Part 1

Unit Circle to Function Graph

UCTFG1.tex

Exercise 1 *The following table contains the values of a circular function g.*

x	0	4.5	9	13.5	18	22.5	27	31.5	36
$g(x)$	2	5	2	-1	2	5	2	-1	2

The period of g is $\boxed{18}$. *The midline of m is $y =$* $\boxed{2}$. *The amplitude of g is* $\boxed{3}$.

On the interval $(0, 4.5)$, g is (increasing ✓/ decreasing).

On the interval $(22.5, 31.5)$, g is (increasing/ decreasing ✓).

Since g is periodic, $g(-72) =$ $\boxed{2}$.

UCTFG2.tex

Exercise 2 *The London Eye is a Ferris wheel 135 meters in diameter. It is boarded at its lowest point (6 o'clock) from a platform which is 6 meters above ground. The wheel makes one full rotation every 30 minutes, and at time $t = 0$ you board at the loading platform (6 o'clock). Let $h = f(t)$ denote your height above ground in meters after t minutes.*

(a) *The period of the function $h = f(t)$ is* $\boxed{30}$ *minutes.*

(b) *The midline of the function $h = f(t)$ is* $\boxed{73.5}$ *meters.*

(c) *The amplitude of the function $h = f(t)$ is* $\boxed{67.5}$ *meters.*

(d) *Which of the following graphs is the graph of $h = f(t)$?*

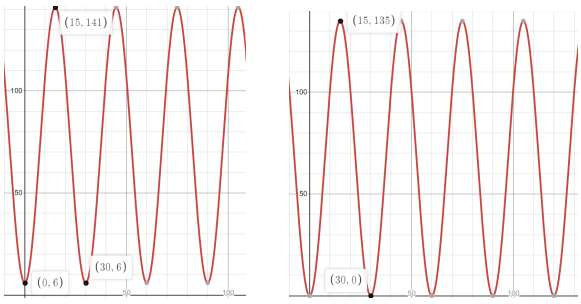

Figure 1: A on the left and B on the right

Multiple Choice:

Acknowledgements: Active Prelude to Calculus (https://activecalculus.org/prelude)

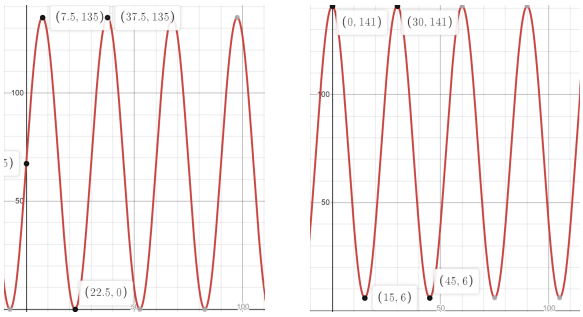

Figure 2: C on the left and D on the right

(i) A ✓
(ii) B
(iii) C
(iv) D

`UCTFG3.tex`

Exercise 3 *The London Eye is a Ferris wheel 135 meters in diameter. It is boarded at its lowest point (6 o'clock) from a platform which is 6 meters above ground. The wheel makes one full rotation every 30 minutes, and at time $t = 0$ you board at the loading platform (6 o'clock). Let $d = g(t)$ denote your horizontal distance from the diameter of the wheel perpendicular to the ground in meters after t minutes.*

(a) *The period of the function $d = g(t)$ is* $\boxed{15}$ *minutes.*

(b) *The midline of the function $d = g(t)$ is* $\boxed{33.75}$ *meters.*

(c) *The amplitude of the function $d = g(t)$ is* $\boxed{33.75}$ *meters.*

(d) *Which of the following graphs is the graph of $d = g(t)$?*

Multiple Choice:

(i) A
(ii) B
(iii) C ✓
(iv) D

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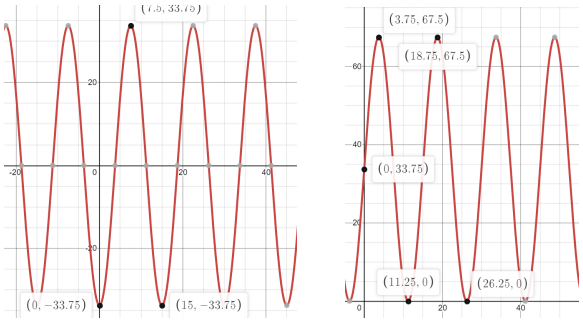

