Finding Zeros of Polynomials

Introduction

This section covers a technique for factoring polynomials like x^3+3x^2+2x+6, which factors as $(x^2+2)(x+3)$. If there are four terms, the technique in this section might help you to factor the polynomial.

Recall that to factor $3x+6$, we factor out the common factor 3:

$$\begin{aligned} 3x+6 &= \overset{\downarrow}{3}x+\overset{\downarrow}{3}\cdot 2 \\ &= 3(x+2) \end{aligned}$$

The "3" here could have been something more abstract, and it still would be valid to factor it out:

$$\begin{aligned} x(a+b)+2(a+b) &= x\overbrace{(a+b)}^{\downarrow}+2\overbrace{(a+b)}^{\downarrow} \\ &= (a+b)(x+2) \end{aligned}$$

In this last example, we factored out the binomial factor $(a+b)$. Factoring out binomials is the essence of this section, so let's see that a few more times:

$$\begin{aligned} x(x+2)+3(x+2) &= x\overbrace{(x+2)}^{\downarrow}+3\overbrace{(x+2)}^{\downarrow} \\ &= (x+2)(x+3) \end{aligned}$$

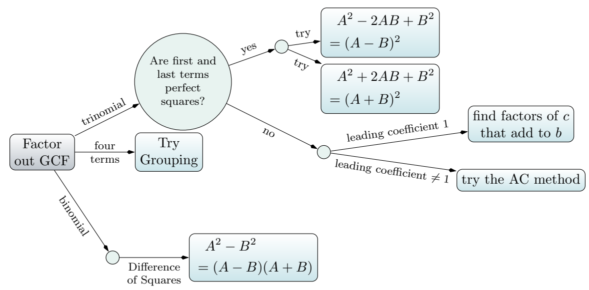
$$\begin{aligned} z^2(2y+5)+3(2y+5) &= z^2\overbrace{(2y+5)}^{\downarrow}+3\overbrace{(2y+5)}^{\downarrow} \\ &= (2y+5)(z^2+3) \end{aligned}$$

And even with an expression like $Q^2(Q-3)+Q-3$, if we re-write it in the right way using a 1 and some parentheses, then it too can be factored:

$$\begin{aligned} Q^2(Q-3)+Q-3 &= Q^2(Q-3)+1(Q-3) \\ &= Q^2\overbrace{(Q-3)}^{\downarrow}+1\overbrace{(Q-3)}^{\downarrow} \\ &= (Q-3)(Q^2+1) \end{aligned}$$

The truth is you are unlikely to come upon an expression like $x(x+2)+3(x+2)$, as in these examples. Why wouldn't someone have multiplied that out already? Or factored it all the way? So far in this section, we have only been looking at a stepping stone to a real factoring technique called **factoring by grouping**.

Learning outcomes:
Author(s):


Factor out GCF
trinomial
Are first and last terms perfect squares?
yes
try
$A^2 - 2AB + B^2 = (A-B)^2$
try
$A^2 + 2AB + B^2 = (A+B)^2$
no
leading coefficient 1
find factors of c that add to b
leading coefficient $\neq 1$
try the AC method
four terms
Try Grouping
binomial
Difference of Squares
$A^2 - B^2 = (A-B)(A+B)$