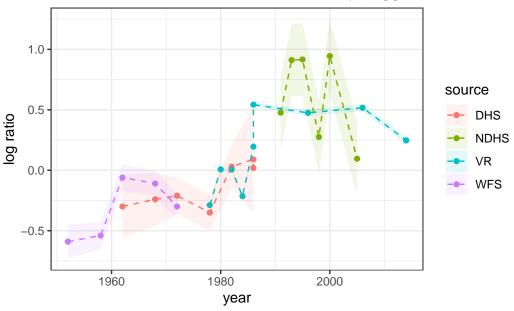
### Week 10: Temporal data

24/03/23

### Child mortality in Sri Lanka

In this lab you will be fitting a couple of different models to the data about child mortality in Sri Lanka, which was used in the lecture. Here's the data and the plot from the lecture:





### Fitting a linear model

Let's firstly fit a linear model in time to these data. Here's the code to do this:

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).

Chain 1: Gradient evaluation took 5.1e-05 seconds

Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.51 seconds.
```

```
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 1: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 1: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 1: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 1: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 1:
Chain 1: Elapsed Time: 0.066 seconds (Warm-up)
Chain 1:
                        0.043 seconds (Sampling)
Chain 1:
                        0.109 seconds (Total)
Chain 1:
SAMPLING FOR MODEL 'anon model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 8e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.08 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 2: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 2: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 2: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 2: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 2:
Chain 2: Elapsed Time: 0.055 seconds (Warm-up)
Chain 2:
                        0.046 seconds (Sampling)
```

```
Chain 2:
                        0.101 seconds (Total)
Chain 2:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 4e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.04 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 3: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 3: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 3: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 3: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 3:
Chain 3: Elapsed Time: 0.061 seconds (Warm-up)
Chain 3:
                        0.054 seconds (Sampling)
Chain 3:
                        0.115 seconds (Total)
Chain 3:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 7e-06 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 4: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 4: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 4: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 4: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 4: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 4: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
```

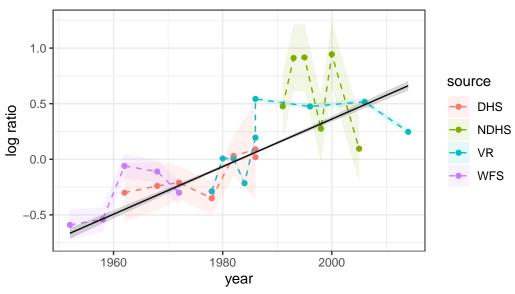
```
Chain 4: Iteration: 1400 / 2000 [ 70%] (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%] (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%] (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%] (Sampling)
Chain 4:
Chain 4: Elapsed Time: 0.051 seconds (Warm-up)
Chain 4: 0.045 seconds (Sampling)
Chain 4: 0.096 seconds (Total)
Chain 4:
```

Extract the results:

```
res <- mod %>%
  gather_draws(mu[t]) %>%
  median_qi() %>%
  mutate(year = years[t])
```

Plot the results:

### Ratio of neonatal to other child mortality (logged), Sri Lanka Linear fit shown in black



### Question 1

Project the linear model above out to 2023 by adding a generated quantities block in Stan (do the projections based on the expected value  $\mu$ ). Plot the resulting projections on a graph similar to that above.

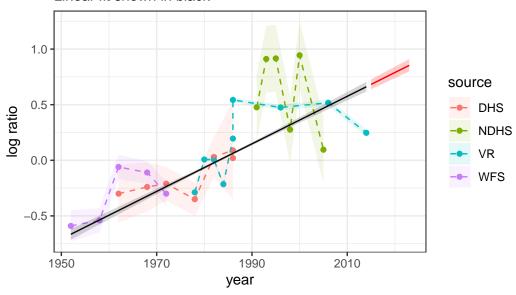
SAMPLING FOR MODEL 'anon\_model' NOW (CHAIN 1).

