

Quantification of Operational Resiliency and Restoration for Power Distribution Network

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Abstract—This paper presents a novel approach for quantifying the operational resiliency and restoration capabilities of power distribution networks. Leveraging principles from graph theory, where we model the network as an undirected graph representing buses, loads, generators and transmission lines. Through this graph-based representation, we develop a comprehensive framework to assess the resiliency of the system under various contingencies.

Furthermore, the paper proposes an innovative algorithm to restore isolated loads resulting from contingencies, ensuring uninterrupted power supply to critical loads. By analyzing the topological structure of the network and employing efficient restoration strategies, our algorithm achieves rapid response times and optimizes resource allocation during restoration operations.

The effectiveness of the proposed approach is demonstrated through extensive simulations on standard IEEE power distribution networks. Results show capability of model in restoring the loads and quantifying the resiliency of the networks which is easy to compute and light on resources, highlighting the practical applicability of the proposed methodology in enhancing the decision making during contingencies by network operators.

Index Terms—Operational Resiliency, Power Distribution Networks, graph theory, graphical representations, load restoration

I. INTRODUCTION

IN recent years, there has been a surge in research interest regarding the quantification of operational resiliency and restoration capabilities for power distribution networks. This growing interest arises from the critical importance of resilient power distribution systems in maintaining essential services and infrastructure during adverse events. Previous studies have laid the groundwork by developing methodologies and tools to evaluate the resiliency of power distribution networks and to optimize restoration strategies in the event of disruptions.

One avenue of research has delved into the application of graph theory for modeling power distribution systems and scrutinizing their structural characteristics. Notable studies, such as [1], [2] and [3], have probed the utility of graph-based representations to encapsulate the topology of power distribution networks, facilitating the identification of critical nodes and vulnerabilities. Additionally, works like [4] and [5] have introduced algorithms grounded in graph theory to streamline the placement of protective devices, thereby bolstering the reliability of power distribution systems.

Moreover, significant strides have been made in crafting algorithms and techniques for the restoration of power distribution networks post-disruption. Research endeavors exemplified by [6] and [7] have concentrated on devising heuristic

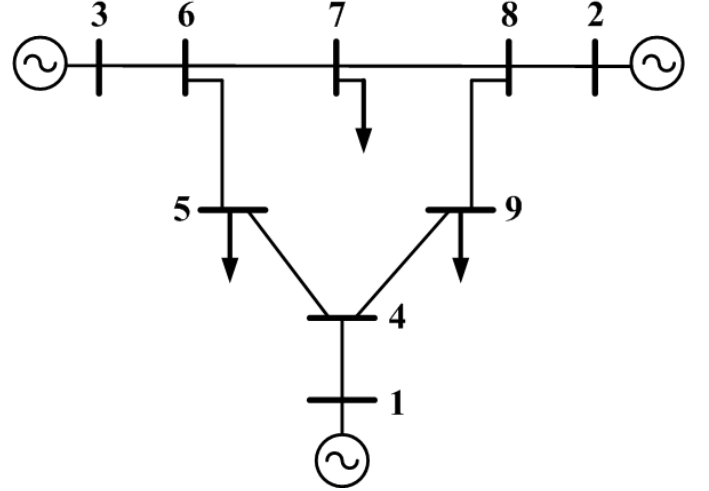


Fig. 1. Single line diagram of 9 bus, 3 generator distribution network [10]

and optimization-driven approaches to prioritize restoration endeavors and allocate resources judiciously. Furthermore, the integration of machine learning and artificial intelligence methodologies has augmented restoration strategies' precision and efficacy [8], [9].

Despite these advancements, challenges persist in accurately quantifying power distribution networks' operational resiliency and devising robust restoration plans. Many extant methodologies hinge on oversimplified models and assumptions that may fail to encapsulate the complexity and dynamism of real-world power systems. Furthermore, the burgeoning interdependence and intricacy of modern power grids pose novel hurdles for resiliency assessment and restoration planning.

