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Analysis of solar sailing as a means of orbit maneuvering for nanosatellites in low earth orbit

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The paper presents a detailed analysis of the feasibility and effectiveness of solar sailing as a means of orbit maneuvering for nano satellites orbiting in low-earth orbits. Demonstrating orbit maneuvering using solar sails is the scientific objective of COEPSAT-2, a satellite being developed by undergraduate students of College of Engineering, Pune. Solar sailing proves to be a cost effective and energy efficient means of orbit maneuvering for nanosats considering their mass, size and energy constraints. A comparative study of solar sails with thrusters, which are a common means of orbit maneuvering, has also been carried out. Solar sailing works on the principle of momentum transfer due to solar radiation. The aim of the Attitude Determination and Control System (ADCS) of satellite is to orient the satellite in such a way that the thrust in the direction of velocity is maximized and the drag force is minimized over the orbit. Consequently, at higher altitudes, the average thrust due to solar radiation pressure will be greater than the average drag, resulting in increase in kinetic energy of satellite. This increase in energy will cause a rise in the satellite's orbit. De-orbiting of the satellite will be carried out on reaching maximum desired altitude, by using another set of orientations. These orientations will use drag and solar radiation pressure both, to generate maximum thrust in the direction opposite to the orbital velocity. The paper presents sail orientations for orbit raising and de-orbiting along with simulation results to support their effectiveness. As solar sailing involves continuous thrust, a numerical propagator was used for simulations. Precise models to determine solar thrust and drag force have been incorporated in this simulation. Standard atmosphere models like MSISE-90 and Jacchia-Roberts were used to calculate the atmospheric density and drag forces at different altitudes. The paper also presents a comparison between solar thrust and drag force for orbits of various altitudes. Simulations were carried out for different initial values of inclination, semi-major axes and area of sail to observe their effects on the values of thrust, drag and the final shape of the orbit. The analysis and simulations presented in the paper give us a preliminary assurance of successful orbit maneuvering using solar sail.

Nomenclature

α	Angle between sail normal and sun line
$\bar{n}$	Sun to earth vector
$\bar{r}$	Satellite position vector
$\bar{u}$	Sun to sail vector
$\bar{v}$	Satellite velocity vector
$\hat{k}$	Sail normal
Ω	Right Ascension of Ascending Node
ω	Argument of Perigee
θ	Angle between sun line and position vector
a	Semimajor Axs
N	Normal component of perturbing force
S	Radial component of perturbing force
T	Transverse component of perturbing force
sun line	Projection of line joining the sun and earth in the plane of the orbit

I. Introduction

The COEP satellite team is an undergraduate research group which is currently involved in designing and developing a solar sailing nanosatellite COEPSAT-2. The scientific objective of the mission is to demonstrate orbit maneuvering by using an appropriate solar sail orientation control law. The proposed mission will involve raising the orbit from 700km to 1000km above the surface, followed by deorbiting. The mission proposes to use this maneuver to measure charged particle densities at various altitudes. The proposed nanosatellite will have a mass of $9kg$, a sail area of $40m^2$ and an initial orbit of $700-800km$. At this altitude, atmospheric drag will still be a significant perturbation.

The main idea of the paper is a sail orientation control law for orbit raising in an environment where air drag is significant, and a part of the orbit is eclipsed by the earth Simulation results, along with an analysis of the suitability of the orientation is presented in sections 5 and 6 respectively. The paper also analyses simulation results for various orbits to determine parameters TODO (initial orbit, sail area, mass, orientations) for which effective raising will take place.

A solar sail is a thin and highly reflective film. When solar radiation strikes the surface of the sail, it gets reflected. This results in a momentum transfer which develops a thrust on the satellite.The magnitude of the thrust depends upon the distance between the sun and the sail, and the angle between the sail normal and the sun to sail vector and it's area. The direction is normal to the sail due to reflected photons, and along the $\bar{u}$ vector for absorbed photons. By controlling the orientation of the sail force due to solar pressure can be controlled.

The paper also describes the method used for carrying out solar sailing orbit trajectory simulations. The simulations were carried out using NASA's General Mission Analysis Tool, with *FiniteBurn* being used to model the effect of solar pressure and drag. GMAT does not provide an attitude model, so a custom python module, called using GMAT's Python interface is used to calculate the forces which depend on the satellite's orientation.