Figure 3: A on the left and B on the right

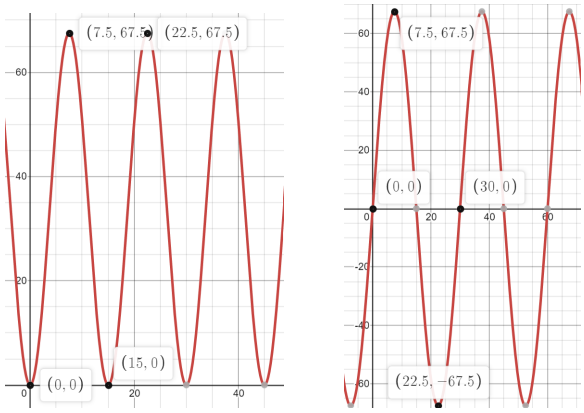

Figure 4: C on the left and D on the right

`UCTFG4.tex`

Exercise 4 (a) *The average rate of change of the sine function on the interval* $\left[0, \frac{\pi}{2}\right]$ *is* $AV_{\left[0,\frac{\pi}{2}\right]} = \boxed{\frac{2}{\pi}}$.

(b) *The average rate of change of the sine function on the interval* $\left[0, \frac{5\pi}{2}\right]$ *is* $AV_{\left[0,\frac{5\pi}{2}\right]} = \boxed{\frac{2}{5\pi}}$.

(c) *The average rate of change of the sine function on the interval* $\left[\frac{\pi}{2}, \pi\right]$ *is*

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$AV_{[\frac{\pi}{2},\pi]} = \boxed{-\frac{2}{\pi}}$.

(d) *The average rate of change of the sine function on the interval* $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$ *is* $AV_{[\frac{\pi}{2},\frac{5\pi}{2}]} = \boxed{0}$.

(e) *Select all intervals on which the sine function is increasing.*

Select All Correct Answers:

(i) $\left(\frac{\pi}{2}, \pi\right)$

(ii) $\left(0, \frac{\pi}{2}\right)$ ✓

(iii) $(0, \pi)$

(iv) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ✓

(v) $\left(0, \frac{5\pi}{2}\right)$

`UCTFG5.tex`

Exercise 5 (a) *The average rate of change of the cosine function on the interval* $\left[0, \frac{\pi}{2}\right]$ *is* $AV_{[0,\frac{\pi}{2}]} = \boxed{-\frac{2}{\pi}}$.

(b) *The average rate of change of the cosine function on the interval* $\left[0, \frac{5\pi}{2}\right]$ *is* $AV_{[0,\frac{5\pi}{2}]} = \boxed{-\frac{2}{5\pi}}$.

(c) *The average rate of change of the cosine function on the interval* $\left[\frac{\pi}{2}, \pi\right]$ *is* $AV_{[\frac{\pi}{2},\pi]} = \boxed{-\frac{2}{\pi}}$.

(d) *The average rate of change of the cosine function on the interval* $[0, \pi]$ *is* $AV_{[0,\pi]} = \boxed{-\frac{2}{\pi}}$.

(e) *Select all intervals on which the cosine function is decreasing.*

Select All Correct Answers:

(i) $\left(\frac{\pi}{2}, \pi\right)$ ✓

Acknowledgements: Active Prelude to Calculus
(https://activecalculus.org/prelude)

(ii) $\left(0, \frac{\pi}{2}\right)$ ✓

(iii) $(0, \pi)$ ✓

(iv) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(v) $\left(0, \frac{5\pi}{2}\right)$

UCTFG6.tex

Exercise 6 *For each interval of real numbers below, select the quadrant of the unit circle in which the corresponding angles lie.*

(a) $\left(0, \frac{\pi}{6}\right)$

Multiple Choice:

(i) *Quadrant I* ✓

(ii) *Quadrant II*

(iii) *Quadrant III*

(iv) *Quadrant IV*

(b) $\left(\frac{11\pi}{7}, \frac{17\pi}{9}\right)$

Multiple Choice:

(i) *Quadrant I*

(ii) *Quadrant II*

(iii) *Quadrant III*

(iv) *Quadrant IV* ✓

(c) $(3.5, 4)$

Multiple Choice:

(i) *Quadrant I*

(ii) *Quadrant II*

(iii) *Quadrant III* ✓

(iv) *Quadrant IV*

(d) $\left(-\frac{11}{10}, -\frac{1}{10}\right)$

Multiple Choice:

(i) *Quadrant I*
(ii) *Quadrant II*
(iii) *Quadrant III*
(iv) *Quadrant IV* ✓

(e) $(2, 3)$

Multiple Choice:

(i) *Quadrant I*
(ii) *Quadrant II* ✓
(iii) *Quadrant III*
(iv) *Quadrant IV*

`UCTFG7.tex`

We will algebraically find a candidate for the period of the tangent function, defined by $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$.