In this paper, we build upon prior research works to address these challenges by proposing a comprehensive framework for quantifying the operational resiliency and restoration capabilities of power distribution networks. Leveraging graph theory and innovative algorithmic approaches, our aim is to shed light on power distribution systems' structural properties and develop efficient restoration strategies to mitigate disruptions. Through exhaustive simulations and case studies, we showcase the effectiveness of our proposed methodologies in fortifying the reliability and robustness of power distribution infrastructures.

II. MODELLING POWER DISTRIBUTION NETWORKS

Consider the single line diagram of the network in Fig. 1. The network consists of 9 buses and 3 generators. To model

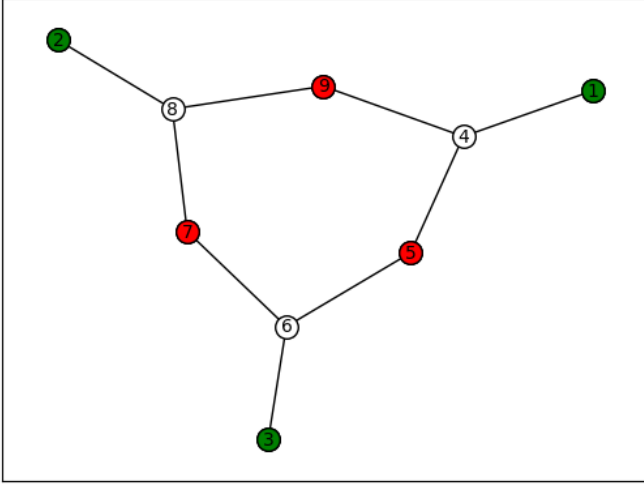


Fig. 2. Modelled graph for 9 bus, 3 generator system

the network for analysis we have represented the buses with nodes in the graph and transmission lines as edges between nodes. Since our study is focused on critical loads *CLs* for quantification and restoration we represent the critical loads with red nodes and generators as green nodes. To model the network, the bus and generator data is taken from case files provided in MatPower [11].

Fig. 2 shows the graph representing the 9 bus, 3 generator power distribution system.

III. PREPROCESSING GRAPH DATA

To quantify resiliency of the network we define three important terms which will be used in calculation further.

A. Path list

Path list is the collection of all possible paths from each critical node to one of the sources in faulted network, so in the list of paths, each path contains one critical node and a source node. To find the path list we iterate through each of the critical loads in the network and use depth first search algorithm (DFS) [12] to find a path from critical load to a source.

B. Path combinations

To find path combinations for the given faulted network, we select one path for each critical load and combine them to form a network which could be either connected or disconnected.

If there are N critical loads in the network then the path list contains all the paths from each of the critical loads to source node. Let's assume that the paths for critical load i is denoted by set CL_i then path combination is of the form,

$$PC = \{q = \bigcup_{i=1}^N p_i | \forall p_i \in CL_i, 1 \leq i \leq N\} \quad (1)$$

C. Feasible Networks

Feasible networks are the reconfiguration of the network suggested by our model if there is a contingency in the current network. To find feasible networks we use the path combinations. For each of the path combination we form a network by taking union on the set of edges in path combination and current network. Since path combinations prioritize critical loads, we ensure that if there is a mismatch in the status of the switch in path combination and the current network for any edge, we take the status of the switch as in the path combination.

After finding all feasible networks, we also verify the feasibility of operation of the network using power flow analysis. If a network does not satisfy power flow restrictions, we remove this network from feasible network list. After this we get our final list of feasible networks.

IV. PARAMETER CALCULATION

To quantify resiliency we will use the following five parameters.

- 1) Ratio of critical loads to sources, *rsl*
- 2) Switching operations, *ops*
- 3) Aggregated degree centrality, *cen*
- 4) Overlapping branches, *obs*
- 5) Average path length, *apl*

Now we will define each of the parameters one by one.