```
Chain 1:
Chain 1: Gradient evaluation took 3.8e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.38 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration: 1 / 2000 [ 0%]
                                         (Warmup)
Chain 1: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 1: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 1: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 1: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 1:
Chain 1: Elapsed Time: 0.062 seconds (Warm-up)
Chain 1:
                        0.057 seconds (Sampling)
                        0.119 seconds (Total)
Chain 1:
Chain 1:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 8e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.08 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration: 1 / 2000 [ 0%]
                                         (Warmup)
Chain 2: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 2: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 2: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 2: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
```

```
Chain 2:
Chain 2: Elapsed Time: 0.062 seconds (Warm-up)
Chain 2:
                        0.051 seconds (Sampling)
Chain 2:
                        0.113 seconds (Total)
Chain 2:
SAMPLING FOR MODEL 'anon model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 1.5e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.15 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 3: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 3: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 3: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 3: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 3:
Chain 3:
         Elapsed Time: 0.072 seconds (Warm-up)
Chain 3:
                        0.062 seconds (Sampling)
Chain 3:
                        0.134 seconds (Total)
Chain 3:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 1e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.1 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 4: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 4: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 4: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 4: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
```

```
Chain 4: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 4: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 4: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 4:
Chain 4: Elapsed Time: 0.06 seconds (Warm-up)
                        0.051 seconds (Sampling)
Chain 4:
Chain 4:
                        0.111 seconds (Total)
Chain 4:
  res1a <- mod2 %>%
    gather_draws(mu[t]) %>%
    median_qi() %>%
    mutate(year = years[t])
  res1b <- mod2 %>%
    gather_draws(mu_p[p]) %>%
    median_qi() %>%
    mutate(year = years[nyears]+p)
  ggplot(lka, aes(year, logit_ratio)) +
    geom_point(aes( color = source)) +
    geom_line(aes( color = source), lty = 2) +
    geom_ribbon(aes(ymin = logit_ratio - se,
                    ymax = logit_ratio + se,
                    fill = source), alpha = 0.1) +
    theme_bw()+
    geom_line(data = res1a, aes(year, .value)) +
    geom_ribbon(data = res1a, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2)+
    geom_line(data = res1b, aes(year, .value), col = "red") +
    geom_ribbon(data = res1b, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2, fi
    theme_bw()+
    labs(title = "Ratio of neonatal to other child mortality (logged), Sri Lanka",
         y = "log ratio", subtitle = "Linear fit shown in black")
```

# Ratio of neonatal to other child mortality (logged), Sri Lanka Linear fit shown in black



### Random walks

#### Question 2

Code up and estimate a first order random walk model to fit to the Sri Lankan data, taking into account measurement error, and project out to 2023.

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).

Chain 1:

Chain 1: Gradient evaluation took 0.000107 seconds

Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 1.07 seconds.

Chain 1: Adjust your expectations accordingly!

Chain 1:

Chain 1:

Chain 1: Iteration: 1 / 2000 [ 0%] (Warmup)

Chain 1: Iteration: 200 / 2000 [ 10%] (Warmup)
```

```
Chain 1: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 1: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 1: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 1:
Chain 1: Elapsed Time: 0.383 seconds (Warm-up)
Chain 1:
                        0.234 seconds (Sampling)
                        0.617 seconds (Total)
Chain 1:
Chain 1:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 2.5e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.25 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 2: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 2: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 2: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 2: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 2:
Chain 2: Elapsed Time: 0.312 seconds (Warm-up)
Chain 2:
                        0.284 seconds (Sampling)
Chain 2:
                        0.596 seconds (Total)
Chain 2:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
```

```
Chain 3: Gradient evaluation took 9e-06 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.09 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 3: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 3: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 3: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 3: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 3:
Chain 3:
          Elapsed Time: 0.313 seconds (Warm-up)
Chain 3:
                        0.262 seconds (Sampling)
                        0.575 seconds (Total)
Chain 3:
Chain 3:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 1e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.1 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 4: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 4: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 4: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 4: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 4: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 4: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 4: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 4:
```

