Relations and Graphs: Famous Functions

We list important relations and functions that will be explored throughout this course and used often in Calculus.

Throughout this course and in Calculus you will study certain relations in depth. These relations are usually functions, and they are the functions that come up often in real world applications. In this section, we will give you a list of some of these functions, including their algebraic equation, their graph, and a table of some of their most important values. You may never have seen some of these functions before and you might not know what their algebraic expressions mean. That is ok. We will learn more about them throughout this course. Remember, though, that a relation or function can be given by a graph. For now, familiarize yourself with these graphs. They will come up in examples throughout the course.

Author(s): Elizabeth Miller

Linear Functions

Of the most important types of functions is a linear function. The graph of a linear function is a line.

Example 1. *A prototypical example of a linear function is*

$$y = x.$$
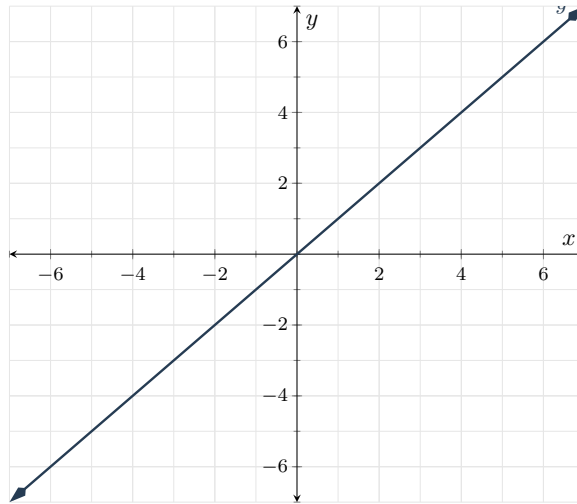
Important Values of $y = x$	
x	y
-2	-2
-1	-1
0	0
1	1
2	2

In general, linear functions can be written as $y = mx + b$ where m and b can be any numbers. You can play with changing the values of m and b on the graph using Desmos and see how that changes the line.

Desmos link: https://www.desmos.com/calculator/japnhapzvn

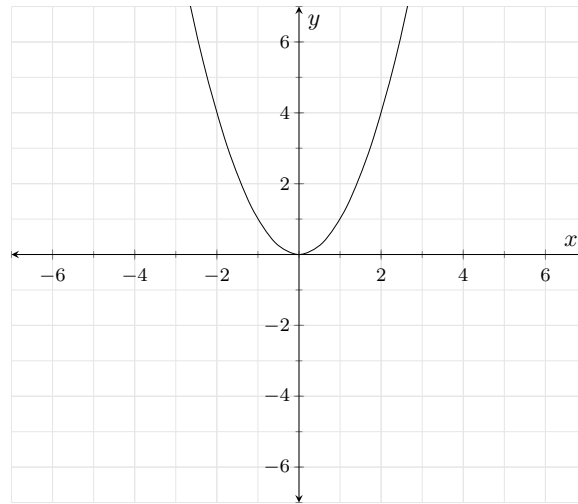## Quadratic Functions

Another very important type of function is the quadratic function. The graph of a quadratic function is a parabola.

Example 2. *A prototypical example of a quadratic function is*

$$y = x^2.$$

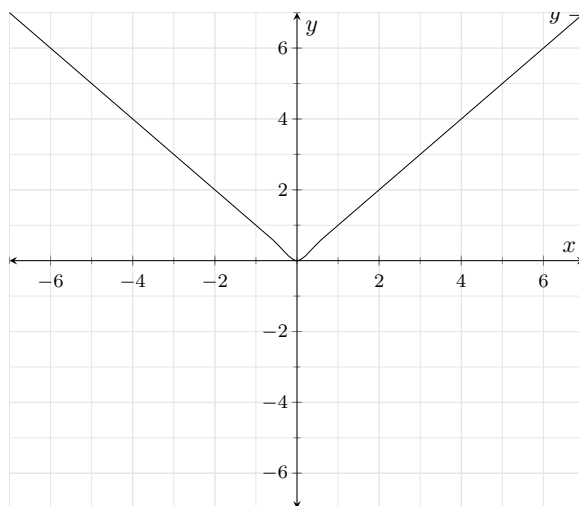| Important Values of $y = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

In general, quadratic functinos can be written as $y = ax^2 + bx + c$ where a, b, and c can be any numbers. You can play with changing the values of a, b, and c on the graph using Desmos and see how that changes the parabola.

Desmos link: https://www.desmos.com/calculator/nmlghfrws9

Absolute Value

Another important type of function is the absolute value function. This is the function that takes all y-values and makes them positive. The absolute value function is written as
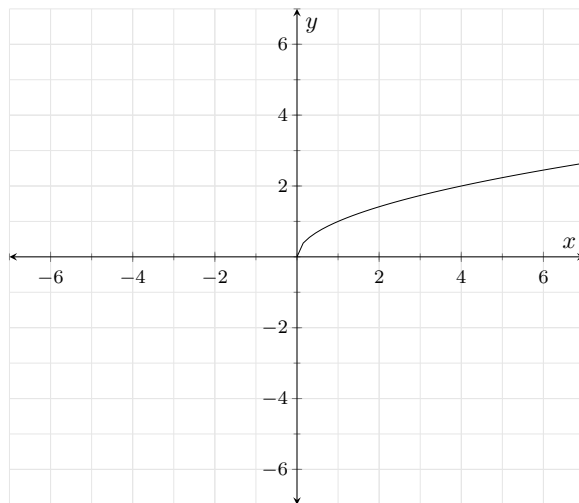
$$y = |x|.$$

