

1. Find the volume, \mathcal{V} , of a ball of radius R in four dimensions. Hint #1: In general, in n dimensions (for any positive integer n), we of course trivially have that $\mathcal{V} = \int_{\mathcal{B}} dV$, if \mathcal{B} is the ball-shaped region whose volume we are seeking and dV is the n -dimensional volume element. Hint #2: For any fixed n , then, we obtain \mathcal{V} by first explicitly writing dV in a choice of coordinates, then secondly knowing the bounds of \mathcal{B} in our choice of coordinates, and then thirdly by actually performing the n -dimensional integration of dV over \mathcal{B} . Hint #3: First do the $n = 2$ case (where volume is also called “area” and a ball is also called a “disk”), then do the $n = 3$ case. (The $n = 1$ case works as well, but is less instructive.) Can you find the answer for all n ?
2. Find explicit analytical expressions for the inverse hyperbolic trigonometric functions $\cosh^{-1}(x)$, $\sinh^{-1}(x)$, and $\tanh^{-1}(x)$. That is, invert $y = \cosh(x) := \frac{1}{2}(e^x + e^{-x})$ to solve for y , and then do likewise for $y = \sinh(x) := \frac{1}{2}(e^x - e^{-x})$ and $y = \tanh(x) := \cosh(x)/\sinh(x)$ (or use the previous two answers). Caution: pay attention to domains of definition.