Part 1

Function Properties

FP1.tex

Exercise 1 *Let f be a function defined as follows.*

$$f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$$

(a) *Compute $f(1)$.*
$f(1) = \boxed{1}$

(b) *Compute $f(-1)$.*
$f(-1) = \boxed{1}$

(c) *The calculations in parts (a) and (b) above show that f is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even but not odd.*
(iii) *odd but not even.*
(iv) *both even and odd.*
(v) *not odd, but f may not be even.* ✓
(vi) *not even, but f may not be odd.*

(d) *Compute $f(2)$.*
$f(2) = \boxed{2}$

(e) *Compute $f(-2)$.*
$f(-2) = \boxed{4}$

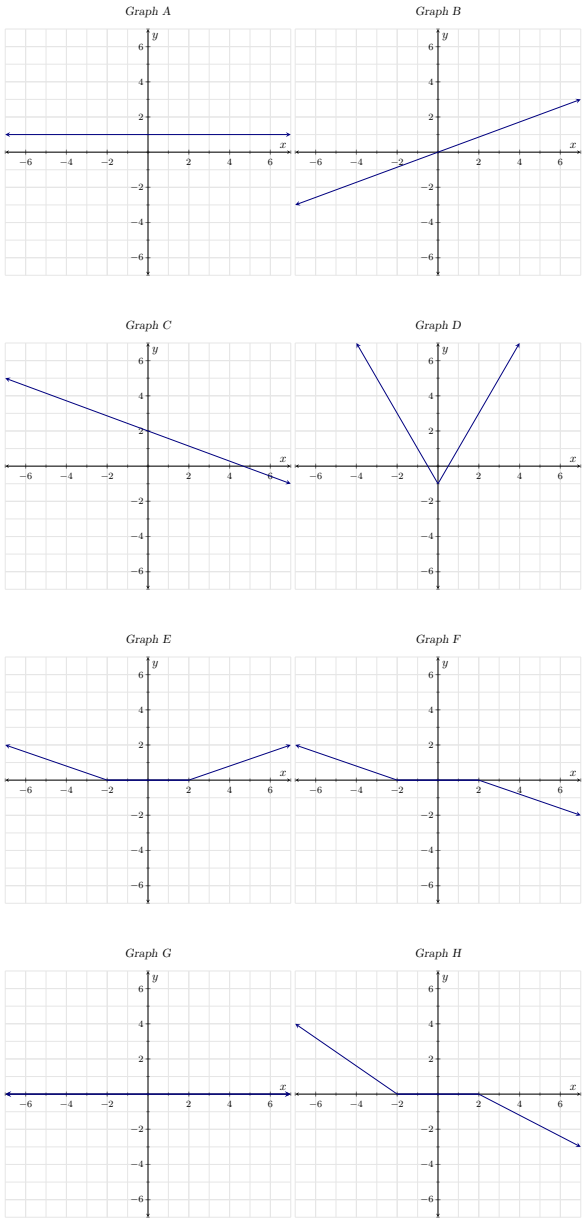
(f) *The calculations in parts (d) and (e) above show that f is*

Multiple Choice:

(i) *neither even nor odd.* ✓
(ii) *even, but not odd.*
(iii) *odd, but not even.*
(iv) *both even and odd.*
(v) *The calculations do not say anything about whether f is even or odd.*

FP2.tex

Exercise 2 *Look at the following graphs of functions. Assume that all the important behavior of the functions is shown on the graphs below.*



(a) *The function corresponding to Graph A is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even, but not odd.* ✓
(iii) *odd, but not even.*
(iv) *both even and odd.*

(b) *The function corresponding to Graph B is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even, but not odd.*
(iii) *odd, but not even.* ✓
(iv) *both even and odd.*

(c) *The function corresponding to Graph C is*

Multiple Choice:

(i) *neither even nor odd.* ✓
(ii) *even, but not odd.*
(iii) *odd, but not even.*
(iv) *both even and odd.*

(d) *The function corresponding to Graph D is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even, but not odd.* ✓
(iii) *odd, but not even.*
(iv) *both even and odd.*

(e) *The function corresponding to Graph E is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even, but not odd.* ✓
(iii) *odd, but not even.*
(iv) *both even and odd.*

(f) *The function corresponding to Graph F is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even, but not odd.*
(iii) *odd, but not even.* ✓
(iv) *both even and odd.*

(g) *The function corresponding to Graph G is*

Multiple Choice:

(i) *neither even nor odd.*
(ii) *even, but not odd.*
(iii) *odd, but not even.*
(iv) *both even and odd.* ✓

(h) *The function corresponding to Graph H is*

Multiple Choice:

(i) *neither even nor odd.* ✓
(ii) *even, but not odd.*
(iii) *odd, but not even.*
(iv) *both even and odd.*

`FP3.tex`

Exercise 3 (a) *The function f defined by $f(x) = 12x$ is*

Multiple Choice:

(i) *even.*
(ii) *odd.* ✓
(iii) *neither even nor odd.*
(iv) *both even and odd.*

(b) *The function f defined by $f(x) = 12x + 2$ is*

Multiple Choice:

(i) *even.*
(ii) *odd.*
(iii) *neither even nor odd.* ✓
(iv) *both even and odd.*

(c) *The function f defined by $f(x) = 12$ is*

Multiple Choice:

(i) *even.* ✓
(ii) *odd.*
(iii) *neither even nor odd.*
(iv) *both even and odd.*

(d) *The function f defined by $f(x) = 5x^2 - 4$ is*

Multiple Choice:

(i) *even.* ✓
(ii) *odd.*
(iii) *neither even nor odd.*
(iv) *both even and odd.*

(e) *The function f defined by $f(x) = 3x^3 - 5x$ is*

Multiple Choice:

