Linear Modeling

We explore how real-world applications of linear equations.

Let's see how these linear functions can help us in some "real world" contexts.

Example 1. *Your friends are trying to get an idea of how many people they can invite to their wedding. The venue they're looking at costs about $2,000 to rent and the event coordinator suggested that they budget about $100 per attendee into the costs.*

(a) *Write a function that represents an estimate for how much this will cost if they have x attendees.*

(b) *Now rewrite your function from part (a) with units for each of the values (including the variables).*

(c) *Suppose the venue has a maximum capacity of 250 people. What is an estimated maximum cost for using that venue?*

(d) *Suppose your friends are trying to stick to about $20,000 for their budget. About how many guests can they invite?*

Explanation

(a) $y = 100x + 2000$ where y is the total cost and x is the number of attendees.

(b) $y \quad \text{dollars} = \dfrac{\$100}{1 \quad \text{attendee}} x \quad \text{attendees} + 2000 \quad \text{dollars}$

(c) $y = 100 \times 250 + 2000$
y = 25000 + 2000
y = 27000
The maximum cost is $27000.

(d) $20000 = 100x + 2000$
18000 = 100x
180 = x
The maximum amount of guests they can invite for $20000 would be 180 people.

Example 2. *Now let's look closer at how we use $y = mx + b$ for the previous example.*

Learning outcomes:
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(a) *What is the "b-value" in your equation? What does it mean in this context?*

(b) *What is the "m-value" in your equation?*

(c) *What are the units for the m-value? Explain why it makes sense that the "m-value" would have these units based on the* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *values' units.*

(d) *What does this m-value mean in this context?*

Explanation

(a) The "b-value" is 2000. This is also called the y-intercept, because it is the point that the line crosses the y-axis. In the context of this situation, it means that if 0 people attend the wedding the total cost will be \$2000.

(b) The "m-value" is 100. This is the slope or rate of change of the line.

(c) In this context, it is \$100 per attendee or $\frac{\$100}{1 \text{ attendee}}$. The units for x are attendees and the units for y are dollars. Since slope is defined by the change in y over the change in x, this makes sense that the units for m are dollars per attendee.

(d) In this context, it means that for each person that attends, the final cost will increase by \$100.

Exploration Below is some information for how electricity usage is billed in Columbus

KWH = Kilowatt Hour **ELECTRICITY RATES Effective January 1, 2018** PCRA = Power Cost Reserve Adjustment

RATE	DESCRIPTION	CHARGES
Residential Schedule A **KW10** (1163.04)	Apartments and dwellings providing domestic accommodations for an individual family. Applicable to residential users with summer usages exceeding 700 KWH in any month.	Customer: $10.70 Energy (per KWH): .0873 *PCRA varies each month based on actual purchase power costs.
Residential Schedule A-1 *(Small User)* **KW11** (1163.05)	Apartments and dwellings providing domestic accommodations for an individual family. Applicable to residential users with summer usages less than 700 KWH in any month.	Customer: $10.70 Energy (per KWH): .0724 *PCRA varies each month based on actual purchase power costs.

https://www.columbus.gov/Templates/Detail.aspx?id=2147500472

(a) Write a function, or set of functions, which determine how much your electricity bill will be if you use x KWH in a month. Don't worry about the variable PCRA rates. (Note: the 0.0873 and 0.0724 are dollar amounts, i.e. \$0.0873 and \$0.0724)

(b) Suppose exactly 700 KWH are used in a month. Use Schedule A information to calculate a cost for the bill.

(c) Suppose exactly 700 KWH are used in a month. Now use Schedule A-1 information to calculate a cost for the bill.

Exploration According to True Car (www.truecar.com), a 2018 Toyota Camry (conventional) (29/41 MPG city/hwy) sells for an average of $22,030, and a Camry Hybrid (51/53 MPG city/hwy) sells for an average of $26,247. Currently gas prices are in the upper $2 per gallon, so let's estimate about $2.80 per gallon.

(a) Write a linear function that will estimate the cost of driving a conventional Camry x miles, given the information above. (Hint: Think about what the units are for x and what the units should be for y. Then use the units of the information given to help you figure out what should be multiplied and what should be divided in order to give those desired units).

(b) Write a linear function that will estimate the cost of driving a Hybrid Camry x miles, given the information above. (Hint: the function will be very similar to part(a)).

(c) What are the "b-values" in these expressions? What do they represent in this context?

(d) What are the "m-values" in these expressions (write them with their units)? What do these mean in this context?

(e) What other factors could we be considering when comparing the "costs" between these two vehicles?

Summary

- When writing linear equations, consider the units being used in the situation. That can go a long way to properly writing the equation and fully understanding the context.
- context