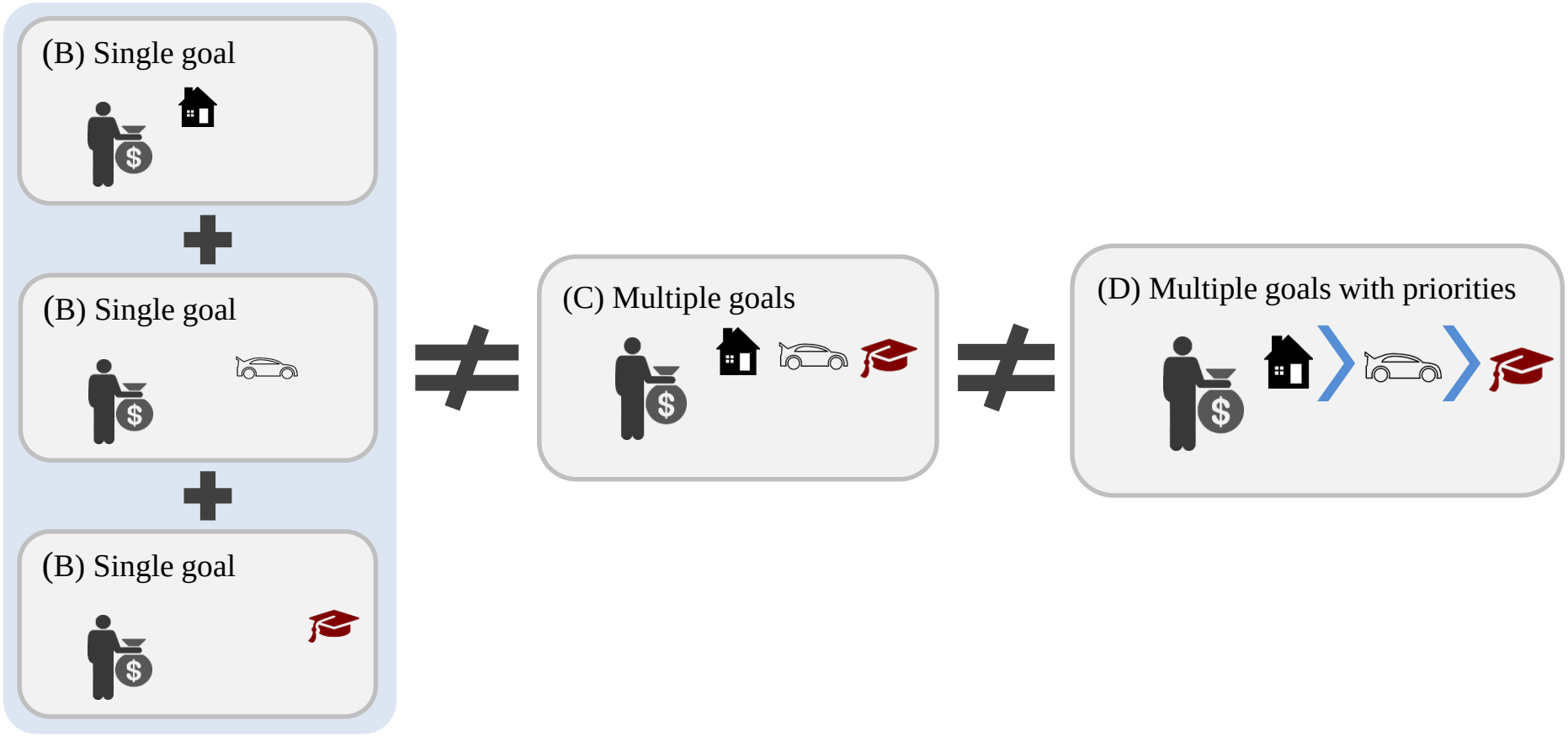


Case (D): Multiple Goals with Priorities

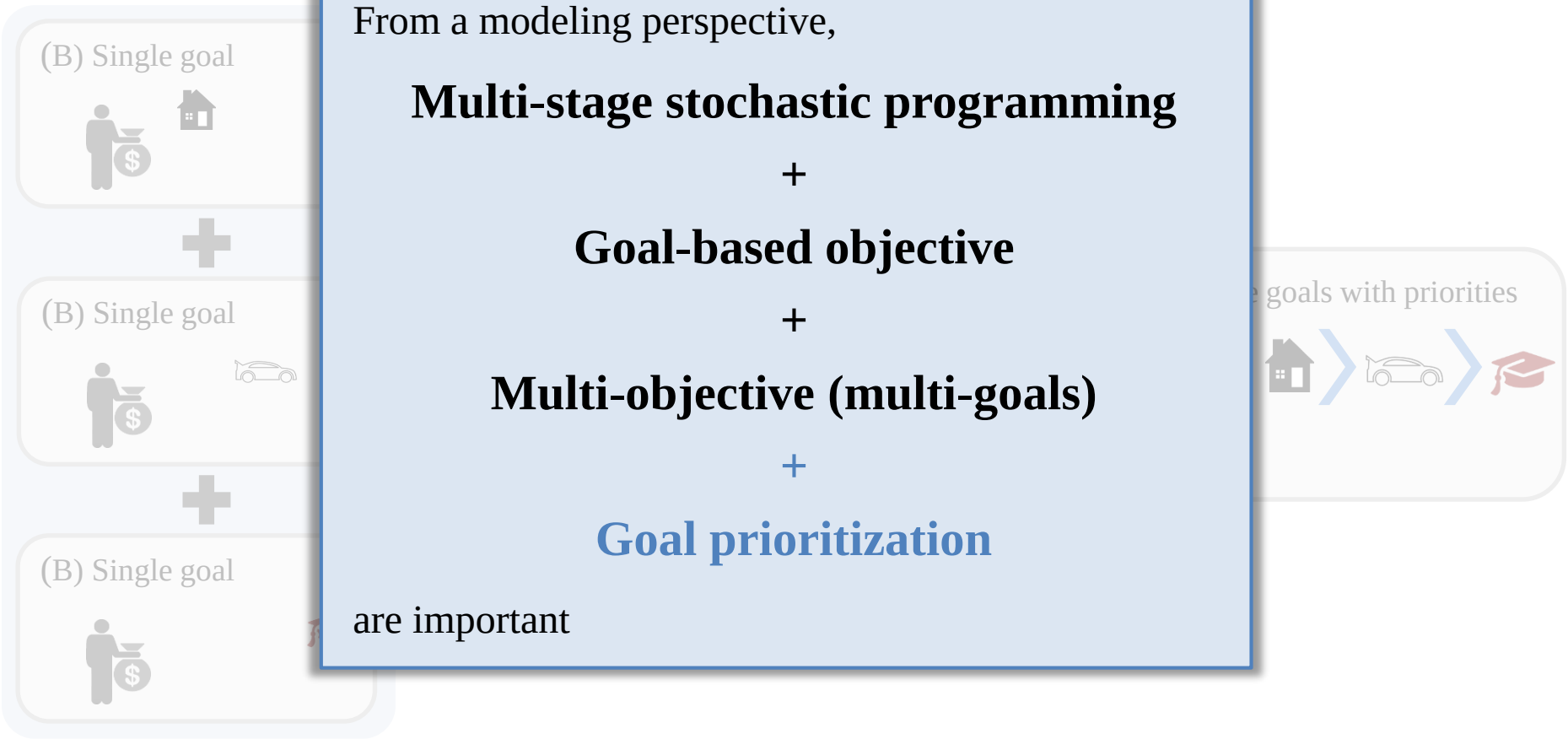
- Investors have preferences among their goals
  - For example, purchasing a house can be more important than purchasing a car



Case (D): Multiple Goals with Priorities



Case (D): Multiple Goals with Priorities



From a modeling perspective,

Multi-stage stochastic programming

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Goal-based objective

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Multi-objective (multi-goals)