II. Models Used

The solar pressure model used for analysis is from [2]. The solar pressure is described as

$$\bar{F} = -K(r) \iint_S B(\theta)\, \bar{ds} - C_2 K(r)\bar{u} \iint_S cos(\theta)\, ds$$

WHERE this = that, and so on

The drag model used was MSISE-90. The atmospheric drag force is given by

FORMULA according to [JPL Doc]

For both cases, the area of the satellite body is neglected in comparison with the large area of the sail.

The satellite's orbit is described in terms of the Keplerian elements (6 tuple here). The aim of orbit raising is to maximise the semimajor axis. In an environment with drag, only a rise in semimajor axis will not be sufficient for continuous orbit raising. If the perigee moves closer to the earth, the magnitude of drag will eventually increase, causing a net decrease in a. Thus, the perigee should also be regulated.

[1] provides Lagrange equations which describe instantaneous rates of change of the orbital elements in terms of Transverse, Radial and Normal components of perturbing forces. The radial direction is the direction of the $\bar{r}$. *Normal* refers to the $\bar{r}X\bar{v}$ direction, whereas the Transverse direction is the direction of the vector in the orbital plane perpendicular to the radius vector (completing the right handed system). Note that the transverse direction is *not* the direction of the satellite's velocity. However, for low eccentric-

ity orbits these two directions can be considered to be the same.

The rate of change of semimajor axis is given as $\frac{da}{df} = \frac{2pr^2}{u(1-e^2)^2}[Se\sin f + T\frac{p}{r}] Tomaximisethisrate, thet$ ... $maximised.$

III. Problems with existing control laws

Conventional solar sailing theory provides control laws for sun centred orbits. McInnes[1] also provides laws for earth centred orbits, but suggests that satellites should be launched at a higher orbit. An educational nanosatellite will not get a higher orbit. Also, the large amount of power required for communication at such an orbit is not feasible for small satellites. These laws aim to maximise the semimajor axis, and consequently the transverse force.

In LEO, as mentioned above, atmospheric density is significant, and drag is comparable to the force due to solar thrust. Also, drag acts in the direction opposite to the velocity vector, making it more potent. As a result, simply maximising the transverse component of solar thrust will not be suitable for orbit raising in LEO. (as such a strategy will involve orienting the sail along the transverse direction)

Three main sail control laws have been presented in McInnes[1]. The on off control law simply orients the sail perpendicular to solar radiation when the satellite is moving away from the sun, and parallel when moving towards the sun. Lightsail2 uses this control law. While this has resulted in rise in it's apogee, it is accompanied by a falling perigee. Lightsail2 predicts that eventually drag will dominate and will cause an overall decrease in the semimajor axis.

The orbit rate steering law involves the sail rotating through a full 180 degree in the orbit, with the angle being a linear function of the angle between the satellite position vector and the sun line. The assumption here is that for an orbit with a large semimajor axis, a very negligible part will be eclipsed by the earth, and there will be solar radiation incident on the sail for the entire orbit. However, in LEO, a large part of the orbit lies in the eclipse region. The orbit rate steering law will prove to be unsuitable as drag will be present and solar thrust absent in the eclipse region of the orbit.

The third law presented is the locally optimal control law, for which the sail is oriented to maximise the transverse component of the solar force at any position. This too, does not work for LEO, for the reasons mentioned above.

All three laws have been presented assuming that the sun is in the plane of the orbit, and ignoring perturbations other than solar radiation pressure. Thus, these prove inadequate for LEO.

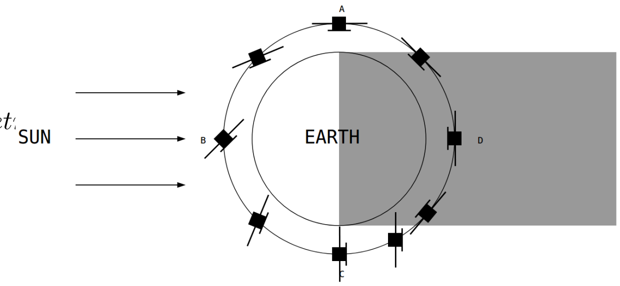

Fig. 1: Illustration of the sail orientation control law. Sail rotates about the pitch axis

IV. The modified control law

The law ensures that the transverse component due to perturbations is dominated by solar thrust (proof- diagram), and ensures that drag is 0 in the eclipse region of In this paper, we present a solar sail control law which takes drag into account, and does not assume that the sun is in the orbital plane. The proposed law, which is a modification of the orbit rate steering law results in a continuous and rapid rise in semimajor axis for an initial orbit of $750km$ or higher, and rise in apogee for lower orbits.

