Part 1

Zeros of Famous Functions

ZoFF1.tex

Exercise 1 *Which of the following are polynomial functions?*

Select All Correct Answers:

(a) $f(x) = 0$ ✓

(b) $f(x) = -9$ ✓

(c) $f(x) = 3x + 1$ ✓

(d) $f(x) = x^{1/2} - x + 8$

(e) $f(x) = -4x^{-3} + 5x^{-1} + 7 - 18x^2$

(f) $f(x) = (x+1)(x-1) + e^x - e^x$ ✓

(g) $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$

(h) $f(x) = x^7 - 32x^6 - \pi x^3 + 45/84$ ✓

Exercise 2 *Which of the following are rational functions?*

Select All Correct Answers:

(a) $f(x) = 0$ ✓

(b) $f(x) = \dfrac{3x + 1}{x^2 - 4x + 5}$ ✓

(c) $f(x) = e^x$

(d) $f(x) = \dfrac{\sin(x)}{\cos(x)}$

(e) $f(x) = -4x^{-3} + 5x^{-1} + 7 - 18x^2$ ✓

(f) $f(x) = x^{1/2} - x + 8$

(g) $f(x) = \dfrac{\sqrt{x}}{x^3 - x}$

ZoFF2.tex

Let f be the function given by the formula $f(x) = 5x^2 + 2$.

Exercise 3 • *The graph of f is:*

Multiple Choice:

(a) *a straight line.*
(b) *a parabola opening upward.* ✓
(c) *a parabola opening downward.*
(d) *a growing exponential.*
(e) *a decaying exponential.*

- *The domain of f is* $\left(\boxed{-\infty}, \boxed{\infty}\right)$.
- *The range of f is* $\left[\boxed{2}, \boxed{\infty}\right)$.
- *How many zeros does f have?* $\boxed{0}$
- *The value of $f(2)$ is* $\boxed{22}$
- *The formula for $f(2+t)$ is:* $\boxed{5}t^2 + \boxed{20}t + \boxed{22}$
- *The formula for* $\dfrac{f(2+t)-f(2)}{t}$ *is:* $\boxed{5}t + \boxed{20}$

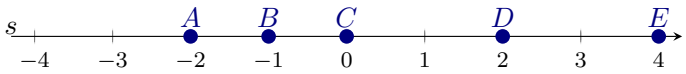
`ZoFF3.tex`

Exercise 4 *An object is moving along a horizontal line. Its position in meters is given by*

$$s(t) = t^2 - 2$$

where t is in seconds.

Consider the points on the line below.

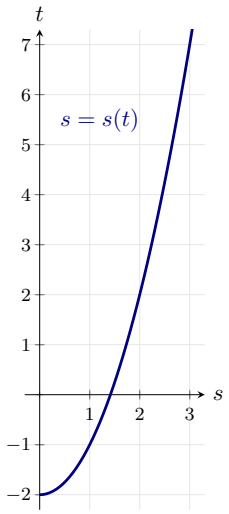


The point that corresponds to the position of the particle at $t = 1$ is $\boxed{B}$.

Exercise 4.1 *The average velocity of the object on the interval* $[1, 3]$. *That is, calculate the average rate of change,* $AV_{[1,3]}$.

$$AV_{[1,3]} = \boxed{4}\, m/s.$$

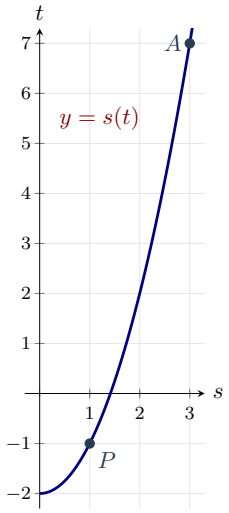
Exercise 4.1.1 *The graph of* $y = s(t)$ *is given below.*



Assume that P *and* A *are points on the graph of* $s(t)$. *Then*

$$P = \left(1, \boxed{-1}\right) \text{ and } A = \left(3, \boxed{7}\right).$$

Exercise 4.1.1.1 *The secant line through* P *and* A *is shown in the figure below.*



The slope of the secant line containing the points P and A is

$$m_{sec} = \boxed{4}.$$

What is the connection between the slope of this secant line and the average velocity of the object over the interval $[1, 3]$?

