

UNIT - 1

A Decimal no. such as 7392 represents a quantity equal to 7 thousand plus 3 hundreds, plus 9 tens plus 2 units. The thousands, hundreds, etc are powers of 10 implied by the position of the coefficients. To be more exact, 7392 should be written as -

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

There are numbers with different bases -

- i) Decimal (base 10)
- ii) Binary (base 2)
- iii) Octal (base 8)
- iv) Hexadecimal (base 16)

TABLE - No. with different bases

| DECIMAL (BASE 10) | BINARY (BASE 2) | OCTAL (BASE 8) | HEXADECIMAL BASE |
|----------------------|--------------------|-------------------|---------------------|
| 00 | 0000 | 00 | 0 |
| 01 | 0001 | 01 | 1 |
| 02 | 0010 | 02 | 2 |
| 03 | 0011 | 03 | 3 |
| 04 | 0100 | 04 | 4 |
| 05 | 0101 | 05 | 5 |
| 06 | 0110 | 06 | 6 |
| 07 | 0111 | 07 | 7 |
| 08 | 1000 | 10 | 8 |
| 09 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |

Q. Convert $(24.25)_{10}$ In base 2, 4, 8, 16.

$$(24.25)_{10}$$

i)

$$(24.25)_2 = \begin{array}{r} 2 | 24 \\ 2 | 12 \quad 0 \\ 2 | 6 \quad 0 \\ 2 | 3 \quad 0 \\ 2 | 1 \quad 1 \\ \hline & 1 \end{array}$$

$$0.25 \times 2 = 0.50$$

$$0.50 \times 2 = 1.00$$



$$(24.25)_2 = (11000.01)_2$$

ii)

$$(24.25)_4 = \begin{array}{r} 4 | 24 \\ 4 | 6 \quad 0 \\ 4 | 1 \quad 2 \\ \hline & 1 \end{array}$$

$$0.25 \times 4 = 1.00$$



$$(24.25)_4 = (120.1)_4$$

iii)

$$(24.25)_8 = \begin{array}{r} 8 | 24 \\ 8 | 3 \quad 0 \\ \hline & 3 \end{array}$$

$$0.25 \times 8 = 2.0$$

$$= (30.2)_8$$

$$\text{iv) } (24.25)_{16} = \begin{array}{r} 16 \longdiv{24} \\ 16 \underline{-} 1 \\ 8 \end{array} \quad \boxed{1}$$

$$0.25 \times 16 = 4.00 \downarrow$$

$$= (18.4)_{16}$$

The given no. may be a combination of integer and fractional part. In this case we divide the integer by the given base in which we have to convert the no. and we multiply the fractional part of no. with the base.

$$\begin{array}{ccc} (N)_{10} & = & (\)_r \\ \downarrow & & \\ (n) + (m) & & \\ \downarrow & \downarrow & \\ \text{Int} & \text{frac.} & \end{array}$$

Ex-

$$(63.24)_{10} = (\)_5$$

$$(63.24)_5 = \begin{array}{r} 5 \longdiv{63} \\ 5 \underline{-} 12 \\ 3 \end{array} \quad \begin{array}{l} \text{LESS SIGNIFICANT BITS} \\ \uparrow \end{array}$$

$$\begin{array}{r} 5 \longdiv{12} \\ 5 \underline{-} 2 \\ 2 \end{array} \quad \begin{array}{l} \text{MOST SIGNIFICANT BITS} \\ \uparrow \end{array}$$

$$\begin{array}{rcl} 0.24 \times 5 = 1.20 & & \\ 0.20 \times 5 = 1.00 & \downarrow & \\ & & = (223.11)_5 \end{array}$$

$$\text{Ex- } (1101 \cdot 100)_2 = (?)_{10}$$

$$(321 \cdot 21)_4 = (?)_{10}$$

$$(625 \cdot 25)_8 = (?)_{10}$$

$$(A4)_{16} = (?)_{10}$$

Conversion of Binary Numbers Into decimals

$$(1101.100)_2$$

$$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} \\ + 0 \times 2^{-3}$$

$$8 + 4 + 0 + 1 + 0.5 + 0 + 0$$

$$13 + 0.5$$

$$(13.5)_{10}$$

$$(321.21)_4$$

$$3 \times 4^2 + 2 \times 4^1 + 1 \times 4^0 + 2 \times 4^{-1} + 1 \times 4^{-2}$$

$$48 + 8 + 1 + 0.5 + 0.625$$

$$57 + 0.6 + 0.5$$

$$(58.625)_{10}$$

$$(625.25)_8$$

$$6 \times 8^2 + 2 \times 8^1 + 5 \times 8^0 + 2 \times 8^{-1} + 5 \times 8^{-2}$$

$$384 + 16 + 5 + 0.25 + 0.007$$

$$(405.32)_{10}$$

(A.4)₁₆

$$A \times 16^1 + 4 \times 16^0$$

$$160 + 4$$

$$(160.25)_{10}$$

OCTAL & HEXADECIMAL NUMBERS -

$$(3212.212)_4 = (?)_2$$

$$= (11100110.100110)_2$$

$$(56.45)_8 = (111101110.100101)_2$$

$$(A2C)_{16} = (101000101101)_2$$

$$(10212)_{16} =$$

$$\left(\frac{0}{1} \frac{1}{1} \frac{0}{1} \frac{1}{1} \frac{1}{3} \cdot \frac{1}{2} \frac{1}{3} \right)_2 = (1113.23)_4$$

$$\left(\frac{0}{1} \frac{0}{2} \frac{1}{7} \frac{1}{5} \frac{1}{4} \right)_2 = (127.54)_8$$

$$\left(\frac{0}{5} \frac{1}{7} \frac{0}{8} \frac{1}{B} \right) = (57.B)_{16}$$

COMPLIMENTS -

$$\tau^{\text{is}} \text{ complement} = (\tau^n)_{10} - (N)_\tau$$

$$(\tau-1)^{\text{is}} \text{ complement} = (\tau^n)_{10} - (\tau^{-m})_{10} - (N)_\tau$$

$$\begin{array}{c} (N)_\tau \\ \diagdown \\ n+m \end{array}$$

For ex - $(65.84)_{10}$

$$\tau = 10$$

$$n = 2$$

$$m = 2$$

$$\begin{aligned} 10^{\text{s}} \text{ complement} &= (10^3)_{10} - (65.84)_{10} \\ &= (100)_{10} - (65.84)_{10} \\ &= (34.16)_{10} \end{aligned}$$

$$\begin{aligned} 9^{\text{s}} \text{ complement} &= (10^2)_{10} - (10^{-2})_{10} - (65.84)_{10} \\ &= (100)_{10} - \left(\frac{1}{100}\right)_{10} - (65.84)_{10} \\ &= (100)_{10} - (0.01)_{10} - (65.84)_{10} \\ &= (34.15)_{10} \end{aligned}$$

$$(65.34)_8$$

$$\pi = 8$$

$$n = 2$$

$$m = -2$$

$$\begin{aligned}8's \text{ complement} &= (8^2)_{10} - (65.34)_8 \\&= (64)_{10} - (65.34)_8 \\&= (100)_8 - (65.34)_8 \\&= (12.44)_8\end{aligned}$$

Here the bases were not same so we converted the base which was not given in the problem.

as -

$$\begin{array}{r} 8 \underline{(64)} \\ 8 \underline{\mid} 8 \quad 0 \\ 8 \underline{\mid} \quad 0 \quad \uparrow \\ \quad \quad \quad 1 \end{array}$$

$$\begin{aligned}7's \text{ complement} &= (8^2)_{10} - (8^{-2})_{10} - (65.34)_8 \\&= (64)_{10} - \left(\frac{1}{64}\right)_{10} - (65.34)_8 \\&= (100)_8 - \left(\frac{1}{100}\right)_8 - (65.34)_8 \\&= (100)_8 - (0.01)_8 - (65.34)_8 \\&= (12.43)_8\end{aligned}$$

(671)₉

$$\begin{aligned}(9-1)^5 \text{ complement} &= (9^3)_{10} - (1)_{10} - (671)_9 \\&= (729)_{10} - (1)_{10} - (671)_9 \\&= (1000)_9 - (1)_9 - (671)_9 \\&= (217)_9\end{aligned}$$

$$\begin{aligned}(1010)_2 &= (2)_4^4 - (1010)_2 \\&= (16)_{10} - (1010)_2 \\&= (10000)_2 - (1010)_2 \\&= (0110)_2\end{aligned}$$

ADDITION & SUBTRACTION & MULTIPLICATION

$$\begin{array}{rcl} (234)_5 & - & \text{Augend} \\ + (444)_5 & - & \text{Addend} \\ \hline (1233)_5 & - & \text{Sum} \end{array} \quad \begin{array}{r} (845)_{11} \\ + (764)_{11} \\ \hline 14 A 9 \end{array}$$

$$\begin{array}{rcl} (4B69)_{13} & & (765)_9 - \text{Minuend} \\ + (38C4)_{13} & & - \underline{(388)_9} - \text{Subtrahend} \\ \hline (8760)_{13} & & (366)_9 \end{array}$$

$$\begin{array}{r}
 (45)_7 \\
 \times (56)_7 \\
 \hline
 402 \\
 324 \\
 \hline
 3642
 \end{array}$$

- Multiplicand
- Multiplier

$$\begin{array}{r}
 (683)_9 \\
 \times (786)_9 \\
 \hline
 4550 \\
 6136 \\
 \hline
 5343 \\
 \hline
 611320
 \end{array}$$

SUBTRACTION USING 2's complement -

If the given expression is of the form -

M - Minuend
- N - Subtrahend

Then first we find the 2's complement of Subtrahend, after it we add it to the minuend and if we get the sum on more bits than in the problem then we discard the extra bits from the left. If carry is not generated then again we find the 2's complement and write it with negative sign.

Carry will generate only when $M \geq N$ i.e when $M < N$ happen then no carry will generate.

Ex -

$$M = 10101$$

$$N = \underline{10001}$$

2's complement of 10001 =

$$\begin{array}{r} 10001 \\ 01110 \\ + \hline 1 \\ 01111 \end{array}$$

Now

$$\begin{array}{r} 10101 \\ + 01111 \\ \hline 100100 \end{array}$$

Discard

Result is 00100

Again If

$$M = 10001$$

$$N = 10101$$

2's complement of 10101 = 01011

Now

$$\begin{array}{r} 10001 \\ + 01011 \\ \hline 11100 \end{array}$$

carry not generated, Then

2's complement of 11100 =

$$\begin{array}{r} 11100 \\ 00011 \\ + \hline 1 \\ 00100 \end{array}$$

The result is -00100.

Ex -

$$M = 1010$$

$$- \quad N = 0111$$

$$\begin{array}{r} \text{2's complement of } 0111 = \\ \begin{array}{r} 0111 \\ + 1000 \\ \hline 1001 \end{array} \end{array}$$

Then

$$\begin{array}{r} 1010 \\ + 1001 \\ \hline 0011 \end{array}$$

Discard $\leftarrow \oplus 0011$

The result is 0011

$$M = (9876)_{10}$$

$$N = (6384)_{10}$$

$$\begin{aligned} 10^8 \text{ complement} &= (10^4)_{10} - (6384)_{10} \\ &= 10000 - 0384 \\ &= 9616 \end{aligned}$$

Then

$$\begin{array}{r} 9876 \\ + 9616 \\ \hline 9492 \end{array} \qquad \underline{\text{Ans}}$$

Gray Code -

| w | x | y | z |
|-------|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| <hr/> | | | |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 |
| <hr/> | | | |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| <hr/> | | | |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 |

Binary to Gray code -

Right \rightarrow left

same - zero

different = one

$$\begin{array}{r}
 101 \\
 111 \\
 \hline
 100
 \end{array}
 \rightarrow \text{Gray code}$$

$$\begin{array}{ccccccc}
 1 & \rightarrow & 0 & \rightarrow & 1 & \rightarrow & 0 \rightarrow 1 \\
 \downarrow & & & & & & \\
 1 & \leftrightarrow & 1 & \leftrightarrow & 1 & \leftrightarrow & 1 \leftrightarrow 1 \\
 \downarrow & & & & & & \\
 1 & 0 & 0 & 0 & 0 & &
 \end{array}$$

Gray Code to Binary Code -

$$\begin{array}{ccc}
 \begin{matrix} 1 & 0 & 0 \\ \downarrow & \nearrow & \nearrow \\ 1 & 1 & 1 \end{matrix} & &
 \begin{matrix} 0 & 1 & 0 \\ \downarrow & \nearrow & \nearrow \\ 0 & 1 & 1 \end{matrix}
 \end{array}$$

Boolean Functions -

| | OR | AND | NOT |
|---------------------|---------|-----------------|-------|
| $+ = x + y$ | $0+0=0$ | $0 \cdot 0 = 0$ | $1=0$ |
| $\cdot = x \cdot y$ | $0+1=1$ | $0 \cdot 1 = 0$ | $0=1$ |
| $- = x' \cdot y'$ | $1+0=1$ | $1 \cdot 0 = 0$ | |
| | $1+1=1$ | $1 \cdot 1 = 1$ | |

$$x+y = y+x$$

$$x+1 = 1$$

$$0+0 = 0+0$$

$$0+1 = 1$$

$$0+1 = 1+0$$

$$1+1 = 1$$

$$1+0 = 0+1$$

$$x \cdot 1 = x$$

$$1+1 = 1+1$$

$$0 \cdot 1 = 0$$

$$x' = x'$$

$$1 \cdot 1 = 1$$

$$1+x' = 1$$

$$x \cdot \bar{x} = 0$$

$$x + yz = (x+y)(x+z)$$

| $x \cdot z$ | $y \cdot z$ | $x+y+z$ | $x+y$ | $x+z$ | $(x+y)(x+z)$ |
|-------------|-------------|---------|-------|-------|--------------|
| 0 0 0 | 0 | 0 | 0 | 0 | 0 |
| 0 0 1 | 0 | 0 | 0 | 1 | 0 |
| 0 1 0 | 0 | 0 | 1 | 0 | 0 |
| 0 1 1 | 1 | 1 | 1 | 1 | 1 |
| 1 0 0 | 0 | 1 | 1 | 1 | 1 |
| 1 0 1 | 0 | 1 | 1 | 1 | 1 |
| 1 1 0 | 0 | 1 | 1 | 1 | 1 |
| 1 1 1 | 1 | 1 | 1 | 1 | 1 |

$$\begin{aligned} x + x' &= 1 \\ 0 + 1 &= 1 \\ 1 + 0 &= 1 \end{aligned} \quad \left. \begin{array}{l} x = 0 \\ x = 1 \end{array} \right\}$$

$$x \cdot x' = 0$$

$$x + x = x$$

$$0 \cdot 0 = 0$$

$$x \cdot x = x$$

or

$$1 \cdot 0 = 0$$

$$x + xy = 0$$

$$x(1+y) = x$$

$$x(x+y) = x \cdot x + x \cdot y = x + xy = x(1+y) = x$$

$$\text{Duality} - \text{i}) \quad x + xy = x$$

$$x \cdot (x+y) = x$$

$$\text{ii}) \quad x + (y+z)$$

$$x \cdot (y+z)$$

$$\text{iii}) \quad x + 1 = x \cdot 0$$

AND \rightarrow OR

0 \rightarrow 1

Replaces

OR \rightarrow AND

1 \rightarrow 0

De - Morgan's law -

$$\text{i}) \quad \overline{x+y} = \overline{x} \cdot \overline{y}$$

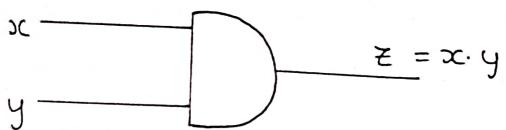
$$\text{ii}) \quad \overline{xy} = \overline{x} + \overline{y}$$

| $\overline{x} \cdot \overline{y}$ | $\overline{x+y}$ | x | y | \overline{x} | \overline{y} | $x \cdot y$ | \overline{xy} | $\overline{x+y}$ | $\overline{x} + \overline{y}$ | \overline{xy} |
|-----------------------------------|------------------|---|---|----------------|----------------|-------------|-----------------|------------------|-------------------------------|-----------------|
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |

GATES

Basic Gates -

i) AND - $Z = x \cdot y$



| TRUTH-TABLE | | |
|-------------|---|-----------------|
| INPUT | | OP |
| x | y | $z = x \cdot y$ |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

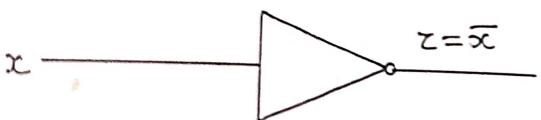
ii) OR - $Z = x + y$



TRUTH-TABLE

| INP | | OP |
|-----|---|-------------|
| x | y | $z = x + y$ |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

iii) NOT - $Z = \bar{x}$

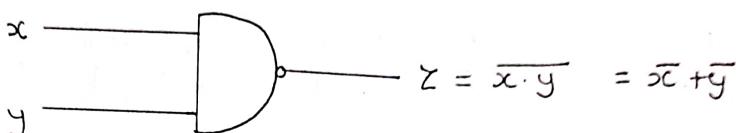
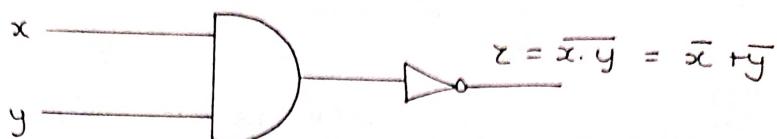