The diagram shows solar radiation in the plane of the orbit. However, this is not neccessary. Hence we present a law which works irrespective of the sun position. The position vector will be obtained using GPS, whereas the velocity vector may be estimated using GPS and Accelerometer readings. Thus, $\bar{r}X\bar{v} = \bar{n}$ gives the orbit normal. The earth to sun vector $\bar{s}$ can be calculated using the julian date, tracked using the RTC.

By taking the component of $\bar{s}$ along $\bar{r}X\bar{v}$, the projection of $\bar{s}$ in the plane of the orbit (say "$\bar{s'}$" can be obtained. This can be used as a reference. Now orientations can be defined as a function of the angle between $\bar{r}$ and $\bar{s'}$.

Additionally $\bar{s}$, $\bar{r}$ and the earth's radius can be used to determine whether the satellite lies in the eclipse region of the orbit. The control law, in terms of this information is given as-

$$\alpha(\theta) = \begin{cases} -\theta & 0 < \theta < \pi/2 \text{ or eclipse} \\ \text{-}\pi/2 + (\theta - \pi/2)/2 & \pi/2 < \theta < 3\pi/2 \\ 0 & \text{otherwise} \end{cases}$$

The law presented is derived from the orbit rate steering law presented in McInnes[1]. The law could be modified to be locally optimal. However simulations and verification has been carried out only for the law presented above.

The law ensures that the transverse component due to perturbations is dominated by solar thrust that drag is 0 in the eclipse region of the orbit. This ensures that da is always positive.

Additionally, we explain a phenomenon we observed in simulations. The gravitational perturbation causes the perigee of the orbit to rotate, which results in uniform orbit raising, with very small changes to the initial eccentricity of the orbit.

The simulation results and analytical treatment of the orientation control law have been presented in the next section, along with an explanation for uniform orbit raising.

V. Simulation

The orbit simulation was done in GMAT, using the Prince-Dormand-78 numerical propagator. GMAT has inbuilt models for solar radiation pressure and drag, but it lacks an attitude model. The SRP and Drag models consider the satellite to be spherical. However, forces due to both drag and SRP depend on the area and orientation of the sail. Thus, a custom Python module was used for modelling these forces. The module takes the PV state vector, atmospheric density and Julian Day as input. It calculates $\bar{s}$ based on J, and calculates the sail normal vector according to the process described in 2. This vector is the *ideal* normal direction. In the actual satellite system, this vector will be used by the orientation control algorithm as the reference vector to determine error.

The module does not simulate rotational dynamics, and assumes that ideal sail orientation is achieved. Accordingly, it calculates the forces due to SRP and drag acting on the sail. The net perturbing force is returned to the calling GMAT script.

The propagation takes place in steps. Every iteration, the python module calculates perturbing forces. Then a FiniteBurn of the calculated magnitude is applied by firing 3 out of 6 virtual ẗhrusterss̈placed along the 6 directions of the earth centred inertial frame to simulate the effect of the perturbing forces. With the *FiniteBurn* active, the satellite is propagated through a time step of 10 seconds.

While this method has not been formally proven, extensive verification has been carried out. Highlights are presented below.

1. Trajectories obtained by considering the sail to be perpendicular to the velocity vector closely resembled those which used GMAT's internal Drag model which considers a spherical satellite of the same cross sectional area.

2. Single impulses approximated using high magnitude finite burns resulted in orbits having keplerian elements equal to those calculated from the equivalent state vector $(\bar{r}, \bar{v} + \bar{dv})$ which would have been the result of the impulse. The keplerian elements corresponding to a particular P-V state can be calculated using the method described in [**wikipedia**].

3. McInnes[1] provides simulation results for the three solar sailing control laws described above. These results are for simulations of orbits which are geosynchronous (no drag), and consider the sun to be at infinity in the plane of the orbit. These conditions were simulated using the method described, giving exactly the same results.

4. Additionally, in every iteration of the simulation, the variational equations were used to calculate the instantaneous rates of change of semimajor axis and eccentricity. (da/df and de/df). These rates were used to calculate da and de for that iteration. The sum of these small changes was maintained, to give an *analytical* estimate of orbital elements. These estimates were compared with the orbital elements provided by GMAT, and were found to resemble each other.

VI. Results

The orbit trajectory was simulated for the sail control law described above. The results for nearly circular launch orbits at a height of $600km, 700km, 800km$ above the earth's surface are presented below. It was observed that while apogee may rise for orbits having initial approximately $700km$ or above, net increase in semimajor axis only occurs for orbits above $770km$.

