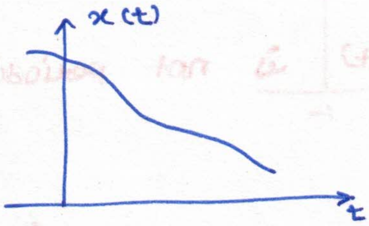
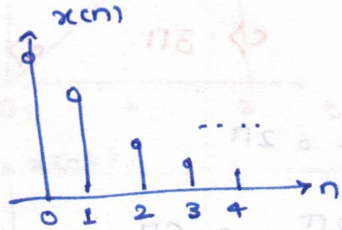
* signal classification :

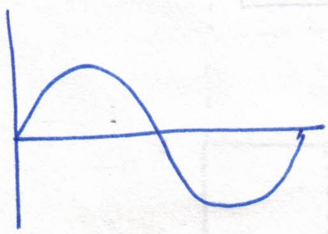
* signal classification
* Basic operations on signals
* Basic signals.
* Mathematical representation of signals.
* system classification.

1. * Analog and Discrete signals:



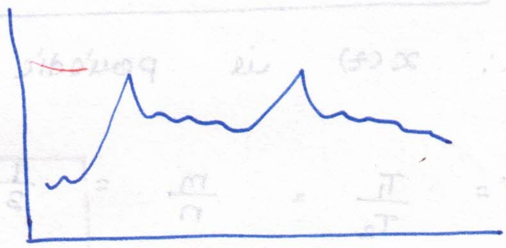


2. * Deterministic and Random signal:



$x(t) = A_m \sin 2\pi f_m t$

Mathematical expression exist.



No mathamatical expression exist.

ex. : Noise, voice.

3. * Periodic and Aperiodic signal:

$x(t) = x(t + T_0)$ ⇐ periodic

$f_0 = \frac{1}{T_0}$

$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ rad/sec

$x(t) = x(t + mT_0)$ $m = 1, 2, \ldots$

Periodicity of signal $x_1(t) + x_2(t)$

$x(t) = x_1(t) + x_2(t)$

is periodic if $\frac{T_1}{T_2} = \frac{m}{n}$ = a rational no.

Period of $x(t) \Rightarrow T = nT_1 = mT_2$

or $T = \text{LCM}(T_1, T_2)$

Q. Period of the following signal will be

$$x(t) = 2\cos t + 3\cos\left(\frac{t}{3}\right)$$

a) $\frac{1}{3}$ b) 6π c) 3π d) $x(t)$ is not periodi

→ $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{1} = 2\pi$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{1/3} = 6\pi$

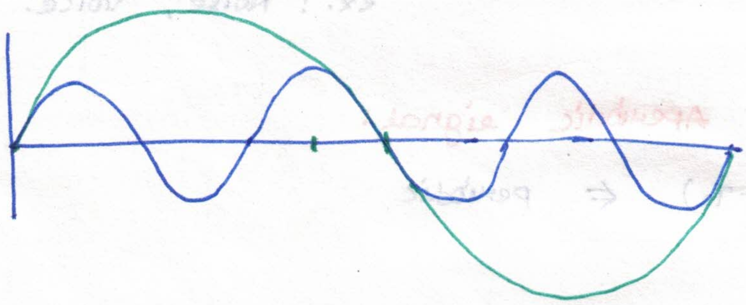
$$\boxed{\frac{T_1}{T_2} = \frac{2\pi}{6\pi} = \frac{1}{3} = \text{rational no.}}$$

∴ $x(t)$ is periodic

$T = \frac{T_1}{T_2} = \frac{m}{n} = \frac{1}{3}$

∴ $T = T_2$ or $3T_1$

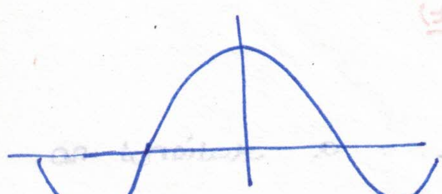
$= 6\pi$

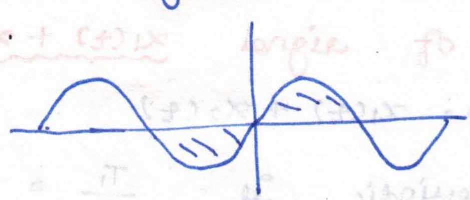


4.
* Even and Odd signal:

$x(t) = x(-t)$ ← even signal

$x(t) = -x(-t)$ ← odd signal





$x(t) = x_e(t) + x_o(t)$

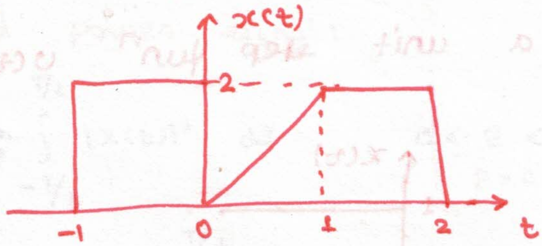
$x(-t) = x_e(-t) + x_o(-t)$

$x(-t) = x_e(t) - x_o(t)$

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

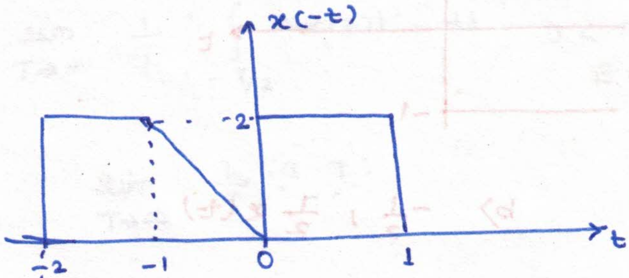
$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

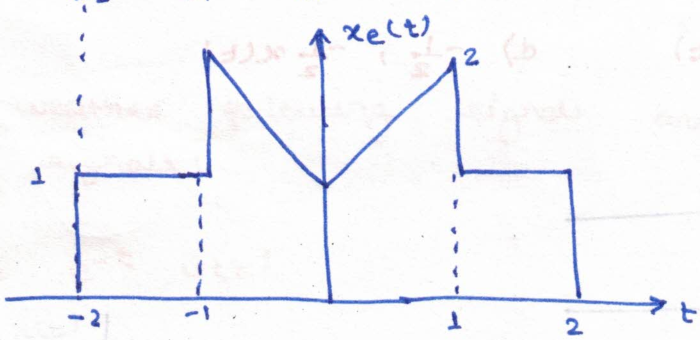
Q. Find even part of $x(t)$



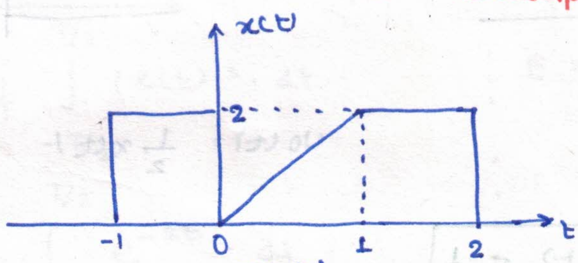
-2^{-1} : $\frac{0+2}{2} = 1$

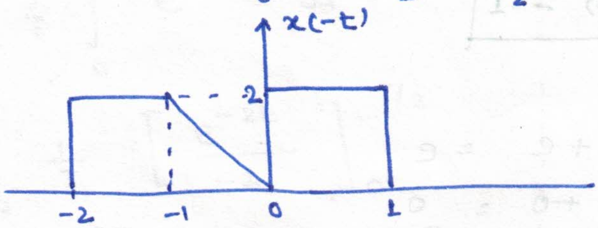
-2^{1} : $\frac{0+2}{2} = 1$

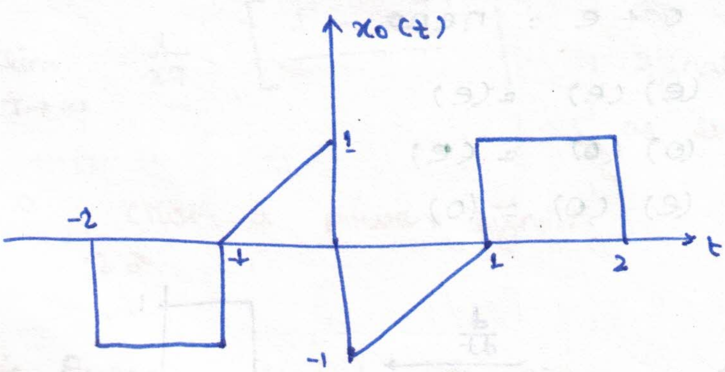




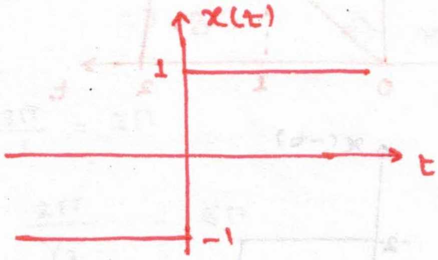
Q. Find odd part in above example:





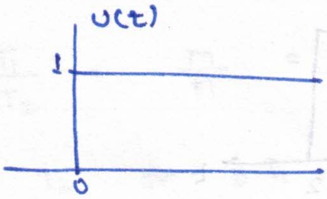


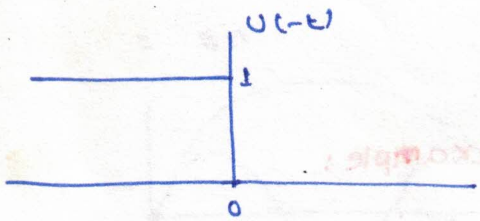
Q. 2005 The funⁿ $x(t)$ is shown in the figure. Even & odd part of a unit step funⁿ $u(t)$ are respectively

