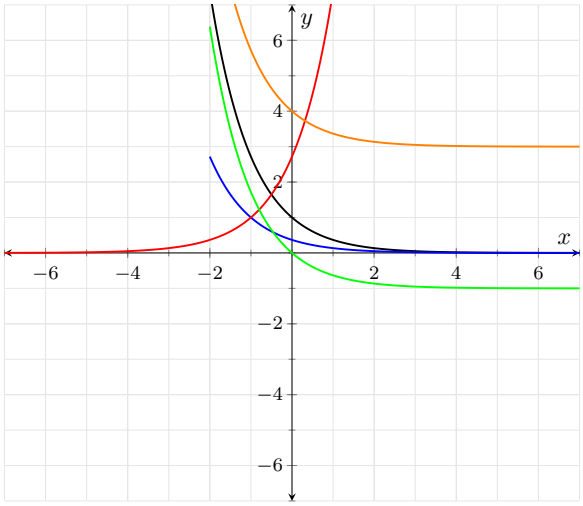
Part 1

Function Transformations

FT1.tex

Exercise 1 *If the graph of $y = f(x) = e^{-x}$ is given in black below, which of the following graphs could be the graph of $y = f(x) + 3$?*



Multiple Choice:

(a) *Blue graph*

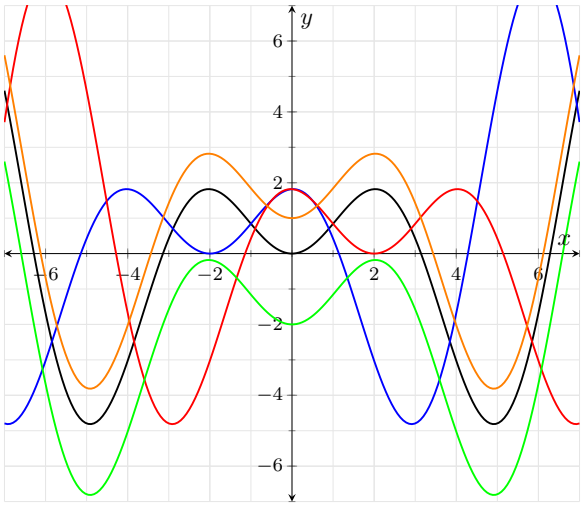
(b) *Red graph*

(c) *Orange graph* ✓

(d) *Green graph*

FT2.tex

Exercise 2 *If the graph of $y = f(x) = x\sin(x)$ is given in black below, which of the following graphs could be the graph of $y = f(x - 2)$?*



Multiple Choice:

(a) *Blue graph*

(b) *Red graph* ✓

(c) *Orange graph*

(d) *Green graph*

`FT3.tex`

Exercise 3 *Write the quadratic function* $f(x) = 4x^2+44x+33$ *in vertex-form, by completing the squares:*

$$f(x) = \boxed{4}\left(x - \boxed{-\frac{11}{2}}\right)^2 + \boxed{-88}.$$

The coordinates of the vertex are:

$$(h,k) = \left(\boxed{-\frac{11}{2}}, \boxed{-88}\right).$$

`FT4.tex`

Exercise 4 *Write the quadratic function $f(x) = -2x^2 + 10x + 13$ in vertex-form, by completing the squares:*

$$f(x) = \boxed{-2}\left(x - \boxed{\frac{5}{2}}\right)^2 + \boxed{\frac{51}{2}}.$$

The coordinates of the vertex are:

$$(h, k) = \left(\boxed{\frac{5}{2}}, \boxed{\frac{51}{2}}\right).$$

`FT5.tex`

Exercise 5 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT6.tex`

Exercise 6 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT7.tex`

Exercise 7 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT8.tex`

Exercise 8 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT9.tex`

Exercise 9 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT10.tex`

Exercise 10 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT11.tex`

Exercise 11 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT12.tex`

Exercise 12 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5 + 4x^4 - 8x^3}{8x^9 - 45x^5 + 9x + 15}\right)$

(f) $\dfrac{4x + 5}{\sqrt{x^6 + 15}}$

`FT13.tex`

Exercise 13 *Let $A = f(r)$ be the area of a circle of radius r.*

a. *Write a formula for $f(r) =$* $\boxed{\pi r^2}$.

b. *Which expression represents the area of a circle whose radius is increased by 5%?*

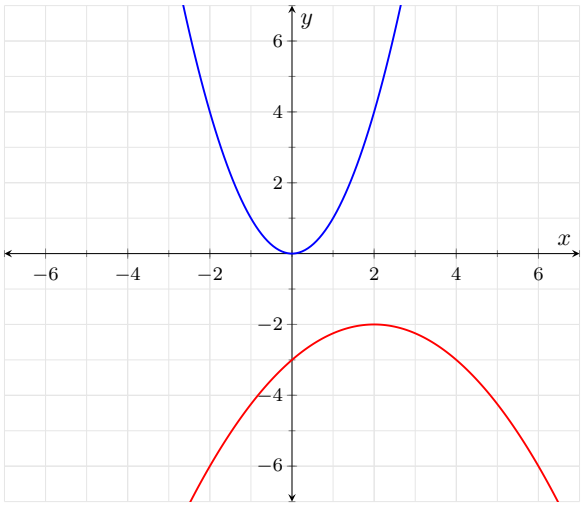
Multiple Choice:

(i) $f(r + 0.05)$
(ii) $0.05f(r)$
(iii) $f(r) + 0.05$
(iv) $f(5 + r)$
(v) $f(1.05r)$ ✓

c. *By what percent does the area increase if the radius is increased by 5%? Round to the nearest 0.01%. Answer:* $\boxed{10.25}$%.

`FT14.tex`

Exercise 14 *Consider the functions $f(x) = x^2$ and $g(x) = -\dfrac{x^2}{4} + x - 3$. Their graphics are below — $y = f(x)$ in blue and $y = g(x)$ in red.*



To produce the graph of g in terms of the graph of f, in which order should you perform the following steps? Enter the numbers 1, 2, 3, and 4, accordingly.

Hint: *Finding the concrete relation $g(x) = af(bx - c) + d$ might be helpful.*

- *Horizontal shift right 1 unit.* $\boxed{3}$
- *Vertical shift up 2 units.* $\boxed{1}$
- *Reflection across the x-axis.* $\boxed{2}$
- *Horizontal stretching by a factor of 2.* $\boxed{4}$.

`FT15.tex`

Exercise 15 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4 - x + 1}{x^2 + 2x + 1}$ ✓

(b) $\dfrac{\sin(x^6 - 4x^3 + 7)}{x^8 - 10x^3 + 10x^2}$

(c) $\dfrac{x^{1000}}{x^{10} - x^9}$ ✓

(d) $x^7 - 34x^6 + 5x^2 + 10$ ✓

(e) $\cos\left(\dfrac{3x^5+4x^4-8x^3}{8x^9-45x^5+9x+15}\right)$

(f) $\dfrac{4x+5}{\sqrt{x^6+15}}$

FT16.tex

Exercise 16 *Select all expressions below which define rational functions:*

Select All Correct Answers:

(a) $\dfrac{x^4-x+1}{x^2+2x+1}$ ✓

(b) $\dfrac{\sin(x^6-4x^3+7)}{x^8-10x^3+10x^2}$

(c) $\dfrac{x^{1000}}{x^{10}-x^9}$ ✓

(d) $x^7-34x^6+5x^2+10$ ✓

(e) $\cos\left(\dfrac{3x^5+4x^4-8x^3}{8x^9-45x^5+9x+15}\right)$

(f) $\dfrac{4x+5}{\sqrt{x^6+15}}$