A. Ratio of critical loads to sources

rsl is defined as the ratio between total number of critical loads $|CLs|$ to total number of sources $|DERs|$ in the network.

$$rsl = \frac{|CLs|}{|DERs|} \quad (2)$$

B. Switching operations

ops is defined as the total number of switching operations we would have to make to reconfigure our network after a contingency.

$$E_{ops} = \{e \in E_{PC} \cap E_{network} | \forall e_1 \in E_{PC}, \forall e_2 \in E_{network}, status(e_1) \neq status(e_2)\}, \quad (3)$$

$$ops = |E_{ops}| \quad (4)$$

where E_{ops} is the set of edges which are changing status, i.e. there is a switching operation between the two nodes forming the edge. E_{PC} , and $E_{network}$ represent set of edges in path combination and in the network respectively. *status* represents current status of switch in the edge which is 0 for open and 1 for closed.

C. Aggregated degree centrality

cen is defined as the average of degree centrality of each node in graph. For a graph with N nodes the *cen* is given by,

$$cen = \frac{1}{N} \sum_{j=1}^N \frac{deg(v_j)}{N-1} \quad (5)$$

where $deg(v_j)$ represents degree of node v_j

D. Overlapping branches

obs is defined as the ratio between the number of edges which overlap with each other while combining path from each of the critical loads to sources from path list to the total number of edges in the network. The expression for *obs* is given by,

$$obs = \frac{|E_{PC}|}{|E_{network}|} \quad (6)$$

where E_{PC} is the set of edges in the path combination and $E_{network}$ is the set of edges in the network.

E. Average path length

apl is defined as the average of path length for each of the critical node to source path taken from path list to form a path combination.

Let the paths in the path combination is denoted by a set such that,

$$PC = \{p_1, p_2, \dots, p_N\} \quad (7)$$

where p_j is a set of edges representing a path for a critical load to source. Then *apl* is given by,

$$apl = \frac{1}{N} \sum_{j=1}^N |p_j| \quad (8)$$

V. QUANTIFYING RESILIENCY

To quantify resiliency of the network we will use the parameters defined in previous section. To get a numerical value from all of the parameters we will use weighted mean as the aggregator function. Further to restrict the range of resiliency value we will use an inverse exponent function. By performing these steps we will get a numerical value for resiliency of the system in the range of [0, 1].

We define the parameter matrix which is of the form,

$$P = [rsl, ops, cen, obs, apl] \quad (9)$$

To calculate weighted mean we find the weight matrix,

$$W = [w_1, w_2, w_3, w_4, w_5] \quad (10)$$

where,

$$\sum_{j=1}^5 w_j = 1 \quad (11)$$

Weighted mean is given by the dot product of parameter and weight matrix,

$$M = P.W = PW^T \quad (12)$$

So the resiliency of the system is given by the equation,

$$R = e^{-M} \quad (13)$$

A. N-1 Branch Resiliency

To calculate N-1 Branch resiliency of the distribution network, we simulate fault in each line between any two buses in the network one by one. After each fault we re-model our graphical network and find all feasible networks. For each of the network we calculate resiliency using eq. 13. The expression for N-1 branch resiliency of the network is given below,

Let's assume the feasible networks for the fault in i^{th} edge is given by the set FN_i then,

$$R_{N-1} = \frac{\sum_{i=1}^N \sum_{j=1}^{|FN_i|} R_{ij}}{\sum_{i=1}^N |FN_i|} \quad (14)$$

where, R_{ij} represents resiliency of the j^{th} network for the set FN_i .

B. N-2 Branch Resiliency

To calculate N-2 Branch resiliency of the distribution network, we simulate fault in each line of the network taken two at a time such that all combination of edges are covered. Similar to N-1 branch resiliency we re-model our network after each fault and find all feasible networks. For each of the network we calculate resiliency using eq. 13. The expression for N-2 branch resiliency of the network can be represented by the following equation,

Let's assume there is a fault in i^{th} and j^{th} edge. Then the set of feasible networks for the fault is given by FN_{ij} where $i \neq j$. Then,

$$R_{N-1} = \frac{\sum_{i=1}^N \sum_{j=1, i \neq j}^N \sum_{k=1}^{|FN_{ij}|} R_{ijk}}{\sum_{i=1}^N \sum_{j=1, i \neq j}^N |FN_{ij}|} \quad (15)$$

VI. SIMULATION RESULTS

To test the model we will run the simulation for three different cases in three different networks.