Chain 4: Elapsed Time: 0.321 seconds (Warm-up)
Chain 4: 0.208 seconds (Sampling)
Chain 4: 0.529 seconds (Total)
Chain 4:

mod3

Inference for Stan model: anon\_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

2.5% 25% 50% 75% 97.5% n\_eff Rhat mean se\_mean sd mu[1] -0.66 -0.57 -0.48 -0.31 -0.570.00 0.13 -0.834090 mu[2] -0.56 0.00 0.19 -0.94 -0.69 -0.56 -0.43 -0.17 3234 1 mu[3] -0.55-1.00 -0.69 -0.55 -0.40 -0.11 2918 0.00 0.22 1 mu[4] -0.540.00 0.23 -1.00 -0.69 -0.53 -0.39 -0.10 2875 1 mu[5] 0.22 -0.98 2809 -0.520.00 -0.67 -0.52 -0.38 -0.09 1 mu[6] -0.51 0.00 0.18 -0.87 -0.63 -0.51 -0.39 -0.17 3368 1 mu[7] -0.500.00 0.10 -0.70-0.57 -0.50 -0.43 -0.30 4203 1 mu[8] -0.400.00 0.17 -0.75-0.51 -0.40 -0.29 -0.07 3913 1 -0.69 mu[9] -0.310.00 0.19 -0.43 -0.31 -0.19 0.06 3787 1 mu[10] -0.220.00 0.17 -0.56-0.33 -0.22 -0.11 0.11 4266 1 0.09 -0.32 mu[11] -0.13 0.00 -0.20 -0.13 -0.07 0.05 3825 1 mu[12] -0.14 0.00 0.18 -0.49-0.25 -0.14 -0.02 0.21 3255 1 mu[13] -0.14 0.00 0.22 -0.56 -0.28 -0.13 0.01 0.29 2962 1 0.22 mu[14] -0.140.00 -0.60-0.28 -0.140.01 0.30 2906 1 mu[15] -0.140.00 0.21 -0.54-0.28 -0.14 0.00 0.29 2870 1 -0.14-0.480.22 mu[16] 0.00 0.18 -0.25 -0.14 -0.033559 1 mu[17] -0.140.00 0.08 -0.31 -0.20 -0.14 -0.08 0.02 4048 1 mu[18] -0.17-0.49-0.28 -0.17 -0.07 4115 0.00 0.16 0.15 1 mu[19] -0.20 0.00 0.19 -0.57-0.32 -0.20 -0.08 0.17 3915 1 mu[20] -0.23 0.00 0.17 -0.56-0.34 -0.23 -0.13 0.10 4711 1 mu[21] -0.43-0.32 -0.26 -0.20 -0.09 -0.260.00 0.09 5448 1 mu [22] -0.270.00 0.17 -0.61-0.38 -0.26 -0.16 0.09 3730 1 mu[23] -0.270.00 0.21 -0.70-0.40 -0.27 -0.140.13 3334 1 mu [24] -0.280.00 0.22 -0.72-0.41 -0.27 -0.140.14 3349 1 mu [25] -0.280.00 0.20 -0.69-0.41 -0.28 -0.15 0.12 3546 1 -0.60 mu[26] -0.28 0.00 0.16 -0.38 -0.28 -0.18 0.03 4366 1 mu[27] -0.290.00 0.01 -0.31-0.30 -0.29 -0.28 -0.26 5851 1 mu[28] -0.14 -0.37 -0.22 -0.14 -0.06 0.00 0.12 0.10 5958 1 mu[29] -0.020.04 0.01 0.00 0.02 0.00 0.01 0.02 6682 1 mu[30] 0.01 0.00 0.12 -0.23 -0.07 0.01 0.08 0.25 5771 1

