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Precalculus with Review 1

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Contents

Part 1

Variables and CoVariation

1.1 Quantitative Reasoning

Learning Objectives

- Estimations
  - Reminders of roles of addition, subtraction, multiplication, division and how to choose each one
- Units
  - Formalize the canceling units as fractions
  - Quick review of properties of fractions, adding, subtracting, multiplying, dividing fractions
- Percentages
  - Formula for finding percent (portion is given by amount*percent)
  - Percent increase and then decrease. What is the whole?

1.1.1 Quantitative Reasoning: Estimates

Tips for Doing Rough Estimates

- Estimations are **NOT** guesses. They can sometimes be educated guesses, but estimations, at least in this course, will always still require some sort of calculation(s).
- Rough estimations are meant to be estimations which can be calculated mentally, meaning without calculators, or even pen and paper. You will still be expected to record/write down your work and thought process for your instructor, but it should be work that you are able to do mentally.
- Everyone has different skill and comfort levels with their mental calculations – you may need to round values a lot to make them something you are comfortable doing in your head. This is completely fine, but the important thing is that in your solutions/write-ups you explain what numbers you rounded and why.

Exploration [In Class Activity]Pizza Party: Now let's give this a try but working with your group to determine what you would buy for a pizza party in the following scenario: You and your roommate are going to have some people over later and so you go to the grocery store to grab some snacks. Everyone has agreed to pitch in $10 to pay for pizza and snacks for the night, and you think about 12 people are coming over. Here are the prices of various snacks from the grocery store:

- Bag of tortilla chips - $3.99
- Salsa - $3.79
- Bag of "normal" chips - $2.99
- Dozen cookies from bakery - $4.99
- Veggie Tray - $14.99
- 12 pack of soda - $5.79

You want to get at least one of each of these items, but you're also going to order some pizza and breadsticks. Here are the prices from your pizza place:

- Cheese:
  - Medium - $9.99
  - Large - $11.99
- Pepperoni:

    - Medium - \$11.49
    - Large - \$13.49
- Breadsticks:
    - 5 for \$4.99
- \$2.49 delivery charge. Don't forget tip!

You are a good person and do not plan to pocket any of the money that your friends are going to give you to pay for these pizzas and snacks. Plus, you and your roommate are going to pay for your fairshare (also each pitch in \$10). Decide what you're going to get!

Example 1. *Roughly estimate how many gallons of gasoline you might use to drive from here to Chicago, IL.*

Explanation Distance to Chicago, IL: about 350 miles (google maps) My vehicle gets about 30 miles per gallon (30 miles = 1 gallon), so 350 mi $\times \frac{1gal}{30mi}$ is about $\frac{360}{30} = 12$ gallons

Exploration Roughly estimate how many seconds you've been alive.

- Determine a number that would definitely be way too low for even a rough estimate for each question, but still requires some calculation. Explain why you think this would be unreasonably low. (Still do not use a calculator, use mental arithmetic.)
- Determine a number that would definitely be way too high for even a rough estimate for each question. Explain why you think this would be unreasonably high.

- For each of those problems, do you think your original rough estimation is an underestimate or overestimate? Explain why you think this based off of your calculations. Or if you're not sure if it's under or over, explain why you are unsure.
- If possible, determine a more exact value by using a calculator and not rounding values to get an exact number, or see if google has any estimate(s). Compare this with your original estimations: were you under or over estimating, or can't tell? If you have values to compare, how different are the two values?

1.1.2 Quantitative Reasoning: Units

Units

Is 12 the same as 1? As a mathematical value, no 12 and 1 are not the same value. But if we give these values units, they actually can represent the same thing!
What units can we give to 12 and to 1 so that they are equal?

$$12 \boxed{?} = 1 \boxed{?}$$

Units in everyday life are often used, even if we don't necessarily think of them as units. For example, you wouldn't say "I went to the grocery store and bought a dozen." A dozen what? You've given how many you've bought (the value), but not how many of what you have bought (the units). Instead you would say "I went to the grocery store and bought a dozen eggs."

Example 2. *A typical bottle of wine holds about 24.5 ounces. How much is this in gallons?*
Explanation If we take the original 24.5 ounces in 1 bottle we can set it up as a fraction:

$$\frac{24.5 \quad ounces}{1 \quad bottle}$$

There are 128 ounces in 1 gallon. This can be set up as the following fraction:

$$\frac{1 \quad gallon}{128 \quad ounces}$$

These two fractions have ounces in common. We can set the fractions up in such a way that the ounces will "cancel out."

$$\frac{24.5 \quad ounces}{1 \quad bottle} \times \frac{1 \quad gallon}{128 \quad ounces} = \frac{24.5 \quad \cancel{ounces}}{1 \quad bottle} \times \frac{1 \quad gallon}{128 \quad \cancel{ounces}} = \frac{0.1914 \quad gallon}{1 \quad bottle}$$

So there are 0.1914 gallons in 1 bottle of wine.

Exploration

- "How many ounces are in a yard?" Explain why this question does not make sense.
- "How many feet are in an acre?" Explain why this question does not make sense.