(i) *even.*
(ii) *odd.* ✓
(iii) *neither even nor odd.*
(iv) *both even and odd.*

(f) *The function f defined by $f(x) = 0$ is*

Multiple Choice:

(i) *even.*
(ii) *odd.*
(iii) *neither even nor odd.*
(iv) *both even and odd.* ✓

FP4.tex

Exercise 4 *The set of integers is the set* $\{\ldots, -3, -2, -1, 0, 1, 2, 2, \ldots\}$ *consisting of all counting numbers, their negatives, and zero.*

The floor *of* x*, denoted* $\lfloor x \rfloor$ *is defined to be the largest integer* k *with* $k \leq x$*. For example,* $\lfloor 5.2 \rfloor = 5$*,* $\lfloor -99.9 \rfloor = -100$ *and* $\lfloor -3 \rfloor = -3$*.*

(a) *The function* f *defined by* $f(x) = \lfloor x \rfloor$ *is*

Multiple Choice:

(i) *odd.*
(ii) *even.*
(iii) *neither odd nor even.* ✓
(iv) *both odd and even.*

(b) *The function* f *defined by* $f(x) = \lfloor x \rfloor$ *is*

Multiple Choice:

(i) *one-to-one.*
(ii) *not one-to-one.* ✓

FP5.tex

Exercise 5 *When studying trigonometry, you will learn that both* sin *and* cos *are periodic functions with period* 2π*. This means that for all inputs* x*,* $\sin(x + 2\pi) = \sin(x)$ *and* $\cos(x + 2\pi) = \cos(x)$ *.*

(a) *Consider the function* f *defined by* $f(x) = 2\sin(x)$*.* f *is*

Multiple Choice:

(i) *not periodic.*
(ii) *periodic with period* π*.*
(iii) *periodic with period* 2π*.* ✓
(iv) *periodic with period* 3π*.*
(v) *periodic with period* 4π*.*

(b) *Consider the function* g *defined by* $g(x) = \cos(x + 5)$*.* g *is*

Multiple Choice:

(i) *not periodic.*
(ii) *periodic with period* π.
(iii) *periodic with period* 2π. ✓
(iv) *periodic with period* 3π.
(v) *periodic with period* 4π.

(c) *Consider the function* h *defined by* $h(x) = \sin(x) + 2\cos(x)$. h *is*

Multiple Choice:

(i) *not periodic.*
(ii) *periodic with period* π.
(iii) *periodic with period* 2π. ✓
(iv) *periodic with period* 3π.
(v) *periodic with period* 4π.

`FP6.tex`

Exercise 6 *When studying trigonometry, you will learn that* sin *is an odd function and* cos *is an even function. This means that for all inputs* x, $\sin(-x) = -\sin(x)$ *and* $\cos(-x) = \cos(x)$. *Additionally,* sin *is not even, and* cos *is not odd.*

(a) *Consider the function* f *defined by* $f(x) = 7.2\sin(x)$. f *is*

Multiple Choice:

(i) *even.*
(ii) *odd.* ✓
(iii) *neither even nor odd.*

(b) *Consider the function* g *defined by* $g(x) = \cos(x) + 308$. g *is*

Multiple Choice:

(i) *even.* ✓
(ii) *odd.*
(iii) *neither even nor odd.*

(c) *Consider the function* h *defined by* $h(x) = \sin(x) + \cos(x)$. *For reference, here is a graph of* h *on Desmos:*

Desmos link: https://www.desmos.com/calculator/t0r1zihobf

h is

Multiple Choice:

(i) *even.*

(ii) *odd.*

(iii) *neither even nor odd.* ✓

FP7.tex

Exercise 7 *When studying trigonometry, you will learn that both* sin *and* cos *are periodic functions with period* 2π.

Many functions that can be built out of sin *and* cos *are also periodic. In this exercise, we'll use Desmos to explore how the period can change.*

(a) *Consider the function* f *defined by* $f(x) = \sin(3x)$. *For reference, here is a graph of* f *on Desmos:*

Desmos link: https://www.desmos.com/calculator/uc3meehtpv

The period of f *is*

Multiple Choice:

(i) π.

(ii) 2π.

(iii) 3π.

(iv) 6π.

(v) $\frac{\pi}{2}$.

(vi) $\frac{2\pi}{3}$. ✓

(b) *Consider the function* g *defined by* $g(x) = \cos\left(\frac{x}{3}\right)$. *For reference, here is a graph of* g *on Desmos:*

Desmos link: https://www.desmos.com/calculator/364oqkoauu

The period of g *is*

Multiple Choice:

(i) π.
(ii) 2π.
(iii) 3π.
(iv) 6π. ✓
(v) $\frac{\pi}{2}$.
(vi) $\frac{2\pi}{3}$.

(c) *Consider the function h defined by $h(x) = \sin(2x - \pi)$. For reference, here is a graph of h on Desmos:*

Desmos link: https://www.desmos.com/calculator/wha8ccbi93

The period of h is

Multiple Choice:

(i) π. ✓
(ii) 2π.
(iii) 3π.
(iv) 6π.
(v) $\frac{\pi}{2}$.
(vi) $\frac{2\pi}{3}$.

FP8.tex

Exercise 8 *In each part, an invertible function f will be defined. For each function, find its inverse.*

(a) $f(x) = 2x - 6$
$f^{-1}(x) = \boxed{\frac{6+x}{2}}$

(b) $f(x) = 29 - x$
$f^{-1}(x) = \boxed{29 - x}$

(c) $f(x) = \dfrac{x-3}{2} + 3$

$f^{-1}(x) = \boxed{2x - 3}$

(d) $f(x) = \sqrt{5x-1} + 3$

$f^{-1}(x) = \boxed{\dfrac{(x-3)^2 + 1}{5}}$