TRUTH-TABLE

| I/P | O/P |
|-----|---------------|
| x | $z = \bar{x}$ |
| 0 | 1 |
| 1 | 0 |

Universal Gates -

i) NAND -

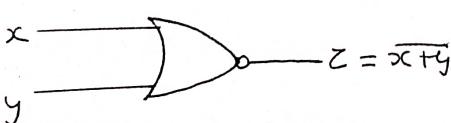
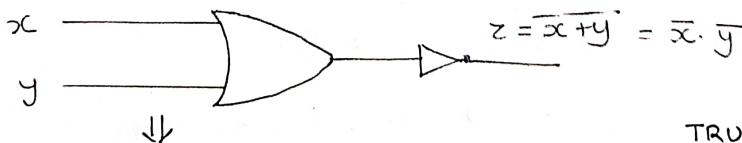


TRUTH TABLE

I/P O/P

| x | y | $z = \overline{xy}$ |
|---|---|---------------------|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

ii) NOR -



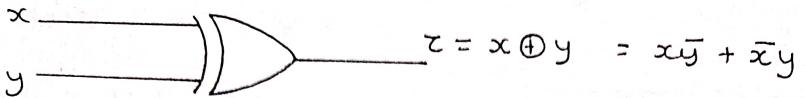
TRUTH - TABLE

| x | y | $z = \overline{xy}$ |
|---|---|---------------------|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

Some - other Gates -

\oplus - Exclusive

i) XOR -

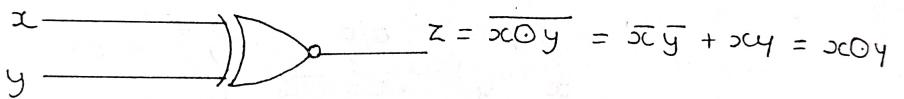


$$z = x \oplus y = xy + x\bar{y}$$

TRUTH TABLE

$$\begin{array}{cc|c} x & y & z = x \oplus y \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$$

ii) XNOR -

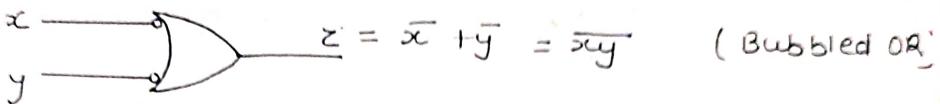
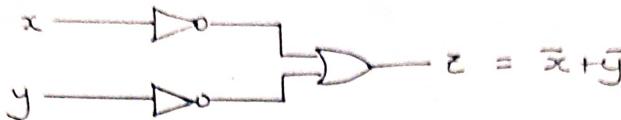
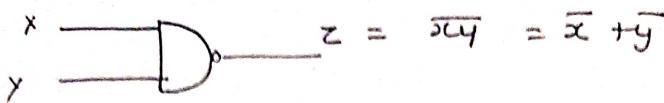


$$z = \overline{x \oplus y} = \overline{xy} + \overline{x}\bar{y} = \overline{xy} + \overline{x}\bar{y}$$

TRUTH TABLE

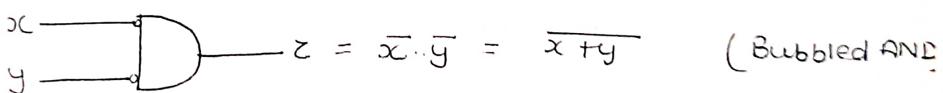
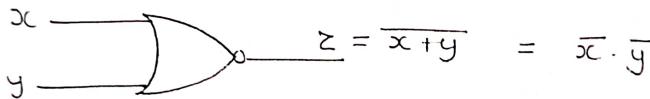
| x | y | z |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

NAND GATE -



Bubbled OR \sim NAND

NOR GATE -

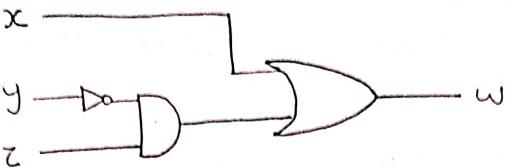


Bubbled AND \sim NOR

Bubbled OR \sim NAND

Q.

$$\omega = \bar{x} + y'z$$

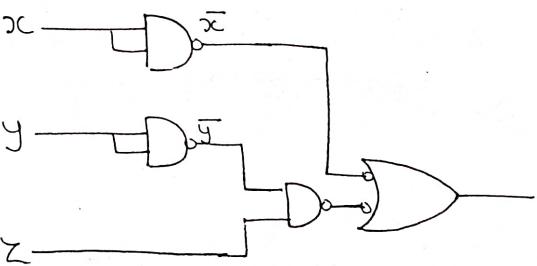


$$\bar{x} \cdot x = 0$$

$$\bar{x} \cdot x = 0$$

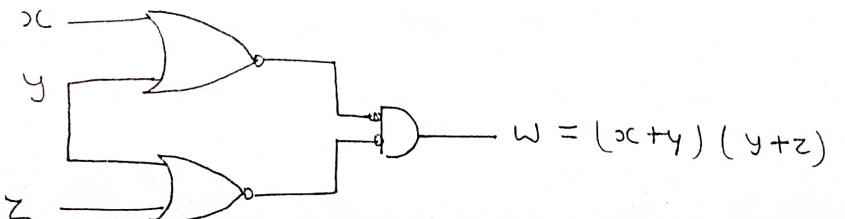
$$\bar{x} + \bar{x} = \bar{x}$$

$$\bar{x} + \bar{x} = \bar{x}$$



Q.

$$\omega = (x+y)(y+z)$$



Some Important Points -

$$x + 0 = x$$

$$x \cdot 1 = x$$

$$x + x' = 1$$

$$x \cdot x' = 0$$

$$x + x = x$$

$$x \cdot x = x$$

$$x + 1 = 1$$

$$x \cdot 0 = 0$$

$$(x')' = x$$

$$(x')' = x$$

$$x+y = y+x$$

$$x \cdot y = y \cdot x$$

$$x + (y+z) = (x+y) + z$$

$$x \cdot (yz) = (x \cdot y) \cdot z$$

$$x(y+z) = xy + xz$$

$$x + (yz) = (x+y) \cdot (x+z)$$

$$\overline{x+y} = \overline{x} \cdot \overline{y}$$

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\overline{xy} = \overline{x} + \overline{y}$$

$$\overline{x+y} = \overline{x} \cdot \overline{y}$$

$$x + xy = x$$

$$x \cdot (x+y) = x$$

| x | y | xy | x+xy |
|------------|---|------------|------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| \uparrow | | \uparrow | |

$$x + xy = x(1+y)$$

$$= x \cdot 1$$

$$x + xy = x$$

$$x + 0 = x \quad \text{duality} \quad x \cdot 1 = x$$

$$x + yz$$

↓

$$xz + xyz$$

↑

$$x'z' + xyz$$

Literals = Simple & Complement

Q. Simplify the following Expressions —

i) $x + x'y$ ii) $x(x' + y)$ iii) $xy + x'z + yz$

Solⁿ i) $x + x'y = (x + x')(x + y)$

$$= 1 \cdot (x + y)$$

$$= (x + y)$$

ii) $x(x' + y) = xx' + xy$

$$= 0 + xy$$

$$= xy$$

(iii)

$$xy + x'z + yz$$

$$xy + x'z + (x+x')(yz) \quad \left\{ x+x'=1 \right\}$$

$$xy + x'z + z'xz + x'yz$$

$$xy + (1+z) + x'z(1+y)$$

$$xy + x'z$$

Q. Find the Dual & complement of each literals

$$f_1' = x'y'z' + x'y'z$$

Dual - $f_1 = (x'+y+z') (x'+y'+z)$

Then complement of each literals

$$f_1 = (x+y'+z) (x+y+z')$$

Minterm = Sum of Product , Maxterm = Product of sum.

Maxterm \Rightarrow 0 - Normal , 1 - complement

Minterm \Rightarrow 1 - Normal , 0 - complement

| x | y | z | Minterm | | Maxterm | |
|---|---|---|----------|-------------|-------------|-------------|
| | | | Term | Designation | Term | Designation |
| 0 | 0 | 0 | $x'y'z'$ | m_0 | $(x+y+z)$ | M_0 |
| 0 | 0 | 1 | $x'y'z$ | m_1 | $(x+y+z')$ | M_1 |
| 0 | 1 | 0 | $x'y'z'$ | m_2 | $(x+y'+z)$ | M_2 |
| 0 | 1 | 1 | $x'yz$ | m_3 | $(x+y'+z')$ | M_3 |
| 1 | 0 | 0 | $x'y'z'$ | m_4 | $(x'+y+z)$ | M_4 |
| 1 | 0 | 1 | $x'y'z$ | m_5 | $(x'+y+z')$ | M_5 |
| 1 | 1 | 0 | $x'yz'$ | m_6 | $(x'+y'+z)$ | M_6 |

Q. Express the Boolean Function

$$f = A + BC \text{ in a sum of Minterm.}$$

Solⁿ

$$f = A + BC$$

$$\begin{aligned} A(B+B') &= AB + AB' \\ &= AB(C+C') + AB'(C+C') \\ &= ABC + ABC' + AB'C + AB'C' \end{aligned}$$

Term = Total no. of variables are must

$$\begin{aligned} BC &= (A+A') BC \\ &= ABC + A'BC \end{aligned}$$

Now,

$$\begin{aligned} f &= ABC + ABC' + AB'C + AB'C' + A'BC + A'BC' \\ &= ABC + ABC' + AB'C + AB'C' + A'BC \\ &= M_7 + M_6 + M_5 + M_4 + M_3 \end{aligned}$$

If un term occurs more than one time, then we count that term as one.

$$= \sum(7, 6, 5, 4, 3) = \sum(3, 4, 5, 6, 7)$$

Q. $xy + x'z$ in the form of product of maxterm.

Solⁿ-

$$xy + x'z$$

$$A + x'z$$

$$\text{Let } A = xy$$

$$= (A+x') (A+z)$$

$$= (\bar{a}y + x') (\bar{a}y + z)$$

$$= (\bar{x}' + \bar{a}c) (\bar{a}c' + y) (z + x) (z + y)$$

$$\vdash . (x' + y) (\bar{a}c + z) (y + z)$$

$$= (\bar{a}c' + y + zz') (x + z + yy') (y + z + xx')$$

$$= (\bar{a}c' + y + z) (x' + y + z') (x + z + y) (x + z + y')$$

$$(y + z + x) (y + z + x')$$

$$= (x' + y + z) (x' + y + z') (\bar{a}c + y + z) (\bar{a}c + y' + z) (\bar{a}c + y + z)$$

$$(\bar{a}c' + y + z)$$

$$= (x' + y + z) (x' + y + z') (\bar{a}c + y + z) (x + y' + z)$$

$$= M_4 \cdot M_5 \cdot M_0 \cdot M_2$$

$$= \pi(0, 2, 4, 5)$$

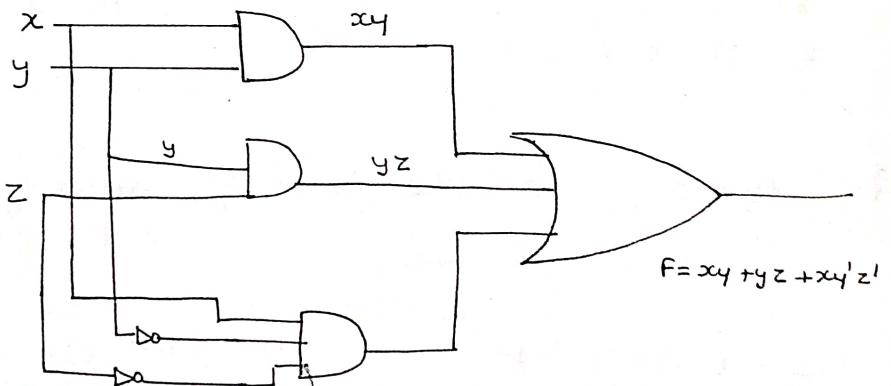
Q. $\bar{a}cy + x'e'z$ in the form of sum of minterm

Q. $F = xy + yz + x'y'z'$

Solution -

Inputs

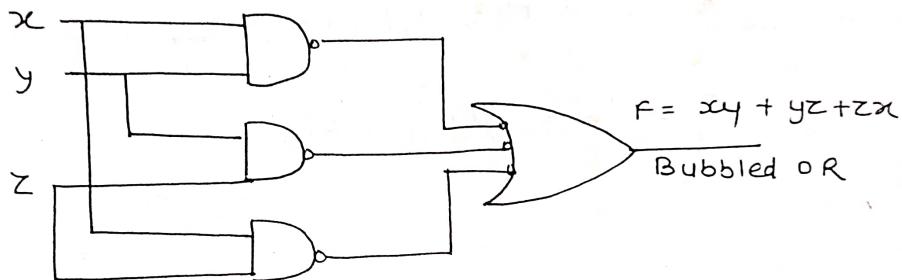
| x | y | z | xy | yz | y' | z' | $x'y'z'$ | F |
|---|---|---|------|------|------|------|----------|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |



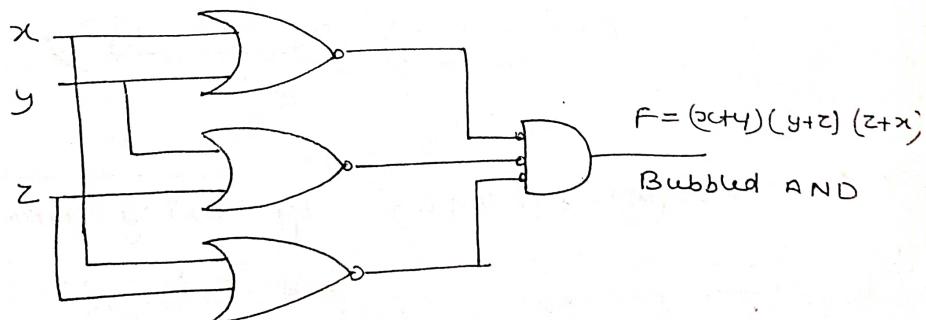
Q. i) If $F = xy + yz + zx$ is given then it is solved generally with NAND GATE because it is given in the form of sum of product.

If ii) $F = (x+y)(y+z)(z+x)$ then it is solved by using NOR gate because of Product of sum.

i) $F = xy + yz + zx$ by using NAND gates



ii) $F = (x+y)(y+z)(z+x)$ by using NOR gates



Q. Design $f = xy + yz$ by using NOR Gate.

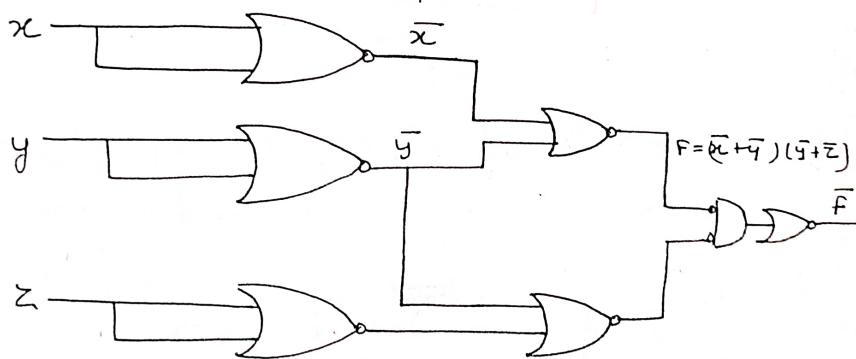
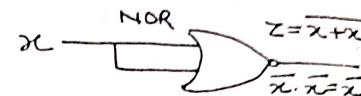
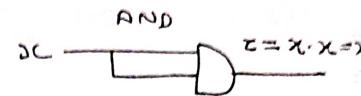
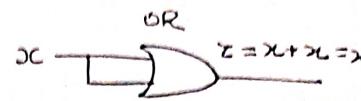
$$F = \overline{\overline{xy} + yz}$$

$$F = \overline{(\overline{xy}) \cdot (\overline{yz})}$$

$$F = \overline{(\overline{x} + \overline{y}) \cdot (\overline{y} + \overline{z})}$$

$$F = \overline{(\overline{x} + \overline{y})} + \overline{(\overline{y} + \overline{z})}$$

$$\bar{F} = (\overline{x} + \overline{y}) + (\overline{y} + \overline{z})$$



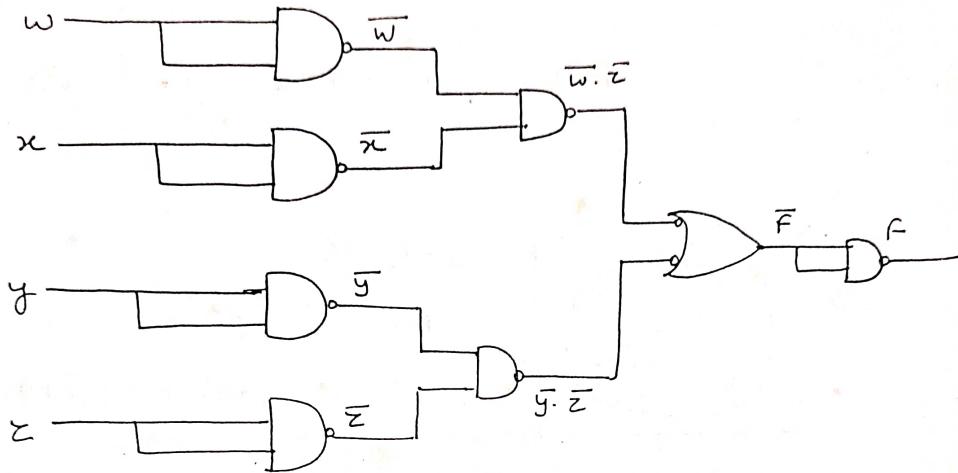
Q. $F = (w+x)(y+z)$ by using NAND gates.