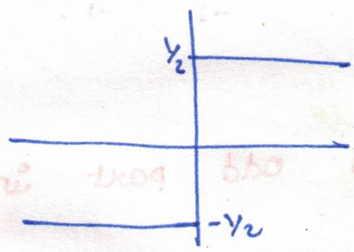


a) $\frac{1}{2}, \frac{1}{2}x(t)$ ✓   b) $-\frac{1}{2}, \frac{1}{2}x(t)$

c) $\frac{1}{2}, -\frac{1}{2}x(t)$ d) $-\frac{1}{2}, -\frac{1}{2}x(t)$







$u_e(t) = \frac{1}{2}$ $u_o(t) = \frac{1}{2}x(t)$

$\boxed{sgn(t) = 2u(t) - 1}$

e + e = e

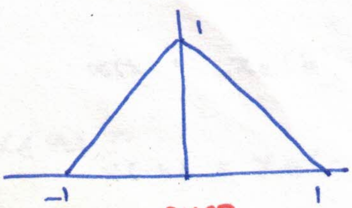
o + o = o

o + e = neno

(e) (e) = (e)

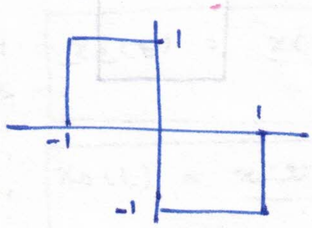
(o) (o) = (e)

(e) (o) = (o)



even

$\xrightarrow{\frac{d}{dt}}$



odd

Differentiation of an even signal is an odd signal and vice-versa.

5. Energy and power signal:

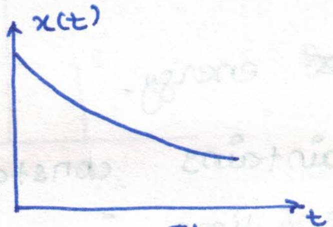
- $E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$ $0 < E < \infty$, $P = 0$ N.P signals are energy signals

- $P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$ $0 < P < \infty$, $E = \infty$ ~~Power signal~~ Periodic signals are power signals. However all power signal need not pe periodic.

$\lim_{T\to\infty} \frac{1}{T} a\, T$

Q. Find weather following signals are energy or power signals:

(i) $x(t) = e^{-t}\, u(t)$



$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$

$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} e^{-2t} \cdot dt$

$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{e^{-2t}}{-2} \right]_0^{T/2}$

$= \lim_{T\to\infty} \frac{1}{2T} \left[1 - e^{-T} \right]$

$= 0$ (not a power signal)

$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$

$= \lim_{T\to\infty} \frac{1}{2} \left[1 - e^{-T} \right]$

$= \frac{1}{2}$ (constant)

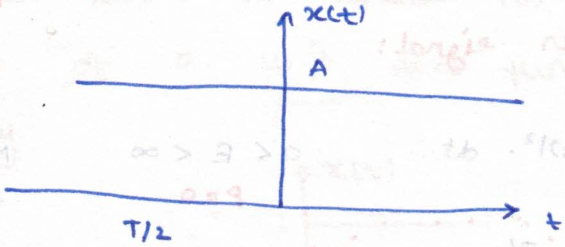
So, its a energy signal.

as it is non-periodic

NOTE: 1) Energy signal has always zero power.

2) $\left.\begin{array}{l} t \to \pm\infty \\ \text{amp} \to 0 \end{array}\right\}$ energy signal.

(ii) $x(t) = A$



- $$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{T} A^2 \cdot \left[\frac{T}{2} + \frac{T}{2}\right]$$

$$= A^2$$ $\therefore$ its a power signal.

- $E = \infty$

$\therefore$ 1. Power signals has ∞ energy.

2. A signal which maintains constant amplitude over infinite time is power signal.

(iii) $x(t) = A \cos(\omega_0 t + \phi)$

$$\text{Power} = (\text{r.m.s})^2 = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

All periodic signals are power signals.

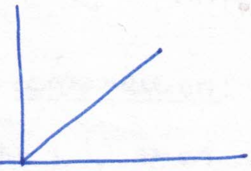
$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cdot \cos^2 \cos^2 \omega_0 t \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{T} A^2 \cdot \int_{-T/2}^{T/2} \left[\frac{1 + \cos 2\omega_0 t}{2}\right] \cdot dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \left[t + \frac{\sin 2\omega_0 t}{2\omega_0}\right]_{-T/2}^{T/2} = \frac{A^2}{2}$$

$$2\omega_0 t$$
$$2 \times \frac{2\pi}{\cancel{T}} \cdot \frac{\cancel{T}}{2} = 2\pi$$

(iv) $x(t) = t\,u(t)$

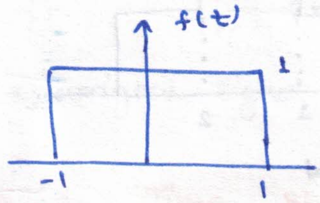


$\left.\begin{array}{l} t \to \infty \\ amp \to \infty \end{array}\right\}$ unbounded signal NENP

