* System classification:

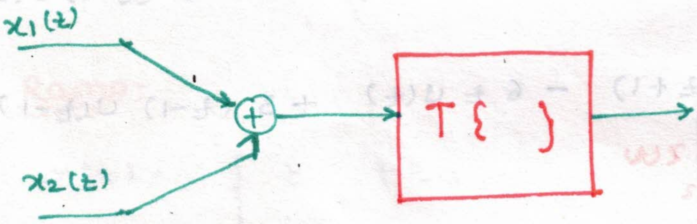
1. linear / Non-linear.
2. ~~Tive~~ Time variant / Time-invariant
3. causal / Non-causal.
~~4. Dynamic~~
~~5. stable & un-stable.~~
4. Dynamic / static (with memory / without mem
5. stable / un-stable.

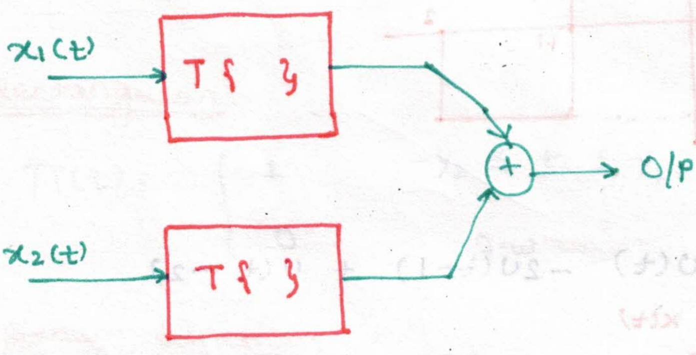
1. linear / Non-linear:

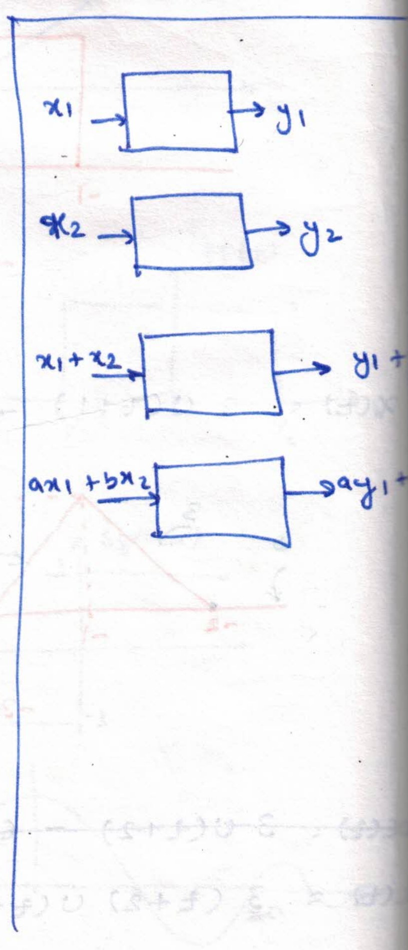
It is based on superposition theorem:
= additivity + scaling

(i) Additivity:

$$T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\}$$







(ii) scaling:

$$T\{a\,x(n)\} = a\,T\{x(n)\}$$

Q. Which of the following system is a linear system?

a) $y(t) = e^{x(t)}$

b) $y(t) = x(t) + K$; K is a constant

c) $y(t) = x(t^2)$

d) $y(t) = x^2(t)$

a) $T\{x_1(t) + x_2(t)\} = e^{\{x_1(t) + x_2(t)\}}$

$= e^{x_1(t)} \cdot e^{x_2(t)}$ — (1)

$T\{x_1(t)\} + T\{x_2(t)\} = e^{x_1(t)} + e^{x_2(t)}$ — (2)

$(1) \neq (2)$

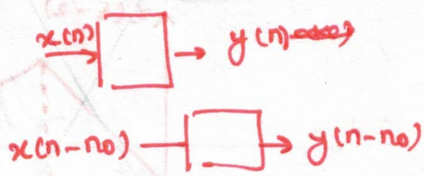
T.V N.L system.