$$F = (w+x)(y+z)$$

$$F = \overline{(w+x)(y+z)}$$

$$= \overline{(\bar{w}+x)} + \overline{(y+z)}$$

$$f = \overline{\bar{w} \cdot \bar{x}} + \overline{y \cdot z}$$

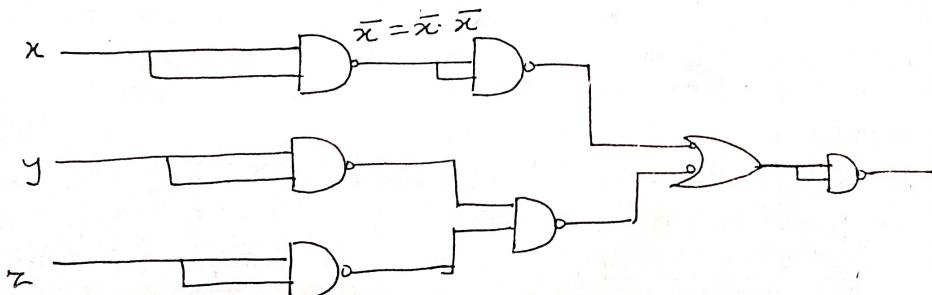


Q. $F = x(y+z)$ using NAND Gates.

$$F = \overline{\bar{x}(y+z)}$$

$$= \overline{\bar{x} + (\bar{y} + z)}$$

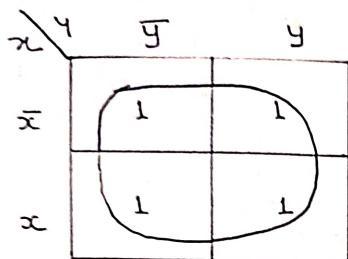
$$= \bar{x} + (\bar{y} \cdot \bar{z})$$



K-MAP (KARNAUGH MAP) -

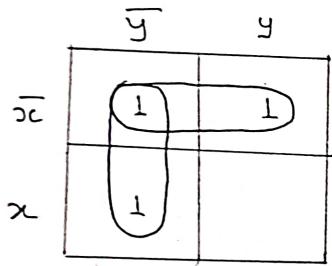
$2^2 = 4$ (Two variables)

| | | y - column | |
|-----------|---------------|------------------------|------------------|
| | | $\bar{y} = 0$ | $y = 1$ |
| x (row) | $\bar{x} = 0$ | 00 $\bar{x}\bar{y}$ | 01 $\bar{x}y$ |
| | $x = 1$ | 10 $x\bar{y}$ | 11 xy |



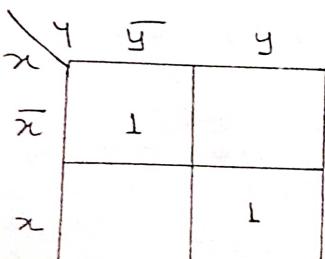
1-quad.

$$\begin{aligned}
 F &= \bar{x}\bar{y} + \bar{x}y + x\bar{y} + xy \\
 &= \bar{x}(\bar{y}+y) + x(\bar{y}+y) \\
 &= \bar{x} + x \\
 F &= 1
 \end{aligned}$$



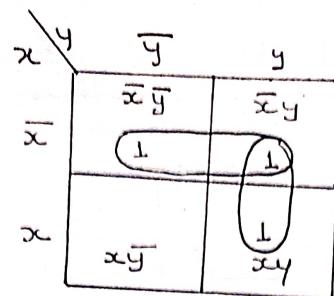
2-pairs

$$\begin{aligned}
 F &= \bar{x} \cdot (\bar{y}+y) & \bar{y}(\bar{x}+x) \\
 &= \bar{x} \cdot 1 & \bar{y} \cdot 1 \\
 F &= \bar{x} & \bar{y}
 \end{aligned}$$



$$\begin{aligned}
 F &= \bar{x}\bar{y} + xy \\
 &= \overline{x \oplus y}
 \end{aligned}$$

Q. $f = \bar{x}\bar{y} + \bar{x}y + xy$ using K-MAP.



$$f = \bar{x}(\bar{y} + y) + (\bar{x} + x)y$$

$$f = \bar{x} \cdot 1 + 1 \cdot y$$

$$f = \bar{x} + y$$

Some Cases - (Three variables) $2^3 = 8$

i)

| $\bar{x}\bar{y}\bar{z}$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-------------------------|------------------|------------|------|------------|
| \bar{x} | 1 | 1 | 1 | 1 |
| x | 1 | 1 | 1 | 1 |

$$f = \bar{x}(\bar{y}\bar{z} + \bar{y}z + yz + y\bar{z})$$

$$f = \bar{x} \cdot 1$$

$$f = \bar{x}$$

$$f = x \text{ (2nd row)}$$

ii)

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

Q. $f(x, y, z) = \sum (2, 3, 4, 5)$

| $x\bar{y}z$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-------------|------------------|------------|------|------------|
| \bar{x} | | | 1 | 1 |
| x | 1 | 1 | | |

$$f = \bar{x}y + xy$$

Q. $f(x, y, z) = \sum (3, 4, 6, 7)$

| $x\bar{y}z$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-------------|------------------|------------|------|------------|
| \bar{x} | | | 1 | |
| x | 1 | | 1 | 1 |

$$f = yz + x\bar{z}$$

Q. $f(x, y, z) = \sum (0, 2, 4, 5, 6)$

| $x\bar{y}z$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-------------|------------------|------------|------|------------|
| \bar{x} | 1 | | 1 | 1 |
| x | 1 | 1 | | 1 |

$$f = \bar{z} + xy$$

$$Q. \quad f(x, y, z) = \sum (1, 2, 3, 5, 7)$$

| $\bar{x}yz$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|-------------|------------------|------------|------|------------|
| \bar{x} | 1 | 1 | 1 | |
| x | | | | |
| \bar{x} | 1 | 1 | | |

$$F = z + \bar{x}y$$

K-MAP using ~~Three~~ ^{Four} variables - $(wxyz)$, $2^4 = 16$

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|------|--------------------|--------------------|--------------------|--------------------|
| $\bar{w}\bar{x}$ | 00 | 0000 0000 0 | 0001 0001 1 | 0011 0011 3 | 0010 0010 2 |
| $\bar{w}x$ | 01 | 0100 0100 4 | 0101 0101 5 | 0111 0111 7 | 0110 0110 6 |
| $w\bar{x}$ | 11 | 1100 1100 12 | 1101 1101 13 | 1111 1111 15 | 1110 1110 14 |
| wx | 10 | 1000 1000 8 | 1001 1001 9 | 1011 1011 11 | 1010 1010 10 |

$$f = 1$$

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|------|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 1 | 1 | 1 | 1 | 1 |
| $\bar{w}x$ | 1 | 1 | 1 | 1 | 1 |
| $w\bar{x}$ | 1 | 1 | 1 | 1 | 1 |
| wx | 1 | 1 | 1 | 1 | 1 |

Q. Simplify the function $F(w,x,y,z) = \sum(0,1,2,4,5,$

6, 8, 9, 12, 13, 14)

Soln

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|----|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 1 | 1 | | | 1 |
| $\bar{w}x$ | 1 | | 1 | | 1 |
| wx | 1 | | 1 | | 1 |
| $w\bar{x}$ | 1 | 1 | | | |

$$F = \bar{y} + \bar{w}\bar{z} + x\bar{z}$$

Q. $F(w,x,y,z) = \sum(0,1,2,6,8,9,10)$

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|----|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 1 | 1 | | | 1 |
| $\bar{w}x$ | | | | | 1 |
| wx | | | | | |
| $w\bar{x}$ | 1 | 1 | | | 1 |

$$F = \bar{x}\bar{z} + \bar{x}\bar{y} + \bar{w}\bar{z}y$$

Q. Simplify the following function

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

| | $\bar{C}D$ | $\bar{C}\bar{D}$ | CD | $C\bar{D}$ |
|------------------|------------|------------------|------|------------|
| $\bar{A}\bar{B}$ | 1 | 1 | | 1 |
| $\bar{A}B$ | | | | 1 |
| $A\bar{B}$ | | | | |
| AB | 1 | 1 | | 1 |

$$F = B'C' + B'C + A'CD' \quad (\text{Sum of Product})$$

K- MAP (By Product of Sum) or (Product of Maxterm)

| | $y+z$ | $y+\bar{z}$ | $\bar{y}+z$ | $\bar{y}+\bar{z}$ |
|-------------------|-------|-------------|-------------|-------------------|
| $w+x$ | 0000 | 0001 | 0011 | 0010 |
| $w+\bar{x}$ | 0100 | 0101 | 0111 | 0110 |
| $\bar{w}+\bar{x}$ | 1100 | 1101 | 1111 | 1110 |
| $\bar{w}+x$ | 1000 | 1001 | 1011 | 1010 |

Q. Simplify the following by using i) SOP ii) POS

$$F(w, x, y, z) = \sum(0, 1, 2, 4, 8, 9, 10)$$

| | $\bar{y}z$ | $\bar{y}\bar{z}$ | $y\bar{z}$ | yz |
|------------|------------|------------------|------------|------|
| $w\bar{x}$ | 1 | 1 | 0 | 1 |
| $\bar{w}x$ | 1 | 0 | 0 | 0 |
| wx | 0 | 0 | 0 | 0 |
| $w\bar{x}$ | 1 | 1 | 0 | 1 |

$$SOP = \bar{y}\bar{z} + \bar{x}y + \bar{w}\bar{y}\bar{z}$$

$$POS = (\bar{w} + \bar{x}) (\bar{y} + \bar{z}) (\bar{x} + \bar{z}) (\bar{x} + \bar{y})$$

Q. $F(A, B, C, D) = \sum(0, 1, 2, 5, 8, 9, 10)$

| AB | $\bar{C}D$ | $\bar{C}D$ | $\bar{C}D$ | CD | CD | |
|------------------|---------------|---------------|---------------------|---------------|------|---------------------|
| $\bar{A}\bar{B}$ | 1 | 1 | 0 | 1 | | $A + B$ |
| $\bar{A}B$ | 0 | 1 | 0 | 0 | 0 | $A + \bar{B}$ |
| $A\bar{B}$ | 0 | 0 | 0 | 0 | 0 | $\bar{A} + \bar{B}$ |
| AB | 1 | 1 | 0 | 0 | 1 | $\bar{A} + B$ |
| | $\bar{B} + D$ | $C + \bar{D}$ | $\bar{C} + \bar{D}$ | $\bar{C} + D$ | | |

$$SOP = \bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}\bar{C}D$$

$$POS = (\bar{A} + \bar{B}) (\bar{C} + \bar{D}) (\bar{B} + D)$$

Q. $F = (A' + B' + D) (A' + D') (A' + B' + D') (A + B' + C + D)$

| AB | $\bar{C}D$ | $\bar{C}D$ | CD | CD | | |
|------------------|---------------|---------------|---------------------|---------------|---|---------------------|
| $\bar{A}\bar{B}$ | 1 | 0 | 0 | 1 | | $A + B$ |
| $\bar{A}B$ | 0 | 1 | 1 | 1 | 1 | $A + \bar{B}$ |
| $A\bar{B}$ | 0 | 0 | 0 | 0 | | $\bar{A} + \bar{B}$ |
| AB | 1 | 0 | 0 | 1 | | $\bar{A} + B$ |
| | $\bar{B} + D$ | $C + \bar{D}$ | $\bar{C} + \bar{D}$ | $\bar{C} + D$ | | |

$$SOP = \bar{B}\bar{D} + \bar{A}BD + \bar{A}BC$$

$$POS = (\bar{A} + \bar{B})(B + \bar{D})(\bar{B} + C + D)$$

DON'T CARE CONDITION -

| wx | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ | |
|------------------|------------------|------------|------|------------|-------------------|
| $\bar{w}\bar{x}$ | X | 1 | 1 | X | $w+x$ |
| $\bar{w}x$ | 0 | 0 | 1 | 1 | $w+\bar{x}$ |
| wx | X | 0 | 0 | 0 | $\bar{w}+\bar{x}$ |
| $w\bar{x}$ | 1 | 0 | X | 1 | $\bar{w}+x$ |

$$SOP = \bar{w}y + \bar{x}\bar{z} + \bar{w}\bar{x}$$

$$POS = (\bar{w} + \bar{x})(\bar{x} + y)(\bar{w} + \bar{z})$$

Q. $f(w, x, y, z) = \sum(1, 3, 7, 11, 15)$

$$d(w, x, y, z) = \sum(0, 2, 5)$$

| wx | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ | |
|------------------|------------------|------------|------|------------|-------------------|
| $\bar{w}\bar{x}$ | X | 1 | 1 | X | $w+x$ |
| $\bar{w}x$ | 0 | X | 1 | 0 | $w+\bar{x}$ |
| wx | 0 | 0 | 1 | 0 | $\bar{w}+\bar{x}$ |
| $w\bar{x}$ | 0 | 0 | 1 | 0 | $\bar{w}+x$ |

$$Q. \quad F(A, B, C, D) = \sum (5, 6, 7, 12, 14, 15)$$

$$d(A, B, C, D) = \sum (3, 9, 11)$$

| | $\bar{C}D$ | $\bar{C}D$ | CD | $C\bar{D}$ | |
|------------|------------|-------------|-------------------|-------------|-------------------|
| $\bar{A}B$ | 0 | 0 | X | 0 | $A+B$ |
| $\bar{A}B$ | 0 | 1 | 1 | 1 | $A+\bar{B}$ |
| AB | 1 | 0 | 1 | 1 | $\bar{A}+\bar{B}$ |
| $A\bar{B}$ | 0 | X | X | 0 | $\bar{A}+B$ |
| | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ | |

$$\text{SOP} = CB + D\bar{A}B + AB\bar{D}$$

$$\text{POS} = B(\bar{A} + C + D)(A + C + D)$$

UNIT-2**COMBINATIONAL & SEQUENTIAL CIRCUITS**

Combinational- Output depends upon input.

Sequential- Output depends upon input as well as memory element.

| INP | C | S | INP | B | D |
|---------|---|---|---------|---|---|
| 0+0 = 0 | 0 | 0 | 0-0 = 0 | 0 | 0 |
| 0+1 = 0 | 1 | | 0-0 = 0 | 1 | |
| 1+0 = 0 | 1 | | 0-1 = 1 | 1 | |
| 1+1 = 1 | 0 | | 1-1 = 0 | 0 | 0 |

Half Adder- This circuit needs two binary inputs and two binary outputs.

