Part 1

Range

R1.tex

In each part, an invertible function f will be defined. For each function, find its inverse.

Exercise 1 $f(x) = 5x + 3$

$$f^{-1}(x) = \boxed{\frac{x-3}{5}}$$

Exercise 2 $f(x) = \dfrac{x-4}{7} - 2$

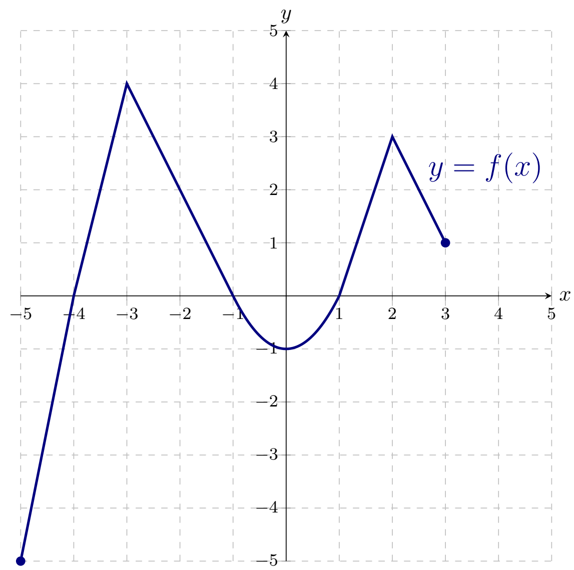
$$f^{-1}(x) = \boxed{7(x+2)+4}$$

Exercise 3 $f(x) = \sqrt[3]{3-x} + 1$

$$f^{-1}(x) = \boxed{3-(x-1)^3}$$

R2.tex

The entire graph of a function f is given below. Use the graph of f to answer the questions.



Exercise 4 *Find the range of f.*

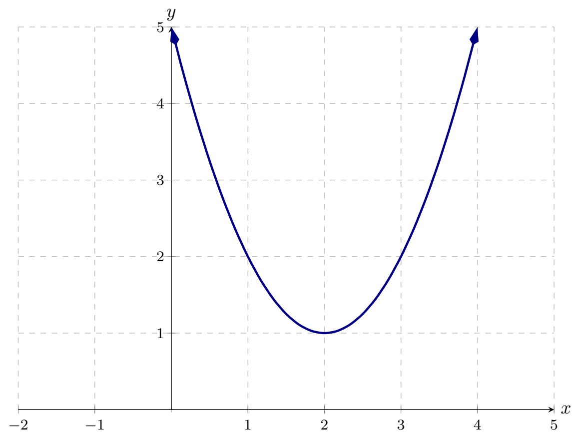
$$[\boxed{-5}, \boxed{4}]$$

Exercise 5 *List the x-values of the x-intercepts of f. (List your answers from least to greatest)*

$\boxed{-4}$, $\boxed{-1}$, *and* $\boxed{1}$

`R3.tex`

Exercise 6 *The function given by $f(x) = 3(x-2)^2 + 1$ (graphed below) is not a one-to-one function on $(-\infty, \infty)$. If we restrict the domain, however, it can be made to be one-to-one.*



Find a formula for $f^{-1}(x)$ when f is restricted to $(-\infty, 2]$.

$$f^{-1}(x) = \boxed{2 - \sqrt{\frac{x-1}{3}}}$$

Hint: *We're starting with $y = f(x)$, so that's:*

$$y = \boxed{3(x-2)^2 + 1}$$

Swap x and y.

$$x = \boxed{3(y-2)^2 + 1}$$

Solving for y you find two solutions. They are:

$$y = 2 - \sqrt{\boxed{\frac{x-1}{3}}}$$

$$y = 2 + \sqrt{\boxed{\frac{x-1}{3}}}$$

The domain of f was restricted to $(-\infty, 2]$, which means we want the range of f^{-1} to be $(-\infty, 2]$. Which of the two solutions you found give outputs which are not greater than 2?

`R4.tex`

The function f is invertible and takes the following values.

x	$f(x)$
0	5
1	2
2	4
3	1
4	3

Exercise 7 *Evaluate.*

$$f^{-1}(1) = \boxed{3}.$$

Exercise 8 *Solve the equation*

$$f^{-1}(x) = 2.$$

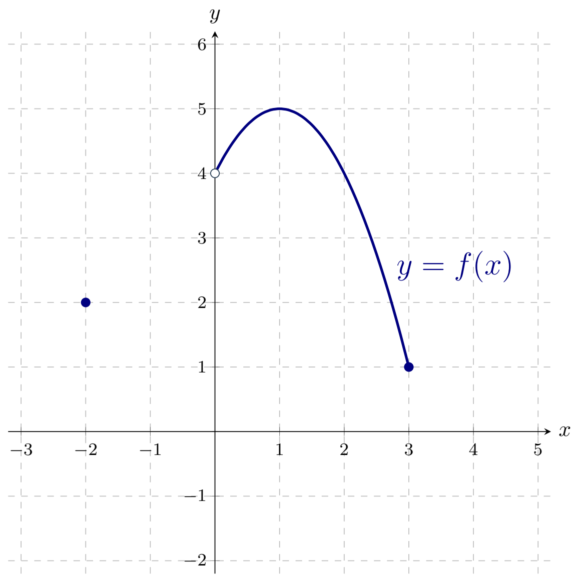
$$x = \boxed{4}$$

(If there is no answer, type DNE)