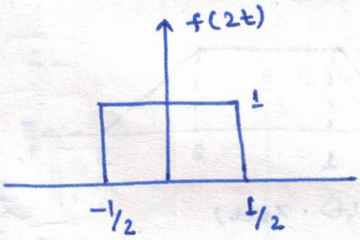
GATE 2001

Q. If the signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

a) E ~~b)~~ $\frac{E}{2}$ c) $2E$ d) $4E$.



$$E = \int_{-1}^{1} (1)^2 \cdot dt = 2 \qquad E$$



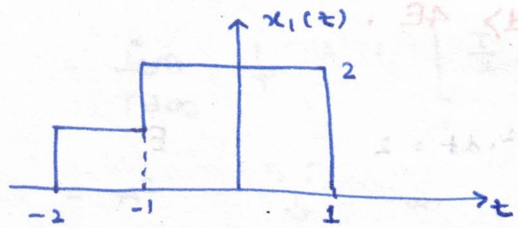
$$E = \int_{-1/2}^{1/2} (1)^2 \cdot dt = 1 \qquad \frac{E}{2}$$

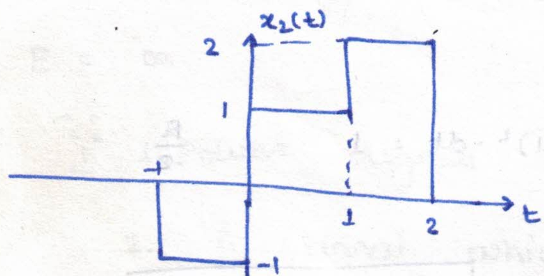
In general $\boxed{f(at) \leftrightarrow \frac{E}{|a|}}$

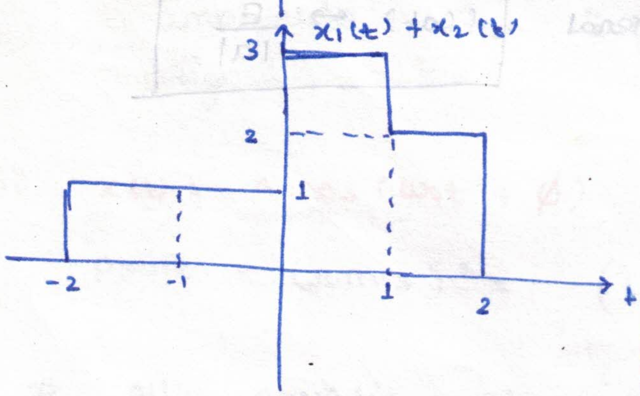
* refer last page.

