Quantitative Reasoning: Units

We build some "real-world" math skills by considering unit conversions. Units conversions will come up later in the course in the context of other problems we want to solve.

Units

Is 12 the same as 1? As a mathematical value, no 12 and 1 are not the same value. But if we give these values units, they actually can represent the same thing!
What units can we give to 12 and to 1 so that they are equal?

$$12 \boxed{\text{inches}} = 1 \boxed{\text{foot}}$$

Units in everyday life are often used, even if we don't necessarily think of them as units. For example, you wouldn't say "I went to the grocery store and bought a dozen." A dozen what? You've given how many you've bought (the value), but not how many of what you have bought (the units). Instead you would say "I went to the grocery store and bought a dozen eggs."

Example 1. *A typical bottle of wine holds about 24.5 ounces. How much is this in gallons?*

Explanation. *If we take the original 24.5 ounces in 1 bottle we can set it up as a fraction:*

$$\frac{24.5 \quad ounces}{1 \quad bottle}$$

There are 128 ounces in 1 gallon. This can be set up as the following fraction:

$$\frac{1 \quad gallon}{128 \quad ounces}$$

These two fractions have ounces in common. We can set the fractions up in such a way that the ounces will "cancel out."

$$\frac{24.5 \quad ounces}{1 \quad bottle} \times \frac{1 \quad gallon}{128 \quad ounces} = \frac{24.5 \quad \cancel{ounces}}{1 \quad bottle} \times \frac{1 \quad gallon}{128 \quad \cancel{ounces}} = \frac{0.1914 \quad gallon}{1 \quad bottle}$$

So there are 0.1914 gallons in 1 bottle of wine.

Learning outcomes:
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Exploration 1

- *"How many ounces are in a yard?" Explain why this question does not make sense.*
- *"How many feet are in an acre?" Explain why this question does not make sense.*

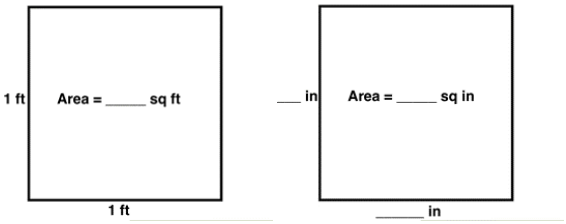
Let's talk a little more about this idea highlighted in the question above (about feet and acre units). It is important to note that feet, square feet, and cubic feet are all different units. They are not measuring the same thing! Feet measures length (one dimension), square feet measure area (two dimensions), and cubic feet measure volume (three dimensions).

Q. *We* ***can*** *convert between* ***cubic feet*** *and which units from previous examples? Why?*

We can convert between feet and inches, or feet and miles. Can we convert between square/cubic feet and square/cubic inches, or square/cubic feet and square/cubic miles? To answer this, we have to ask if these pairs of units are measuring the same property. If the answer is yes, then yes we can convert between them!

- *Do square feet, square inches, and square miles measure the same property?*
- *Do cubic feet, cubic inches, and cubic miles measure the same property?*

In order to convert between units, we need some sort of equivalence. Just like knowing 4 quarts = 1 gallon, or 16 oz = 1 pound, these are equivalences that we can then use to convert from quarts to gallons or vice versa, or ounces to pounds or vice versa. Let's determine the equivalence for square feet and square inches.



Fill in the blanks above (assume that the squares are the exact same size). Since the squares are the exact same size, we can say that 1 *sq ft* = $\boxed{144}$ *sq in. Use this same reasoning to determine the equivalence between square miles and square feet:*

1 *sq mi* = $\boxed{27878400}$ *sq ft*