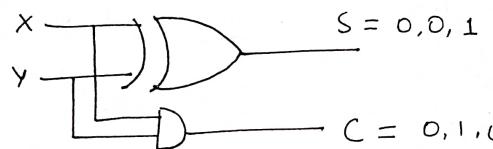
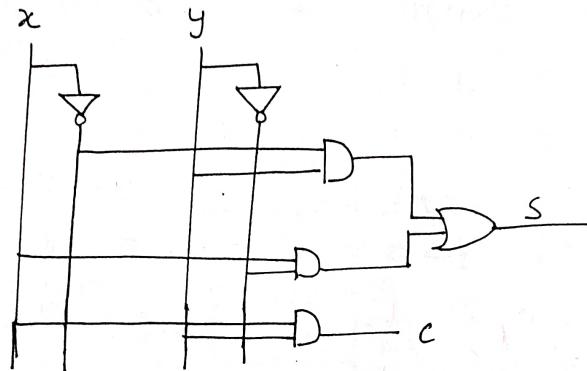
| INPUTS | | OUTPUT | |
|--------|---|--------|---|
| X | Y | C | S |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$$S = \bar{x}y + xy \quad (\text{sum of minterms})$$

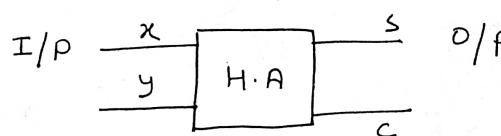
$$S = \bar{x} \oplus y$$

$$C = \bar{x}y$$

Logic Diagram -



Symbolic Diagram -



Full Adder- A Full Adder is a combinational circuit that forms the arithmetic sum of three bits. It consists of three inputs and two outputs.

| INPUT | | | OUTPUT | |
|-------|---|---|--------|---|
| x | y | z | c | s |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$S = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z}$$

$$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$$

$$S = \bar{x}(y \oplus z) + x(\bar{y} \oplus z)$$

$$S = \bar{x}f + xf$$

$$S = \bar{x} \oplus f$$

$$S = x \oplus (y \oplus z)$$

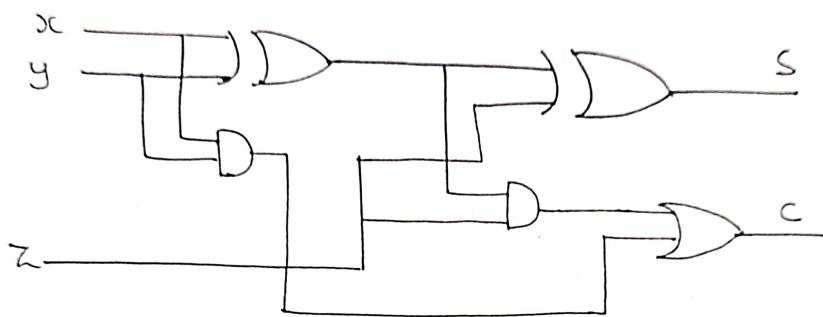
$$S = x \oplus y \oplus z$$

$$C = \bar{x}yz + xc\bar{y}z + xc\bar{y}\bar{z} + x\bar{y}c$$

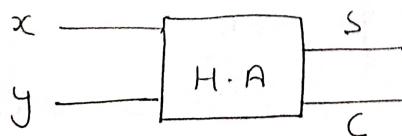
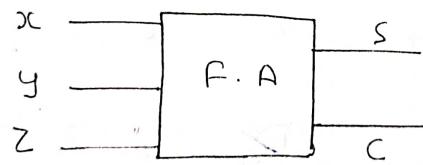
$$C = z(\bar{x}y + x\bar{y}) + xy(\bar{z} + z)$$

$$C = (x \oplus y)z + xy$$

logic Diagram -



Symbolic Diagram -



Half subtractor -

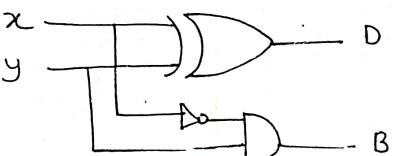
| INPUT | | OUTPUT | |
|-------|---|--------|---|
| x | y | B | D |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

$$D = \bar{x}y + x\bar{y}$$

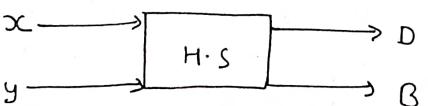
$$D = x \oplus y$$

$$B = \bar{x}y$$

logic Diagram -



Symbolic Diagram -



Full - Subtractor-

| INPUT | | | OUTPUT | |
|-------|---|---|--------|---|
| x | y | z | B | D |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$D = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + xy\bar{z}$$

$$= \bar{x}(\bar{y}\bar{z} + y\bar{z}) + x(\bar{y}\bar{z} + y\bar{z})$$

$$= \bar{x}(y \oplus z) + x(\bar{y} \oplus z)$$

$$D = x \oplus y \oplus z$$

$$B = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + xy\bar{z}$$

$$= \bar{x}(\bar{y}\bar{z} + y\bar{z}) + yz(x + \bar{x})$$

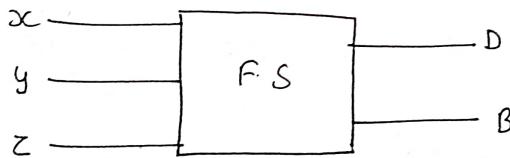
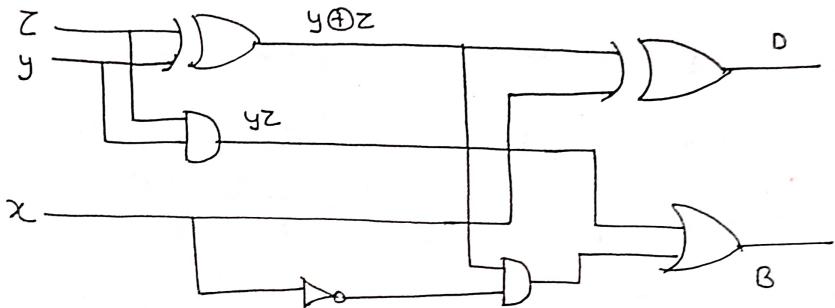
$$= \bar{x}(y \oplus z) + yz$$

OR

$$B = \bar{x}z(y + \bar{y}) + \bar{x}y(\bar{z} + z) + yz(\bar{x} + x)$$

$$B = \bar{x}\bar{z} + \bar{x}y + yz$$

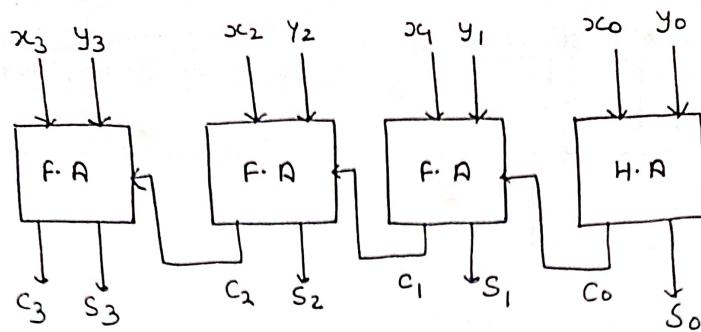
Logic Diagram -



Binary Adder - Binary Adder consists one half Adder and (n) Full Adder.

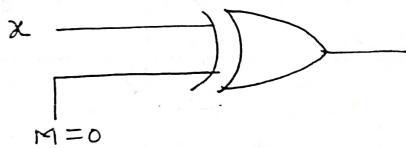
| | c_2 | c_1 | c_0 | |
|-------|-------|-------|-------|-------|
| $x =$ | x_3 | x_2 | x_1 | x_0 |
| $y =$ | y_3 | y_2 | y_1 | y_0 |
| | c_3 | s_3 | s_2 | s_1 |
| | F.A | F.A | F.A | A.D |

Symbolic Diagram -

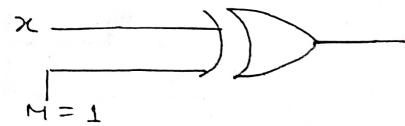


★ when we take '0' It gives the output as same as the input.

★ when we take '1' It gives the opposite of the input. i.e complement

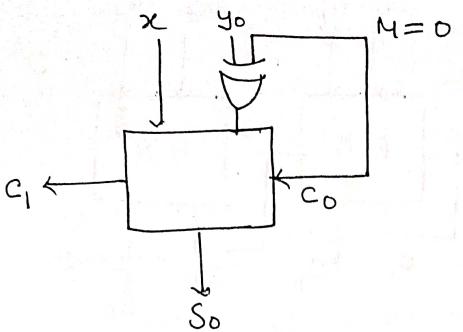


$$\begin{aligned}
 f &= xy + \bar{x}y \\
 &= x \cdot \bar{y} + \bar{x} \cdot M \\
 &= x \cdot 0 + \bar{x} \cdot 0 \\
 &= x \cdot 1 \\
 f &= x
 \end{aligned}$$

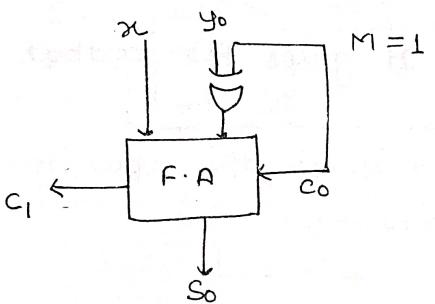


$$\begin{aligned}
 f &= x \cdot \bar{y} + \bar{x} \cdot M \\
 &= x \cdot \bar{1} + \bar{x} \cdot 1 \\
 &= x \cdot 0 + \bar{x} \\
 &= 0 + \bar{x} \\
 f &= \bar{x}
 \end{aligned}$$

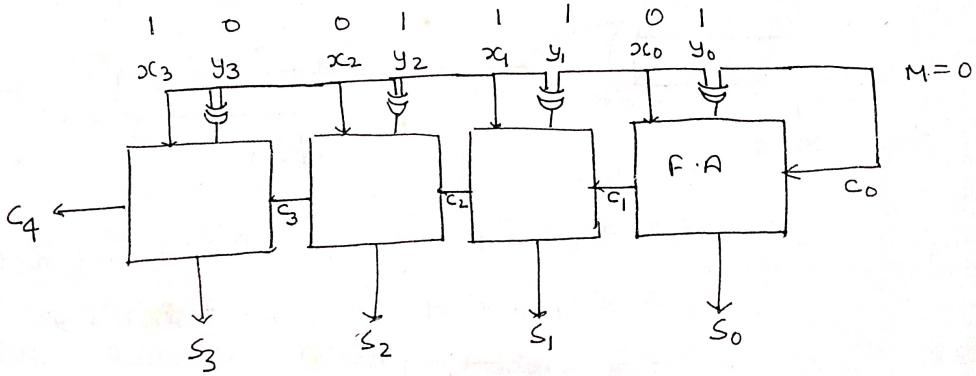
Symbolic Diagram -



$$\bar{x} + \bar{y}$$



$$x + \bar{y} + 1 = x - y$$



$M=0$ (Addition)

$M=1$ (Subtraction)

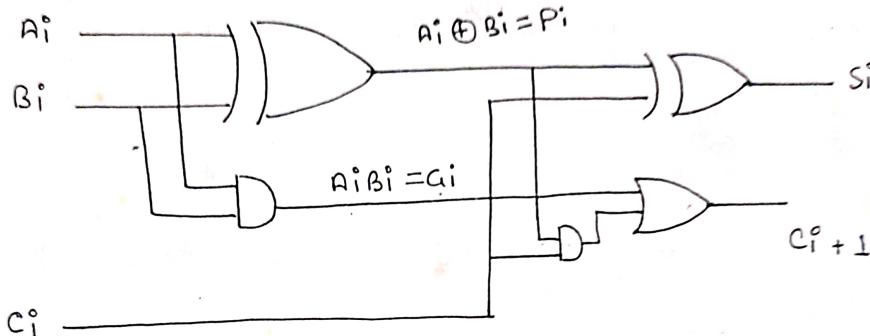
10001

23.08.16

CARRY PROPOGATION -

$$S = A \oplus B \oplus C$$

$$C = (A \oplus B) C + AB$$



$$S^o = A^i \oplus B^i \oplus C^i = P^i \oplus G^i$$

$$C^o + 1 = (A^i \oplus B^i) C^i + A^i B^i = P^i C^i + G^i$$

$i=0$

$$C_1 = P_0 C_0 + G_0$$

$i=1$

$$C_2 = P_1 C_1 + G_1$$

$$= P_1 (P_0 C_0 + G_0) + G_1$$

$$= \underbrace{P_1 P_0 C_0}_{\text{AND}} + \underbrace{P_1 G_0}_{\text{AND}} + G_1$$

$\boxed{\text{OR}}$

$$i = 2$$

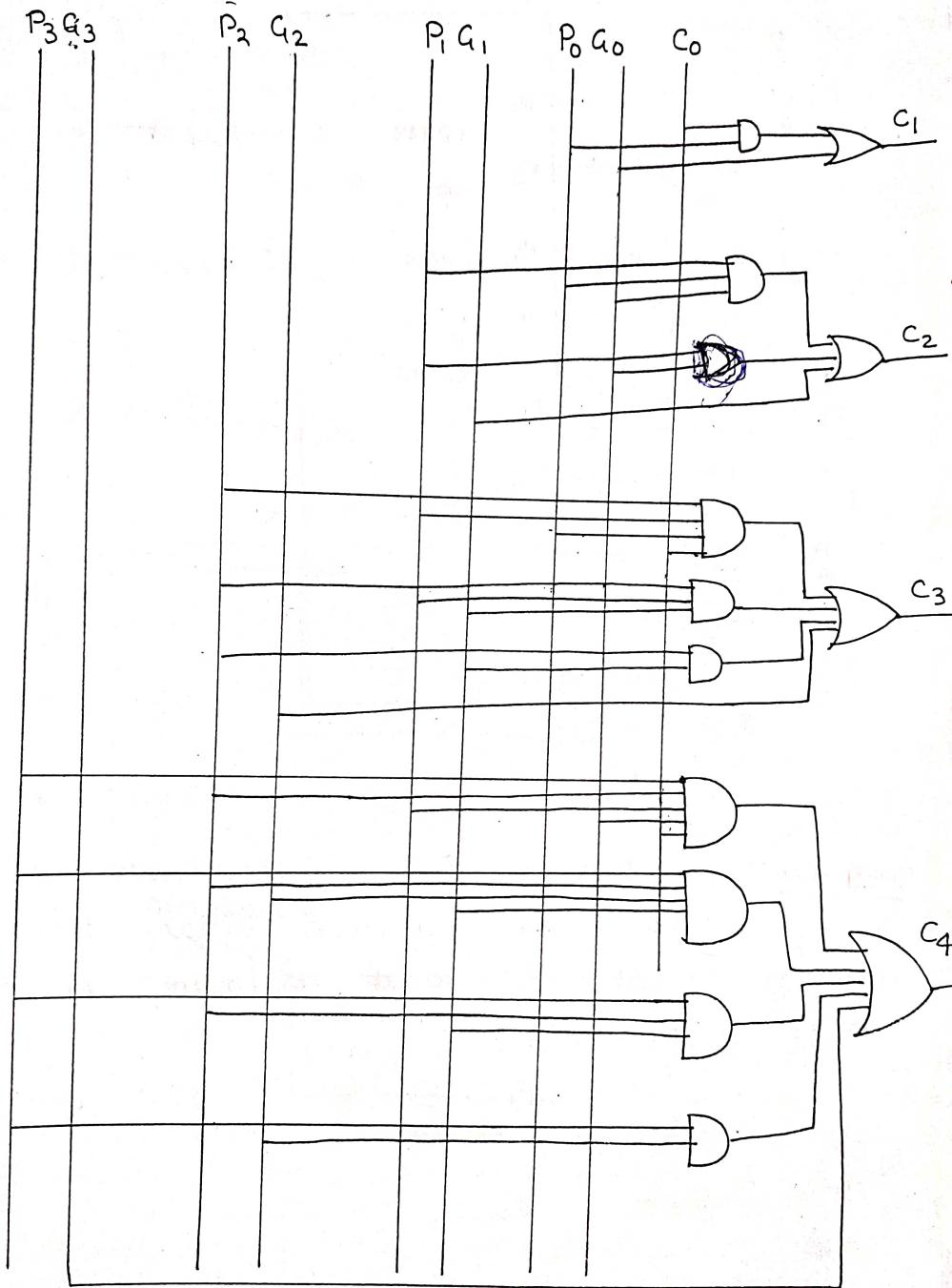
$$\begin{aligned}C_3 &= P_2 C_2 + G_2 \\&= P_2 (P_1 P_0 C_0 + P_1 G_0 + G_1) + G_2 \\&= \underbrace{P_2 P_1 P_0 C_0}_{\text{AND}} + \underbrace{P_2 P_1 G_0}_{\text{AND}} + \underbrace{P_2 G_1}_{\text{AND}} + G_2 \\&\quad \text{OR}\end{aligned}$$

$$i = 3$$

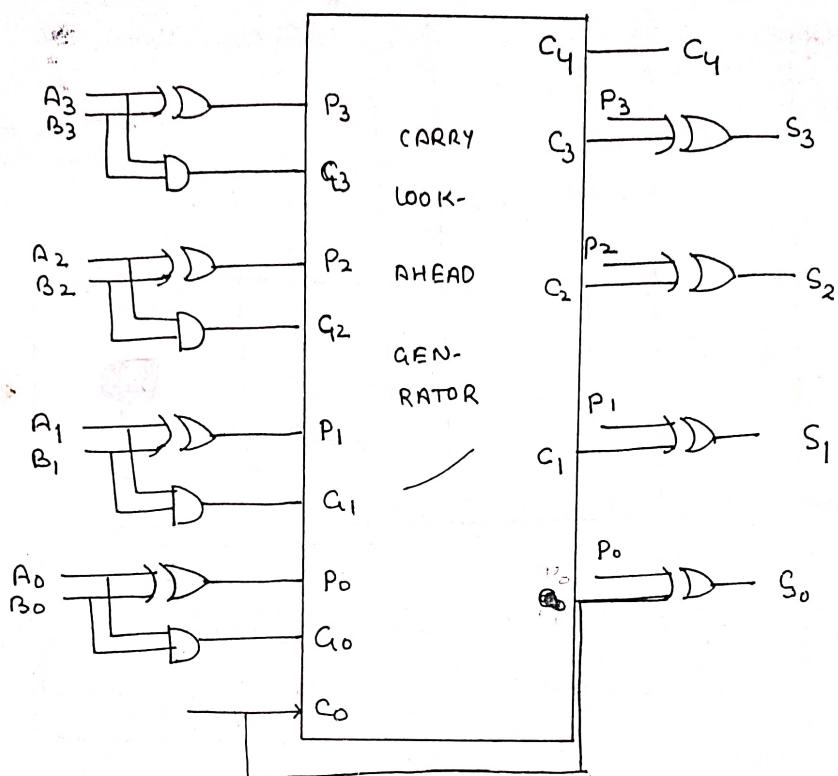
$$\begin{aligned}C_4 &= P_3 C_3 + G_3 \\&= P_3 (P_2 P_1 P_0 C_0 + P_2 P_1 G_0 + P_2 G_1 + G_2) + G_3 \\&= \underbrace{P_3 P_2 P_1 P_0 C_0}_{\text{AND}} + \underbrace{P_3 P_2 P_1 G_0}_{\text{AND}} + \underbrace{P_3 P_2 G_1}_{\text{AND}} + \underbrace{P_3 G_2 + G_3}_{\text{AND}} \\&\quad \text{OR}\end{aligned}$$

★ Carry Propagation solution increases complexity of circuit and decreases the time delay.