1. Addition.
2. Multiplication.
3. Amplitude scaling
4. Time scaling.
5. Time shifting
~~e folding~~
6. Folding.

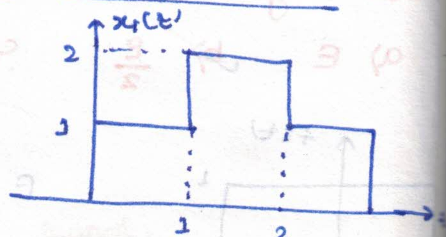
1. **Addition:**

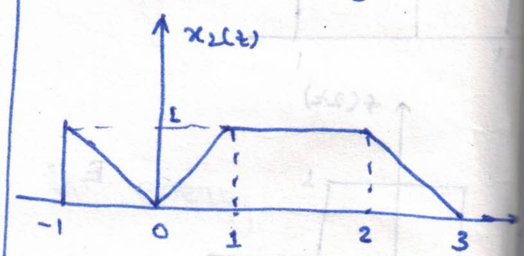


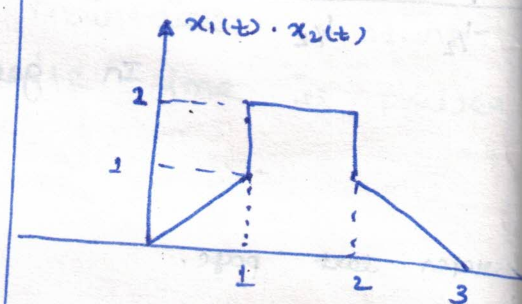




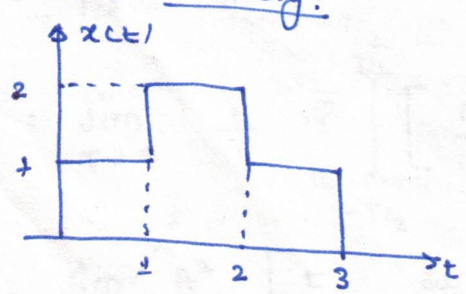
2. **Multiplication:**



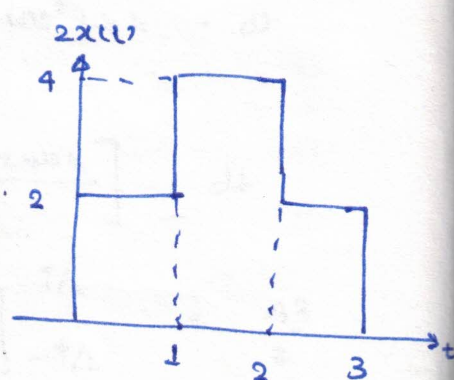




3. **Amplitude scaling:**



$2x(t)$
$\Rightarrow$



4. Time scaling:

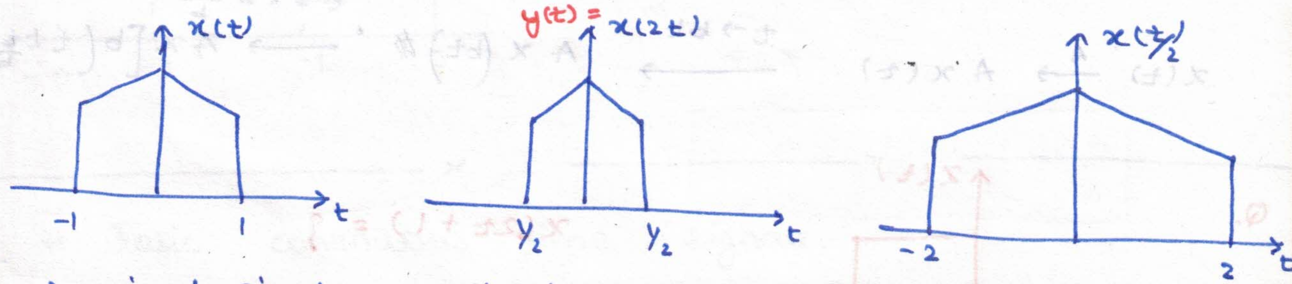
$x(at)$ is time-scaled version of $x(t)$

- Time-compression:

if $|a| > 1$, then $x(at)$ is a compressed version of $x(t)$

- Time-Expansion:

if $|a| < 1$, then $x(at)$ is an expanded version of $x(t)$

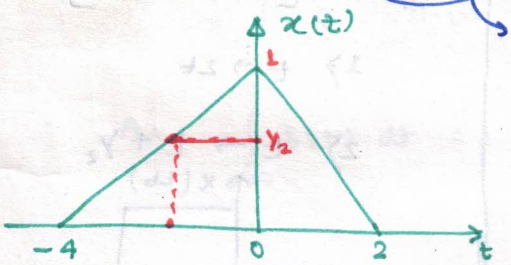


a) orignal signal. b) Time-compression c) Time-expansion.

5./6. Time shifting / Folding:

→ delay

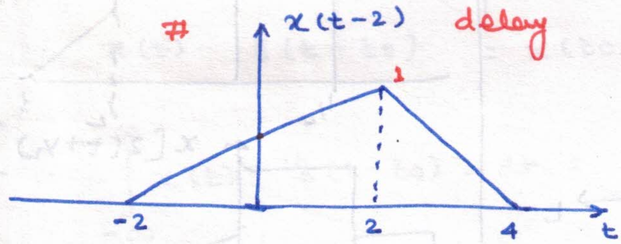
→ advancement

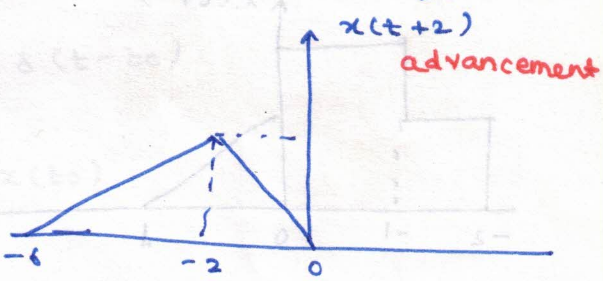


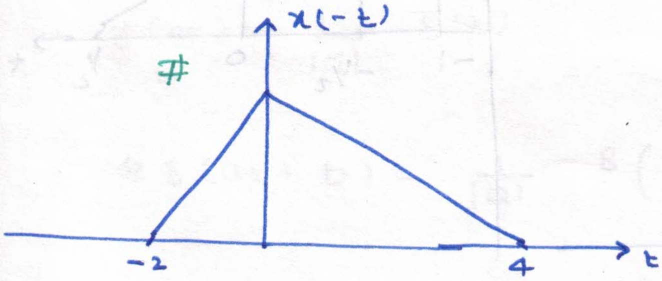
$x(t-2) = ?$ ($=0$, $t=2$)

$x(2-t) = ?$

$x[-(t-2)]$ ($=0$, $t = 2$)