A. Case 1: Two transmission lines are faulted

For this case we use the IEEE-9 bus, 3 generator system. Single line diagram for the system and modelled graph network is shown in fig. 1, fig. 2 respectively.

To show the working of our model we modify the network without affecting its stable operation. In this case we add a tie-line switch between bus 1 and bus 9. So the modified network can be represented by the graph shown in Fig. 3. Dashed line in network represents a tie-line switch.

To simulate the contingency in the network we remove the line between bus (9 and 8), (4 and 9), resulting in islanding of bus 9. The visualization of network after the contingency is shown in Fig. 4.

The output of model for this case is shown in Fig. 5.

The visualization of the feasible network having highest resiliency is shown in Fig. 6.

Comparison between line data before reconfiguration and after configuration is shown in Fig. 7

As evident from Fig. 6, critical load at bus 9 is successfully restored.

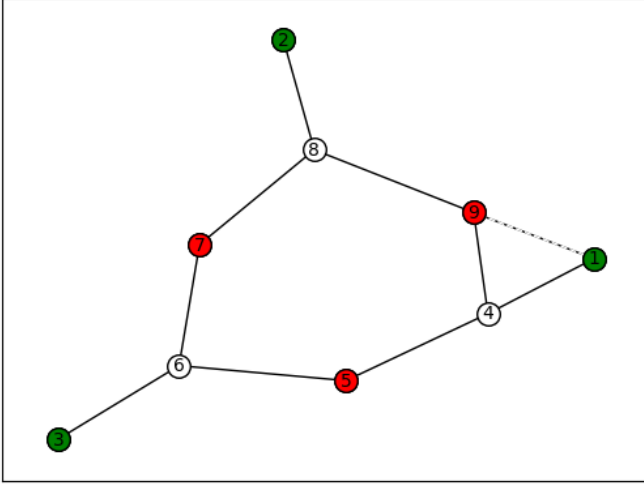


Fig. 3. Modified graph network for case 1 network

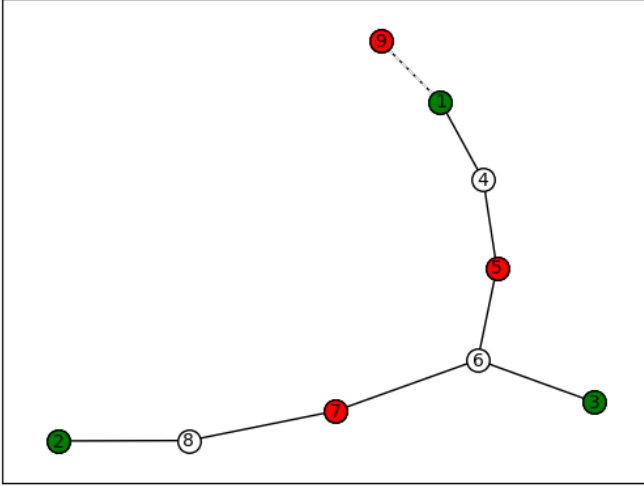


Fig. 4. Network with contingency for case 1

Resiliency of network considering N-1 branches: 0.3772483916753444
 Resiliency of network considering N-2 branches: 0.3768110702415141

Resiliency and parameters of feasible networks for case 1:

	rsl	ops	cen	obs	apl	wtd_mean	resiliency
FN4	1.0	1.0	0.178571	0.0	1.666667	0.875595	0.416614
FN1	1.5	1.0	0.238095	0.0	1.666667	1.028571	0.357517
FN2	1.5	1.0	0.238095	0.0	1.666667	1.028571	0.357517
FN3	1.5	1.0	0.266667	0.2	1.666667	1.080000	0.339596