The position of the sun with respect to the orbit decides whether raising will take place. For some inclinations, solar thrust does not exceed drag even for higher orbits. We have not yet developed a method (other than trial and error) to determine the optimum time and inclination for launch. (The position of the sun can be considered to be a function of time).

For the simplified case in which the sun is at infinity in the orbital plane, the increase in semimajor

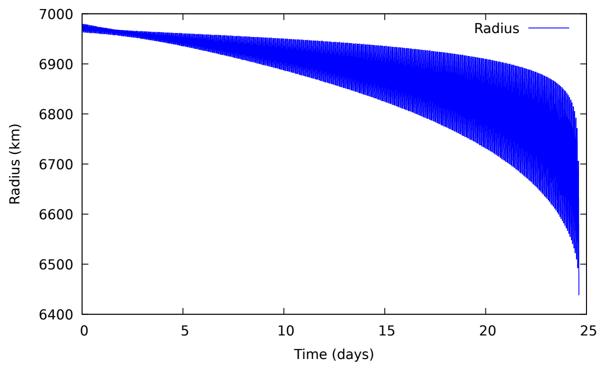

Fig. 2: Radius vs time for orbit with initial radius $6971km$

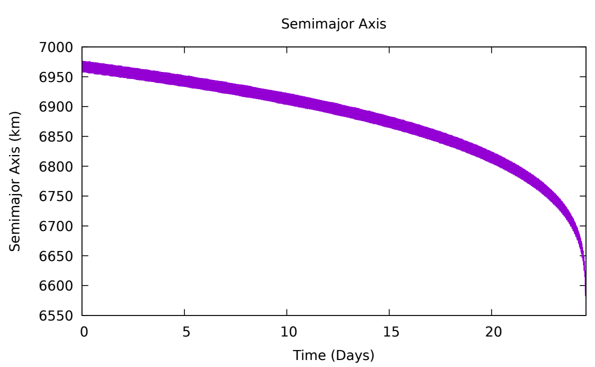

Fig. 3: Semi Major Axis vs time for orbit with initial radius $6971km$

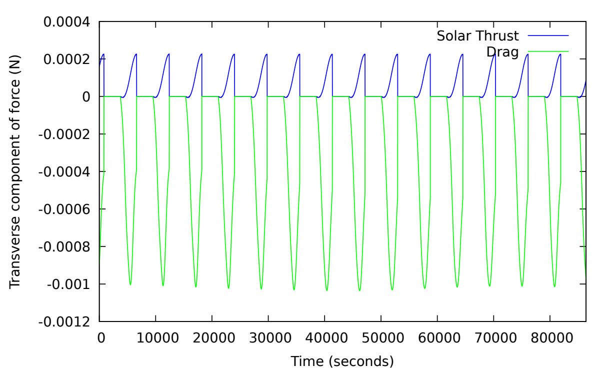

Fig. 4: Force vs time for orbit with initial radius $6971km$

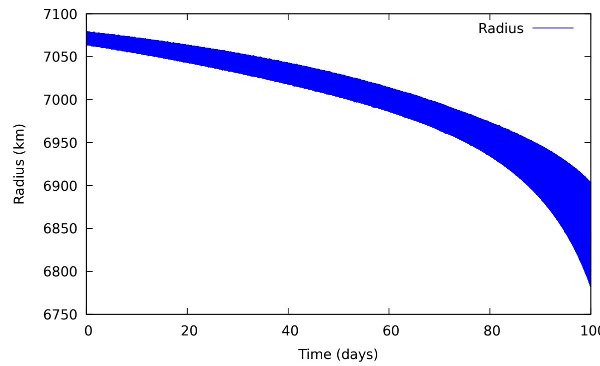

Fig. 5: Radius vs time for orbit with initial radius $7071km$

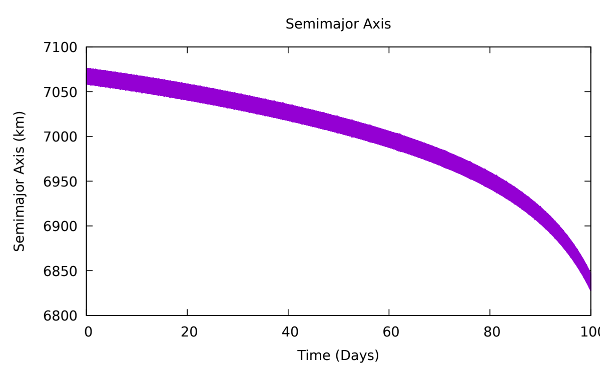

Fig. 6: Semi Major Axis vs time for orbit with initial radius $7071km$

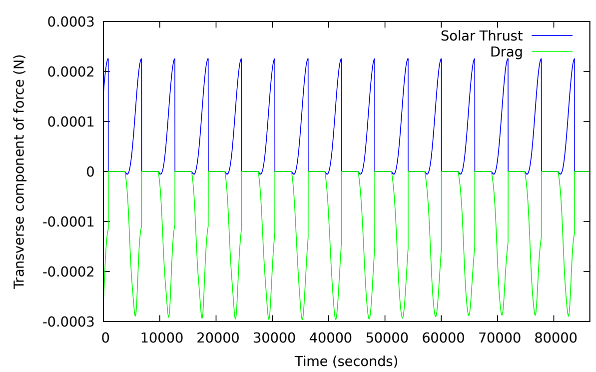

Fig. 7: Force vs time for orbit with initial radius $7071km$

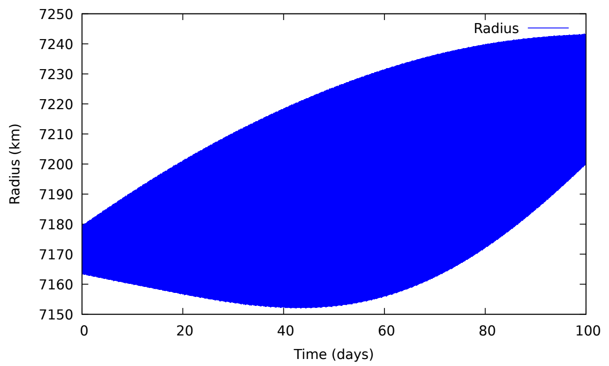

Fig. 8: Radius vs time for orbit with initial radius $7171km$

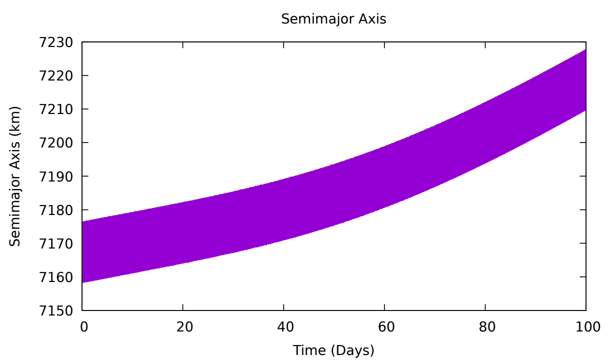

Fig. 9: Semi Major Axis vs time for orbit with initial radius $7171km$

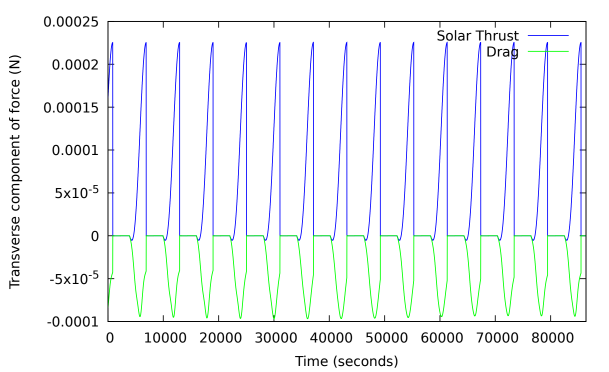

Fig. 10: Force vs time for orbit with initial radius $7171km$

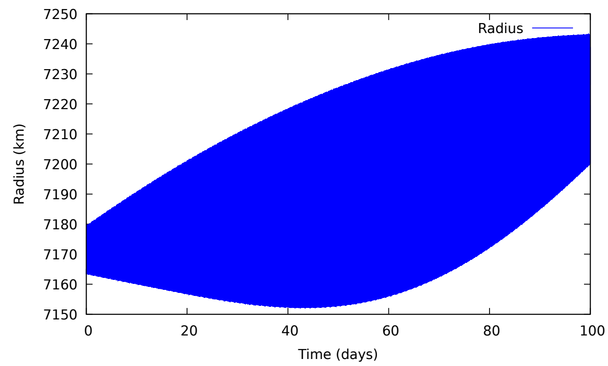