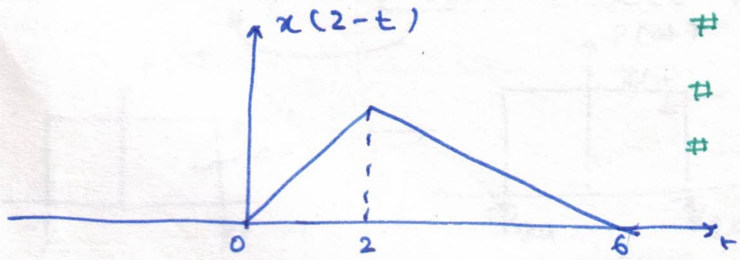




$t = -2 \Rightarrow x(-4) = 0$ (⇓ 0)

$t = 0 \Rightarrow x(-2) =$

$t = 4 \Rightarrow x(2) = 0$



$t = 0 \Rightarrow x(2) = 0$

$t = 2 \Rightarrow x(0) = 1$

$t = 6 \Rightarrow x(4) = 0$

* Multiple operation on signals :

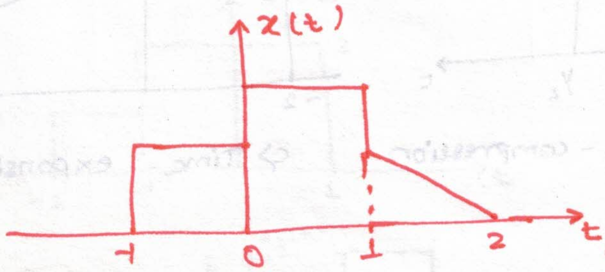
$$x(t) \rightarrow A\, x(bt \pm t_0)$$

$$x(t) \xrightarrow{A} A\,x(t) \xrightarrow{t \rightarrow t \pm t_0} A\,x(t \pm t_0) \xrightarrow{t \rightarrow bt} A\,x(bt \pm t_0)$$

$$x(t) \longrightarrow A\,x\left[b\left(t \pm \frac{t_0}{b}\right)\right]$$

$$x(t) \xrightarrow{A} A\,x(t) \xrightarrow{t \rightarrow bt} A\,x(bt) \xrightarrow{t \rightarrow t \pm \frac{t_0}{b}} A\,x\left[b\left(t \pm \frac{t_0}{b}\right)\right]$$

Q.

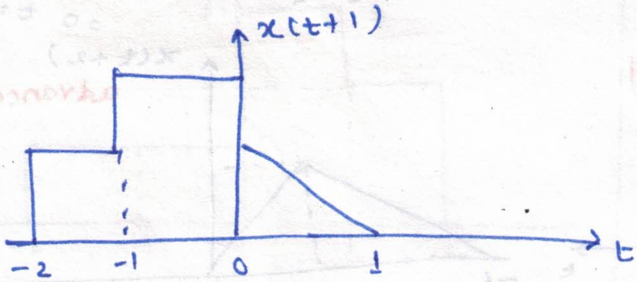


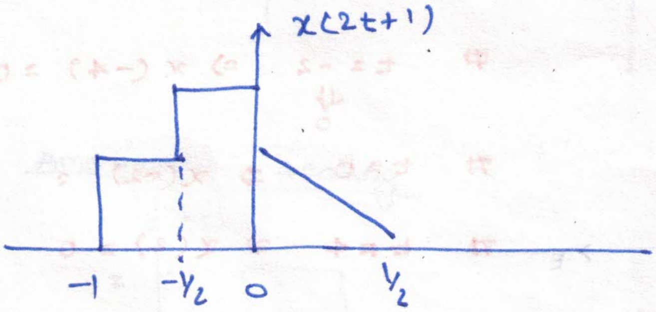
$x(2t+1) = ?$

1) $x[2t+1]$

1) $x(t+1)$ shifting.

2) $t \rightarrow 2t$ scaling

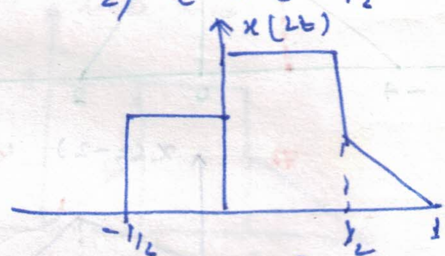


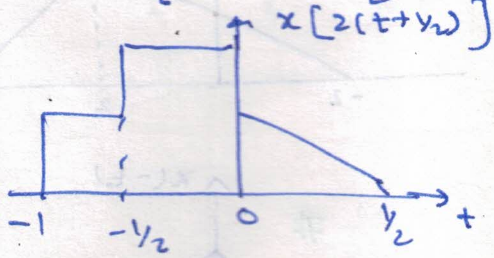


2) $x[2(t+1/2)]$

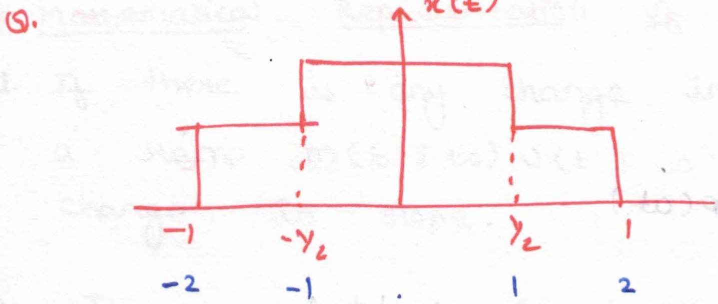
1) $t \rightarrow 2t$

2) $t \rightarrow t + 1/2$



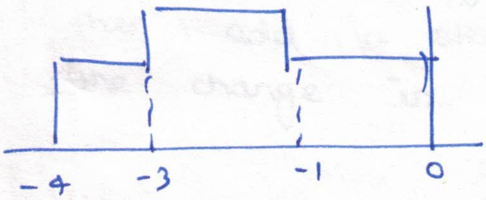


Q.



