

**Exercise 1** Use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.

**Problem 1.1** For  $f(x) = x^2 - x + 1$  and  $g(x) = 3x - 5$

- $(g \circ f)(x) = \boxed{3x^2 - 3x - 2}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{9x^2 - 33x + 31}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{x^4 - 2x^3 + 2x^2 - x + 1}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$

**Problem 1.2** For  $f(x) = x^2 - 4$  and  $g(x) = |x|$

- $(g \circ f)(x) = \boxed{|x^2 - 4|}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{x^2 - 4}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{x^4 - 8x^2 + 12}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$

**Problem 1.3** For  $f(x) = 3x - 5$  and  $g(x) = \sqrt{x}$

- $(g \circ f)(x) = \boxed{\sqrt{3x - 5}}$  with domain  $\left[\boxed{\frac{5}{3}}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{3\sqrt{x} - 5}$  with domain  $\left[\boxed{0}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{9x - 20}$  with domain  $\left(\boxed{-\infty}, \boxed{\infty}\right)$

**Problem 1.4** For  $f(x) = \frac{x}{2x + 1}$  and  $g(x) = \frac{2x + 1}{x}$

- $(g \circ f)(x) = \boxed{\frac{4x + 1}{x}}$  with domain  $\left(\boxed{-\infty}, \boxed{-\frac{1}{2}}\right) \cup \left(\boxed{-\frac{1}{2}}, \boxed{0}\right) \cup \left(\boxed{0}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{\frac{2x + 1}{5x + 2}}$  with domain  $\left(\boxed{-\infty}, \boxed{-\frac{2}{5}}\right) \cup \left(\boxed{-\frac{2}{5}}, \boxed{0}\right) \cup \left(\boxed{0}, \boxed{\infty}\right)$

- $(f \circ f)(x) = \frac{x}{4x+1}$  with domain  $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

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**Problem 1.5** For  $f(x) = |x|$  and  $g(x) = \sqrt{4-x}$

- $(g \circ f)(x) = \sqrt{4-|x|}$  with domain  $[-4, 4]$
  - $(f \circ g)(x) = |\sqrt{4-x}|$  with domain  $[-\infty, 4]$
  - $(f \circ f)(x) = |x|$  with domain  $[-\infty, \infty]$
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