Fig. 11: Radius vs time for orbit with initial radius $7171km$

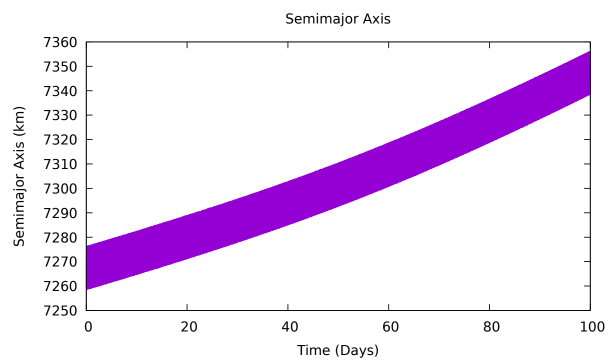

Fig. 12: Semi Major Axis vs time for orbit with initial radius $7271km$

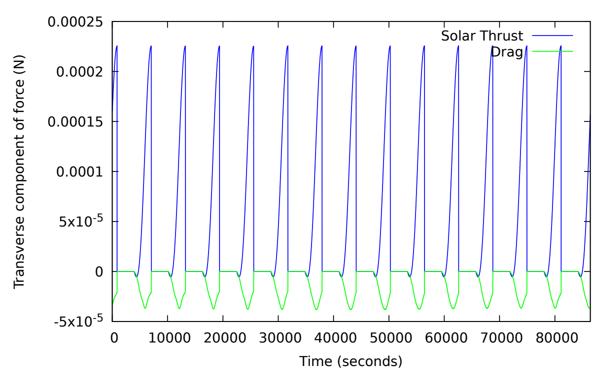

Fig. 13: Force vs time for orbit with initial radius $7271km$

axis per orbit can be calculated by integrating da/df over the orbit.

Ignoring the sunlit region in the third quadrant and drag, the increase turns out to be , which is half of the rate mentioned in McInnes[1]. This is expected considering the nature of the orientations.

Consider the equation 2. For a nearly circular orbit, $e = 0$. As a result, $(1-e^2)^2 = 0$, $Se\sin f = 0$. Additionally, one can consider that $r = p = a$. Thus equation 2 becomes

$$\frac{da}{df} = \frac{2Ta^3}{u}$$

. Now, considering that the sun is at infinity in the orbital plane, the solar thrust acting on the sail can be written as

$$F(f) = k\cos^2(f/2 - 3\pi/4)$$

, where k is the maximum amount of force acting on the sail (when the sail normal is parallel to solar radiation).[2][1]. Then, T can be given as

$$T(f) = k\cos^3(f/2 - 3\pi/4)$$

Integrating this over π radians, the increase in semimajor axis turns out to be $\frac{8ka^3}{3u}$. We integrate over π radians and not the entire orbit because *force acting on the sail will be zero for half the orbit.* Also, in this case, *which interval of π we are considering, and the location of the perigee (which determines f, the true anomaly) does not matter.