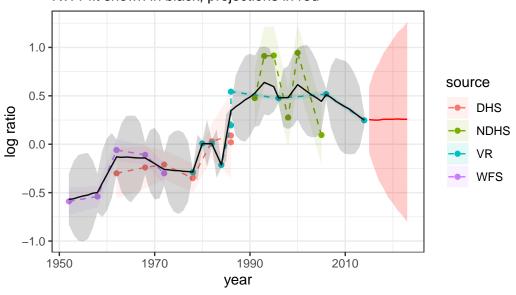
mu[31]	0.01	0.00	0.02	-0.02	-0.01	0.01	0.02	0.04	6399	1
mu[32]	-0.10	0.00	0.12	-0.35	-0.18	-0.10	-0.02	0.15	6307	1
mu[33]	-0.21	0.00	0.02	-0.24	-0.22	-0.21	-0.20	-0.18	5047	1
mu[34]	0.07	0.00	0.12	-0.18	-0.01	0.07	0.15	0.31	6804	1
mu[35]	0.34	0.00	0.01	0.32	0.34	0.34	0.35	0.37	5966	1
mu[36]	0.39	0.00	0.16	0.09	0.28	0.39	0.49	0.71	3663	1
mu[37]	0.42	0.00	0.20	0.04	0.29	0.42	0.56	0.82	3122	1
mu[38]	0.46	0.00	0.22	0.02	0.32	0.46	0.60	0.91	2820	1
mu[39]	0.49	0.00	0.22	0.09	0.35	0.49	0.64	0.93	2877	1
mu[40]	0.53	0.00	0.19	0.18	0.40	0.53	0.66	0.92	3008	1
mu[41]	0.59	0.00	0.20	0.19	0.46	0.58	0.72	1.00	2712	1
mu[42]	0.64	0.00	0.18	0.31	0.52	0.64	0.76	1.01	2682	1
mu[43]	0.62	0.00	0.18	0.30	0.50	0.62	0.74	0.99	2910	1
mu[44]	0.60	0.00	0.14	0.34	0.50	0.59	0.69	0.88	3301	1
mu[45]	0.48	0.00	0.02	0.43	0.46	0.48	0.49	0.53	5142	1
mu[46]	0.48	0.00	0.15	0.18	0.39	0.48	0.58	0.78	4832	1
mu[47]	0.48	0.00	0.16	0.16	0.38	0.48	0.59	0.81	3628	1
mu[48]	0.55	0.00	0.19	0.18	0.43	0.55	0.68	0.93	3173	1
mu[49]	0.62	0.00	0.19	0.26	0.49	0.61	0.75	1.00	2649	1
mu[50]	0.58	0.00	0.22	0.16	0.44	0.58	0.72	1.02	2598	1
mu[51]	0.55	0.00	0.24	0.09	0.39	0.54	0.70	1.03	2890	1
mu[52]	0.51	0.00	0.22	0.07	0.36	0.51	0.65	0.96	2906	1
mu[53]	0.48	0.00	0.20	0.08	0.34	0.48	0.61	0.86	3205	1
mu[54]	0.44	0.00	0.14	0.15	0.34	0.44	0.54	0.71	3753	1
mu[55]	0.51	0.00	0.03	0.45	0.49	0.51	0.53	0.57	5898	1
mu[56]	0.48	0.00	0.16	0.17	0.38	0.48	0.58	0.80	3805	1
mu[57]	0.45	0.00	0.21	0.03	0.32	0.44	0.58	0.89	3185	1
mu[58]	0.41	0.00	0.23	-0.07	0.27	0.41	0.57	0.87	2665	1
mu[59]	0.38	0.00	0.24	-0.10	0.23	0.39	0.54	0.86	2938	1
mu[60]	0.35	0.00	0.23	-0.10	0.20	0.36	0.50	0.81	2733	1
mu[61]	0.32	0.00	0.21	-0.10	0.19	0.32	0.45	0.72	3063	1
mu[62]	0.28	0.00	0.16	-0.03	0.18	0.28	0.39	0.60	3716	1
mu[63]	0.25	0.00	0.03	0.18	0.23	0.25	0.27	0.32	5483	1
sigma	0.17	0.00	0.04	0.11	0.14	0.16	0.19	0.26	522	1
mu_p[1]	0.25	0.00	0.17	-0.09	0.14	0.25	0.36	0.59	3668	1
mu_p[2]	0.25	0.00	0.25	-0.24	0.09	0.25	0.41	0.75	3510	1
mu_p[3]	0.25	0.00	0.30	-0.37	0.06	0.25	0.44	0.83	3902	1
mu_p[4]	0.25	0.01	0.35	-0.45	0.03	0.26	0.48	0.94	4014	1
mu_p[5]	0.25	0.01	0.39	-0.55	0.00	0.26	0.50	1.01	4036	1
mu_p[6]	0.25	0.01	0.42	-0.61	-0.01	0.26	0.53	1.09	4065	1
mu_p[7]	0.25	0.01	0.46	-0.68	-0.04	0.26	0.55	1.16	4180	1
mu_p[8]	0.26	0.01	0.49	-0.74	-0.06	0.26	0.57	1.20	4135	1
mu_p[9]	0.25	0.01	0.52	-0.80	-0.07	0.26	0.58	1.26	4097	1

```
lp_ -6.79 0.58 12.11 -32.48 -14.61 -5.99 1.99 14.34 439 1
```