+

Goal prioritization

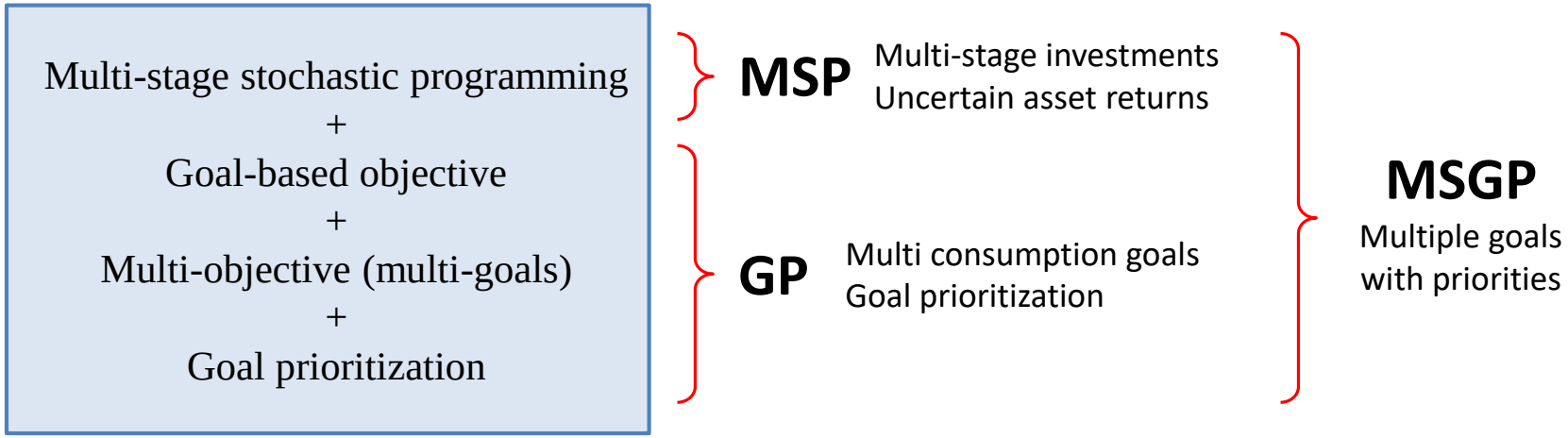
are important

Case (D): Multiple Goals with Priorities

- We propose a GBI model for automated investment management services

- We combine Multi-stage Stochastic Programming and Goal Programming

  → **Multi-stage Stochastic Goal Programming (MSGP)**



Case (D): Multiple Goals with Priorities

- **Goal Programming (GP)**
  - is sequential optimization that solves ***multi-objective*** problems
  - uses ***priority level*** of goals to solve a problem with multiple objectives (*does not* use *importance weights* of goals)

Case (D): Multiple Goals with Priorities

- **Multi-stage Stochastic Goal Programming (MSGP)**
  - Sequential process for considering goal priorities

(1) Maximize probability of achieving highest priority goals

(2) Maximize probability of achieving second highest priority goals

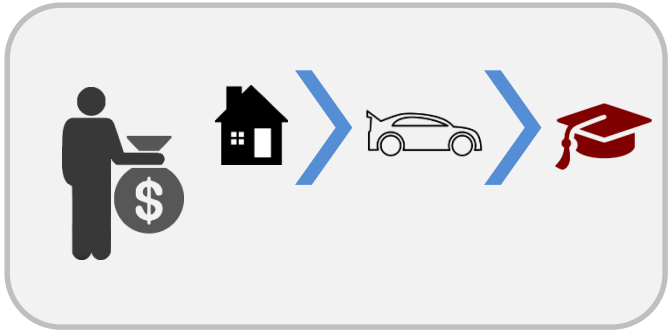
- *while satisfying the maximum probability of (1)*

(3) Maximizes probability of achieving third highest priority goals

- *while satisfying the maximum probability of (1) and (2)*

(4) …

Case (D): Multiple Goals with Priorities



(1) Maximize probability of 

(2) Maximize probability of 

- *while satisfying the maximum probability of* 

(3) Maximize probability of 

- *while satisfying the maximum probability of*  *and* 

Section 3

MULTI-STAGE STOCHASTIC GOAL PROGRAMMING

MSGP: Overview

- Objective: Goal achievement maximization
- Approach: Sequential optimization based on goal priorities
- Formulation: MSGP
  - Multi-stage stochastic programming + Goal programming
  - Linear programming
  - Scenario trees