$x\left(\frac{-t-2}{2}\right)$

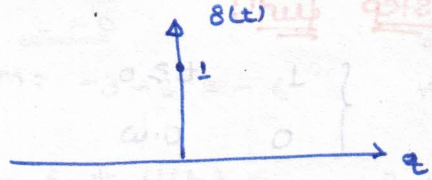
$x\left[-\frac{1}{2}\left[t+2\right)\right]$



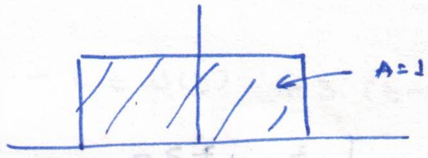
* Basic continuous time signals:

1. Unit Impulse :

- $\delta(t) = \begin{cases} \cancel{\infty}\ 1 & t = 0 \\ 0 & \text{o.w} \end{cases}$



- $A = \int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$



- $x(t) \cdot \delta(t-t_0) = x(t_0)\ \delta(t-t_0)$

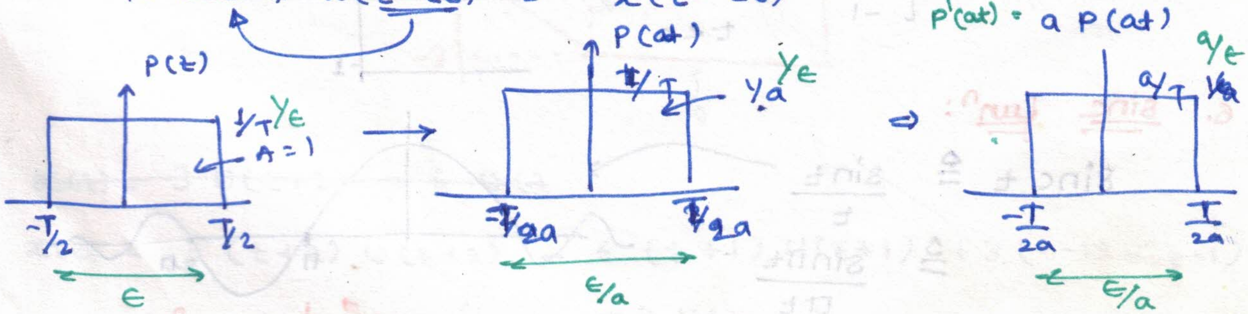
- $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0)$

- $\delta(at) = \frac{1}{|a|}\ \delta(t)$

- $\delta(at+b) = \frac{1}{|a|}\ \delta\left(t+\frac{b}{a}\right)$

- $x(t) * \delta(t-t_0) = x(t-t_0)$

$p'(at) = a\,p(at)$



$$\delta(t) = \lim_{\varepsilon \to 0} p(t)$$

$t \to at$

$$\delta(at) = \lim_{\varepsilon \to 0} p(at)$$

$$= \lim_{\varepsilon \to 0} \cancel{\tfrac{1}{a} \cdot a\, p(at)} \; \frac{1}{a} p'(at)$$

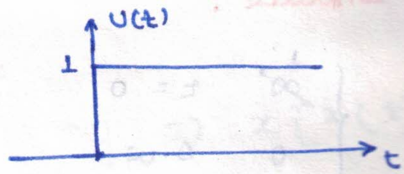
$$= \frac{1}{a} \lim_{\varepsilon \to 0} p'(at)$$