Multiple Choice:

(a) *m_{sec} is greater than $AV_{[1,3]}$.*

(b) *m_{sec} equals $AV_{[1,3]}$.* ✓

(c) *m_{sec} is less than $AV_{[1,3]}$.*

(d) *There is no connection between m_{sec} and $AV_{[1,3]}$.*

`ZoFF4.tex`

Exercise 5 *The following is a rational function.*

$$f(x) = \frac{x^3 - 2x + 1}{x - 1} - \frac{1}{2}x + 1.$$

How many zeros does this function have? $\boxed{2}$

Exercise 5.1 *List them in order from left to right.*

$x = \boxed{-1/2}$ *and* $x = \boxed{0}$

Hint: *Try rewriting the function as a single fraction.*

Exercise 6 *The following is a rational function.*

$$g(x) = \frac{1}{x + 3} + \frac{1}{x - 3} - \frac{x^2 - 3}{x^2 - 9}.$$

How many zeros does this function have? $\boxed{1}$

Exercise 6.1 *It is at* $x = \boxed{-1}$.

Exercise 6.1.1 *Why is* $x = 3$ *NOT a zero of* g*?*

Multiple Choice:

(a) *Because* $g(3)$ *is a nonzero number.*

(b) *Because* $g(3) = 0$.

(c) *Because* $x = 3$ *is not in the domain of* g. ✓

Hint: *Make sure to check your possible solutions are actually solutions.*

Exercise 7 *The following is a rational function.*

$$h(x) = 1 - \frac{x^2 - 2x + 1}{x^3 + x^2 - 2x}.$$

How many zeros does this function have? $\boxed{0}$

Feedback(*`attempt`*): *What are you left with if you rewrite h as a single fraction and simplify? Are there any values of x making that resulting fraction zero?*

`ZoFF5.tex`

Exercise 8 *The following function involves radicals.*

$$f(x) = 3x - 2 + \sqrt{6 - 9x}.$$

How many zeros does this function have? $\boxed{2}$

Exercise 8.1 *List them in order from left to right.*

$x = \boxed{-1/3}$ *and* $x = \boxed{2/3}$

Exercise 9 *The following function involves radicals.*

$$g(x) = x + 1 - \sqrt{3x + 7}.$$

How many zeros does this function have? $\boxed{1}$

Hint: *Make sure to check your possible solutions are actually solutions.*

Exercise 9.1 *It is at* $x = \boxed{3}$.

Exercise 10 *The following function involves radicals.*

$$h(x) = \sqrt{4-x} + 3 - \sqrt{2x+1}.$$

How many zeros does this function have? $\boxed{1}$

Exercise 10.1 *It is at* $x = \boxed{4}$.

`ZoFF6.tex`

Exercise 11 *The following is an exponential function.*

$$f(x) = e^x - 5$$

How many zeros does this function have? $\boxed{1}$

Exercise 11.1 *It is at* $x = \boxed{\ln(5)}$.

Hint: *How do you bring the exponent of an exponential down? Remember that the natural logarithm is the inverse of the natural exponential function.*

Exercise 12 *The following is an exponential function.*

$$g(x) = 4e^x - 9$$

How many zeros does this function have? $\boxed{1}$

Exercise 12.1 *It is at* $x = \boxed{\ln(9/4)}$.

Exercise 13 *The following is an exponential function.*

$$f(x) = 5^x + 8$$

How many zeros does this function have? $\boxed{0}$

ZoFF7.tex

Exercise 14 *The following is a logarithmic function.*

$$f(u) = \log_3(u) - 3$$

How many zeros does this function have? $\boxed{1}$

Exercise 14.1 *It is at* $u = \boxed{27}$.

Hint: *Remember that* $\log_3$ *is the inverse of the exponential function with base* 3.

Exercise 14.1.1 *Suppose* $g(x) = \log_3(x^2 - 6x) - 3$. *The function* g *will have value zero at an* x*-value if* $x^2 - 6x = \boxed{27}$.

Hint: *Notice that* g *is really just* f *with the* u *replaced by* $x^2 - 6x$.

Exercise 14.1.1.1

How many zeros does the function g *have?* $\boxed{2}$

Exercise 14.1.1.1.1 *List them in order from left to right.*

$x = \boxed{-3}$ *and* $x = \boxed{9}$

Exercise 15 *The following is a logarithmic function.*

$$f(u) = \log_7(u) - 1$$

How many zeros does this function have? $\boxed{1}$

Exercise 15.1 *It is at* $u = \boxed{7}$.

Exercise 15.1.1 *Suppose* $g(x) = \log_7\left(\dfrac{14x}{2x-3}\right) - 1$. *The* g *will have value zero at an* x*-value if* $\dfrac{14x}{2x-3} = \boxed{7}$.

Exercise 15.1.1.1

How many zeros does the function g *have?* $\boxed{0}$