* Substituting $0.0003N$ as the maximum force on the sail due to solar radiation pressure, the value of Δa for a single orbit turns out to be 0.085399. For initial a of $7231km$, the number of orbits per day will be 14.02. Thus, the expected increase in a per day is of $1.19806km$.

VII. Uniform raising phenomenon

Consider an orbit where the sun is at infinity, with radiation parallels to the $X-axis$. In such a situation, the solar thrust is directed along the X direction. Thus, an initial hypotheses was that the perigee should always be somewhere in the third quadrant- as the maximum amount of force acting on the satellite is in the third quadrant, and no force acts in the first quadrant. Based on this hypothesis, we believed that the perigee of the orbit should remain more or less fixed, with the apogee increasing continuously. Alternately, if drag is significant, the perigee would decrease at a constant rate, with the apogee increasing at a higher rate.

However, simulation results showed a perigee which decreased for the first few days, and increased later (refer to 14. This was completely unexpected, and caused concern about the validity of the simulation.

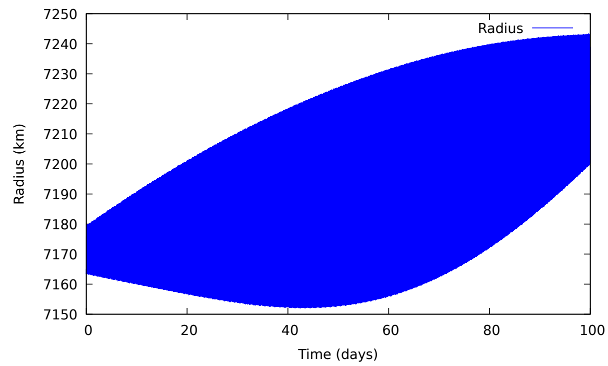

Fig. 14: Radius vs time for orbit with initial radius $7171km$. The lower boundary of the graph is the perigee. It is observed that the perigee decreases for the first 40 days, then increases.

