Relations and Graphs: Relations

We define relations and graph examples of examples.

In the last section, we discussed graphing points using the Cartesian coordinate system. While individual points can be useful, we often want to study collections of points.

> **Definition** A **relation** is a collection of points of the form (x, y). If the point (x_0, y_0) is in the relation, then we say x_0 and y_0 are **related**.

This might seem like a strange definition, but hopefully a few examples will you see the relationship (pun intended!) between the mathematical definitions of the words "relation" and "related" and the way we often use these words in everyday speech.

Example 1. *Let's look at the relationship between the number of chicken nuggets you can buy in a single container at a local fast food store and the price for that container of nuggets.*

Amount of Nuggets	*Price*
4	1.99
6	2.99
10	4.49
20	5.00
40	8.99

Explanation Now this table defines a relation because we can think of these as the points $(4, 1.99)$, $(6, 2.99)$, $(10, 4.49)$, $(20, 5.00)$, and $(40, 8.99)$. We say that 4 nuggets is related to \$1.99. And 40 nuggets is related to \$8.99. We can also represent this relationship using a graph.

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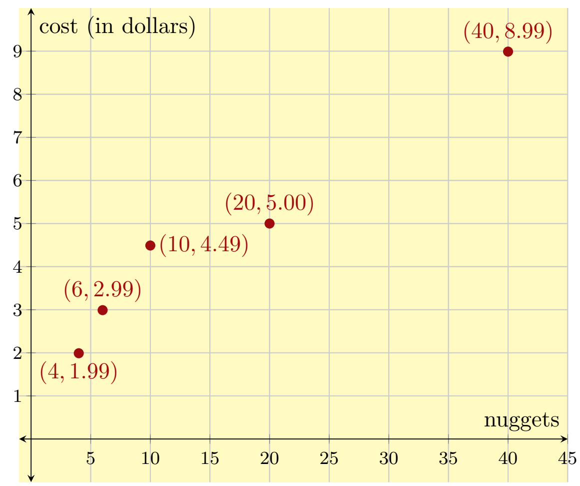


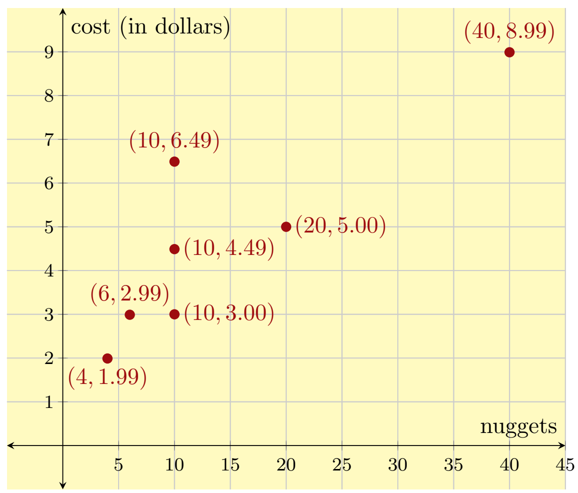
We could ask lots of mathematical questions about this relation. One such example might be, "What's the cheapest way to buy 100 nuggets?" But for now, its enough to know it is a relation and to know you can represent that relation in multiple ways such as a table, a list, or a graph.

Example 2. *It's important to note that nothing about our definition of relation restricts what points can be included. Assume that in our chicken nugget example above, there is a coupon that allows you to buy 10 chicken nuggets for \$3.00. Assume there is also an option to buy a chicken nugget meal which includes 10 chicken nuggets (and fries and a drink but we don't care about those) for \$6.49.*

Explanation Then we could modify the table of our relation to be:

Amount of Nuggets	Price
4	1.99
6	2.99
10	4.49
20	5.00
40	8.99
10	3
10	6.49

Now this table still defines a relation and we can say that 10 nuggets is related to \$4.49 and \$3.00 and \$6.49. Here is the graph of this relation.