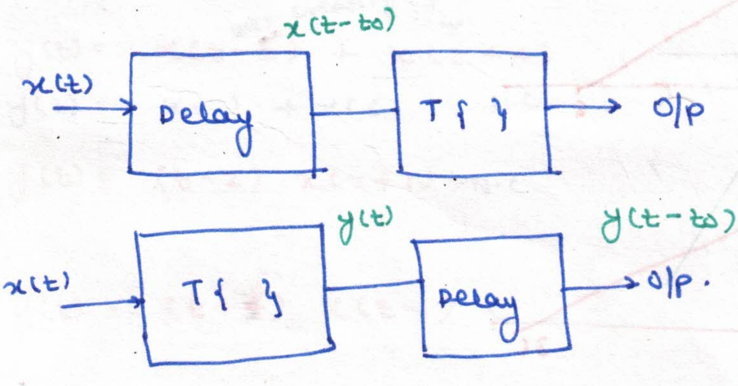
b) N.L system.

c) linear.

d) N.L system.

2. Time Variant:





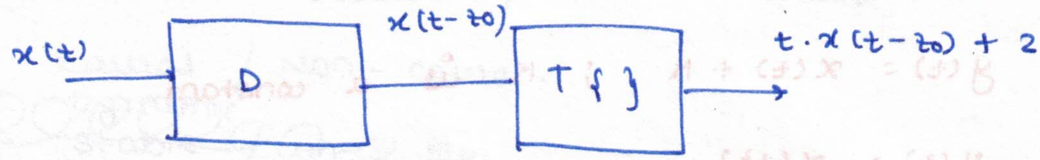
a) $y(t) = t\,x(t) + 2$

b) $y(t) = x(t^2)$

c) $y(t) = e^{x(t)}$

a) The Response ~~to~~ of delayed excitation:

$T\{x(t-t_0)\} = t \cdot x(t-t_0) + 2 \quad -(1)$



The delayed Response :

$y(t-t_0) = (t-t_0)\, x(t-t_0) + 2 \quad -(2)$

(1) $\neq$ 2

T.V

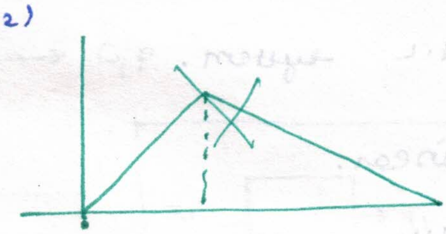
b) The response ~~to~~ of delayed excitation:

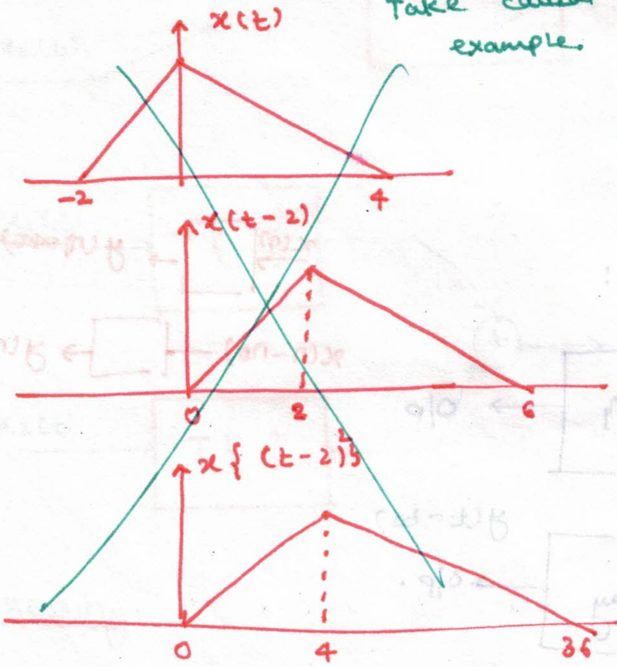
$T\{x(t-t_0)\} = x[(t-t_0)^2] \quad -(1)$

The delayed response is

$y(t-t_0) = x(t^2 - t_0) \quad -(2)$

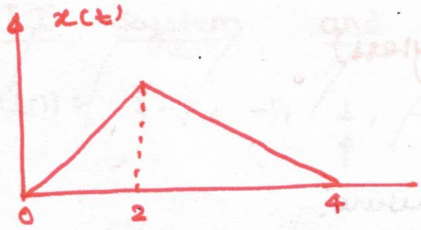
Take causal example.

