0/1 Knapsack Problem | Dynamic Programming | Example

Design & Analysis of Algorithms

Knapsack Problem-

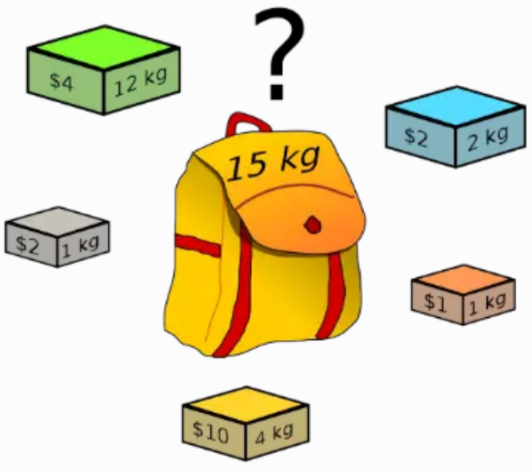
You are given the following-

- A knapsack (kind of shoulder bag) with limited weight capacity.
- Few items each having some weight and value.

The problem states-

Which items should be placed into the knapsack such that-

- The value or profit obtained by putting the items into the knapsack is maximum.
- And the weight limit of the knapsack does not exceed.



Knapsack Problem

Knapsack Problem Variants-

Knapsack problem has the following two variants-

- Fractional Knapsack Problem
- 0/1 Knapsack Problem

0/1 Knapsack Problem

In 0/1 Knapsack Problem,

- As the name suggests, items are indivisible here.
- We can not take the fraction of any item.
- We have to either take an item completely or leave it completely.
- It is solved using dynamic programming approach.

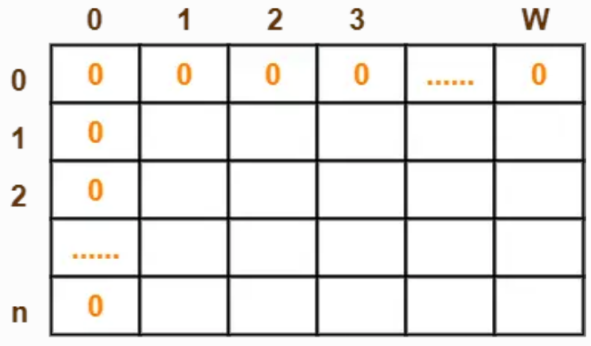
0/1 Knapsack Problem Using Dynamic Programming-

Consider-

- Knapsack weight capacity = w
- Number of items each having some weight and value = n

0/1 knapsack problem is solved using dynamic programming in the following steps-

Step-01:

- Draw a table say ‘T’ with (n+1) number of rows and (w+1) number of columns.
- Fill all the boxes of 0th row and 0th column with zeroes as shown-
-
- 

	0	1	2	3	.....	W
0	0	0	0	0	.....	0
1	0					
2	0					
	.....					
n	0					

T-Table

Step-02:

Start filling the table row wise top to bottom from left to right.

-

Use the following formula-

T (i , j) = max { T (i-1 , j) , valuei + T(i-1 , j – weighti) }

- Here, T(i , j) = maximum value of the selected items if we can take items 1 to i and have weight restrictions of j.

- This step leads to completely filling the table.
- Then, value of the last box represents the maximum possible value that can be put into the knapsack.

Step-03:

To identify the items that must be put into the knapsack to obtain that maximum profit,

- Consider the last column of the table.
- Start scanning the entries from bottom to top.
- On encountering an entry whose value is not same as the value stored in the entry immediately above it, mark the row label of that entry.
- After all the entries are scanned, the marked labels represent the items that must be put into the knapsack.

Time Complexity-

- Each entry of the table requires constant time θ(1) for its computation.
- It takes θ(nw) time to fill (n+1)(w+1) table entries.
- It takes θ(n) time for tracing the solution since tracing process traces the n rows.

- Thus, overall θ(nw) time is taken to solve 0/1 knapsack problem using dynamic programming.

PRACTICE PROBLEM BASED ON 0/1 KNAPSACK PROBLEM-

Problem-

For the given set of items and knapsack capacity = 5 kg, find the optimal solution for the 0/1 knapsack problem making use of dynamic programming approach.