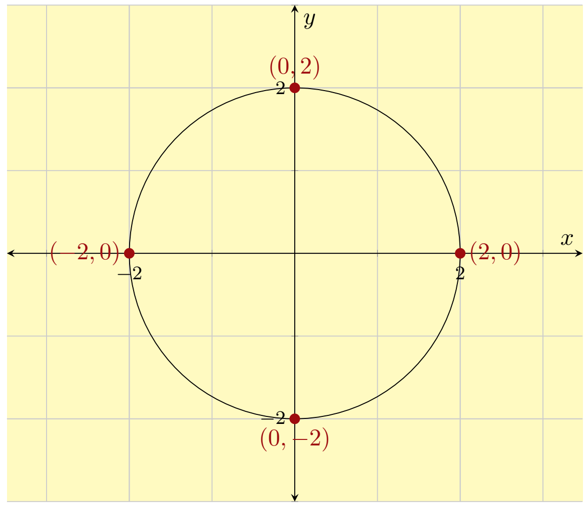


> **Remark** There is a special type of relation called a **function** where each x-coordinate is only allowed to be related to one unique y-coordinate. In the examples above, the first relation is a function but the second relation which includes the coupon and a meal is not a function because 10 nuggets is related to more than one cost. Functions are going to be extremely important and we will come back to them later in the course.

The two examples we have seen so far have been a relations given by a list of points. This does not have to be the case. Relations can contain an infinite number of points. Some of the relations we will be studying the most are given by an equation relating two variables.

Example 3. *Let's consider the relation that is the collection of all points* (x, y) *where* $x^2 + y^2 = 4$.

Explanation Some points contained in this relation are $(2, 0)$, $(0, 2)$, $(-2, 0)$, and $(0, -2)$, but these are not all the points in this relation. Often, one of best ways to think about a relation given by an equation is with a graph. The graph of $x^2 + y^2 = 4$ is the circle of radius 2 centered at the origin.

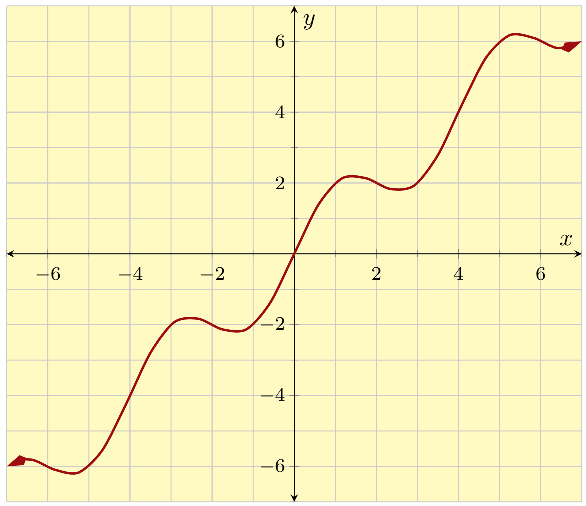


Example 4. *Verify algebraically that the point* $(2, 0)$ *is on the curve* $x^2+y^2 = 4$.

Explanation Note that this is the same question as, "Verify that the point $(2, 0)$ is a member of the relation given by $x^2 + y^2 = 4$." We want to show that this point satisfies the condition. The point $(2, 0)$ means that the $x = 2$ and $y = 0$. If we plug these values in for x and y in the equation, we want to check that both sides of the equation are equal.

$$x^2 + y^2 = 4 \tag{1}$$
$$(2)^2 + (0)^2 = 4 \tag{2}$$
$$4 + 0 = 4 \tag{3}$$
$$4 = 4 \tag{4}$$

Example 5. *A relation can also be intitally given by a graph. For example, this is a relation.*



Explanation We can list some of the points on this graph. It looks like $(0, 0)$ and $(1, 2)$ are points on this curve, but there are many other points we cannot explicitly list.