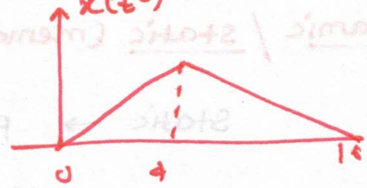


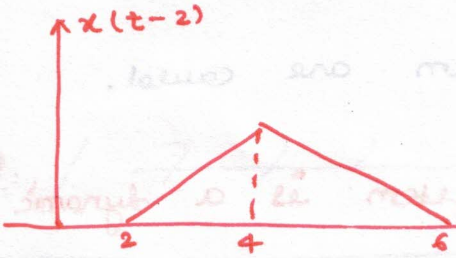


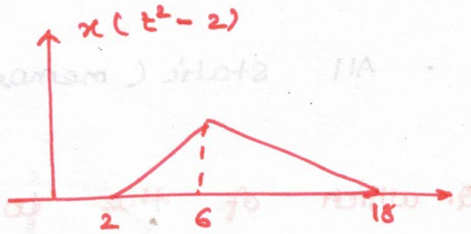
(1) $\neq$ (2)

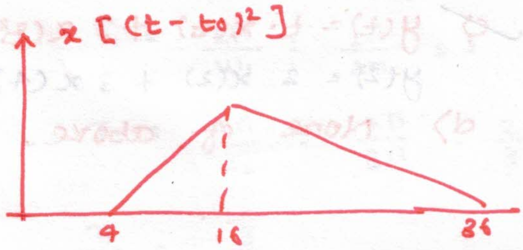
Time variant.





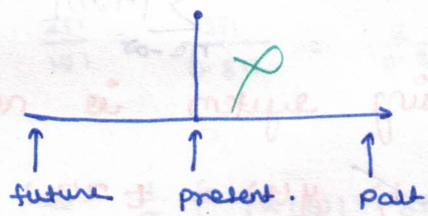


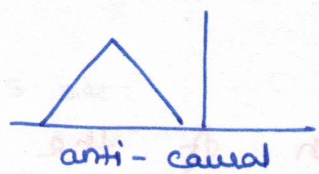


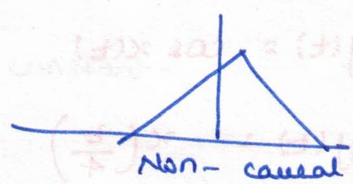


c) T.I.V.

3. causal / Non- causal:





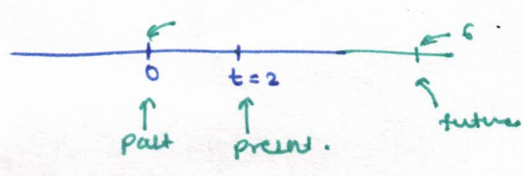


causal → Present + past

no future

Q. $y(t) = x(t-2) + \underbrace{x(t+4)}_{\text{creating problem}}$

$y(2) = x(0) + x(6)$ N.C



$y(t) = (t-2)\ x(t+1)$ N.C

$y(t) = (t+4)\ x(t-1)$ C

$y(t) = (t+5)\ x(t+5)$ N.C

/ Static (memory / memoryless)

Static → Present

no past, no future.

- All static (memoryless) system are causal.

Q. which of the following system is a dynamic system ?

a) $y(t) = 2\,x(t)$

b) $y(t) = t\,x(t) + 6\,x^3(t)$

~~c)~~ $y(t) = t \cdot x(t) + 3\,x(t^2)$

$y(2) = 2\,x(2) + 3\,x(4)$

d) None of above.

6. Stable and un-stable system:

Stable → BIBO

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Q.

Q. Which of the following system is not stab

a) $y(t) = \cos x(t)$

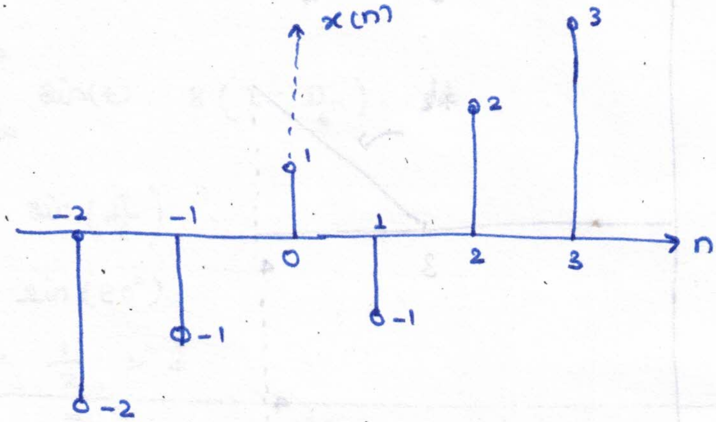
~~b)~~ $y(t) = t\,x(t)$

c) $y(t) = x\left(\frac{t}{4}\right)$

d) $y(t) = |x(t)|$

* LTI system and convolution *

$x(n) = \{-2, -1, \underset{\uparrow}{1}, -1, 2, 3\}$



positions.

