

1. Find the volume,  $\mathcal{V}$ , of a ball of radius  $R$  in four dimensions. Hint #1: In general, in  $n$  dimensions (for any positive integer  $n$ ), we of course trivially have that  $\mathcal{V} = \int_{\mathcal{B}} dV$ , if  $\mathcal{B}$  is the ball-shaped region whose volume we are seeking and  $dV$  is the  $n$ -dimensional volume element. Hint #2: For any fixed  $n$ , then, we obtain  $\mathcal{V}$  by first explicitly writing  $dV$  in a choice of coordinates, then secondly knowing the bounds of  $\mathcal{B}$  in our choice of coordinates, and then thirdly by actually performing the  $n$ -dimensional integration of  $dV$  over  $\mathcal{B}$ . Hint #3: First do the  $n = 2$  case (where volume is also called “area” and a ball is also called a “disk”), then do the  $n = 3$  case. (The  $n = 1$  case works as well, but is less instructive.) Can you find the answer for all  $n$ ?
2. Find explicit analytical expressions for the inverse hyperbolic trigonometric functions  $\cosh^{-1}(x)$ ,  $\sinh^{-1}(x)$ , and  $\tanh^{-1}(x)$ . That is, invert  $y = \cosh(x) := \frac{1}{2}(e^x + e^{-x})$  to solve for  $y$ , and then do likewise for  $y = \sinh(x) := \frac{1}{2}(e^x - e^{-x})$  and  $y = \tanh(x) := \sinh(x)/\cosh(x)$  (or use the previous two answers). Caution: pay attention to domains of definition.