Fig. 5. Output of the model for case 1

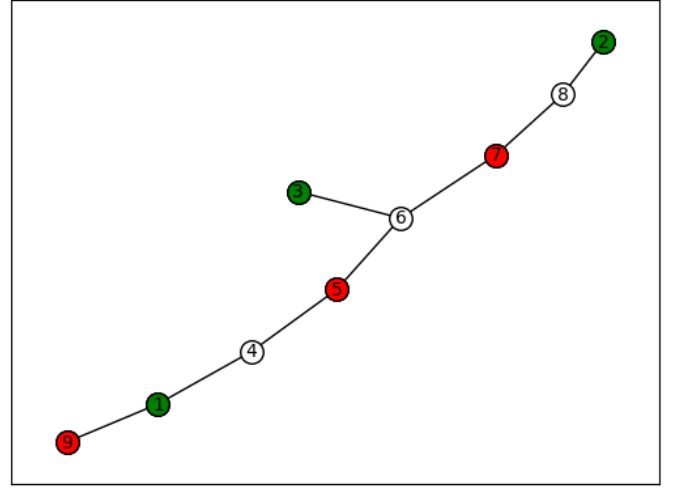


Fig. 6. Feasible Network for the reconfiguration of network in case 1

Line data after contingency						Line data after reconfiguration					
fbus	tbus	r	x	status		fbus	tbus	r	x	status	
0	1	4	0.000000	0.057600	1	0	1	4	0.000000	0.057600	1
1	4	5	0.017000	0.092000	1	1	4	5	0.017000	0.092000	1
2	5	6	0.039000	0.170000	1	2	5	6	0.039000	0.170000	1
3	3	6	0.000000	0.058600	1	3	3	6	0.000000	0.058600	1
4	6	7	0.011900	0.100800	1	4	6	7	0.011900	0.100800	1
5	7	8	0.008500	0.072000	1	5	7	8	0.008500	0.072000	1
6	8	2	0.000000	0.062500	1	6	8	2	0.000000	0.062500	1
9	1	9	0.000000	0.057600	0	9	1	9	0.000000	0.057600	1

Fig. 7. Line data of the network before and after reconfiguration for case 1

B. Case 2: One generator and two lines are faulted

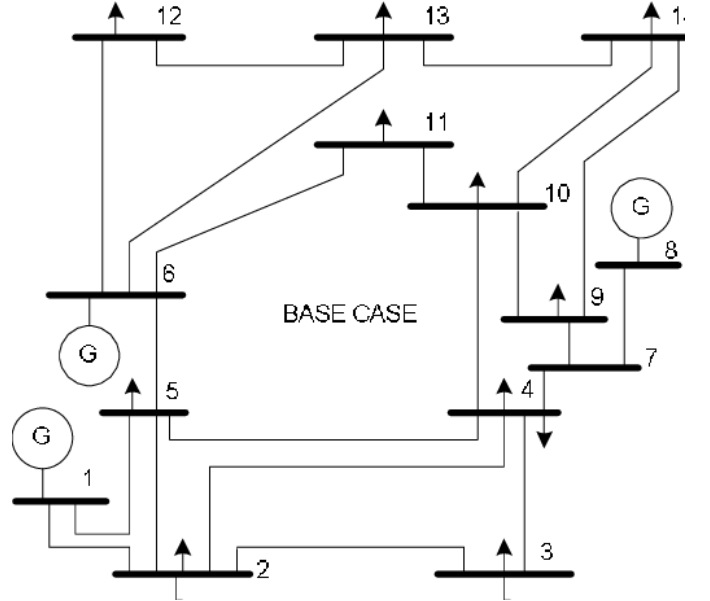


Fig. 8. Single line diagram of IEEE-14 bus distribution network [13]

For this case we use the IEEE-14 bus system. Single line diagram for the system is shown in fig. 8.

