

## 1.3.1 EXERCISES

In Exercises 1 - 12, determine whether or not the relation represents  $y$  as a function of  $x$ . Find the domain and range of those relations which are functions.

1.  $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$

2.  $\{(-3, 0), (1, 6), (2, -3), (4, 2), (-5, 6), (4, -9), (6, 2)\}$

3.  $\{(-3, 0), (-7, 6), (5, 5), (6, 4), (4, 9), (3, 0)\}$

4.  $\{(1, 2), (4, 4), (9, 6), (16, 8), (25, 10), (36, 12), \dots\}$

5.  $\{(x, y) \mid x \text{ is an odd integer, and } y \text{ is an even integer}\}$

6.  $\{(x, 1) \mid x \text{ is an irrational number}\}$

7.  $\{(1, 0), (2, 1), (4, 2), (8, 3), (16, 4), (32, 5), \dots\}$

8.  $\{\dots, (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), \dots\}$

9.  $\{(-2, y) \mid -3 < y < 4\}$

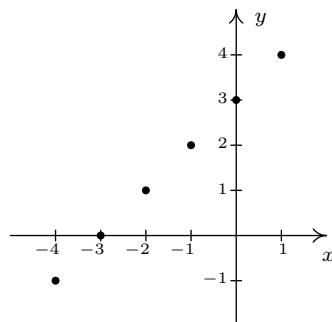
10.  $\{(x, 3) \mid -2 \leq x < 4\}$

11.  $\{(x, x^2) \mid x \text{ is a real number}\}$

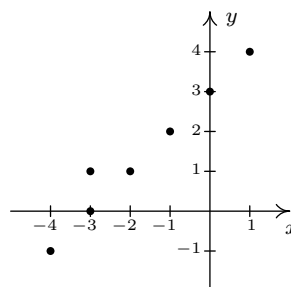
12.  $\{(x^2, x) \mid x \text{ is a real number}\}$

In Exercises 13 - 32, determine whether or not the relation represents  $y$  as a function of  $x$ . Find the domain and range of those relations which are functions.