MSGP: Advantages

- Optimality:
  - Holistic optimization
- Simplicity:
  - Intuitive investor inputs: current wealth, investment amount, and consumption goals (with priority levels)
- Efficiency:
  - Linear structure of the model
  - Prebuilt scenario trees that are independent of investor inputs
- Flexibility:
  - Add constraints: allocation restrictions, transaction cost, CVaR
  - Market models

Allow trial-and-error interaction with investors

MSGP: Model Output

- Optimal financial plan
  - Optimal investment plan during the investment horizon
  - Optimal portfolio at the beginning of the investment plan

- Projected wealth accumulation

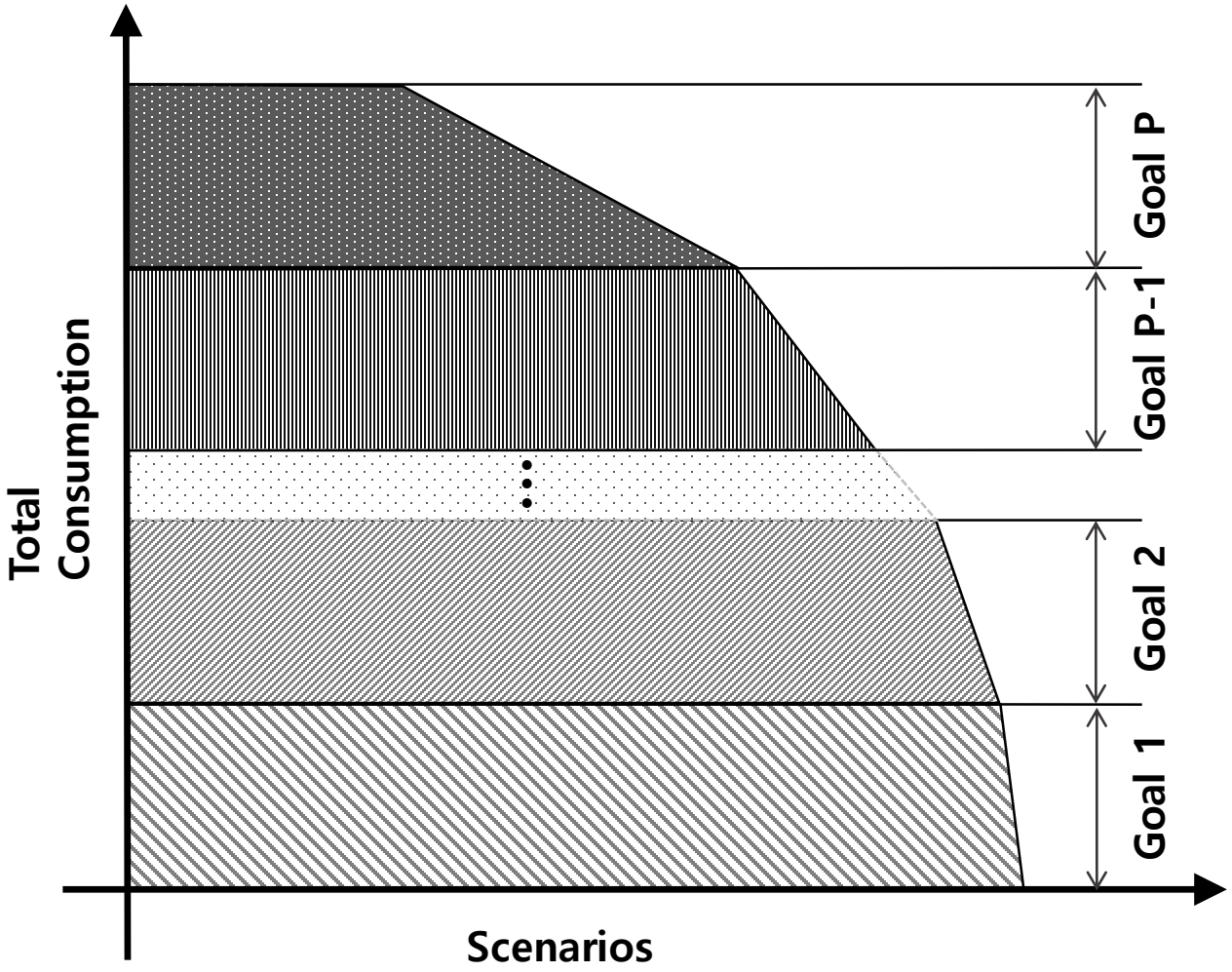
MSGP: Goal Achievement Information

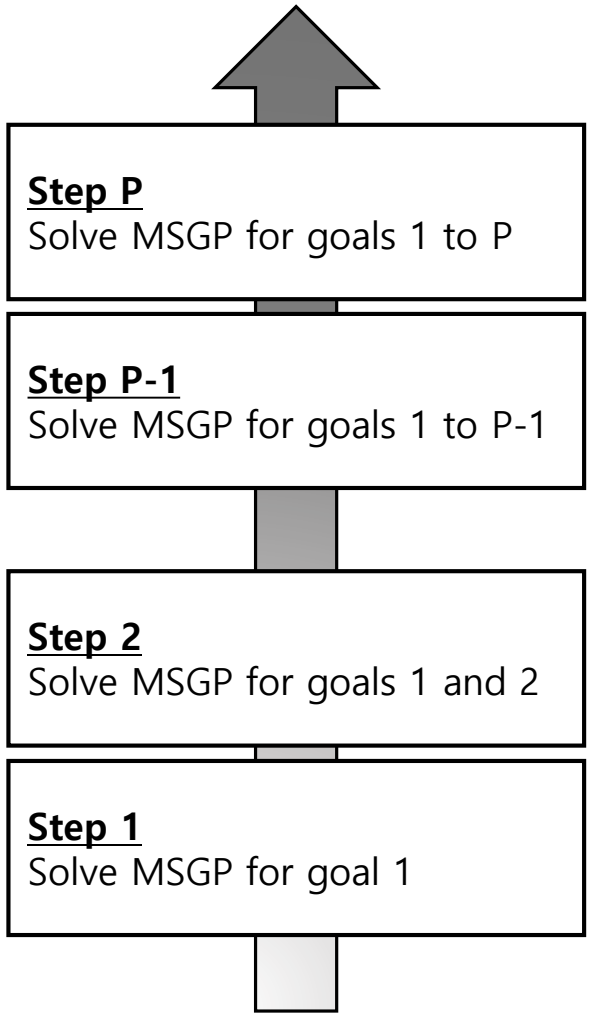
- **Goal achievement probability**
  - *Are my goals achievable?*
  - *Can I reach my goal over 90% of the time?*

- **Effect of savings** on goal achievements
  - *How does my goal probability change if I invest 2% more?*
  - *Can I still reach my goals if I invest less?*

- **Recommend savings level** for achieving a goal
  - *If I want to purchase a house in 10 years, how much more do I need to invest?*

MSGP: Sequential Optimization





Section 4

GOAL-BASED INVESTING EXAMPLES

Investment Condition

- Investable assets

Asset class	Expected return*	Standard deviation*
3-month bond (3-month Treasury Bill)	3.13%	2.46%
10-year bond (10-year Treasury Bond)	7.13%	9.28%
Commodity (S&P GSCI Commodity)	7.45%	26.37%
Real estate (FTSE NAREIT All REITs)	11.62%	19.43%
Developed market (MSCI EAFE)	7.12%	17.77%
U.S. market (S&P 500)	11.52%	17.92%
Emerging market (MSCI EM)	13.70%	24.66%

* Estimated from 1989.01 to 2015.12 (annualized)

Investment Condition

- Stages and decisions

	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Number of years (Age)	0 (30)	10 (40)	10 (50)	10 (60)	20 (80)

- Scenario tree

	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
# of descent nodes	80	10	5	5	-
# of scenarios	1	80	800	4000	**20000**