To show the working of our model we modify the network without affecting its stable operation. In this case we add a tie-

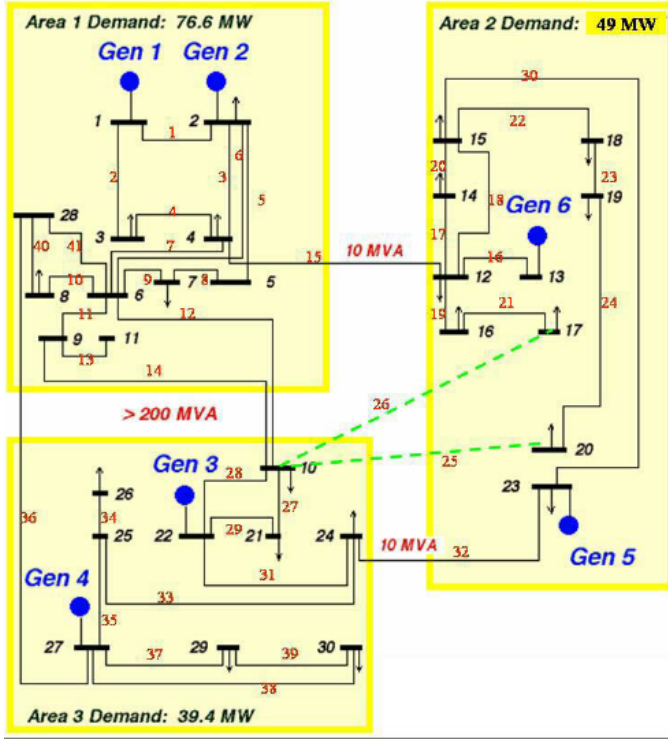


Fig. 14. Single line diagram of 30 bus, 6 generator distribution network [14]

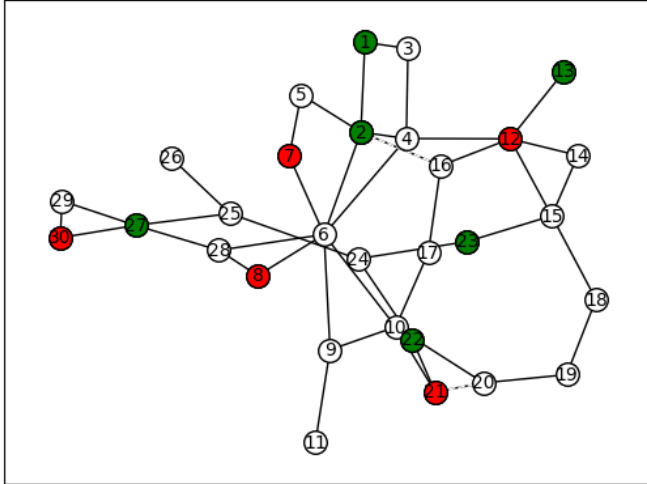


Fig. 15. Modified graph network for case 3 network

Comparison between line data before reconfiguration and after configuration is shown in Fig. 19

As evident from Fig. 18, we have successfully restored area 2.

VII. CONCLUSION

The study presented in this paper underscores the critical importance of quantifying operational resiliency and restoration strategies within power distribution networks. Through comprehensive analysis and modeling, we have demonstrated the significance of proactive measures in mitigating the impact of disruptive events. We have presented a methodology to

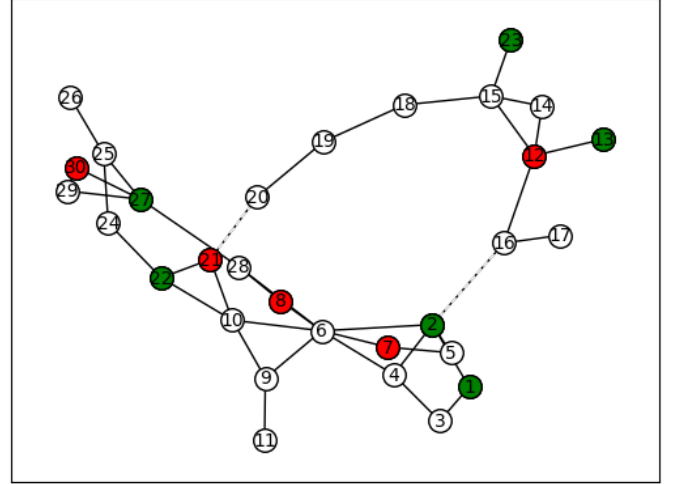


Fig. 16. Network with contingency for case 3

Resiliency of network considering N-1 branches: 0.17919124598924915
Resiliency of network considering N-2 branches: 0.18105920602528797