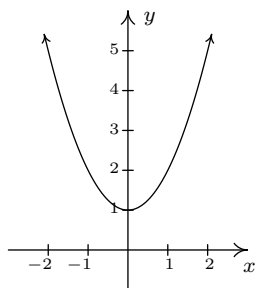
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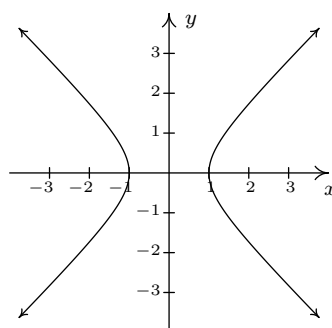
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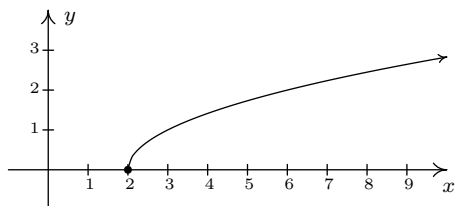
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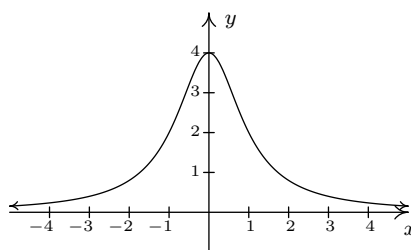
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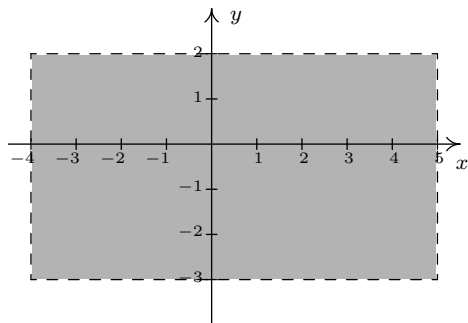
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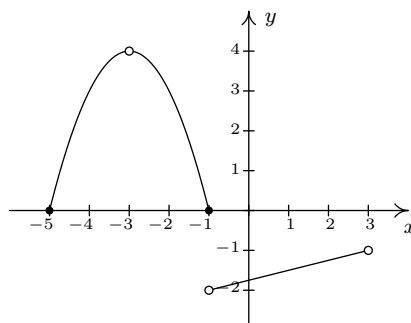
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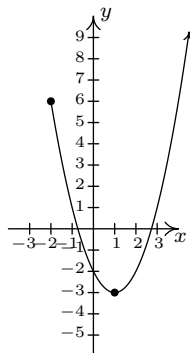
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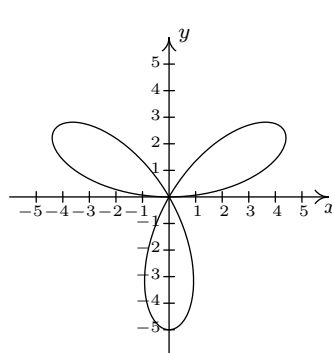
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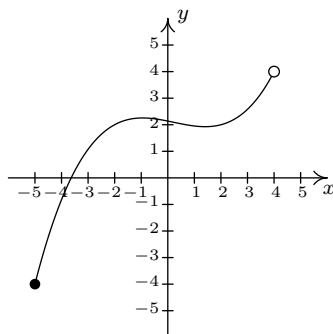
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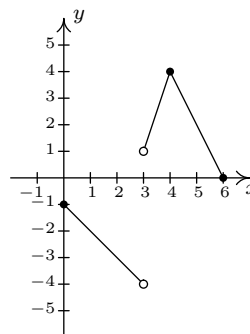
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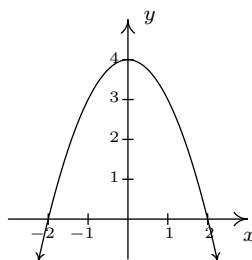
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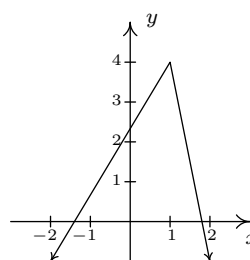
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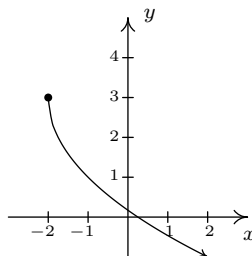
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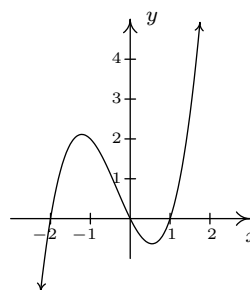
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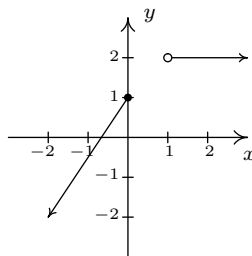
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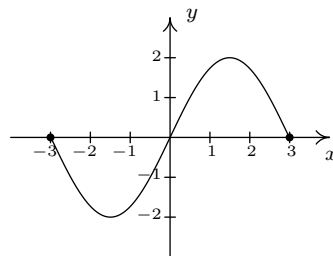
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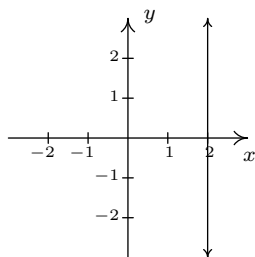
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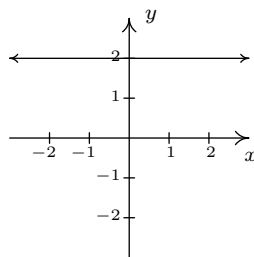
30.



31.



32.



In Exercises 33 - 47, determine whether or not the equation represents  $y$  as a function of  $x$ .

33.  $y = x^3 - x$

34.  $y = \sqrt{x - 2}$

35.  $x^3y = -4$

36.  $x^2 - y^2 = 1$

37.  $y = \frac{x}{x^2 - 9}$

38.  $x = -6$

39.  $x = y^2 + 4$

40.  $y = x^2 + 4$

41.  $x^2 + y^2 = 4$

42.  $y = \sqrt{4 - x^2}$

43.  $x^2 - y^2 = 4$

44.  $x^3 + y^3 = 4$

45.  $2x + 3y = 4$

46.  $2xy = 4$

47.  $x^2 = y^2$

48. Explain why the population  $P$  of Sasquatch in a given area is a function of time  $t$ . What would be the range of this function?