Investor Examples

Investor 1
(*invest less*)

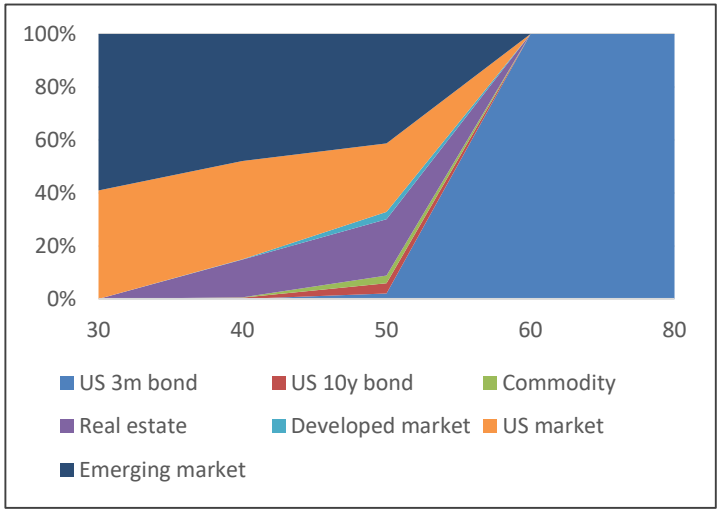
Age		30	40	50	60
Investment		50,000	83,000	110,000	137,000
Goal priority	High				**750,000**
	Med			**50,000**	
	Low				**30,000**

Investor 2
(*invest more*)

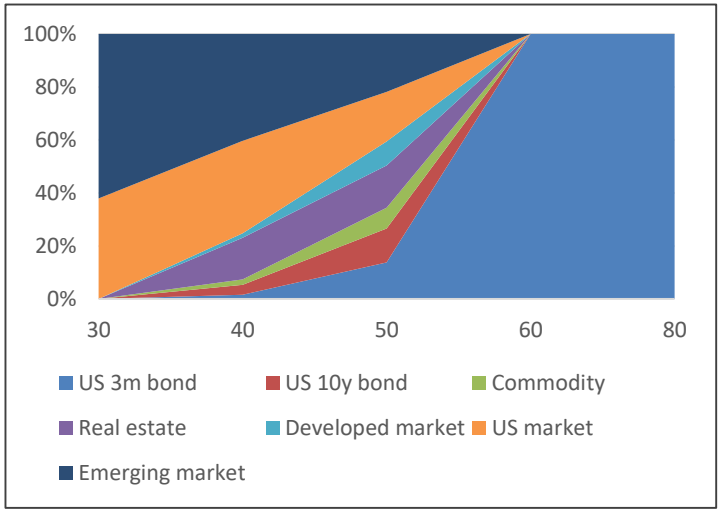
Age		30	40	50	60
Investment		50,000	100,000	132,000	164,000
Goal priority	High				**750,000**
	Med			**50,000**	
	Low				**30,000**