Resiliency and parameters of feasible networks for case 3:

	id	rsl	ops	cen	obs	apl	wtd_mean	resiliency
FN179	178	1.25	0.0	0.115385	0.000000	1.8	0.830769	0.435714
FN180	179	1.25	0.0	0.109890	0.000000	2.0	0.880495	0.414578
FN207	206	1.25	0.0	0.109890	0.000000	2.0	0.880495	0.414578
FN177	176	1.25	0.0	0.109890	0.000000	2.0	0.880495	0.414578
FN1019	1018	1.25	0.0	0.128205	0.000000	2.0	0.881410	0.414198
...
FN1175	1174	2.50	1.0	0.141026	0.214286	2.8	1.660623	0.190021
FN1204	1203	2.50	1.0	0.123810	0.133333	3.0	1.689524	0.184607
FN1176	1175	2.50	1.0	0.131868	0.200000	3.0	1.706593	0.181483
FN1161	1160	2.50	1.0	0.131868	0.200000	3.0	1.706593	0.181483
FN1162	1161	2.50	1.0	0.123810	0.187500	3.2	1.753065	0.173242

[1344 rows x 8 columns]

Fig. 17. Output of the model for case 3

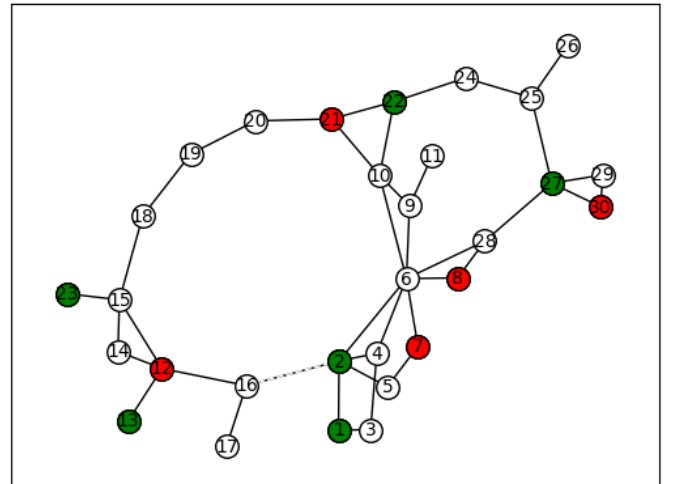


Fig. 18. Feasible Network for the reconfiguration of network in case 3

Line data after contingency						Line data after reconfiguration					
fbus	tbus	r	x	status		fbus	tbus	r	x	status	
0	1	2	0.020000	0.060000	1	0	1	2	0.020000	0.060000	1
1	1	3	0.050000	0.190000	1	1	1	3	0.050000	0.190000	1
2	2	4	0.060000	0.170000	1	2	2	4	0.060000	0.170000	1
3	3	4	0.010000	0.040000	1	3	3	4	0.010000	0.040000	1
4	2	5	0.050000	0.200000	1	4	2	5	0.050000	0.200000	1
5	2	6	0.060000	0.180000	1	5	2	6	0.060000	0.180000	1
6	4	6	0.010000	0.040000	1	6	4	6	0.010000	0.040000	1
7	5	7	0.050000	0.120000	1	7	5	7	0.050000	0.120000	1
8	6	7	0.030000	0.080000	1	8	6	7	0.030000	0.080000	1
9	6	8	0.010000	0.040000	1	9	6	8	0.010000	0.040000	1
10	6	9	0.000000	0.210000	1	10	6	9	0.000000	0.210000	1
11	6	10	0.000000	0.560000	1	11	6	10	0.000000	0.560000	1
12	9	11	0.000000	0.210000	1	12	9	11	0.000000	0.210000	1
13	9	10	0.000000	0.110000	1	13	9	10	0.000000	0.110000	1
15	12	13	0.000000	0.140000	1	15	12	13	0.000000	0.140000	1
16	12	14	0.120000	0.260000	1	16	12	14	0.120000	0.260000	1
17	12	15	0.070000	0.130000	1	17	12	15	0.070000	0.130000	1
18	12	16	0.090000	0.200000	1	18	12	16	0.090000	0.200000	1
19	14	15	0.220000	0.200000	1	19	14	15	0.220000	0.200000	1
20	16	17	0.080000	0.190000	1	20	16	17	0.080000	0.190000	1
21	15	18	0.110000	0.220000	1	21	15	18	0.110000	0.220000	1
22	18	19	0.060000	0.130000	1	22	18	19	0.060000	0.130000	1
23	19	20	0.030000	0.070000	1	23	19	20	0.030000	0.070000	1
26	10	21	0.030000	0.070000	1	26	10	21	0.030000	0.070000	1
27	10	22	0.070000	0.150000	1	27	10	22	0.070000	0.150000	1
28	21	22	0.010000	0.020000	1	28	21	22	0.010000	0.020000	1
29	15	23	0.100000	0.200000	1	29	15	23	0.100000	0.200000	1
30	22	24	0.120000	0.180000	1	30	22	24	0.120000	0.180000	1
32	24	25	0.190000	0.330000	1	32	24	25	0.190000	0.330000	1