$x(n) = -2\,\delta(n+2) - \delta(n+1) + \delta(n) - \delta(n-1) + 2\,\delta(n-2) + 3\,\delta(n-3)$

Amplitude

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)}$$

Position.

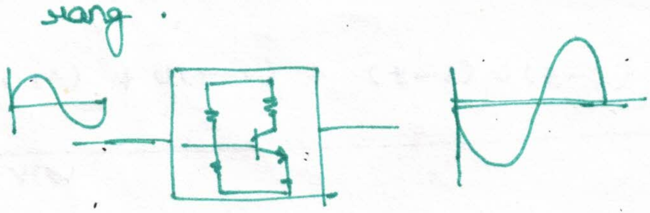
Amplitude

$\delta(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(n)$

impulse response.

Pani da rang .

#



$$x(n) \longrightarrow \boxed{\begin{matrix} h(n) \\ T\{\ \} \end{matrix}} \longrightarrow y(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$y(n) = T\{x(n)\} = T\left\{\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right\}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,T\{\delta(n-k)\}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$\boxed{y(n) = x(n) * h(n)}$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\cdot d\tau$$

$$\delta(n) \longrightarrow \boxed{LTI} \longrightarrow h(n)$$

$$\delta(n-k) \longrightarrow \boxed{LTI} \longrightarrow h(n-k)$$ Time in-variant.

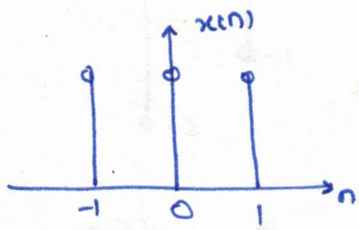
$$\text{i/p}\left\{\begin{array}{l} x(1)\,\delta(n-1) \longrightarrow \boxed{} \longrightarrow x(1)\,\cancel{\delta}h(n-1) \\ x(2)\,\delta(n-2) \longrightarrow \boxed{} \longrightarrow x(2)\,h(n-2) \\ \vdots \\ x(\infty)\,\delta(n-\infty) \longrightarrow \boxed{} \longrightarrow x(k)\,h(n-\infty) \end{array}\right\}\text{o/p}$$

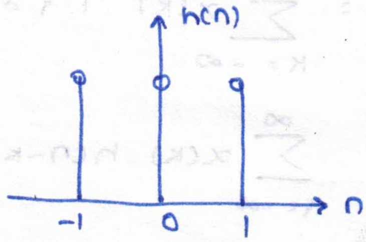
$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \Longrightarrow y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

Q. Find the convolution of two finite duration sequences:

$$x(n) = \begin{cases} 1 & -1 \le n \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$h(n) = \begin{cases} 1 & -1 \le n \le 1 \\ 0 & \text{otherwise.} \end{cases}$$





$y_l = x_l + h_l = -1 + (-1) = -2$

$y_r = x_r + h_r = 1 + 1 = 2$ y(n).

- $\underline{n = -2}$:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(-2) = \sum_{k=-1}^{1} x(k)\, h(-2-k)$$

$$= x(-1)\, h(-1) + x(0)\, \overset{0}{h(-2)} + x(1)\, \overset{0}{h(-3)}$$

$$= 1$$

- $\underline{n = -1}$

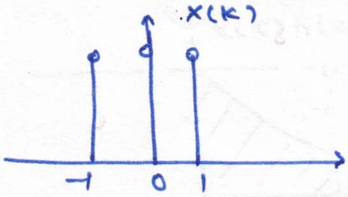
$$y(-1) = \sum_{k=-1}^{1} x(k)\, h(-1-k)$$

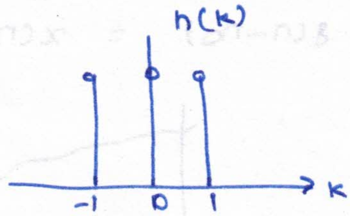
$$= x(-1)\, h(0) + x(0)\, h(-1) + x(1)\, h(-2)$$

$$= 2 \qquad \vdots$$

$$y(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$$

* Graphical Method:



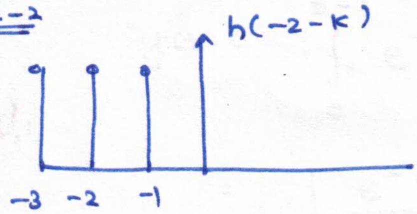


1. Change of variable
2. Fold h(k)
3. shifting of h(k)

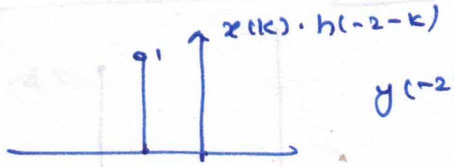
$h(-2-k)$

$h[-(k+2)]$

n=-2

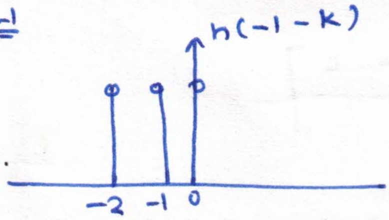


⇒

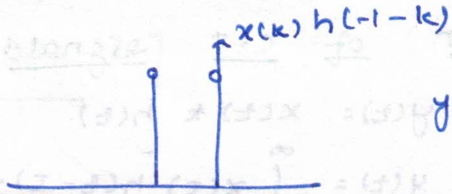


$y(-2) = 1$

n=-1

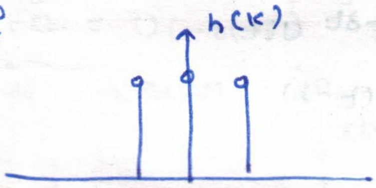


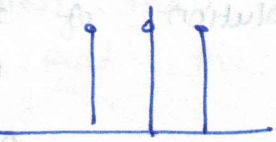
⇒



$y(-1) = 2$

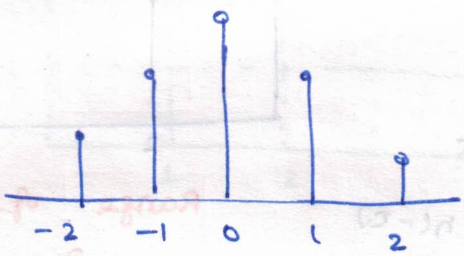
n=0





$y(0) = 3$

⋮

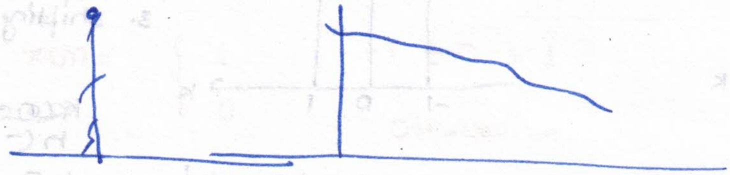


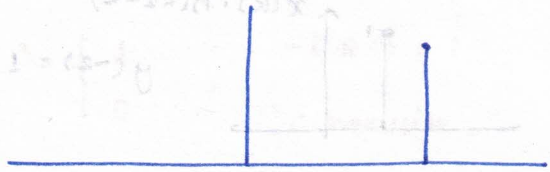
* Tabular Method:

	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

$y(n) = \{1 \quad 2 \quad \underset{\uparrow}{3} \quad 2 \quad 1\}$

$x(n) * \delta(n-n_0) = x(n - n_0)$





* Convolution of C.T signal:

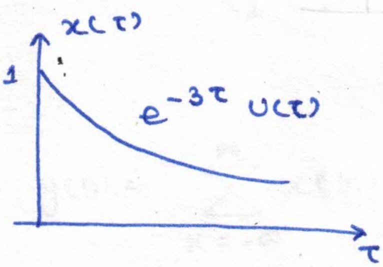
$$y(t) = x(t) * h(t)$$

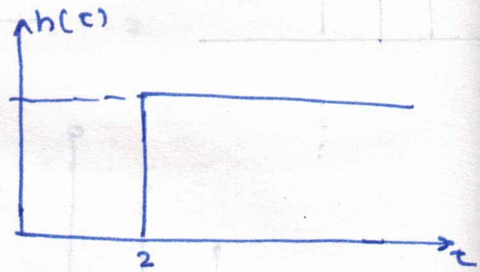
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau) \cdot d\tau$$