Hint: *What happens if we plug both sides of the equation $f^{-1}(x) = 2$ into the function f?*

`R5.tex`

The entire graph of a function f is given below. Use the graph of f to answer the questions.



Exercise 9 *Find the domain of f.*

$$\{\boxed{-2}\} \cup (\boxed{0}, \boxed{3}]$$

Exercise 10 *Find the range of f.*

$$[\boxed{1}, \boxed{5}]$$

Exercise 11 *Find the interval on which f is decreasing.*

$$[\boxed{1}, \boxed{3}]$$

`R6.tex`

Exercise 12 *The function f is defined by the formula $f(x) = 2x + 3$.*

The range of f is $(\boxed{-\infty}, \boxed{\infty})$.

Exercise 13 *The function g is defined by the formula $g(x) = 3x^2 + 5$.*

$$\text{The range of } g \text{ is } [\boxed{5}, \boxed{\infty}).$$

Exercise 14 *The function k is defined by the formula $k(x) = 2 + \ln(x)$.*

$$\text{The range of } k \text{ is } (\boxed{-\infty}, \boxed{\infty}).$$

R7.tex

Exercise 15 *The function f is defined by the formula $f(x) = 3e^x + 1$.*

$$\text{The range of } f \text{ is } (\boxed{1}, \boxed{\infty}).$$

Exercise 16 *The function g is defined by the formula $g(x) = 5\sin(x^2 + 2)$.*

$$\text{The range of } g \text{ is } [\boxed{-5}, \boxed{5}].$$

R8.tex

Suppose an object is dropped from a height of 490 meters, and strikes the ground 10 seconds later. Let $h(t)$ denote the height of the object at time t, with h measured in meters, and t measured in seconds with $t = 0$ corresponding to the instant the object was released.

Exercise 17 *The domain of h is $[\boxed{0}, \boxed{10}]$.*

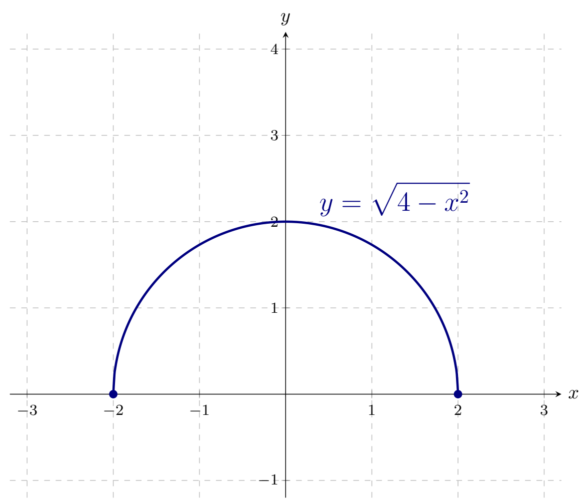
Exercise 17.1 *The range of h is $[\boxed{0}, \boxed{490}]$.*

Exercise 17.1.1 *The average rate of change of h between $t = 0$ and $t = 10$ is $\boxed{-49}$ m/s.*

R9.tex

If R is a positive constant, then the graph of $y = \sqrt{R^2 - x^2}$ is the top half of the circle of radius R centered at the origin.

As an example, this is graphed below for $R = 2$.



Exercise 18 *The domain of the function* $\sqrt{4 - x^2}$ *is* $[\boxed{-2}, \boxed{2}]$ *and the range is* $[\boxed{0}, \boxed{2}]$.

Hint: *This is exactly the function graphed above.*

Exercise 18.1 *The domain of the function* $\sqrt{25 - x^2}$ *is* $[\boxed{-5}, \boxed{5}]$ *and the range is* $[\boxed{0}, \boxed{5}]$.

Hint: *This is* $\sqrt{R^2 - x^2}$ *for* $R = 5$. *The graph of this function is a circle with what radius?*

Exercise 18.1.1 *The domain of the function* $\sqrt{R^2 - x^2}$ *is* $[\boxed{-R}, \boxed{R}]$ *and the range is* $[\boxed{0}, \boxed{R}]$.

Hint: *The graph of this function is a circle with what radius?*