33	25	26	0.250000	0.380000	1	33	25	26	0.250000	0.380000	1
34	25	27	0.110000	0.210000	1	34	25	27	0.110000	0.210000	1
35	28	27	0.000000	0.400000	1	35	28	27	0.000000	0.400000	1
36	27	29	0.220000	0.420000	1	36	27	29	0.220000	0.420000	1
37	27	30	0.320000	0.600000	1	37	27	30	0.320000	0.600000	1
38	29	30	0.240000	0.450000	1	38	29	30	0.240000	0.450000	1
39	8	28	0.060000	0.200000	1	39	8	28	0.060000	0.200000	1
40	6	28	0.020000	0.060000	1	40	6	28	0.020000	0.060000	1
41	2	16	0.060000	0.180000	0	41	2	16	0.060000	0.180000	0
42	21	20	0.010000	0.020000	0	42	21	20	0.010000	0.020000	1

Fig. 19. Line data of the network before and after reconfiguration for case 3

quantify resiliency which is fast and easy to implement. We also demonstrated the ability of the model to successfully restore isolated critical loads during different scenarios. We present each feasible network after the contingency ranked by their resiliency value which can aid the network operator.

VIII. SCOPE FOR FUTURE WORK

Building upon the findings of this paper, future research can aim to further enhance our understanding and implementation of operational resiliency and restoration strategies within power distribution networks. Here are some specific areas for improvement and expansion:

- 1) **Refinement of Model Parameters:** Future studies can focus on refining the parameters used in the model presented in this paper to better reflect the dynamic nature of distribution networks and the complex interactions among various components. Incorporating more granular data and advanced simulation techniques can improve the accuracy of predictions and enable more effective decision-making during disruptive events.
- 2) **Optimization of Restoration Strategies:** Building upon the restoration strategies outlined in the paper, future research can explore optimization techniques to streamline the restoration process and minimize downtime

following disruptive events. This could involve the development of heuristic algorithms, optimization models, and decision support tools tailored to the unique characteristics of distribution networks.

- 3) **Integration of Emerging Technologies:** With the rapid advancement of technologies such as Internet of Things (IoT), artificial intelligence (AI), and blockchain, there is an opportunity to explore how these innovations can be leveraged to enhance operational resiliency and restoration capabilities. Investigating the integration of these technologies into distribution networks and assessing their impact on system performance and reliability can yield valuable insights.

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