$$\boxed{\delta(at) = \frac{1}{a}\,\delta(t)}$$

---

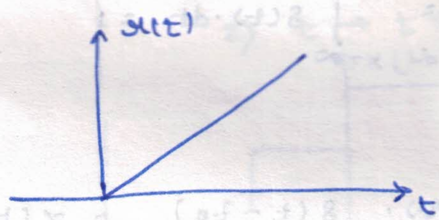
2. **Unit - step funⁿ:**

$$U(t) = \begin{cases} 1 & t \geq 0 \\ 0 & o.w \end{cases}$$



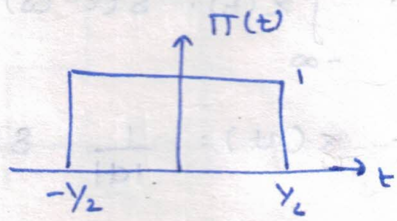
3. **Ramp:**

$$x(t) = \begin{cases} t & t \geq 0 \\ 0 & o.w \end{cases}$$



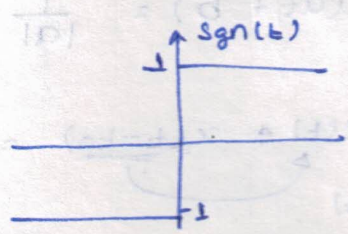
4. **Rectangular:**

$$\Pi(t) = \begin{cases} 1 & -1/2 \leq t \leq 1/2 \\ 0 & o.w \end{cases}$$



5. **Signum function:**

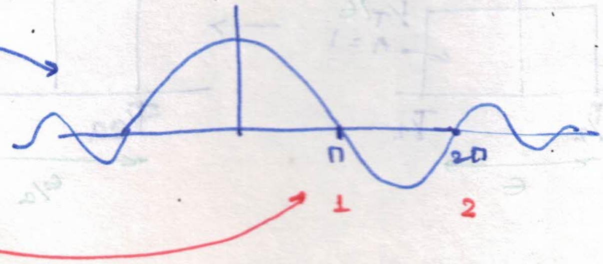
$$\mathrm{Sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



6. **Sinc funⁿ:**

$$\mathrm{sinc}\ t \triangleq \frac{\sin t}{t}$$

$$\triangleq \frac{\sin \pi t}{\pi t}$$



* Mathemat

1. If the
a ra
change

2. If a
then
the ch

Q.

t = -1
m =
3 (t
So x(

Q.

x(t) =

Q.

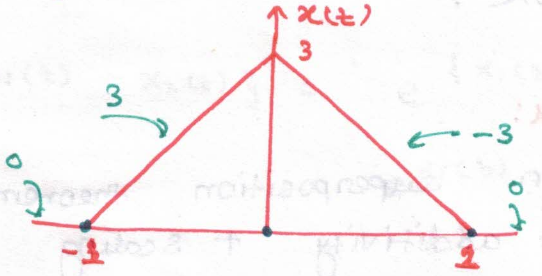
~~x(t)~~
x(t) =

* Mathematical Representation of signal:

1. If there is any change in slope at $t = \pm t_0$, add a ramp $m(t \mp t_0)\, u(t \mp t_0)$, where m is the change in slope.

2. If at $t = \pm t_0$, signal steps up or steps down, then add a step signal $A\, u(t \mp t_0)$, where A is the change in amplitude.

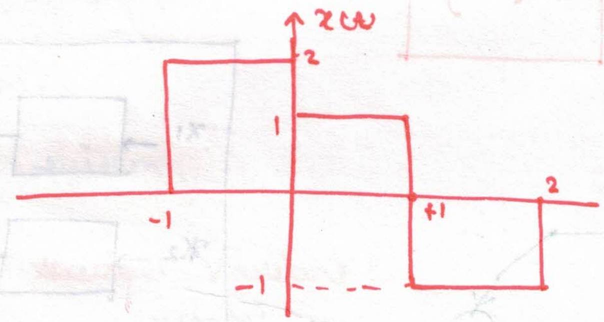
Q.



$\underline{t=-1}$	$\underline{t=0}$	$\underline{t=1}$
$m = 3 - 0 = 3$	$m = -3 - 3 = -6$	$m = 0 + 3 = 3$
$3(t+1)\,u(t+1)$	$-6\,t\,u(t)$	$3(t-1)\,u(t-1)$

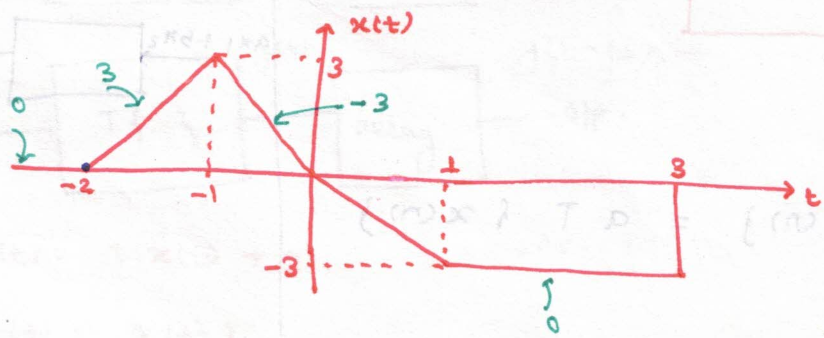
So $x(t) = 3(t+1)\,u(t+1) - 6\,t\,u(t) + 3(t-1)\,u(t-1)$

Q.



$x(t) = 2\,u(t+1) - u(t) - 2u(t-1) + u(t-2)$

Q.



~~x(t) = 3 u(t+2) − 6 u(t~~

$x(t) = 3(t+2)\,u(t+2) - 6(t+1)\,u(t+1) + 3(t-1)\,u(t-1)$
$+ 3\,u(t-3)$