On inspection, the trends observed in the orbital elements were as follows. The semimajor axis showed a continuous increase, which was expected according to the variational equations presented in McInnes[1]. The Ω was changing at a rate of around 1 degree per day, which was expected, as it was a near sun-synchronous orbit.

Two orbital elements, eccentricity and argument of perigee were also showing variations which were unexplainable. The argument of perigee was decreasing continuously (plot in 16). The eccentricity was increasing, then again decreasing (observed in 15).

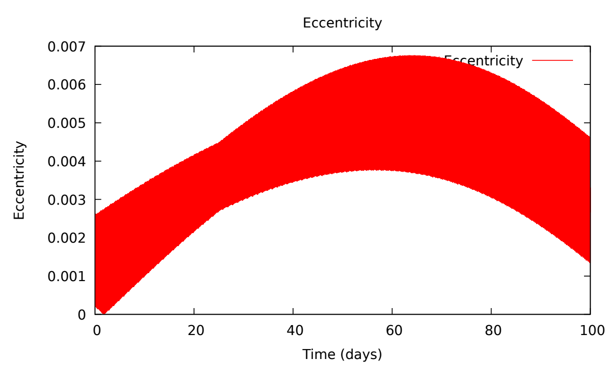

Fig. 15: Eccentricity vs time for orbit with initial radius $7171km$. There is a rise in eccentricity followed by a fall. This corresponds to a fall and rise in perigee as according to Fig. 15

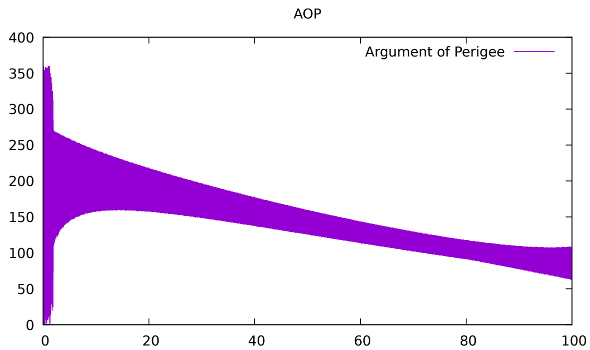

Fig. 16: Argument of Perigee vs time for orbit with initial radius $7171km$. There is a rise in eccentricity followed by a fall. This corresponds to a fall and rise in perigee as according to Fig. 15

The Lagrange variational equations can be used to calculate the instantaneous rates of change of orbital elements with respect to true anomaly. By integrating these rates of change over the orbit, the change in orbital element per orbit can be calculated.

The formula for da, mentioned in 2 indicates that the semimajor axis should rise irrespective of the current argument of perigee. This is because the change in semimajor axis per orbit does not depend on *which* interval of π radians the transverse force is nonzero, and thus on the relative position of the sun with respect to the perigee.

The formula for e is however

$$\frac{de}{df} = \frac{r^2}{u}[S\sin f + T(1+\frac{r}{p})\cos f + T\frac{r}{p}e]$$

Considering the approximations done in section 6, this is reduced to

$$\frac{de}{df} = \frac{r^2}{u}[S\sin f + 2T\cos f]$$

Now, unlike the approximate expression for da/df, the term to be integrated consists of

$$T(f)trig(f)$$

. So, the value of the integral depends on the interval π for which F is nonzero. Thus unlike da/df, the rate of $de/df and d\omega/df$ depend on the current ω.

The formula for rate of change of ω is

$$\frac{d\omega}{df} = -\frac{d\Omega}{df}\cos i + \frac{r^2}{ue}[-S\cos f + T(1+\frac{r}{p})\sin f]$$

The instantaneous rate of change of e and ω involve a non- force cosine true anomaly term. The rate of change of these parameters per orbit thus depend on the current argument of perigee and the position of the sun. Thus, if the relative position of the sun with respect to the perigee changes, the rates of change of eccentricity and ω too will change. This can be demonstrated as follows.