Item**Weight****Value**123234345456

-

OR

Find the optimal solution for the 0/1 knapsack problem making use of dynamic programming approach. Consider-

n = 4

w = 5 kg

(w1, w2, w3, w4) = (2, 3, 4, 5)

(b1, b2, b3, b4) = (3, 4, 5, 6)

OR

A thief enters a house for robbing it. He can carry a maximal weight of 5 kg into his bag. There are 4 items in the house with the following weights and values. What items should thief take if he either takes the item completely or leaves it completely?

ItemWeight (kg)Value ($) Mirror23Silver nugget34Painting45Vase56

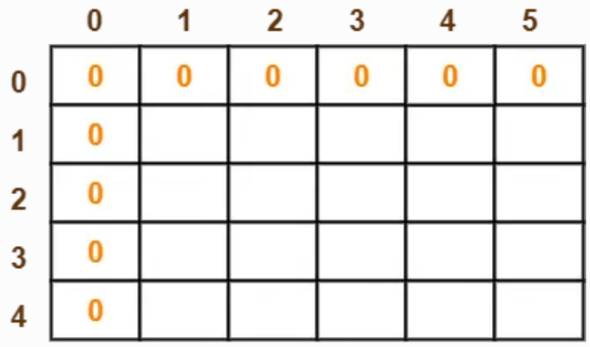
Solution-

Given-

- Knapsack capacity (w) = 5 kg
- Number of items (n) = 4

Step-01:

- Draw a table say ‘T’ with (n+1) = 4 + 1 = 5 number of rows and (w+1) = 5 + 1 = 6 number of columns.
- Fill all the boxes of 0th row and 0th column with 0.

- 

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					
4	0					

T-Table

Step-02:

Start filling the table row wise top to bottom from left to right using the formula-

