Definition of Polynomials

We explore polynomial functions.

A polynomal is a particular type of algebraic expression

(a) A company's sales, s (in millions of dollars), can be modeled by $2.2t + 5.8$, where t stands for the number of years since 2010.

(b) The height of an object from the ground, h (in feet), launched upward from the top of a building can be modeled by $-16t^2 + 32t + 300$, where t represents the amount of time (in seconds) since the launch.

(c) The volume of an open-top box with a square base, V (in cubic inches), can be calculated by $30s^2 - \frac{1}{2}s^2$, where s stands for the length of the square base, and the box sides have to be cut from a certain square piece of metal.

Polynomial Vocabulary

A polynomial is an expression with one or more terms summed together. A term of a polynomial must either be a plain number or the product of a number and one or more variables raised to natural number powers. The expression 0 is also considered a polynomial, with zero terms.

Example 1. *Here are some examples of polynomials*

(a) *Here are three polynomials:* $x^2 - 5x + 2$, $t^3 - 1$, $7y$.

(b) *The expression* $3x^4y^3 + 7xy^2 - 12xy$ *is an example of a polynomial in more than one variable.*

(c) *The polynomial* $x^2 - 5x + 3$ *has three terms:* x^2, $-5x$, *and* 3.

(d) *The polynomial* $3x^4 + 7xy^2 - 12xy$ *also has three terms.*

(e) *The polynomial* $t^3 - 1$ *has two terms.*

Definition 1. *The coefficient (or numerical coefficient) of a term in a polynomial is the numerical factor in the term.*

Example 2. (a) *The coefficient of the term* $\frac{4}{3}x^6$ *is* $\frac{4}{3}$.

(b) *The coefficient of the second term of the polynomial* $x^2 - 5x + 3$ *is* -5.

Learning outcomes:
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(c) *The coefficient of the term* $\frac{y^7}{4}$ *is* $\frac{1}{4}$, *because we can rewrite* $\frac{y^7}{4}$ *as* $\frac{1}{4}y^7$.

A term in a polynomial with no variable factor is called a constant term.

Example 3. *The constant term of the polynomial* $x^2 - 5x + 3$ *is* 3.

Definition 2. *The degree of a term is one way to measure how large it is. When a term only has one variable, its degree is the exponent on that variable. When a term has more than one variable, its degree is the sum of the exponents on the variables. A nonzero constant term has degree* 0.

Example 4.

- *The degree of* $5x^2$ *is* 2.
- *The degree of* $-\frac{4}{7}y^5$ *is* 5.
- *The degree of* $-4x^2y^3$ *is* 5.
- *The degree of* 17 *is* 0. *Constant terms always have* 0 *degree.*

Definition 3. *The* ***degree*** *of a nonzero polynomial is the greatest degree that appears amongst its terms*

Remark 1. *To help us recognize a polynomial's degree, the standard convention at this level is to write a polynomial's terms in order from highest degree to lowest degree. When a polynomial is written in this order, it is written in standard form. For example, it is standard practice to write* $7 - 4x - x^2$ *as* $-x^2 - 4x + 7$ *since* $-x^2$ *is the leading term. By writing the polynomial in standard form, we can look at the first term to determine both the polynomial's degree and leading term.*

Adding and Subtracting Polynomials

Bayani started a company that makes one product: one-gallon ketchup jugs for industrial kitchens. The company's production expenses only come from two things: supplies and labor. The cost of supplies, S (in thousands of dollars), can be modeled by $S = 0.05x^2 + 2x + 30$, where x is number of thousands of jugs of ketchup produced. The labor cost for his employees, L (in thousands of dollars), can be modeled by $0.1x^2 + 4x$, where x again represents the number of jugs they produce (in thousands of jugs). Find a model for the company's total production costs.

Evaluating Polynomial Expressions

Recall that evaluating expressions involves replacing the variable(s) in an expression with specific numbers and calculating the result. Here, we will look at evaluating polynomial expressions.

Example 5. *Evaluate the expression*

$$-12y^3 + 4y^2 - 9y + 2 \textit{ for } y = -5$$

Explanation. *We will replace* y *with* -5 *and simplify the result:*

$$\begin{aligned} 12y^3 + 4y^2 - 9y + 2 \quad &= -12({\color{blue}-5})^3 + 4({\color{blue}-5})^2 - 9({\color{blue}-5}) + 2 \\ &= -12(-125) + 4(25) + 45 + 2 \\ &= 1647 \end{aligned}$$