Design 4 bit carry lookahead Generator (fast adder)



CARRY LOOK AHEAD GENERATOR (ADDER)
4-Bit



Decoder - A Decoder is a combinational circuit that converts binary Info. from n- input lines to a maximum of 2^n unique output lines.

Truth Table -

29/08/16

| x | y | z | D ₀ | D ₁ | D ₂ | D ₃ | D ₄ | D ₅ | D ₆ | D ₇ |
|---|---|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

$$D_0 = \bar{x}\bar{y}\bar{z}$$

$$D_1 = \bar{x}\bar{y}z$$

$$D_2 = \cancel{\bar{x}yz}$$

$$D_3 = \bar{x}yz$$

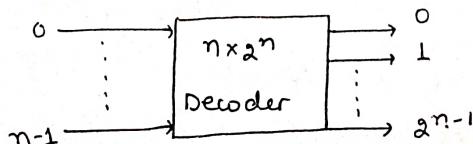
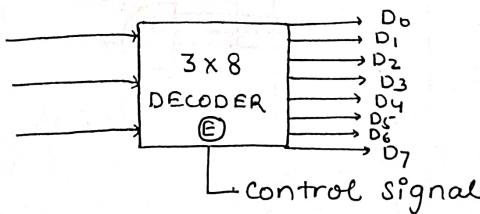
$$D_4 = xy\bar{z}$$

$$D_5 = xy\bar{z}$$

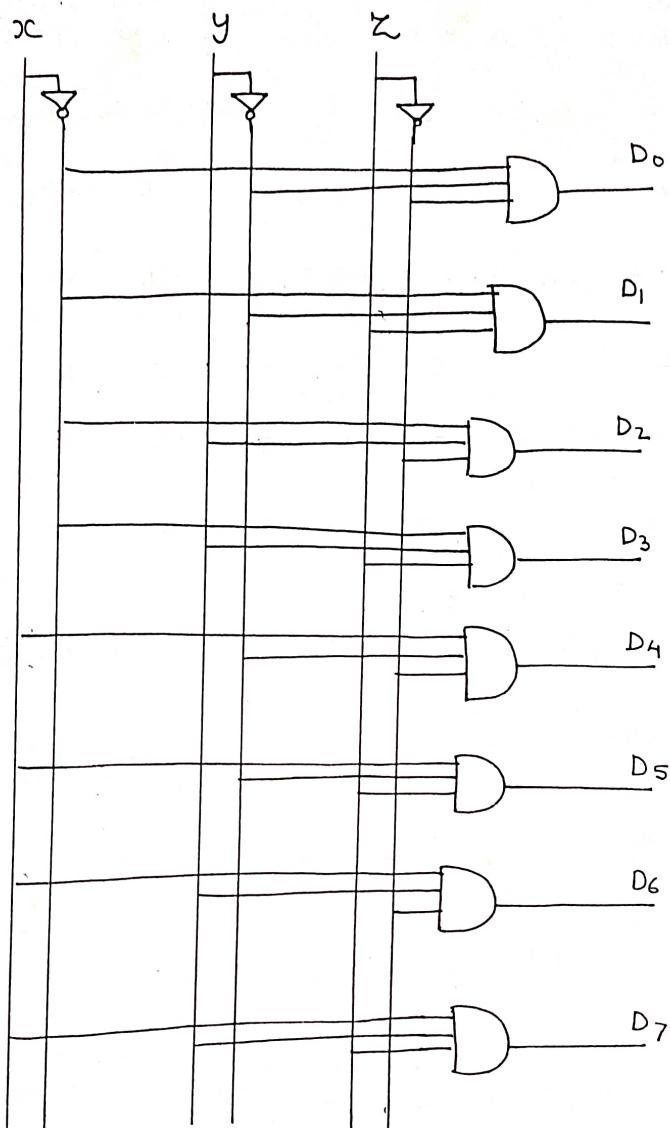
$$D_6 = xy\bar{z}$$

$$D_7 = xyz$$

Symbolic Diagram -



LOGIC DIAGRAMS -



Design BCD Decoder with unused combinations

Binary \rightarrow Decimal

Input

Output

| w x y z | D ₀ | D ₁ | D ₂ | D ₃ | D ₄ | D ₅ | D ₆ | D ₇ | D ₈ | D ₉ |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 0 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 0 0 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 0 1 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 0 1 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 1 0 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 1 0 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 1 1 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 1 1 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 0 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 0 0 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | |
|---------|---|---|---|---|---|---|---|---|---|---|
| 1 0 1 0 | x | x | x | x | x | x | x | x | x | x |
| 1 0 1 1 | x | x | x | x | x | x | x | x | x | x |
| 1 1 0 0 | x | x | x | x | x | x | x | x | x | x |
| 1 1 0 1 | x | x | x | x | x | x | x | x | x | x |
| 1 1 1 0 | x | x | x | x | x | x | x | x | x | x |
| 1 1 1 1 | x | x | x | x | x | x | x | x | x | x |

→ UNUSED OR USED AS DON'T CARE

K-MAP for D_0 -

| wx | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 1 | 0 | 0 | 0 |
| $\bar{w}x$ | 0 | 0 | 0 | 0 |
| wx | x | x | x | x |
| $w\bar{x}$ | 0 | 0 | x | x |

$$D_0 = \bar{w}\bar{x}\bar{y}\bar{z}$$

K-MAP for D_1 -

| wx | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 0 | 1 | 0 | 0 |
| $\bar{w}x$ | 0 | 0 | 0 | 0 |
| wx | x | x | x | x |
| $w\bar{x}$ | 0 | 0 | x | x |

$$D_1 = \bar{w}\bar{x}\bar{y}\bar{z}$$

K-MAP for D_3 -

| wx | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 0 | 0 | 1 | 0 |
| $\bar{w}x$ | 0 | 0 | 0 | 0 |
| wx | x | x | x | x |
| $w\bar{x}$ | 0 | 0 | x | x |

$$D_3 = \bar{x}yz$$

K-MAP for D_2 -

| wx | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|------------------|------------|------|------------|
| $\bar{w}\bar{x}$ | 0 | 0 | 0 | 1 |
| $\bar{w}x$ | 0 | 0 | 0 | 0 |
| wx | x | x | x | x |
| $w\bar{x}$ | 0 | 0 | x | x |

$$D_2 = \bar{x}y\bar{z}$$

K-MAP for D₄ -

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-----|------------------|------------|-----|------------|
| $\bar{w}\bar{x}$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\bar{w}x$ | 1 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| wx | x x | x x | x x | x x | x x |
| w \bar{x} | 0 0 | x x | x x | x x | x x |

$$D_4 = x\bar{y}\bar{z}$$

K-MAP for D₅ -

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-----|------------------|------------|-----|------------|
| $\bar{w}\bar{x}$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\bar{w}x$ | 0 0 | 1 0 | 0 0 | 0 0 | 0 0 |
| wx | x x | x x | x x | x x | x x |
| w \bar{x} | 0 0 | x x | x x | x x | x x |

$$D_5 = x\bar{y}z$$

K-MAP for D₆ -

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-----|------------------|------------|-----|------------|
| $\bar{w}\bar{x}$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\bar{w}x$ | 0 0 | 0 0 | 0 1 | 0 0 | 0 0 |
| wx | x x | x x | x x | x x | x x |
| w \bar{x} | 0 0 | x x | x x | x x | x x |

$$D_6 = \begin{matrix} x\bar{y}\bar{z} \\ 110 \end{matrix}$$

K-MAP for D₇ -

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-----|------------------|------------|-----|------------|
| $\bar{w}\bar{x}$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\bar{w}x$ | 0 0 | 0 0 | 1 0 | 0 0 | 0 0 |
| wx | x x | x x | x x | x x | x x |
| w \bar{x} | 0 0 | 0 0 | 0 0 | 0 0 | x x |

$$D_7 = \begin{matrix} xy\bar{z} \\ 001 \end{matrix}$$

K-Map for D₈

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-----|------------------|------------|-----|------------|
| $\bar{w}\bar{x}$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\bar{w}x$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| wx | x x | x x | x x | x x | x x |
| w \bar{x} | 1 0 | 0 x | x x | x x | x x |

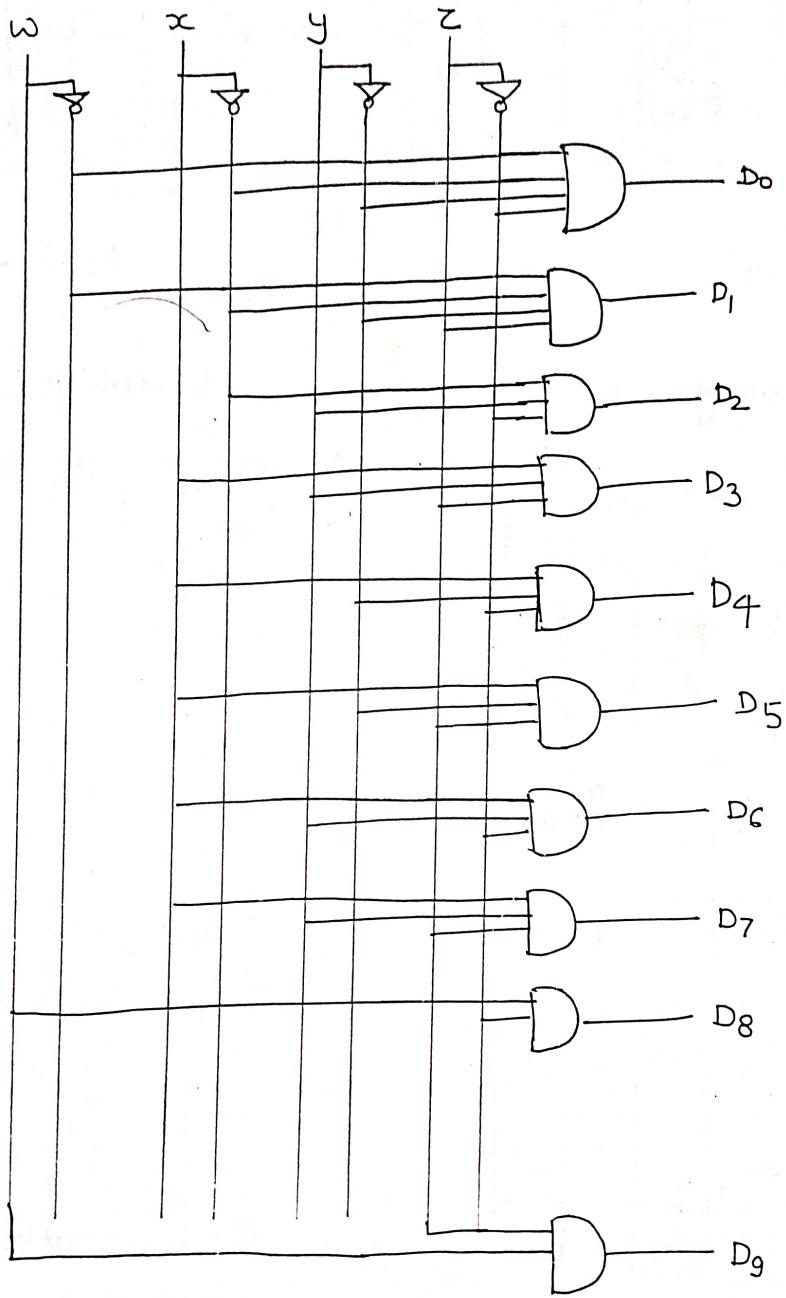
$$D_8 = w\bar{z}$$

K-Map for D₉

| wx | yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | yz | $y\bar{z}$ |
|------------------|-----|------------------|------------|-----|------------|
| $\bar{w}\bar{x}$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\bar{w}x$ | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| wx | x x | x x | x x | x x | x x |
| w \bar{x} | 0 1 | 1 x | x x | x x | x x |

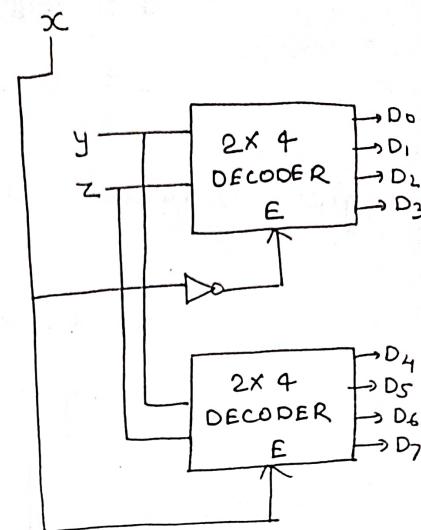
$$D_9 = w\bar{z}$$

Design BCD Decoder using don't care -



Design 3×8 by using 2×4 Decoder

| x | y | z | |
|---|---|---|-------------------|
| 0 | 0 | 0 | $\rightarrow D_0$ |
| 0 | 0 | 1 | $\rightarrow D_1$ |
| 0 | 1 | 0 | $\rightarrow D_2$ |
| 0 | 1 | 1 | $\rightarrow D_3$ |
| 1 | 0 | 0 | $\rightarrow D_4$ |
| 1 | 0 | 1 | $\rightarrow D_5$ |
| 1 | 1 | 0 | $\rightarrow D_6$ |
| 1 | 1 | 1 | $\rightarrow D_7$ |



30/08/16

Encoder - 2^n input & n output

Decoder - n -input & 2^n output

Encoders - An encoder is a digital circuit that performs the inverse operation of a

Decoder.

TRUTH-TABLE

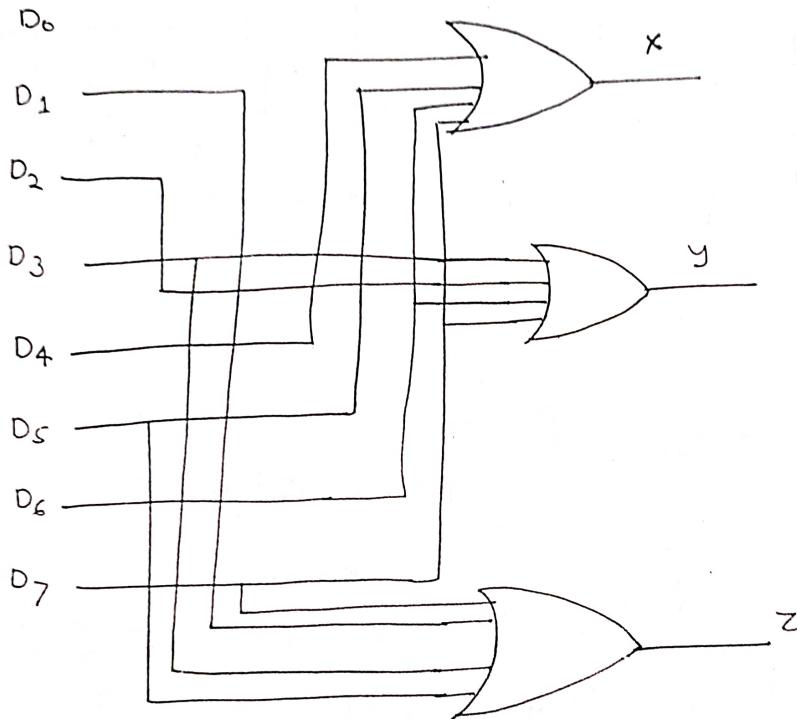
| D ₀ | D ₁ | D ₂ | D ₃ | D ₄ | D ₅ | D ₆ | D ₇ | x | y | z |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

$$x = D_4 + D_5 + D_6 + D_7$$

$$y = D_2 + D_3 + D_6 + D_7$$

$$z = D_1 + D_3 + D_5 + D_7$$

logic Diagram for Encoders -



priority Encoder- A Priority Encoder is an encoder circuit that includes the priority function. The operation of the priority encoder is such that if two or more inputs are equal to 1 at the same time. The input having the highest priority will take precedence.