49. Explain why the relation between your classmates and their email addresses may not be a function. What about phone numbers and Social Security Numbers?

The process given in Example 1.3.5 for determining whether an equation of a relation represents  $y$  as a function of  $x$  breaks down if we cannot solve the equation for  $y$  in terms of  $x$ . However, that does not prevent us from proving that an equation fails to represent  $y$  as a function of  $x$ . What we really need is two points with the same  $x$ -coordinate and different  $y$ -coordinates which both satisfy the equation so that the graph of the relation would fail the Vertical Line Test 1.1. Discuss with your classmates how you might find such points for the relations given in Exercises 50 - 53.

50.  $x^3 + y^3 - 3xy = 0$

51.  $x^4 = x^2 + y^2$

52.  $y^2 = x^3 + 3x^2$

53.  $(x^2 + y^2)^2 = x^3 + y^3$

## 1.3.2 ANSWERS

1. Function  
domain =  $\{-3, -2, -1, 0, 1, 2, 3\}$   
range =  $\{0, 1, 4, 9\}$
2. Not a function
3. Function  
domain =  $\{-7, -3, 3, 4, 5, 6\}$   
range =  $\{0, 4, 5, 6, 9\}$
4. Function  
domain =  $\{1, 4, 9, 16, 25, 36, \dots\}$   
 $= \{x \mid x \text{ is a perfect square}\}$   
range =  $\{2, 4, 6, 8, 10, 12, \dots\}$   
 $= \{y \mid y \text{ is a positive even integer}\}$
5. Not a function
6. Function  
domain =  $\{x \mid x \text{ is irrational}\}$   
range =  $\{1\}$
7. Function  
domain =  $\{x \mid x = 2^n \text{ for some whole number } n\}$   
range =  $\{y \mid y \text{ is any whole number}\}$
8. Function  
domain =  $\{x \mid x \text{ is any integer}\}$   
range =  $\{y \mid y = n^2 \text{ for some integer } n\}$
9. Not a function
10. Function  
domain =  $[-2, 4)$ , range =  $\{3\}$
11. Function  
domain =  $(-\infty, \infty)$   
range =  $[0, \infty)$
12. Not a function
13. Function  
domain =  $\{-4, -3, -2, -1, 0, 1\}$   
range =  $\{-1, 0, 1, 2, 3, 4\}$
14. Not a function
15. Function  
domain =  $(-\infty, \infty)$   
range =  $[1, \infty)$
16. Not a function
17. Function  
domain =  $[2, \infty)$   
range =  $[0, \infty)$
18. Function  
domain =  $(-\infty, \infty)$   
range =  $(0, 4]$
19. Not a function
20. Function  
domain =  $[-5, -3) \cup (-3, 3)$   
range =  $(-2, -1) \cup [0, 4)$

21. Function  
domain =  $[-2, \infty)$   
range =  $[-3, \infty)$

23. Function  
domain =  $[-5, 4)$   
range =  $[-4, 4)$

25. Function  
domain =  $(-\infty, \infty)$   
range =  $(-\infty, 4]$

27. Function  
domain =  $[-2, \infty)$   
range =  $(-\infty, 3]$

29. Function  
domain =  $(-\infty, 0] \cup (1, \infty)$   
range =  $(-\infty, 1] \cup \{2\}$

31. Not a function

22. Not a function

24. Function  
domain =  $[0, 3) \cup (3, 6]$   
range =  $(-4, -1] \cup [0, 4]$

26. Function  
domain =  $(-\infty, \infty)$   
range =  $(-\infty, 4]$

28. Function  
domain =  $(-\infty, \infty)$   
range =  $(-\infty, \infty)$

30. Function  
domain =  $[-3, 3]$   
range =  $[-2, 2]$

32. Function  
domain =  $(-\infty, \infty)$   
range =  $\{2\}$

33. Function

34. Function

35. Function

36. Not a function

37. Function

38. Not a function

39. Not a function

40. Function

41. Not a function

42. Function

43. Not a function

44. Function

45. Function

46. Function

47. Not a function