$h[-(\tau - t)]$

Q. Find the convolution of $x(t) = e^{-3t} u(t)$

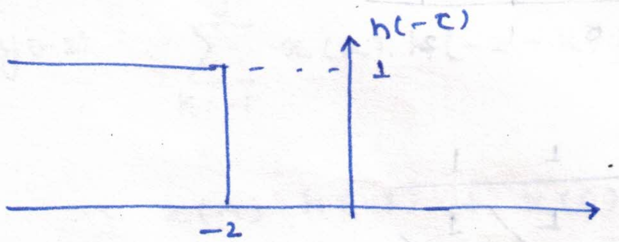
$h(t) = u(t-2)$



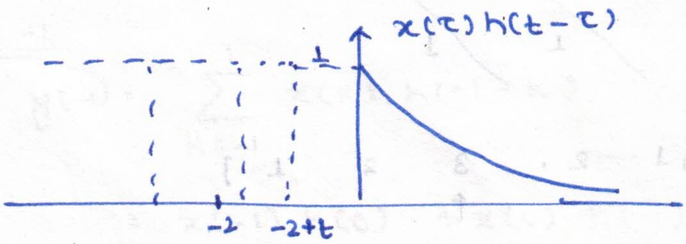


Range of $y(t)$

2 to ∞

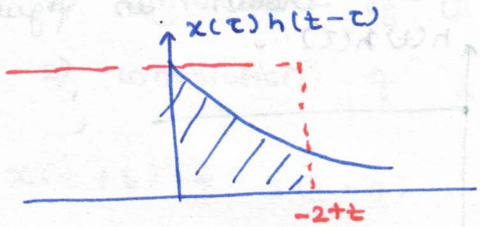


case(i): $t < 2 \Rightarrow t-2 < 0$



$y(t) = 0$

case (ii): $t > 2$

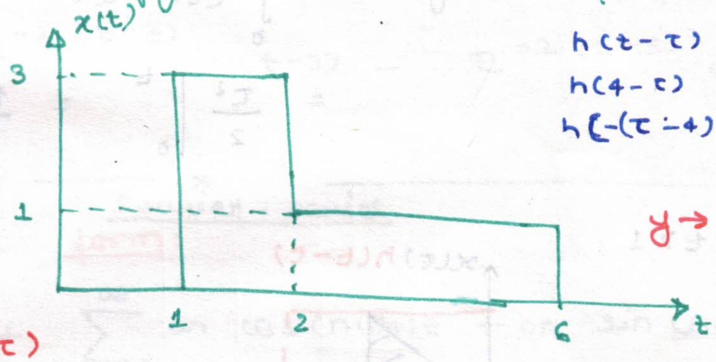


$$y(t) = \int_{0}^{t-2} e^{-3\tau} \cdot d\tau$$

$$= \left[\frac{e^{-3\tau}}{-3} \right]_{0}^{t-2}$$

$$= \left[\frac{1 - e^{-3(t-2)}}{3} \right]$$

Q. An L.T.I system is having an impulse response $h(t) = u(-t-1)$ [$u[-(t+1)]$] for which the i/p signal applied is shown in figure. Find the O/P at $t = 4$