Investor Examples

- Comparison of average allocations

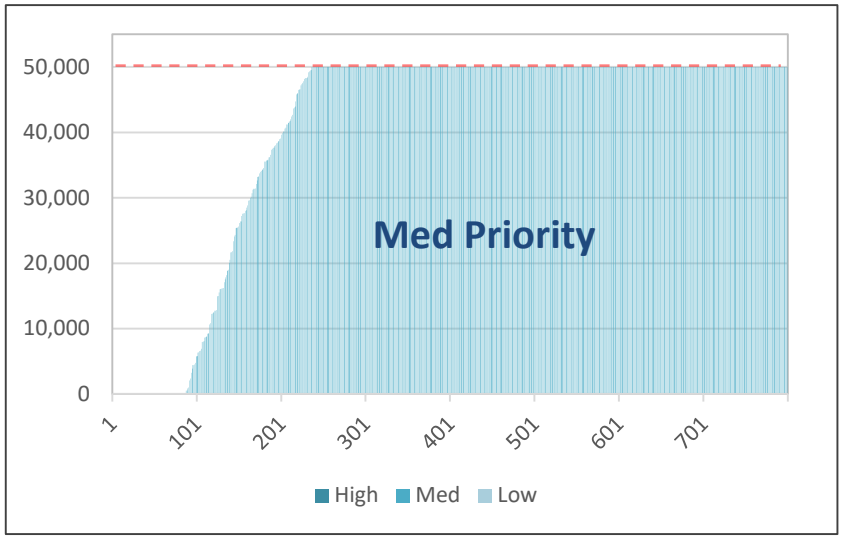


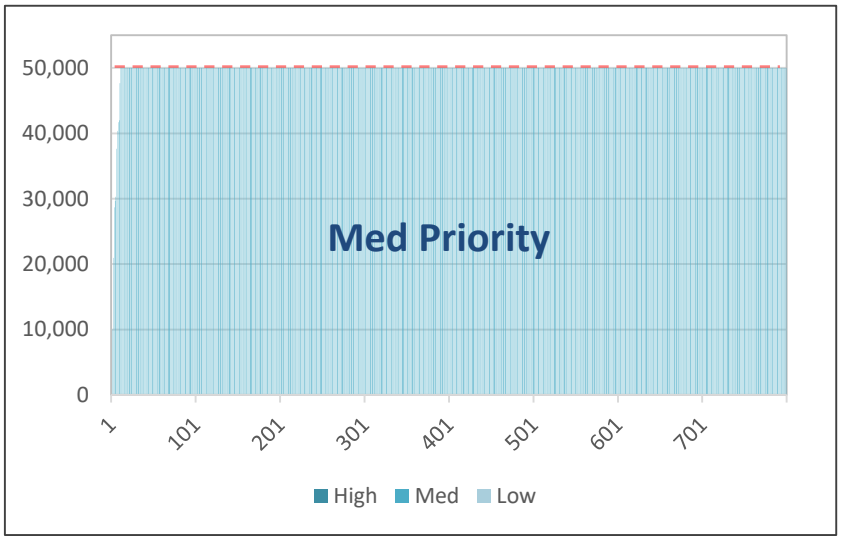
Investor 1
(*invest less*)



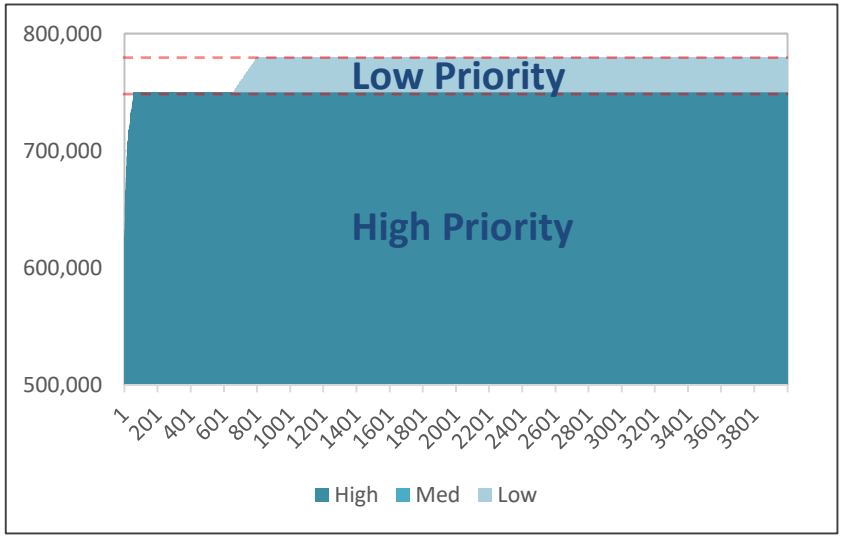
Investor 2
(*invest more*)