Samples were drawn using NUTS(diag\_e) at Fri Mar 24 16:15:35 2023. For each parameter, n\_eff is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat=1).

```
res2a <- mod3 %>%
 gather_draws(mu[t]) %>%
 median_qi() %>%
 mutate(year = years[t])
res2b <- mod3 %>%
  gather_draws(mu_p[p]) %>%
 median_qi() %>%
 mutate(year = years[nyears]+p)
ggplot(lka, aes(year, logit_ratio)) +
  geom_point(aes( color = source)) +
  geom_line(aes( color = source), lty = 2) +
  geom_ribbon(aes(ymin = logit_ratio - se,
                  ymax = logit_ratio + se,
                  fill = source), alpha = 0.1) +
  theme_bw()+
  geom_line(data = res2a, aes(year, .value)) +
  geom_ribbon(data = res2a, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2)+
  geom_line(data = res2b, aes(year, .value), col = "red") +
  geom_ribbon(data = res2b, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2, fi
  theme_bw()+
  labs(title = "Ratio of neonatal to other child mortality (logged), Sri Lanka",
       y = "log ratio", subtitle = "RW1 fit shown in black, projections in red")
```

# Ratio of neonatal to other child mortality (logged), Sri Lanka RW1 fit shown in black, projections in red



#### Question 3

Now alter your model above to estimate and project a second-order random walk model (RW2).

```
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 2.8e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.28 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:
                       1 / 2000 [ 0%]
                                        (Warmup)
Chain 1: Iteration: 200 / 2000 [ 10%]
                                        (Warmup)
Chain 1: Iteration: 400 / 2000 [ 20%]
                                        (Warmup)
Chain 1: Iteration: 600 / 2000 [ 30%]
                                        (Warmup)
```

```
Chain 1: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 1:
Chain 1: Elapsed Time: 1.076 seconds (Warm-up)
Chain 1:
                        1.101 seconds (Sampling)
                        2.177 seconds (Total)
Chain 1:
Chain 1:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 2.7e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.27 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 2: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 2: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 2: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 2: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 2:
Chain 2: Elapsed Time: 0.982 seconds (Warm-up)
Chain 2:
                        0.892 seconds (Sampling)
                        1.874 seconds (Total)
Chain 2:
Chain 2:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 2.2e-05 seconds
```

Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.22 seconds.