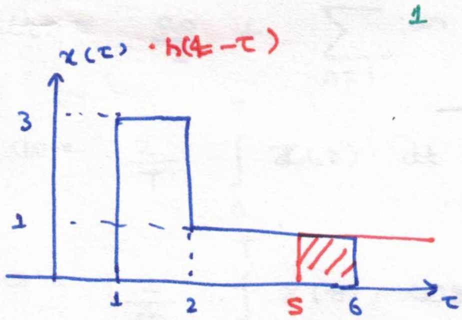


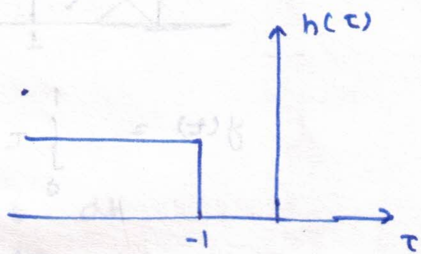
$h(t-\tau)$

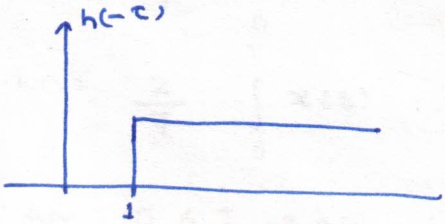
$h(4-\tau)$

$h[-(\tau-4)]$

$y \rightarrow -\infty$ to 5

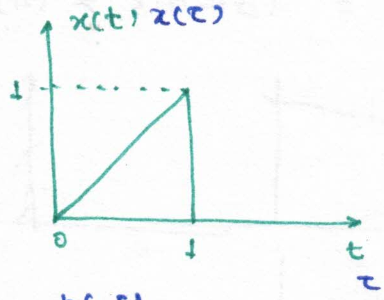


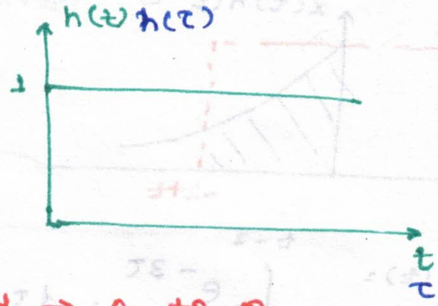


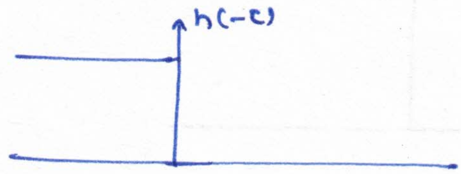


$$y(4) = \int_{5}^{6} (1) \cdot d\tau = 1$$

Q. Find the convolution of signal shown in figure



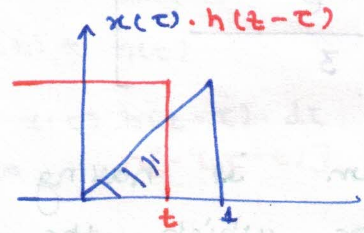




$y \to 0$ to ∞

case (i) : $t < 0$ $\quad y(t) = 0$

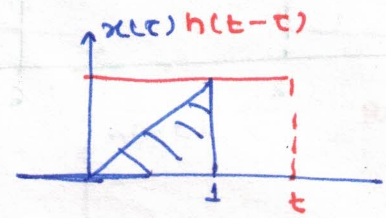
case (ii) : $0 < t < 1$



$$y(t) = \int_0^t (\tau) \cdot d\tau$$

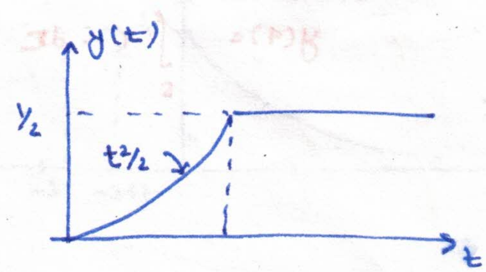
$$= \frac{\tau^2}{2}\Big|_0^t = \frac{t^2}{2}$$

case (iii) : $t > 1$



$$y(t) = \int_0^1 \tau \cdot d\tau$$

$$= \frac{\tau^2}{2}\Big|_0^1 = \frac{1}{2}$$



NOTE: 1) convolution of 2 causal is causal.

2) convolution of 2 anti-causal is anti-causal

Q. $x(t+5) * \delta(t-8)$

$x(t-8+5) = x(t-3)$

Q. $x(t) * \delta(at+b)$

$x(t) * \frac{1}{a}\delta\left(t+\frac{b}{a}\right)$

$\frac{1}{a}x\left(t+\frac{b}{a}\right)$

Q. Given $x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-2t}u(t)$ find $\left[\frac{d}{dt}x(t)\right] * h(t)$

$\frac{d}{dt}x(t) = \delta(t-3) - \delta(t-5)$

$y(t) = e^{-2(t-3)}u(t-3) - e^{-2(t-5)}u(t-5)$

Fourier - series

Trignometric form:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0)t + b_n \sin(n\omega_0)t$$

$$a_0 = \frac{2}{T}\int_0^T x(t)\cdot dt$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cdot \cos(n\omega_0)t\cdot dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\cdot \sin(n\omega_0)t\cdot dt$$

- for $0 \le T \le 2\pi$ $\omega_0 = 1$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x(t)\cdot dt$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x(t)\cdot \cos nt\cdot dt$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x(t)\cdot \sin nt\cdot dt$$