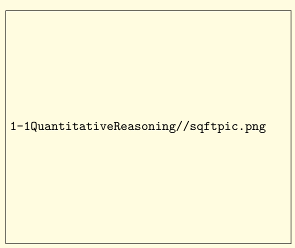
Let's talk a little more about this idea highlighted in the question above (about feet and acre units). It is important to note that feet, square feet, and cubic feet are all different units. They are not measuring the same thing! Feet measures length (one dimension), square feet measure area (two dimensions), and cubic feet measure volume (three dimensions).

Q. We **can** convert between **cubic feet** and which units from previous examples? Why?

We can convert between feet and inches, or feet and miles. Can we convert between square/cubic feet and square/cubic inches, or square/cubic feet and square/cubic miles? To answer this, we have to ask if these pairs of units are measuring the same property. If the answer is yes, then yes we can convert between them!

- Do square feet, square inches, and square miles measure the same property?
- Do cubic feet, cubic inches, and cubic miles measure the same property?

In order to convert between units, we need some sort of equivalence. Just like knowing 4 quarts = 1 gallon, or 16 oz = 1 pound, these are equivalences that we can then use to convert from quarts to gallons or vice versa, or ounces to pounds or vice versa. Let's determine the equivalence for square feet and square inches.



Fill in the blanks above (assume that the squares are the exact same size). Since the squares are the exact same size, we can say that 1 sq ft = $\boxed{?}$ sq in. Use this same reasoning to determine the equivalence between square miles and square feet:
1 sq mi = $\boxed{?}$ sq ft

1.1.3 Quantitative Reasoning: Percents

Percentages

Percentages are everywhere! They are used to describe discounts, markups, commissions, statistics, information, change, and on and on. They are an important topic and so we want to make sure that we really understand them. Let's start by just breaking down the word "percent." "Per" and "cent." There are some specific words that often translate into certain mathematical operations. For example, if a question asks, "how many apples and oranges are there?" what operation are you going to do with the number of apples and oranges? Let's make a short list of which words usually translate into which operation:
If you read a math problem: "There are 5 apples and 6 oranges. How many apples and oranges are there?" What math operation are you going to do with the number of apples (5) and oranges (6)?

"and" often translates to: $\boxed{?}$

Let's change the problem a little bit: "There are 4 apples and 7 oranges. What is the difference between the number of apples and number of oranges?" What math operation are you going to do with the number of apples (4) and oranges (7)?

"difference" often translates to $\boxed{?}$

Let's change it again: "We want each of the 3 people in our group to have 5 apples. How many apples are we going to need?" What math operation are you going to do with the number of people (3) and apples (5)?

"each of" often translates to: $\boxed{?}$

Note: We will also see in the following problems that specifically with percentages, "of" often translates to this same operation.
Another change: "We have a group of 5 people and a total of 15 apples. How many apples per person are there?" What math operation are you going to do with the number of apples (15) and people (5)?

"per" often translates to: $\boxed{?}$

Let's change it one last time: "The total number of apples and oranges is 13. If the number of apples is "x" and the number of oranges is "y," write an equation for the total number of apples and oranges." What math symbol did you put in for the word "is"?

"is" often translates to: $\boxed{?}$

Now "cent." Where have we seen this word before? How many cents are in a dollar? How many years are in a century? How many centimeters are in a meter?

Wherever you see the word "cent", it is likely representing the number $\boxed{?}$

Hence, "percent" can be interpreted as $\boxed{?}$ by $\boxed{?}$

So, for example, $13\% = \boxed{?} = 0.\boxed{?}$

Percent Increase and Decrease

In the problems where percent increases or decreases are calculated, we calculate not only the percent change, but also the "actual value" change as well. There are specific vocabulary terms to describe these two "measures" of changes: absolute change and relative change.

- **Absolute change** is the "actual value" change that occurred. For example, in question 2, the absolute change was \$14.69 – the actual dollar amount that the price changed.
- **Relative change** is the percent change that occurred. For example, again in question 2, the relative change was 30% decrease – the percent or proportion that the dollar amount was changed.

Example 3. *A shop owner raises the price of a \$100 pair of shoes by 50%. After a few weeks, because of falling sales, the owner reduces the price of the shoes by 50%. What is the new price of the shoes (after both percent changes have occurred)?*
Explanation First, we must account for the percent increase. We can find 50% of \$100 and then add it to the original amount

$$0.5 \times \$100 = \$50$$

$$\$100 + \$50 = \$150$$

Now we can deal with the percent decrease. Remember to decrease from the new amount (\$150) and not the original (\$100).

$$0.5 \times \$150 = \$75$$

After subtracting this amount from \$150 we will have the final amount.

$$\$150 - \$75 = \$75$$

After raising the \$100 price by 50% and lowering it be 50% the final price became \$75 an overall decrease of 25%!

Exploration The annual number of burglaries in a town rose by 40% in 2012 and fell by 30% in 2013.

a. What was the total percent change in burglaries over the two years? Let's first try this problem with a specific number of burglaries to start with. In your group, find the % change if there were certain number of burglaries in 2011 (choose a 3 digit number that does NOT end with a 0). Then try to think about how to do this problem without knowing a specific number of burglaries.

b. It might be tempting to say that the change over the two years was a 10% decrease. Why might someone think it is a 10% decrease (i.e., how do you come up with 10% from the numbers in the problem?)

c. Why is 10% incorrect?