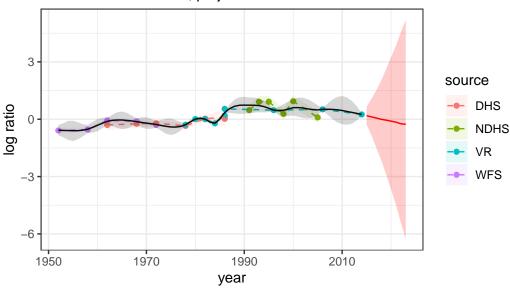
```
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 3: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 3: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 3: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 3: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 3:
Chain 3:
         Elapsed Time: 1.077 seconds (Warm-up)
Chain 3:
                        1.015 seconds (Sampling)
Chain 3:
                        2.092 seconds (Total)
Chain 3:
SAMPLING FOR MODEL 'anon model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 1.5e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.15 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 4: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 4: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 4: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 4: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 4: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 4: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 4: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 4:
Chain 4: Elapsed Time: 1.067 seconds (Warm-up)
Chain 4:
                        1.012 seconds (Sampling)
```

```
Chain 4:
  res3a <- mod4 %>%
    gather_draws(mu[t]) %>%
    median_qi() %>%
    mutate(year = years[t])
  res3b <- mod4 %>%
    gather_draws(mu_p[p]) %>%
    median_qi() %>%
    mutate(year = years[nyears]+p)
  ggplot(lka, aes(year, logit_ratio)) +
    geom_point(aes( color = source)) +
    geom_line(aes( color = source), lty = 2) +
    geom_ribbon(aes(ymin = logit_ratio - se,
                    ymax = logit_ratio + se,
                    fill = source), alpha = 0.1) +
    theme_bw()+
    geom_line(data = res3a, aes(year, .value)) +
    geom_ribbon(data = res3a, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2)+
    geom_line(data = res3b, aes(year, .value), col = "red") +
    geom_ribbon(data = res3b, aes(y = .value, ymin = .lower, ymax = .upper), alpha = 0.2, fi
    theme bw()+
    labs(title = "Ratio of neonatal to other child mortality (logged), Sri Lanka",
         y = "log ratio", subtitle = "RW2 fit shown in black, projections in red")
```

2.079 seconds (Total)

Chain 4:

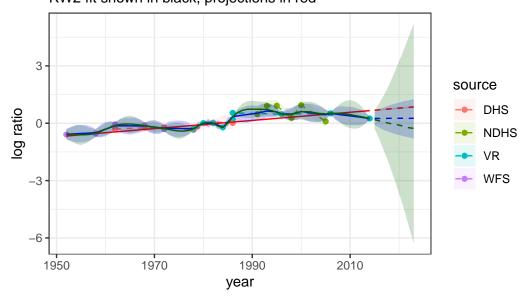
# Ratio of neonatal to other child mortality (logged), Sri Lanka RW2 fit shown in black, projections in red



#### Question 4

Run the first order and second order random walk models, including projections out to 2023. Compare these estimates with the linear fit by plotting everything on the same graph.

# Ratio of neonatal to other child mortality (logged), Sri Lanka RW2 fit shown in black, projections in red



#### **Question 5**

Rerun the RW2 model excluding the VR data. Briefly comment on the differences between the two data situations.