We thus hypothesised that the perturbing forces (solar radiation pressure and drag) cause the perigee of the orbit to rotate in a particular direction. Then, for a particular range of values of ω, the rate of change of eccentricity is positive, and negative for the rest. However, analysis indicated that the orientations specified should cause the perigee to rotate in the direction *opposite* to the one observed from the results. In order to isolate the problem, we conducted simulations which ignored SRP and drag. These showed that the perigee was rotating in the same direction. This led us to conclude that there was another strong perturbation involved. The third perturbation turned out to be the gravitational perturbation.

Note that the objective here is to explain the perigee trend. The perigee p can be written as

$$p = a*(1-e)$$

Taking differentials,

$$dp = da*(1-e) - a*de$$

Assuming low eccentricity,

$$dp = da - a*de$$

Now we know that a is always increasing, so da is always positive. On the other hand, e could be increasing of decreasing so de may be positive or negative. When de is positive, dp would be less positive OR negative (depending on magnitude of da and de. When de is negative, dp would always be positive. Thus, depending on the magnitude and sign of de, the perigee could increase or decrease Considering the fact that ω was constantly changing, change in sign of de was justified. However, the nature of change of ω was not according to predictions made using the variational equations, in first place.

The reason behind the unexpected nature of the change in the argument of perigee is the gravitational perturbation. The ideal keplerian orbit theory considers the earth to be a perfect sphere. However, the earth is Oblate. Thus, the gravitational force due to parts of the earth which are not considered in the Keplerian model are considered to be perturbations to the orbit. While J2 is significant in perturbing Ω, (allowing sun-synchronous orbits), it also perturbs the argument of perigee. King-Hele[3] provides a formula which gives the rate of change of the ω for an orbit of a particular inclination and semimajor axis due to the J2 perturbation. The formula is as follows-

$$\frac{d\omega}{dt} = 5.00(\frac{R}{r})^{3.5}(5\cos^2 i - 1)deg/day$$

The rate of change of ω according to the simulation for orbits without SRP and Drag were found to agree with the formulae from King-Hele[3]. When external forces are present, the rate is lesser in magnitude. However, it is still dictates the direction of rotation of the perigee. Thus the nature of the change in ω can be explained. It is this change in ω which causes the variation in eccentricity, and consequently the perigee.

The exact rate of change of ω cannot be calculated using variational equations. However, the analytical estimate of eccentricity (described in 5) agrees with the results seen in 15. Note that as the perigee rotates, the eccentricity, instead of continuously increasing will fluctuate. The orbit raising therefore will be uniform, and will result in a low eccentricity higher orbit.

VIII. Related Work

The solar sailing control law described in this paper is a variation of the orbit rate steering law from [1]. The Lagrange equations presented in the book were instrumental in analysis of the law and the simulation. The orbit trajectories provided for the three laws described were used for verification of the simulation. The solar radiation pressure model we used was from a NASA Mariner document, but is *equivalent* to the one described in the book. The book's analysis lacks discussion regarding low earth orbits, drag, or the fact that a significant part of the orbit might be eclipsed by the earth, which is what we have attempted to improve upon.

Lightsail2 is a mission similar to ours. Lightsail2 uses the On-Off control law described in the book. However, our analysis and simulations indicate that an orbit of $700km$ is not suitable for uniform increase in semimajor axis. The data provided by the team indicates that Lightsail2's semimajor axis has been decreasing from the time of it's sail deployment. The team however has already acknowledged that the perigee is expected to decrease, and that overall deorbiting will take place after a period of TODO months. The satellite has demonstrated an increase in it's apogee, which beautiful praise here. The paper [] was instrumental in preliminary verification of our simulation.

IX. Conclusions

The analysis and simulations carried out indicate that solar sailing is suitable for overall orbit raising at orbits higher than TODO $770km$ above the surface of the earth. The feasibility of solar sailing however depends on the position of the sun with respect to the orbit. In LEO, force due to drag is comparable to the force due to solar radiation pressure and a major part of the orbit lies in the eclipse region. Hence, a pitch maneuver is required to quickly change the orientation of the sail to become tangent to the orbit as it enters the eclipse region. This minimizes the drag in eclipse.

The gravitational perturbation causes the perigee of the orbit to rotate continuously. A changing perigee results in a near-circular rather than an elliptical orbit. The orbit raising is thus uniform, though solar force acts on the satellite only in parts of the orbit.

The control law described in the paper is not optimum. While CSAT's proposed satellite is limited by it's actuators, and the fact that it's sail can only have a single reflective side, a locally optimal control law could be designed to give a higher rate of orbit raising. Efforts are ongoing in this direction.

References

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