Important Values of $y = |x|$	
x	y
-2	2
-1	1
0	0
1	1
2	2

Square Root

Another famous function is the square root function,

$$y = \sqrt{x}.$$

Important Values of $y = \sqrt{x}$	
x	y
0	0
1	1
4	2
9	3
25	5

Exponential

Another famous function is the exponential growth function,

$$y = e^x.$$

Here e is the mathematical constant known as Euler's number. $e \approx 2.71828..$
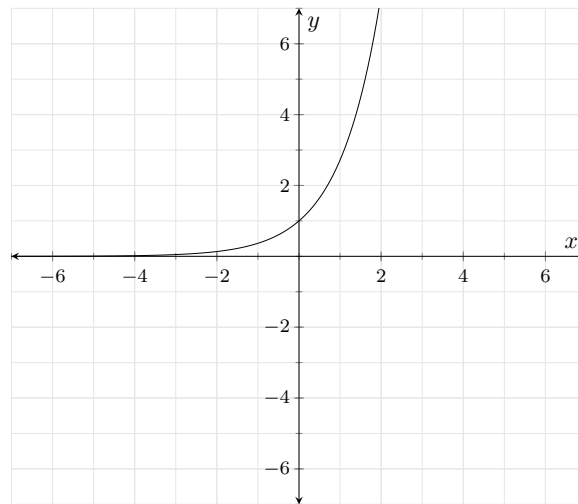
Important Values of $y = e^x$	
x	y
0	1
1	e
-1	$\dfrac{1}{e}$

In general, we can talk about exponential functions of the form $y = b^x$ where b is a positive number not equal to 1. You can play with changing the values of b on the graph using Desmos and see how that changes the graph. Pay particular attention to the difference between $b > 1$ and $0 < b < 1$.
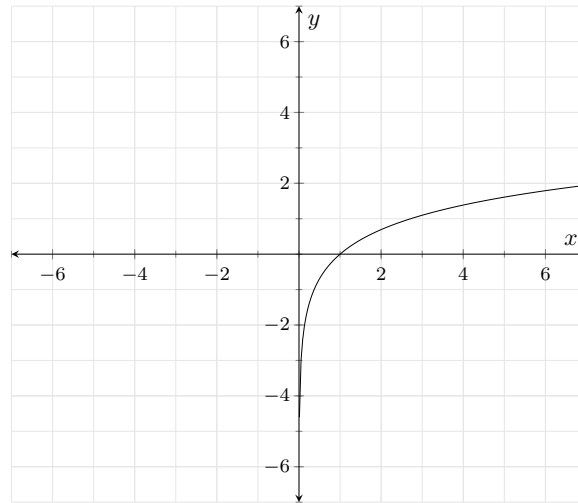
Desmos link: https://www.desmos.com/calculator/qsmvb7tiex

Logarithm

Another group of famous functions are logarithms.

Example 3. *The most famous logarithm function is*

$$y = \ln(x) = \log_e(x).$$

Here e is the mathematical constant known as Euler's number. $e \approx 2.71828$.

Important Values of $y = ln(x)$	
x	y
0	undefined
$\dfrac{1}{e}$	-1
1	0
e	1

You may notice that the table of values for $y = \ln(x)$ and $y = e^x$ are similiar. This is becase these two functions are interconnected. We will explore this more later in the course.

In general, we can talk about logarithmic functions of the form $y = \log_b(x)$ where b is a positive number not equal to 1. You can play with changing the values of b on the graph using Desmos and see how that changes the graph. Pay particular attention to the difference between $b > 1$ and $0 < b < 1$.
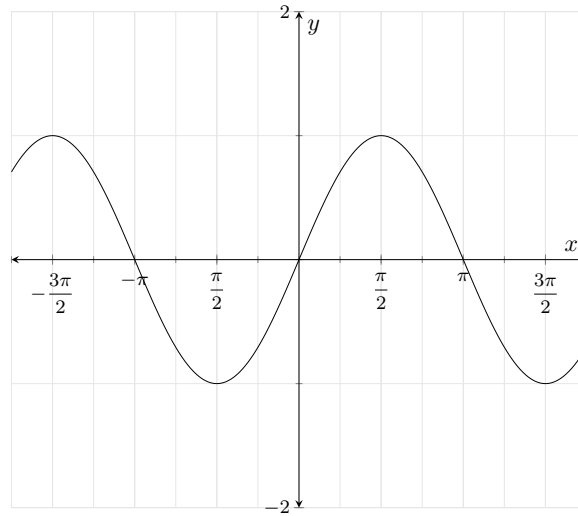
Desmos link: https://www.desmos.com/calculator/lxllnpdi6w

Sine

Another important function is the sine function,

$$y = \sin(x).$$

This function comes from trigonometry. In the table below we will use another mathematical constant, π ("pi" pronounced pie). $\pi \approx 3.14159$.

Important Values of $y = \sin(x)$	
x	y
$-\pi$	0
$-\dfrac{\pi}{2}$	-1
0	0
$\dfrac{\pi}{2}$	1
π	0
$\dfrac{3\pi}{2}$	-1
2π	0

In general, we can consider $y = a\sin(bx)$. You can play with changing the values of a and b on the graph using Desmos and see how that changes the graph.

Desmos link: https://www.desmos.com/calculator/vkxzcfv2aq