-

T (i , j) = max { T (i-1 , j) , valuei + T(i-1 , j – weighti) }

Finding T(1,1)-

We have,

- i = 1
- j = 1
- (value)i = (value)1 = 3
- (weight)i = (weight)1 = 2

Substituting the values, we get-

T(1,1) = max { T(1-1 , 1) , 3 + T(1-1 , 1-2) }

T(1,1) = max { T(0,1) , 3 + T(0,-1) }

-

T(1,1) = T(0,1) { Ignore T(0,-1) }

T(1,1) = 0

Finding T(1,2)-

We have,

- i = 1
- j = 2
- (value)i = (value)1 = 3
- (weight)i = (weight)1 = 2

Substituting the values, we get-

T(1,2) = max { T(1-1 , 2) , 3 + T(1-1 , 2-2) }

T(1,2) = max { T(0,2) , 3 + T(0,0) }

T(1,2) = max {0 , 3+0}

- T(1,2) = 3

Finding T(1,3)-

We have,

- i = 1
- j = 3
- (value)i = (value)1 = 3
- (weight)i = (weight)1 = 2

Substituting the values, we get-

T(1,3) = max { T(1-1 , 3) , 3 + T(1-1 , 3-2) }

T(1,3) = max { T(0,3) , 3 + T(0,1) }

T(1,3) = max {0 , 3+0}

T(1,3) = 3

-

Finding T(1,4)-

We have,

- i = 1
- j = 4
- (value)i = (value)1 = 3
- (weight)i = (weight)1 = 2

Substituting the values, we get-

T(1,4) = max { T(1-1 , 4) , 3 + T(1-1 , 4-2) }

T(1,4) = max { T(0,4) , 3 + T(0,2) }

T(1,4) = max {0 , 3+0}

T(1,4) = 3

Finding T(1,5)-

We have,

- i = 1
- j = 5
- (value)i = (value)1 = 3
- (weight)i = (weight)1 = 2

Substituting the values, we get-

T(1,5) = max { T(1-1 , 5) , 3 + T(1-1 , 5-2) }

-

T(1,5) = max { T(0,5) , 3 + T(0,3) }

T(1,5) = max {0 , 3+0}

T(1,5) = 3

Finding T(2,1)-

We have,

- i = 2
- j = 1
- (value)i = (value)2 = 4
- (weight)i = (weight)2 = 3

Substituting the values, we get-

-

T(2,1) = max { T(2-1 , 1) , 4 + T(2-1 , 1-3) }

T(2,1) = max { T(1,1) , 4 + T(1,-2) }

T(2,1) = T(1,1) { Ignore T(1,-2) }

T(2,1) = 0

Finding T(2,2)-

We have,

- i = 2
- j = 2

- (value)i = (value)2 = 4
- (weight)i = (weight)2 = 3

Substituting the values, we get-

T(2,2) = max { T(2-1 , 2) , 4 + T(2-1 , 2-3) }

T(2,2) = max { T(1,2) , 4 + T(1,-1) }

T(2,2) = T(1,2) { Ignore T(1,-1) }

T(2,2) = 3

Finding T(2,3)-

We have,

- i = 2
- j = 3
- (value)i = (value)2 = 4
- (weight)i = (weight)2 = 3

Substituting the values, we get-

T(2,3) = max { T(2-1 , 3) , 4 + T(2-1 , 3-3) }

T(2,3) = max { T(1,3) , 4 + T(1,0) }

T(2,3) = max { 3 , 4+0 }

T(2,3) = 4

Finding T(2,4)-

We have,

- i = 2
- j = 4
- (value)i = (value)2 = 4
- (weight)i = (weight)2 = 3

Substituting the values, we get-

T(2,4) = max { T(2-1 , 4) , 4 + T(2-1 , 4-3) }

T(2,4) = max { T(1,4) , 4 + T(1,1) }

T(2,4) = max { 3 , 4+0 }

T(2,4) = 4

Finding T(2,5)-

We have,

- i = 2
- j = 5

- (value)i = (value)2 = 4
- (weight)i = (weight)2 = 3

Substituting the values, we get-

T(2,5) = max { T(2-1 , 5) , 4 + T(2-1 , 5-3) }

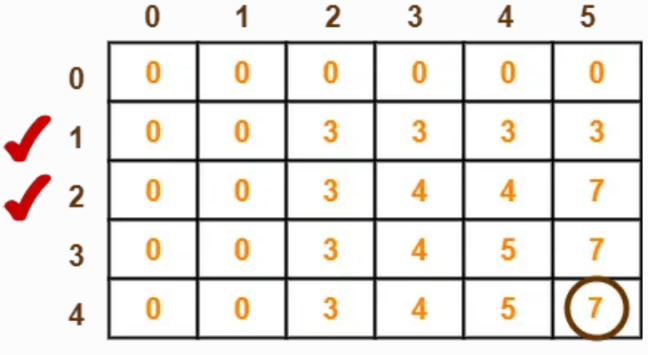
T(2,5) = max { T(1,5) , 4 + T(1,2) }

T(2,5) = max { 3 , 4+3 }

T(2,5) = 7

Similarly, compute all the entries.

After all the entries are computed and filled in the table, we get the following table-

- 

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

T-Table

- The last entry represents the maximum possible value that can be put into the knapsack.
- So, maximum possible value that can be put into the knapsack = 7.

Identifying Items To Be Put Into Knapsack-

Following Step-04,

- We mark the rows labelled “1” and “2”.
- Thus, items that must be put into the knapsack to obtain the maximum value 7 are-

Item-1 and Item-2

-

To gain better understanding about 0/1 Knapsack Problem,

Watch this Video Lecture

Next Article- Travelling Salesman Problem

Get more notes and other study material of **Design and Analysis of Algorithms**.

Watch video lectures by visiting our YouTube channel **LearnVidFun**.

Summary

-

	0	1	2	3	4	5
0	0	0	0	0	0	0
✓ 1	0	0	3	3	3	3
✓ 2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	(7)

T-Table

0/1 Knapsack Problem | Dynamic Programming | Example

Description

0/1 Knapsack Problem is a variant of Knapsack Problem that does not allow to fill the knapsack with fractional items. 0/1 Knapsack Problem solved using Dynamic Programming. 0/1 Knapsack Problem Example & Algorithm.

Fractional Knapsack Problem | Greedy Method | Example

Design & Analysis of Algorithms

Knapsack Problem-

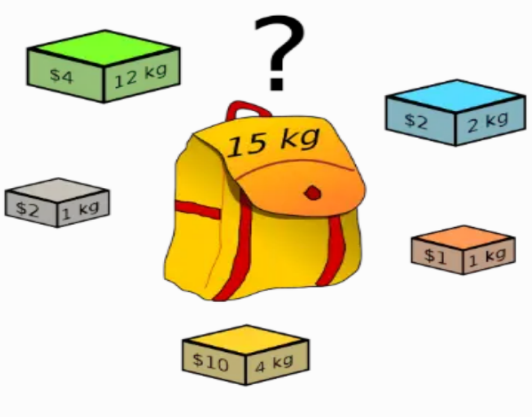
You are given the following-

- A knapsack (kind of shoulder bag) with limited weight capacity.
- Few items each having some weight and value.