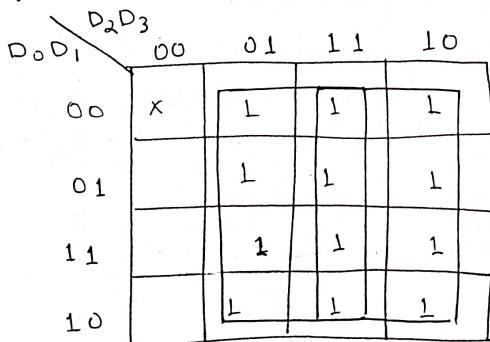
$D_0 \rightarrow$ Lowest

$D_3 \rightarrow$ highest

$D_0 D_1 D_2 D_3$
Low \longrightarrow High Priority

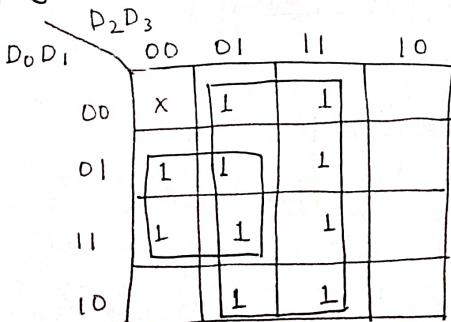
| Input | | | | Output | | |
|-------|-------|-------|-------|--------|---|---|
| D_0 | D_1 | D_2 | D_3 | A | B | V |
| 0 | 0 | 0 | 0 | x | x | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| x | 1 | 0 | 0 | 0 | 1 | 1 |
| x | x | 1 | 0 | 1 | 0 | 1 |
| x | x | x | 1 | 1 | 1 | 1 |

K-Map for A -



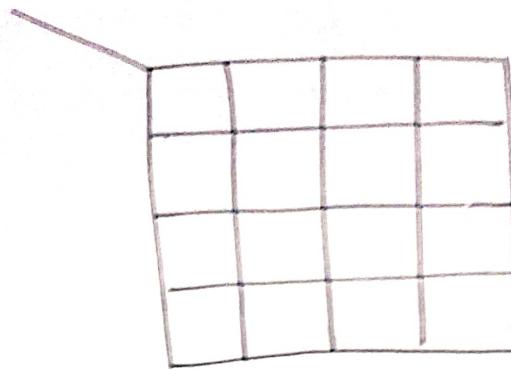
$$A = D_2 + D_3$$

K-MAP for B -



$$B = D_0 \cdot \overline{D_1} \cdot \overline{D_2} \cdot \overline{D_3}$$

K-MAP for V-



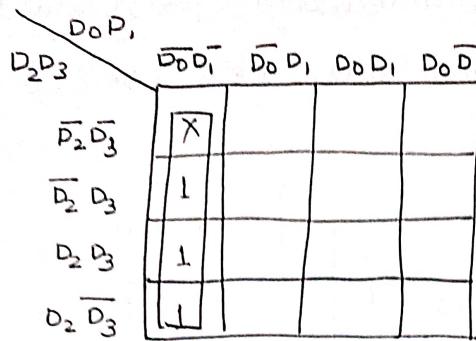
31/08/16

Q. Design 4-bit priority Encoder. $D_3 \rightarrow \text{high}$
 $D_2 \rightarrow \text{low}$

Sol:-

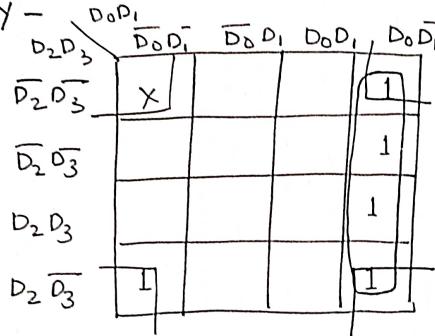
| D_3 | D_2 | D_1 | D_0 | x | y | v |
|-------|-------|-------|-------|-----|-----|-----|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| x | 1 | 0 | 0 | 1 | 0 | 1 |
| x | x | 1 | 0 | 0 | 1 | 1 |
| x | x | x | 1 | 0 | 0 | 1 |

K-MAP for x-



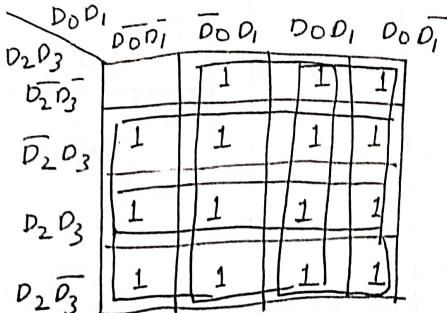
$$X = \overline{D_0} \overline{D_1}$$

K-MAP for Y -



$$Y = D_0 \overline{D_1} + \overline{D_1} \overline{D_3}$$

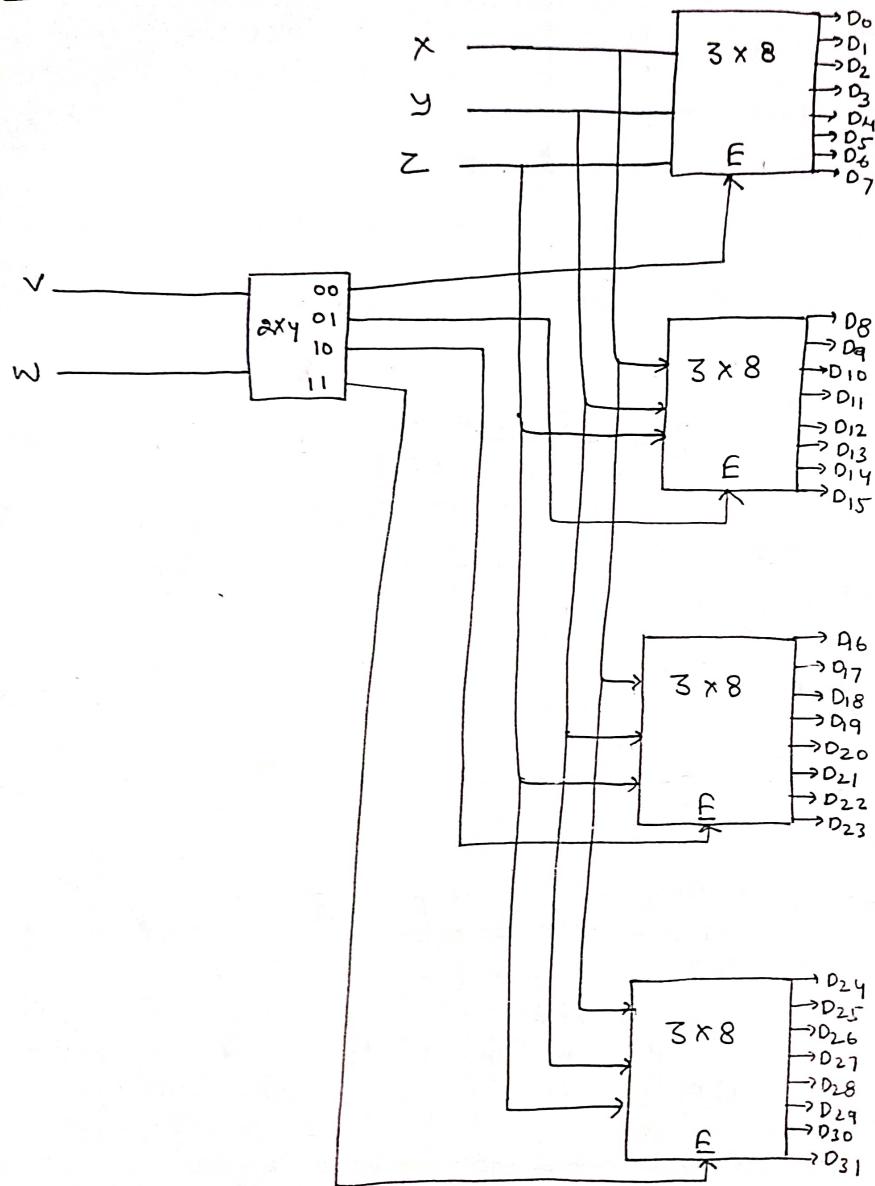
K-MAP for V -



$$V = D_0 + D_1 \overline{D_3} + D_3 + D_2$$

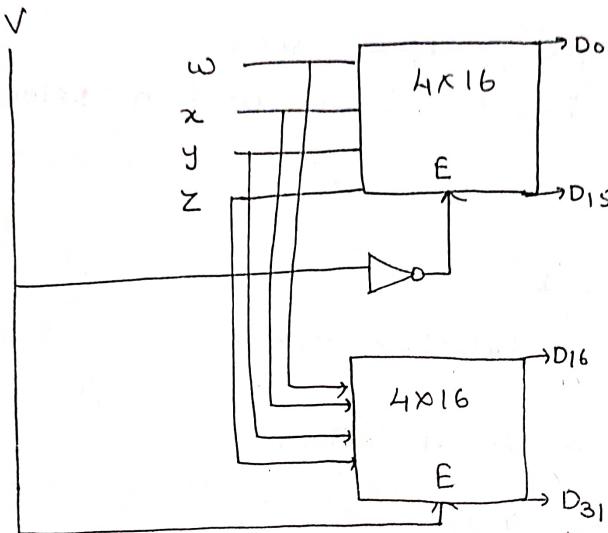
Q. construct 5×32 decoder with enable
and 4 3×8 and one 8×4 decoder.

Solⁿ



Q. Construct 5×32 decoder with enable
and 4×16 (two).

Soln -



| v | w | x | y | z | | |
|---|---|---|---|---|-----|-----------|
| 0 | 0 | 0 | 0 | 0 | → 0 | |
| 0 | 0 | 0 | 0 | 1 | → 1 | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | 1 1 0 0 0 |
| 0 | 0 | 1 | 1 | 1 | → 7 | 1 1 0 0 1 |
| | | 0 | 0 | 0 | | ⋮ ⋮ ⋮ ⋮ ⋮ |
| 0 | 1 | 0 | 0 | 1 | | ⋮ ⋮ ⋮ ⋮ ⋮ |
| 0 | 1 | 0 | 1 | 0 | | 1 1 1 1 1 |
| 0 | 1 | 0 | 1 | 1 | | |
| ⋮ | | | | | | |
| 0 | 1 | 1 | 1 | 1 | | |
| | | 0 | 0 | 0 | | |
| 1 | 0 | 0 | 0 | 1 | | |
| 1 | 0 | 0 | 0 | 1 | | |

05-09-16

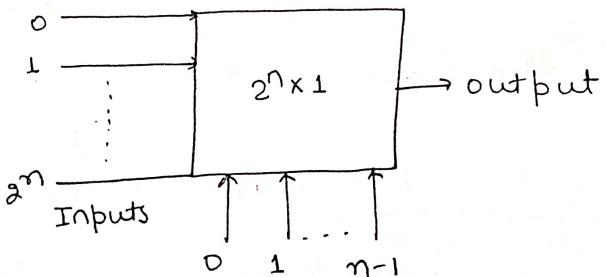
MUX - A Multiplexer is a combinational circuit that selects binary information from one of many input lines and directs it to a single output line. There are 2^n input lines and n -selection lines.

$$2^n \times 1$$

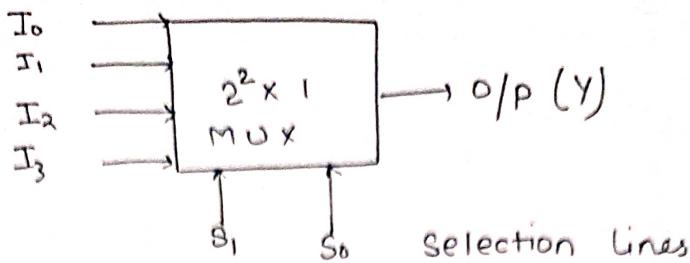
$n \rightarrow$ Selection lines

$2^n \rightarrow$ Input lines

$1 \rightarrow$ Output lines

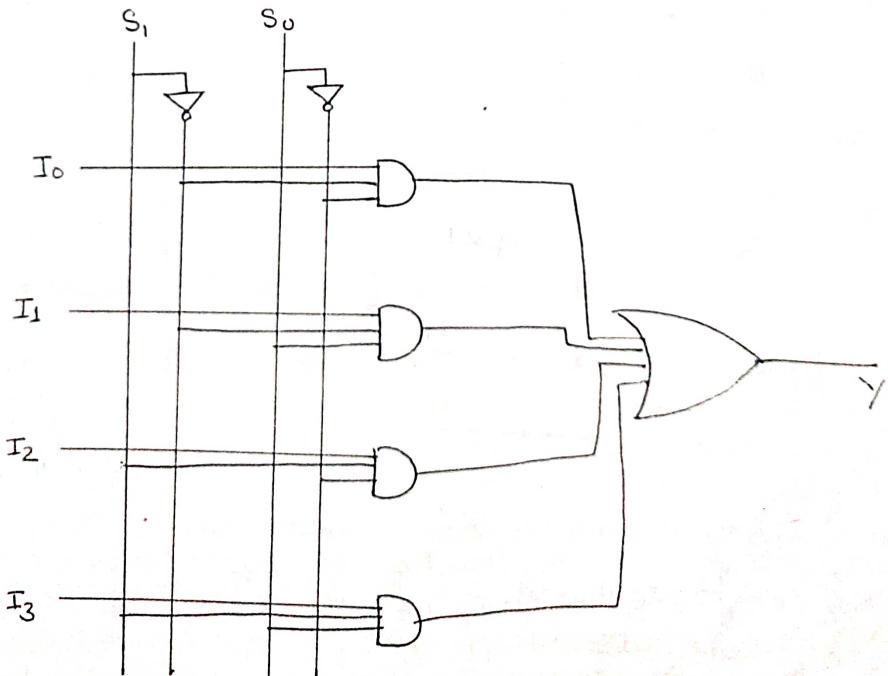


Selection lines

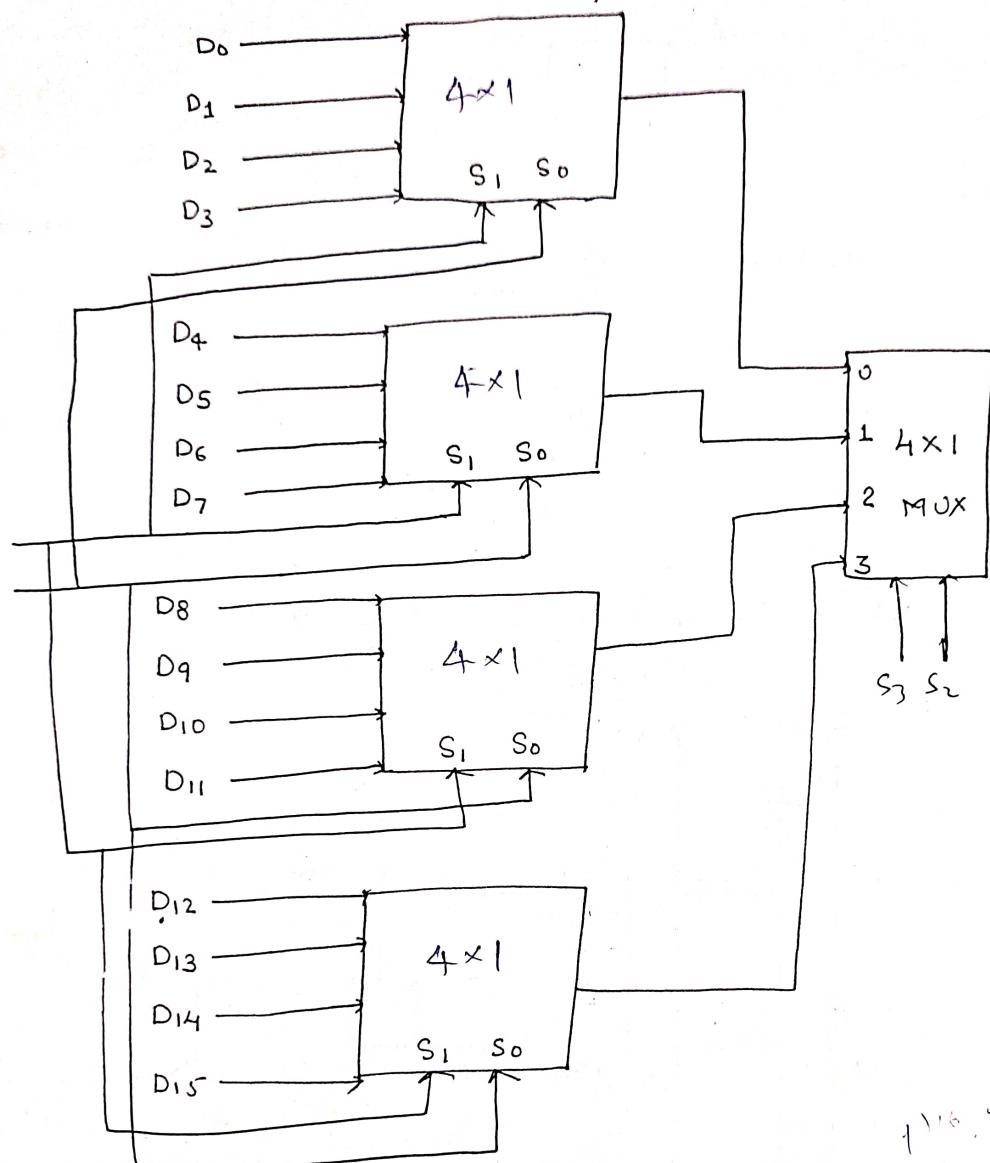


| Inputs (I) | Selection (S_1, S_0) | O/P |
|----------------|-----------------------------|-------|
| I_0 | 0 0 | I_0 |
| I_1 | 0 1 | I_1 |
| I_2 | 1 0 | I_2 |
| I_3 | 1 1 | I_3 |

$$Y = I_0 \bar{S}_1 \bar{S}_0 + I_1 \bar{S}_1 S_0 + I_2 S_1 \bar{S}_0 + I_3 S_1 S_0$$