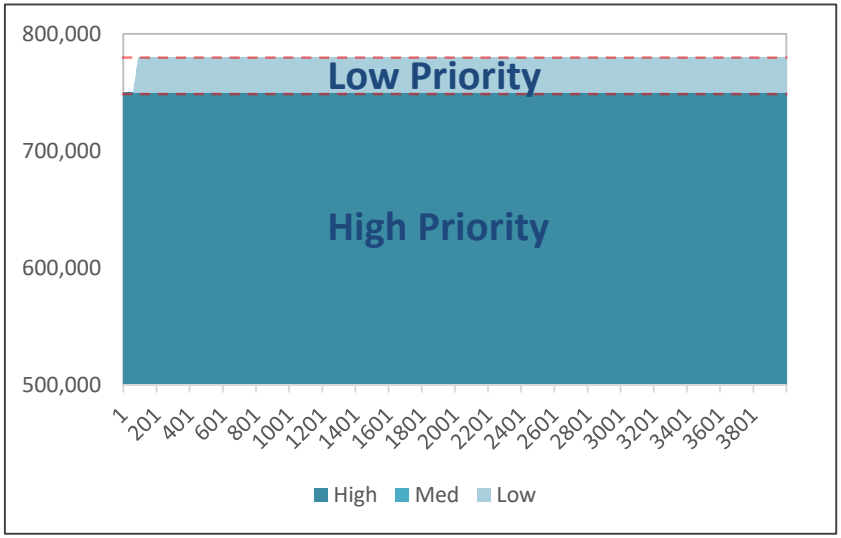
Age 50





Age 60





Investor 1
(*invest less*)

Investor 2
(*invest more*)

Section 5

CONCLUSION

Conclusion

- **Consumption goals are important** in personalized financial planning
- Typical investors have **multiple goals with different priorities**
- **MSGP** is one approach, which combines MSP with GP
- *High demand for personalized wealth management*

  → *Opportunity for interesting research!*

THANK YOU FOR LISTENING!

References

- Bellman, R. (1957). *Dynamic Programming*. Rand Corporation.
- Chhabra, A.B. (2005). Beyond Markowitz: A comprehensive wealth allocation framework for individual investors. *Journal of Wealth Management*, 7(4), 8-34.
- Consiglio, A., Cocco, F., & Zenios, S.A. (2007) Scenario optimization asset and liability modelling for individual investors. *Annals of Operations Research*, 152, 167-191.
- Consigli, G., & Dempster, M. A. H. (1998). Dynamic stochastic programming for asset-liability management. *Annals of Operations Research*, 81, 131-162.
- Dantzig, G. B., & Infanger, G. (1993). Multi-stage stochastic linear programs for portfolio optimization. *Annals of Operations Research*, 45(1), 59-76.
- Dempster, M. A., Kloppers, D., Medova, E., Osmolovsky, I., & Ustinov, P. (2016). Lifecycle goal achievement or portfolio volatility reduction?. *Journal of Portfolio Management*, 42(2), 99.
- Fabozzi, F.J., Gupta, F., & Markowitz, H. (2002). The legacy of modern portfolio theory. *Journal of Investing*, 11(3), 7-22.
- Fowler, G.B., & de Vassal, V. (2006). Holistic asset allocation for private individuals. *Journal of Wealth Management*, 9(1), 18-30.
- Markowitz, H. (1952). Portfolio Selection, *Journal of Finance*, 7(1), 77-91.
- Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*. J. Wiley.
- Martellini, L., & Milhau, V. (2017). Mass Customization versus Mass Production in Retirement Investment Management: Addressing a “Tough Engineering Problem”. *EDHEC-Risk Institute Publication*.
- Medova, E.A., Murphy, J.K., Owen, A.P., & Rehman, K. (2008). Individual asset liability management. *Quantitative Finance*, 8(6), 547-560.
- Mulvey, J.M. (1994). An asset-liability management system. *Interfaces*, 24(3), 22-33.
- Pereira, M. V., & Pinto, L. M. (1991). Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming*, 52(1-3), 359-375.
- Rockafellar, R.T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of Risk*, *2*, 21-42.
- Shefrin, H., & Statman, M. (2000). Behavioral portfolio theory. *Journal of Financial and Quantitative Analysis*, 35, 127-151.
- Ziemba, W. T., & Mulvey, J. M. (1998). *Worldwide Asset and Liability Modeling* (Vol. 10). Cambridge University Press.