The problem states-

Which items should be placed into the knapsack such that-

- The value or profit obtained by putting the items into the knapsack is maximum.
- And the weight limit of the knapsack does not exceed.

- 

Knapsack Problem

Knapsack Problem Variants-

Knapsack problem has the following two variants-

- Fractional Knapsack Problem
- 0/1 Knapsack Problem

Fractional Knapsack Problem-

- In Fractional Knapsack Problem,
- As the name suggests, items are divisible here.
- We can even put the fraction of any item into the knapsack if taking the complete item is not possible.
- It is solved using Greedy Method.

Fractional Knapsack Problem Using Greedy Method-

Fractional knapsack problem is solved using greedy method in the following steps-

Step-01:

For each item, compute its value / weight ratio.

Step-02:

- Arrange all the items in decreasing order of their value / weight ratio.

Step-03:

Start putting the items into the knapsack beginning from the item with the highest ratio.
Put as many items as you can into the knapsack.

Time Complexity-

- The main time taking step is the sorting of all items in decreasing order of their value / weight ratio.
- If the items are already arranged in the required order, then while loop takes O(n) time.
- The average time complexity of **Quick Sort** is O(nlogn).
- Therefore, total time taken including the sort is O(nlogn).

PRACTICE PROBLEM BASED ON FRACTIONAL KNAPSACK PROBLEM-

Problem-

For the given set of items and knapsack capacity = 60 kg, find the optimal solution for the fractional knapsack problem making use of greedy approach.

Item**Weight****Value**153021040315454227752590

OR
Find the optimal solution for the fractional knapsack problem making use of greedy approach. Consider-

n = 5

w = 60 kg

(w1, w2, w3, w4, w5) = (5, 10, 15, 22, 25)

(b1, b2, b3, b4, b5) = (30, 40, 45, 77, 90)

OR
A thief enters a house for robbing it. He can carry a maximal weight of 60 kg into his bag. There are 5 items in the house with the following weights and values. What items should thief take if he can even take the fraction of any item with him?

Solution-

Step-01:

Compute the value / weight ratio for each item-

Items	Weight	Value	Ratio
1	5	30	6
2	10	40	4
3	15	45	3
4	22	77	3.5
5	25	90	3.6

Step-02:

Sort all the items in decreasing order of their value / weight ratio-

- I1 I2 I5 I4 I3

(6) (4) (3.6) (3.5) (3)

Step-03:

Start filling the knapsack by putting the items into it one by one.

Knapsack WeightItems in KnapsackCost60Ø055I13045I1, I27020I1, I2, I5160

-

Now,

- Knapsack weight left to be filled is 20 kg but item-4 has a weight of 22 kg.
- Since in fractional knapsack problem, even the fraction of any item can be taken.
- So, knapsack will contain the following items-

< I1 , I2 , I5 , (20/22) I4 >

Total cost of the knapsack

= 160 + (20/27) x 77

= 160 + 70

= 230 units

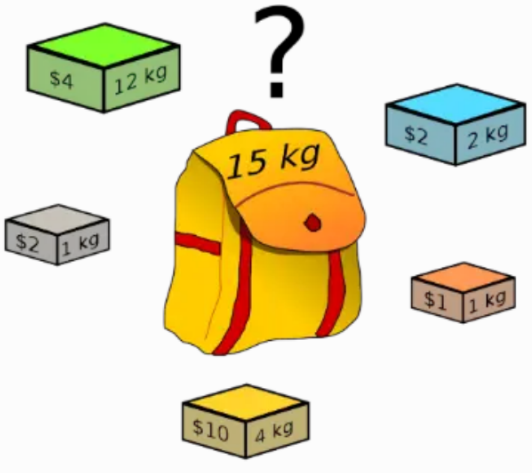
Important Note-

Had the problem been a 0/1 knapsack problem, knapsack would contain the following items-

< I1 , I2 , I5 >

The knapsack's total cost would be 160 units.

Summary



Knapsack Problem

Fractional Knapsack Problem | Greedy Method | Example

Description

Fractional Knapsack Problem is a variant of Knapsack Problem that allows to fill the knapsack with fractional items. Fractional Knapsack Problem solved using Greedy Method. Fractional Knapsack Problem Example & Algorithm.