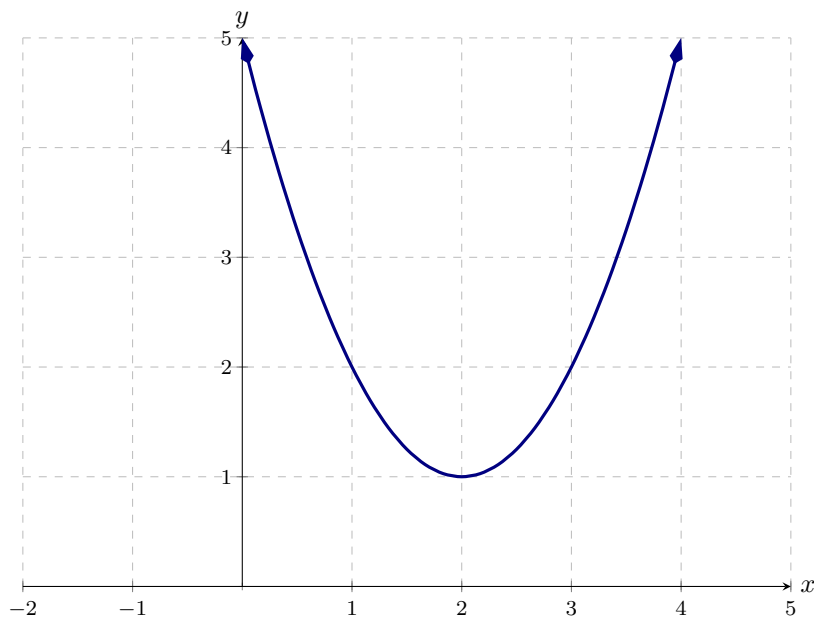


**Exercise 1** The function given by  $f(x) = 3(x - 2)^2 + 1$  (graphed below) is not a one-to-one function on  $(-\infty, \infty)$ . If we restrict the domain, however, it can be made to be one-to-one.



Find a formula for  $f^{-1}(x)$  when  $f$  is restricted to  $(-\infty, 2]$ .

$$f^{-1}(x) = \boxed{2 - \sqrt{\frac{x - 1}{3}}}$$

**Hint:** We're starting with  $y = f(x)$ , so that's:

$$y = \boxed{3(x - 2)^2 + 1}$$

Swap  $x$  and  $y$ .

$$x = \boxed{3(y - 2)^2 + 1}$$

Solving for  $y$  you find two solutions. They are:

$$y = 2 - \sqrt{\frac{x - 1}{3}}$$

$$y = 2 + \sqrt{\frac{x - 1}{3}}$$

The domain of  $f$  was restricted to  $(-\infty, 2]$ , which means we want the range of  $f^{-1}$  to be  $(-\infty, 2]$ . Which of the two solutions you found give outputs which are not greater than 2?

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