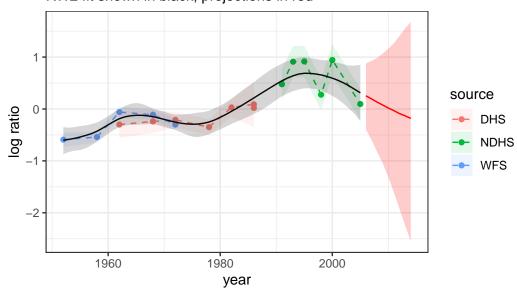
```
lka_noVR <- lka %>%
  filter(source != "VR")
```

```
observed_years <- lka_noVR$year
  years <- min(observed_years):max(observed_years)</pre>
  nyears <- length(years)</pre>
  stan_data <- list(y = lka_noVR$logit_ratio, year_i = observed_years - years[1]+1,
                    T = nyears, years = years, N = length(observed_years),
                    mid_year = mean(years), se = lka_noVR$se,
                    P = 9)
  mod5 <- stan(data = stan_data,</pre>
               file = here("code/models/lab10_3.stan"))
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 1.9e-05 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.19 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration: 1 / 2000 [ 0%]
                                         (Warmup)
Chain 1: Iteration: 200 / 2000 [ 10%]
                                        (Warmup)
Chain 1: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 1: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 1: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 1:
Chain 1: Elapsed Time: 2.299 seconds (Warm-up)
Chain 1:
                       3.34 seconds (Sampling)
                       5.639 seconds (Total)
Chain 1:
Chain 1:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 7e-06 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
```

```
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 2: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 2: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 2: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 2: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 2:
Chain 2:
         Elapsed Time: 2.363 seconds (Warm-up)
Chain 2:
                        1.994 seconds (Sampling)
Chain 2:
                        4.357 seconds (Total)
Chain 2:
SAMPLING FOR MODEL 'anon model' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 1.4e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.14 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 3: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 3: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 3: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 3: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 3: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 3:
Chain 3: Elapsed Time: 2.46 seconds (Warm-up)
Chain 3:
                        2.121 seconds (Sampling)
```

```
Chain 3:
                        4.581 seconds (Total)
Chain 3:
SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 1e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.1 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:
                       1 / 2000 [ 0%]
                                         (Warmup)
Chain 4: Iteration: 200 / 2000 [ 10%]
                                         (Warmup)
Chain 4: Iteration: 400 / 2000 [ 20%]
                                         (Warmup)
Chain 4: Iteration: 600 / 2000 [ 30%]
                                         (Warmup)
Chain 4: Iteration: 800 / 2000 [ 40%]
                                         (Warmup)
Chain 4: Iteration: 1000 / 2000 [ 50%]
                                         (Warmup)
Chain 4: Iteration: 1001 / 2000 [ 50%]
                                         (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]
                                         (Sampling)
Chain 4: Iteration: 1400 / 2000 [ 70%]
                                         (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%]
                                         (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%]
                                         (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%]
                                         (Sampling)
Chain 4:
Chain 4: Elapsed Time: 2.402 seconds (Warm-up)
Chain 4:
                       1.917 seconds (Sampling)
Chain 4:
                       4.319 seconds (Total)
Chain 4:
  res <- mod5 %>%
    gather_draws(mu[t]) %>%
    median_qi() %>%
    mutate(year = years[t])
  res_p <- mod5 %>%
    gather_draws(mu_p[p]) %>%
    median_qi() %>%
    mutate(year = years[nyears]+p)
  ggplot(lka_noVR, aes(year, logit_ratio)) +
    geom_point(aes( color = source)) +
```

# Ratio of neonatal to other child mortality (logged), Sri Lanka RW2 fit shown in black, projections in red



We see that when there is less data the trend loses much of its characteristic linear property. Now it seems as though child mortality might be decreasing, after it increased. This means that our prediction learns this, and the resulting output child mortality is smaller than it was in previous years. This is a good thing, but does not follow the general trend we have seen, so it is a little more unlike the data.

#### Question 6

Briefly comment on which model you think is most appropriate, or an alternative model that would be more appropriate in this context.

Of the models we have tested, the linear model appears to actually do the best in looking at the growth of the logarithm of the ratio. The data grows, and so the linear model is the only one that follows this trend well. An issue with linear models is the unrestricted range, which would not be practical in this example. However, a more specified time series model, like a more complicated AR, MA, might be prudent to try. This is because a first-order or second-order assumption is quite a restrictive assumption that might not be great for the purposes of modeling child mortality. In particular, it's hard to see why Sri Lankan child mortality would be markov of the first order, as there might be more long-term trends that we miss out on.