Q. Design 16×1 mux using 4x1 Inputs.

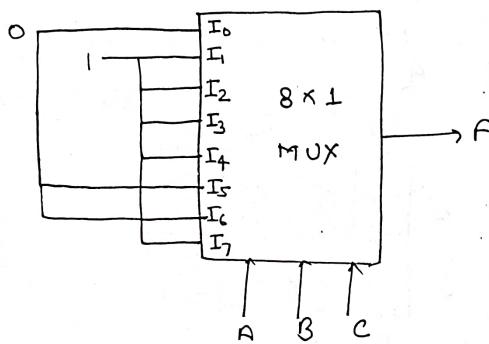


| S ₁ | S ₀ | Y | D ₀ | D ₄ | D ₈ | D ₁₂ |
|----------------|----------------|---|----------------|----------------|-----------------|-----------------|
| 0 | 0 | | D ₀ | D ₄ | D ₈ | D ₁₂ |
| 0 | 1 | | D ₁ | D ₅ | D ₉ | D ₁₃ |
| 1 | 0 | | D ₂ | D ₆ | D ₁₀ | D ₁₄ |
| 1 | 1 | | D ₃ | D ₇ | D ₁₁ | D ₁₅ |

Q. Design $F(A, B, C) = \sum(1, 2, 3, 4, 7)$ using 8×1 .

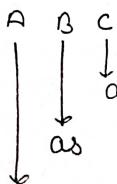
* change last digit in decimal in absence of inputs and count the bits. The no. of bits are 9 inputs.

| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



$$\begin{aligned}
 F &= 1 \cdot \bar{A} \bar{B} \bar{C} + 1 \cdot \bar{A} \cdot B \cdot \bar{C} + 1 \cdot \bar{A} \cdot B \cdot C + 1 \cdot A \bar{B} \bar{C} + 1 \cdot AB \\
 &= \bar{A} \bar{B} C + \bar{A} B \bar{C} + \bar{A} B C + A \bar{B} \bar{C} + A B \bar{C}
 \end{aligned}$$

Q. Design $F(A, B, C) = \sum(1, 2, 3, 4, 7)$ using two $2^2 \times 1$ MUX.



as input $2^0 = 1$ parallel digits use
 as input $2^1 = 2$ parallel digits use
 as input $2^2 = 4$ parallel digits use.

Implementation Table for A, B, C -

for C -

| | I_0 | I_1 | I_2 | I_3 |
|-----------|-------|-------|-----------|-------|
| \bar{C} | 0 | 2 | 4 | 6 |
| C | 1 | 3 | 5 | 7 |
| | C | 1 | \bar{C} | C |

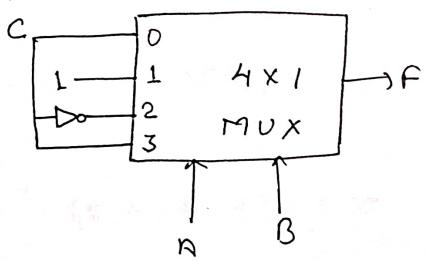
for B -

| | I_0 | I_1 | I_2 | I_3 |
|-----------|-------|-------|-----------|-------|
| \bar{B} | 0 | 1 | 4 | 5 |
| B | 2 | 3 | 6 | 7 |
| | B | 1 | \bar{B} | B |

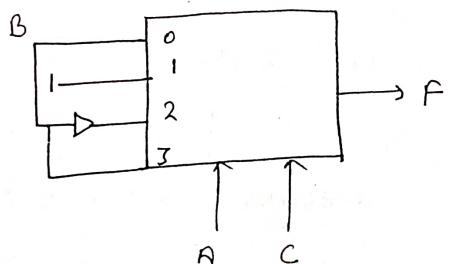
for A -

| | I_0 | I_1 | I_2 | I_3 |
|-----------|-------|-----------|-----------|-------|
| \bar{A} | 0 | 1 | 2 | 3 |
| A | 4 | 5 | 6 | 7 |
| | A | \bar{A} | \bar{A} | 1 |

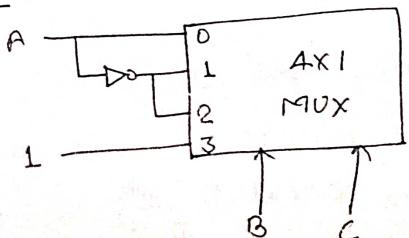
for C -



for B -



for A -



$$\text{For } C - \quad F = \bar{A}\bar{B}C + \bar{A}B\cdot 1 + A\bar{B}\bar{C} + AB\bar{C}$$

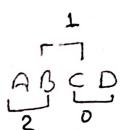
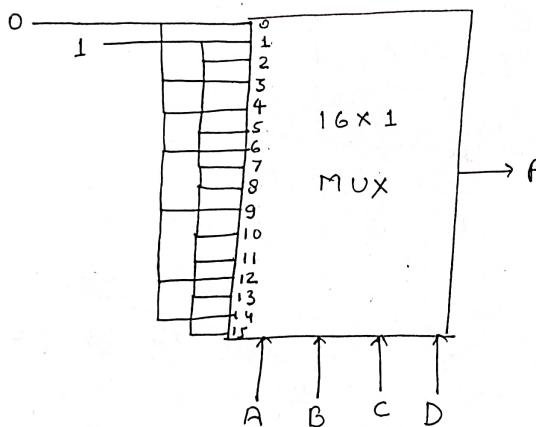
$$\text{For } B - \quad F = \bar{A}\bar{C}B + \bar{A}C\cdot 1 + A\bar{C}\bar{B} + ACB$$

$$\text{For } A - \quad F = \bar{B}\bar{C}A + \bar{B}CA + B\bar{C}\bar{A} + B\cdot 1$$

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Q. $F(A, B, C, D) = \sum(1, 2, 5, 7, 8, 10, 11, 13)$

Design with 16×1 MUX.



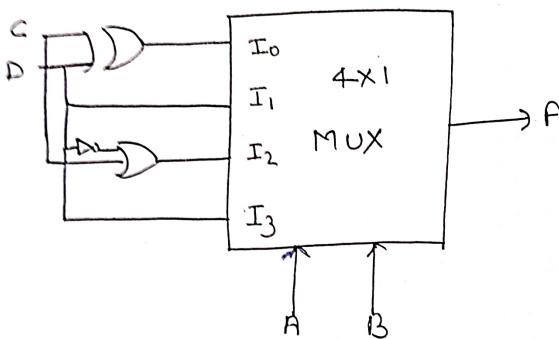
CD - As an input

$2^0 = 1$ parallel digits

Implementation Table for CD -

| | I_0 | I_1 | I_2 | I_3 |
|------------|-------|-------|-------|-------|
| $\bar{C}D$ | 0 | 4 | 8 | 12 |
| $\bar{C}D$ | 1 | 5 | 9 | 13 |
| $C\bar{D}$ | 2 | 6 | 10 | 14 |
| CD | 3 | 7 | 11 | 15 |

$\bar{C}D + C\bar{D}$ D $\bar{C}D + \bar{C}D + CD$ D
 \downarrow \downarrow \downarrow
 $C \oplus D$ D $\bar{D} + C$



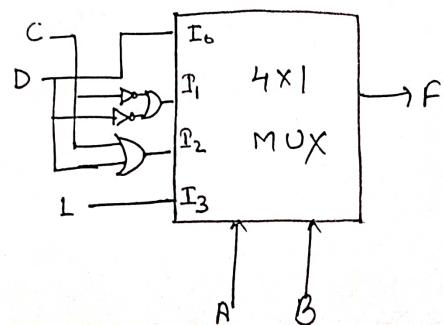
$$F(A, B, C, D) = \sum(1, 3, 4, 11, 12, 13, 14, 15)$$

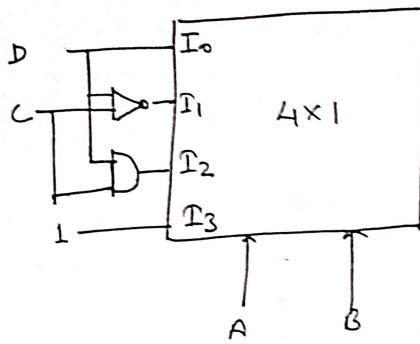
CD - as input

$2^0 = 1$ parallel line

| | I_0 | I_1 | I_2 | I_3 |
|------------|-------|-------|-------|-------|
| $\bar{C}D$ | 0 | (4) | 8 | (12) |
| $\bar{C}D$ | 1 | 5 | 9 | (13) |
| $C\bar{D}$ | 2 | 6 | 10 | (4) |
| CD | 3 | 7 | (11) | (15) |

0 $\bar{C}D$ $C\bar{D}$ 1



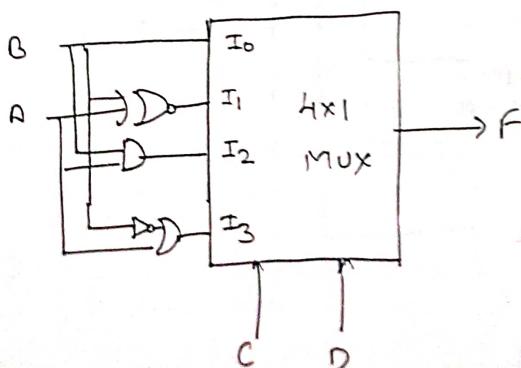


Design again with AB as a input

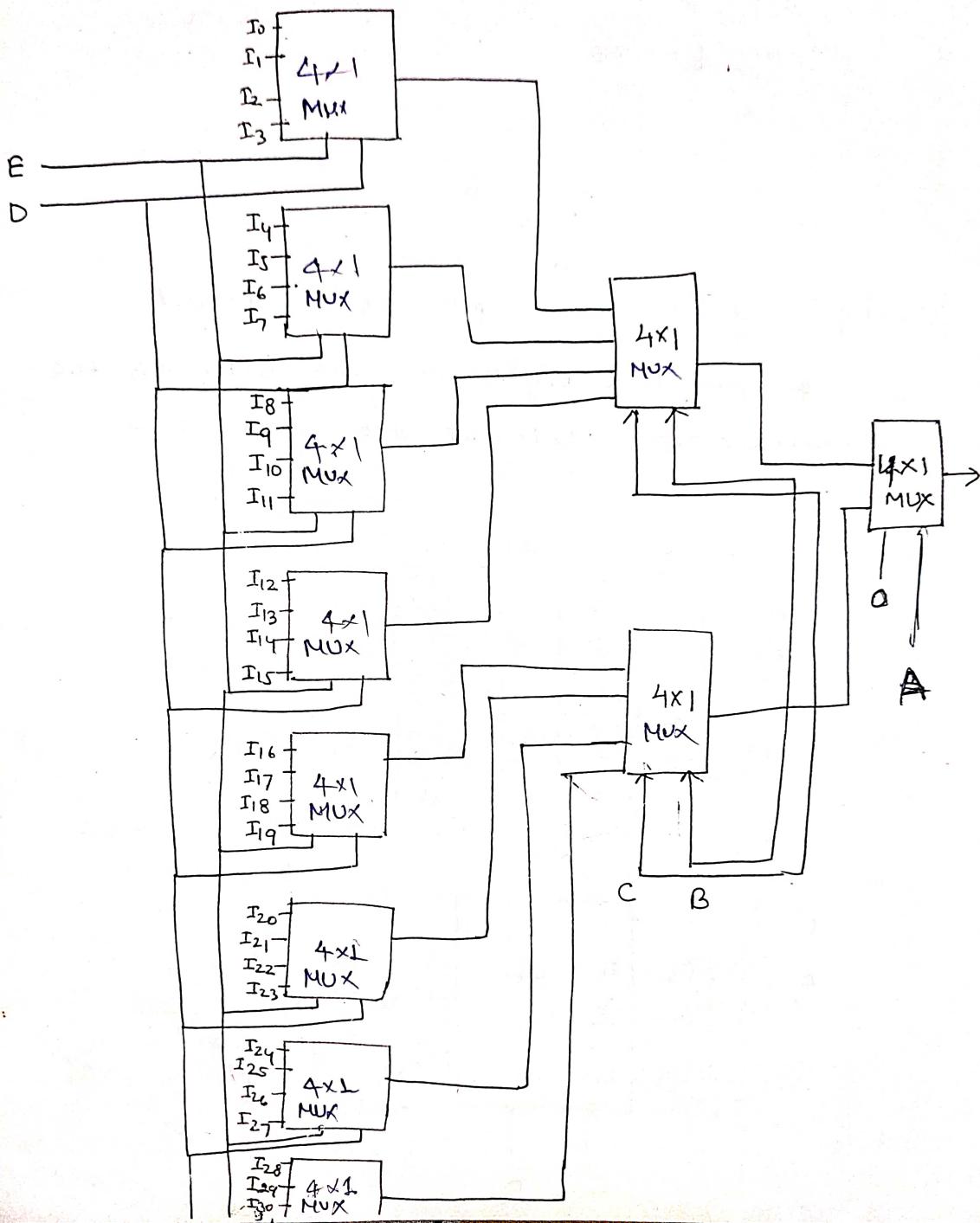
$2^2 = 4$ parallel digits can be used in the Implementation table at the same time.

| | I_0 | I_1 | I_2 | I_3 |
|------------------|-------|-------|-------|-------|
| $\bar{A}\bar{B}$ | 0 | (1) | 2 | (3) |
| $\bar{A}B$ | (4) | 5 | 6 | 7 |
| $A\bar{B}$ | 8 | 9 | 10 | (11) |
| AB | (12) | (13) | (14) | (15) |

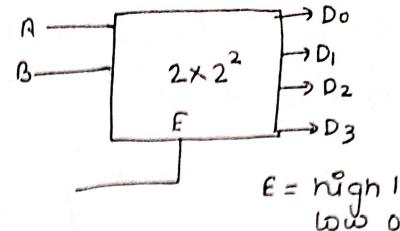
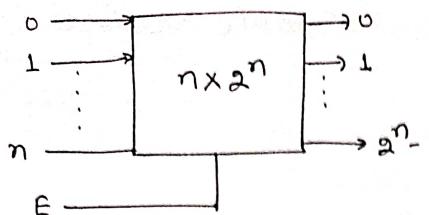
B $A \odot B$ AB $\bar{B} + A$



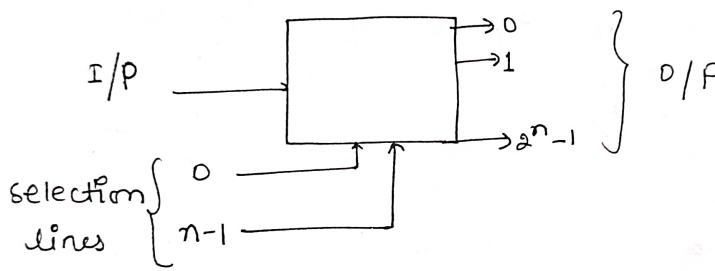
Q. Design 32x1 MUX using four 4x1 MUX.



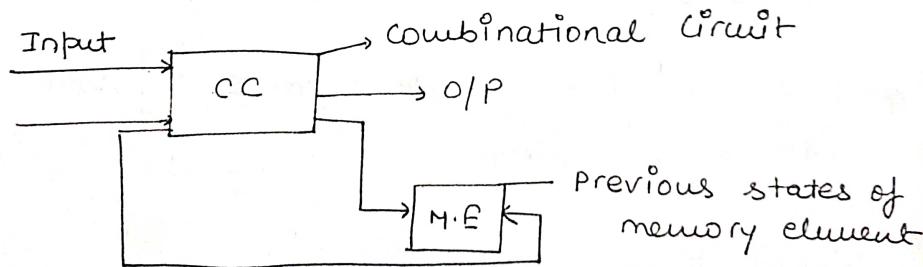
07/09/16



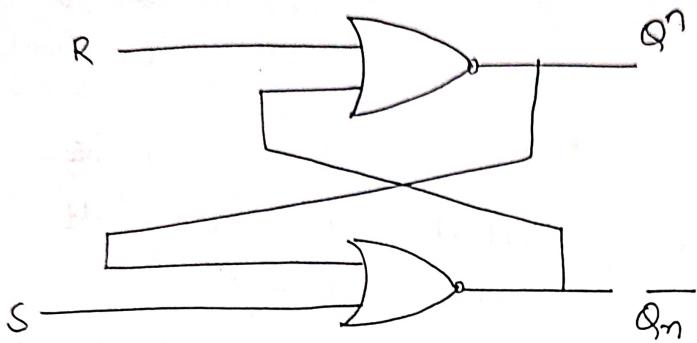
Demuxer - Demux - output - Selection lines
enable - input



Sequential Circuits —



Flip-flop - Flip flop is a memory element used for storing single bit either 0 or 1. Flip-flop is a bistable device.



There are following types of flip-flops -

1. Set Reset (SR)

2. D

3. JK

4. T

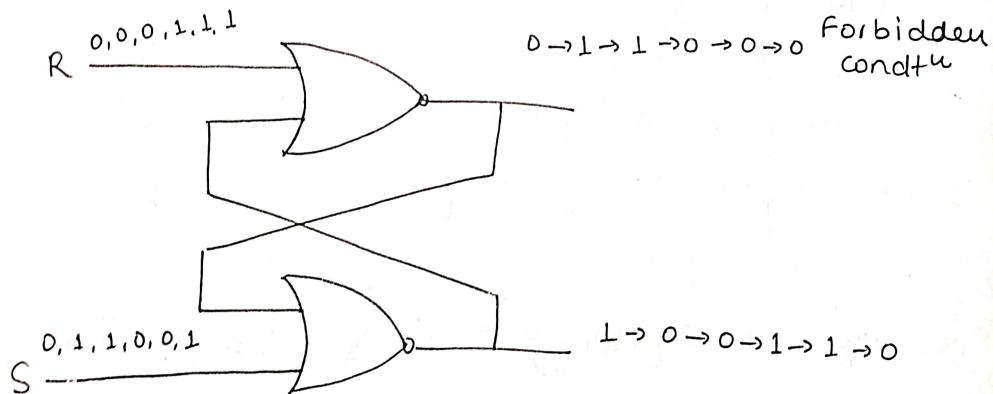
5. Master Slave

★ If we give input - 1 and 0 then one will operate first.

1. SET RESET (SR) -

Truth-Table for SR-

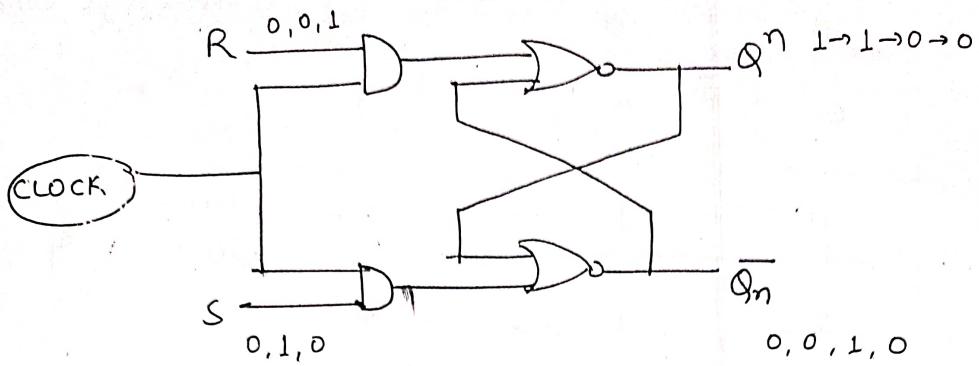
| S | R | Present State Q_n | Next State Q_{n+1} | |
|---|---|------------------------|-------------------------|--------------------------------|
| 0 | 0 | 0 | 0 | Always zero |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 1 | 1 | Always one In case of 1,0 |
| 0 | 1 | 1 | 0 | |
| 0 | 1 | 0 | 0 | Next state will always be 0 |
| 1 | 1 | | 0 | Forbidden condition |



★ If we consider 0 or 1 as a previous state then next state will be 0 or 1.

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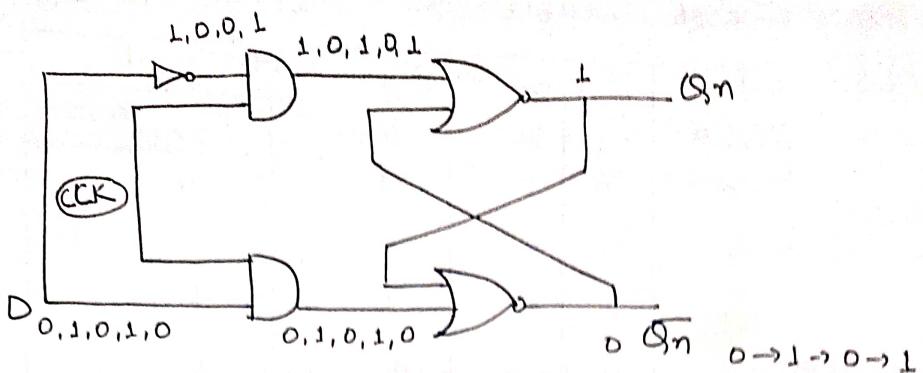
Flip-flop with clock -



Truth Table -

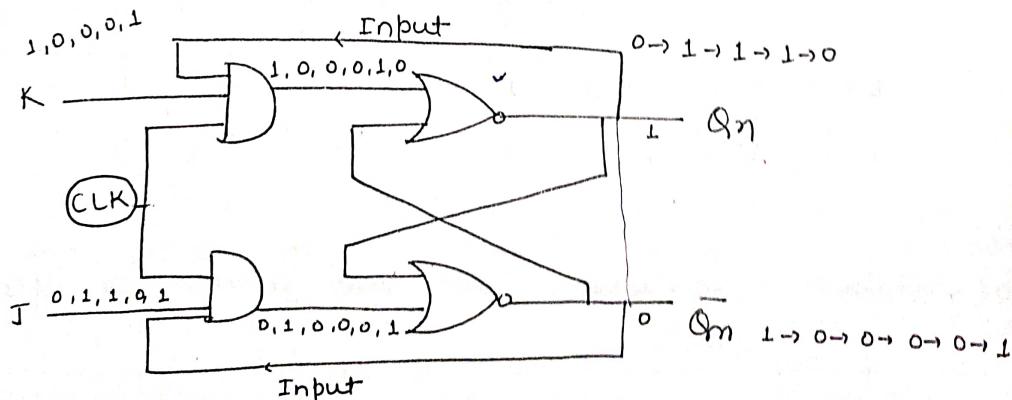
| CLK | S | R | \bar{Q}_m | Q_{m+1} |
|----------|---|---|-------------|-----------|
| (high) ↑ | 0 | 0 | 1 | 1 |
| ↑ | 0 | 1 | 1 | 1 |
| ↑ | 1 | 0 | 1 | 0 |
| ↑ | 1 | 1 | 0 | ? |

D- Flip Flop - (Delay) - It is used for transferring the data from one to another. It can be used for one clock delay.



| Input | P. S | N. S |
|-------|-------|-----------|
| D | Q_n | Q_{n+1} |
| L | 1 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 0 |

J. K Flip-flop - JK flip flop has two inputs and performs all three actions.



Toggling - Output changes in same clock pulse.

Truth-Table for JK -

| | I/P | | P.S | N.S. |
|-----|-----|---|----------------|------------------|
| CLK | J | K | Q _n | Q _{n+1} |
| ↑ | 0 | 1 | 1 | 0 |
| ↑ | 1 | 0 | 0 | 1 |
| ↑ | 1 | 0 | 1 | 1 |
| ↑ | 0 | 0 | 1 | 1 |
| ↑ | 1 | 1 | 1 | 1 → 0 → 1 → 0 |

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Excitation Table -

| P.S | N.S | F / F | | S R | | | | J | K | D | T |
|----------------|------------------|-------|---|-----|---|---|---|---|---|---|---|
| Q _n | Q _{n+1} | S | R | S | R | J | K | D | T | | |
| 0 → 0 | 0 | 0 | x | 0 | x | 0 | x | 0 | 0 | | |
| 0 → 1 | 1 | 0 | 1 | 1 | x | 1 | x | 1 | 1 | | |
| 1 → 0 | 0 | 0 | 1 | x | 1 | 1 | 0 | 0 | 1 | | |
| 1 → 1 | x | 0 | x | x | 0 | 1 | 0 | 1 | 0 | | |

Registers - Registers are the group of flip-flops.

counter - A Register that goes through a prescribed sequence of states upon the application of input pulses is called a counter.

⊕ unused state goes to the initial state.

Design a counter with mod 7 by D-FUL top.

Initial - $S_5 \rightarrow S_4 \rightarrow S_2 \rightarrow S_0 \rightarrow S_3$ Final state
state

| | Q_2 | Q_1 | Q_0 | D_0 | D_1 | D_2 |
|---|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 1 | 0 |
| 5 | 1 | 0 | 1 | 0 | 0 | 1 |
| 6 | 1 | 1 | 0 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 1 | 0 | 1 |

$S_5 - S_4$

| | P.S | N.S | D_0 |
|-------|-------------------|-----|-------|
| Q_0 | $1 \rightarrow 0$ | 0 | |
| Q_1 | $0 \rightarrow 0$ | 0 | |
| | $1 \rightarrow 1$ | 1 | |
| | $0 \rightarrow 1$ | 1 | |

K-MAP for D_0 -

| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | 1 | 1 | 1 | |
| Q_2 | | | | 1 |

$$D_0 = \bar{Q}_2 \bar{Q}_1 + \bar{Q}_2 Q_0 + Q_2 Q_1$$

K-MAP for D_1 -

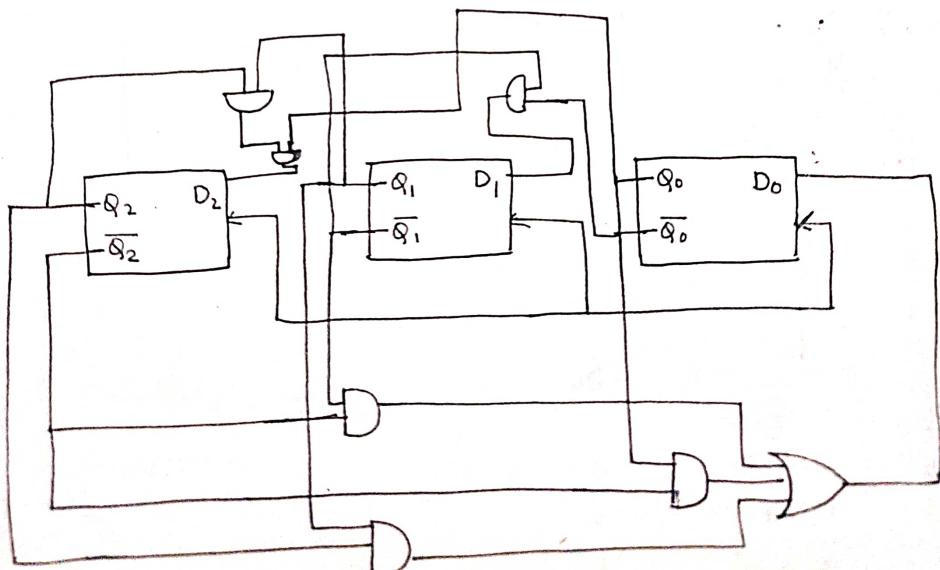
| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | 1 | | | |
| Q_2 | 1 | | | |

$$D_1 = \bar{Q}_1 \bar{Q}_0$$

K-MAP for D_2 -

| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | 1 | 1 | |
| Q_0 | 1 | 1 | 1 |

$$D_2 = Q_0 + Q_2 Q_1$$



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Q. Design counter with mod 6.

| Q_2 | Q_1 | Q_0 |
|-------|-------|-------|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

| J_0 | K_0 | J_1, K_1 | J_2 |
|-------|-------|------------|-------|
| 1 | x | 0x | 0x |
| x | 1 | 1x | 0x |
| 1 | x | x0 | 0x |
| x | 1 | x1 | 1x |
| 1 | x | 0x | x0 |
| x | 1 | 0x | x1 |
| 0 | x | x1 | x1 |
| x | 1 | x1 | x1 |

K-MAP for - J_0

| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | 1 | x | x |
| Q_2 | x | 1 | 1 |

$$J_0 = \bar{Q}_2 + \bar{Q}_1$$

K-MAP for J_1 -

| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | 1 | x | x |
| Q_2 | x | x | x |

$$J_1 = \bar{Q}_2 Q_0$$

K-MAP for K_0

| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | x | 1 | 1 |
| Q_2 | x | 1 | 1 |

$$K_0 = 1$$

K-MAP for K_1 -

| $\bar{Q}_1 \bar{Q}_0$ | $\bar{Q}_1 Q_0$ | $Q_1 \bar{Q}_0$ | $Q_1 Q_0$ |
|-----------------------|-----------------|-----------------|-----------|
| \bar{Q}_2 | x | x | 1 |
| Q_2 | x | x | 1 |

$$K_1 = Q_0 + Q_2$$

K-MAP for J_2 -

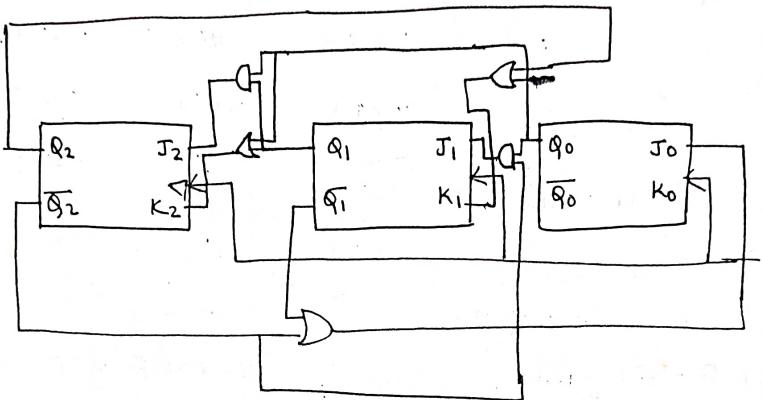
| | | $\bar{Q}_1 \bar{Q}_0$ | | $\bar{Q}_1 Q_0$ | | $Q_1 \bar{Q}_0$ | |
|-------------|-------|-----------------------|-------|-----------------|-------|-----------------|-------|
| | | \bar{Q}_2 | Q_2 | \bar{Q}_2 | Q_2 | \bar{Q}_2 | Q_2 |
| \bar{Q}_2 | Q_2 | X | X | X | X | 1 | |
| | | | | | | | |

$$J_2 = Q_1 Q_0$$

K-MAP for K_2 -

| | | $\bar{Q}_1 \bar{Q}_0$ | | $\bar{Q}_1 Q_0$ | | $Q_1 \bar{Q}_0$ | |
|-------------|-------|-----------------------|-------|-----------------|-------|-----------------|-------|
| | | \bar{Q}_2 | Q_2 | \bar{Q}_2 | Q_2 | \bar{Q}_2 | Q_2 |
| \bar{Q}_2 | Q_2 | X | | X | | X | X |
| | | 0 | 1 | 1 | 1 | 1 | 1 |

$$K_2 = Q_0 + Q_1$$



Design the same with D-Flip-Flop -

| Q_2 | Q_1 | Q_0 | D_0 | D_1 | D_2 |
|-------|-------|-------|-------|-------|-------|
| 0 0 | 0 | 0 | 1 | 0 | 0 |
| 1 0 | 0 | 1 | 0 | 1 | 0 |
| 0 0 | 1 | 0 | 1 | 1 | 0 |
| 0 1 | 1 | 1 | 0 | 0 | 1 |
| 1 1 | 0 | 0 | 1 | 0 | 1 |
| 1 1 | 1 | 0 | 0 | 0 | 0 |
| 0 1 | 1 | 1 | 0 | 0 | 0 |
| 1 1 | 1 | 1 | 0 | 0 | 0 |

K-MAP for D₀ -

| | | Q ₁ Q ₀ |
|--|--|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| | | Q ₂ | Q ₂ | Q ₂ | Q ₂ |
| | | Q ₂ | Q ₂ | Q ₂ | Q ₂ |
| | | 1 | | | 1 |
| | | 1 | | | |

$$D_0 = \bar{Q}_1 Q_0 + \bar{Q}_2 \bar{Q}_0$$

K-MAP for D₁ -

| | | Q ₁ Q ₀ |
|--|--|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| | | Q ₂ | Q ₂ | Q ₂ | Q ₂ |
| | | Q ₂ | Q ₂ | Q ₂ | Q ₂ |
| | | | 1 | | 1 |
| | | | | | |

$$\begin{aligned}D_1 &= \bar{Q}_2 \bar{Q}_1 Q_0 + \bar{Q}_2 Q_1 \bar{Q}_0 \\&= \bar{Q}_2 (Q_1 Q_0 + Q_1 \bar{Q}_0) \\&= \bar{Q}_2 (Q_1 \oplus Q_0)\end{aligned}$$

K-MAP for D₃ -

| | | Q ₁ Q ₀ |
|--|--|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| | | Q ₂ | Q ₂ | Q ₂ | Q ₂ |
| | | Q ₂ | Q ₂ | Q ₂ | Q ₂ |
| | | | | 1 | |
| | | 1 | | | |

$$D_2 = \bar{Q}_2 Q_1 Q_0 + Q_2 \bar{Q}_1 \bar{Q}_0$$