Exercise 7 *Using the angle sum identity, we know that for any real number* x,

$$\sin(x + \pi) = \sin(x)\cos(\boxed{\pi}) + \cos(x)\sin(\boxed{\pi})$$

and

$$\cos(x + \pi) = \cos(x)\cos(\boxed{\pi}) - \sin(x)\sin(\boxed{\pi}).$$

Exercise 7.1 *Using knowledge of famous angles, we can simplify the following expressions as follows:*

$$\sin(x)\cos(\pi) + \cos(x)\sin(\pi) = \boxed{-\sin(x)}$$

and

$$\cos(x)\cos(\pi) - \sin(x)\sin(\pi) = \boxed{-\cos(x)}$$

.

Exercise 7.1.1 *Using the information found above,* $\tan(x + \pi) = \boxed{\tan(x)}$.

Exercise 7.1.1.1 *We conclude that a possible period of the tangent function is* $\boxed{\pi}$.

UCTFG8.tex

Recall that the cosecant function is defined as the reciprocal of the sine function: $\csc(x) = \dfrac{1}{\sin(x)}$. In this problem, we will find some properties of the cosecant function.

Exercise 8 (a) *Recall that* $\sin(x) = 0$ *when* x *is a multiple of* π*:* $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$. *Select the domain of the cosecant function.*

Multiple Choice:

(i) $(-\infty, \infty)$

(ii) $(-\infty, 0) \cup (0, \infty)$

(iii) $\cdots \cup (-2\pi, -\pi) \cup (-\pi, 0) \cup (0, \pi) \cup (\pi, 2\pi) \cup \cdots$ ✓

(iv) $\cdots \cup \left(-\dfrac{5\pi}{2}, -\dfrac{3\pi}{2}\right) \cup \left(-\dfrac{3\pi}{2}, --\dfrac{\pi}{2}\right) \cup \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right) \cup \cdots$

(b) *Recall that sine is an odd function. Cosecant is*

Multiple Choice:

(i) *odd.* ✓

(ii) *even.*

(iii) *odd and even.*

(iv) *neither odd nor even.*

(c) *On the interval* $\left(0, \dfrac{\pi}{2}\right)$*, cosecant is*

Multiple Choice:

(i) *increasing.*

(ii) *decreasing.* ✓

(iii) *neither increasing nor decreasing.*

(d) *Using knowledge of famous angles,* $\csc\left(\dfrac{\pi}{3}\right) = \boxed{\dfrac{2}{\sqrt{3}}}$.

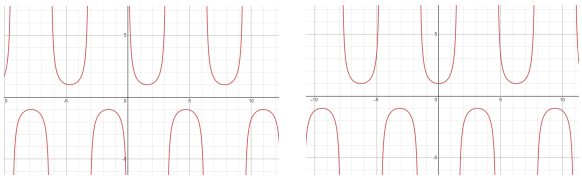

Figure 5: A on the left and B on the right

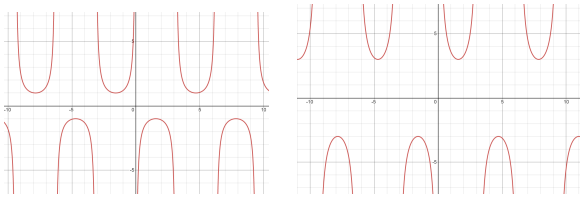

Figure 6: C on the left and D on the right

(e) *Which of the following graphs is the graph of* $\csc(x)$*?*

Multiple Choice:

(i) *A* ✓

(ii) *B*

(iii) *C*

(iv) *D*

`UCTFG9.tex`

Recall that the secant function is defined as the reciprocal of the cosine function: $\sec(x) = \dfrac{1}{\cos(x)}$. In this problem, we will find some properties of the secant function.

Exercise 9 (a) *Recall that* $\cos(x) = 0$ *when* x *is an odd multiple of* $\dfrac{\pi}{2}$: $\ldots, -\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2}, \ldots$. *Select the domain of the cosecant function.*

Multiple Choice:

(i) $(-\infty, \infty)$

(ii) $(-\infty, 0) \cup (0, \infty)$

(iii) $\cdots \cup (-2\pi, -\pi) \cup (-\pi, 0) \cup (0, \pi) \cup (\pi, 2\pi) \cup \cdots$

(iv) $\cdots \cup \left(-\frac{5\pi}{2}, -\frac{3\pi}{2}\right) \cup \left(-\frac{3\pi}{2}, --\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \cdots$ ✓

(b) *Recall that cosine is an even function. Secant is*

Multiple Choice:

(i) *odd.*

(ii) *even.* ✓

(iii) *odd and even.*

(iv) *neither odd nor even.*

(c) *On the interval* $\left(0, \frac{\pi}{2}\right)$*, secant is*

Multiple Choice:

(i) *increasing.* ✓

(ii) *decreasing.*

(iii) *neither increasing nor decreasing.*

(d) *Using knowledge of famous angles,* $\sec\left(\frac{\pi}{3}\right) = \boxed{2}$.

(e) *Which of the following graphs is the graph of* $\sec(x)$*?*

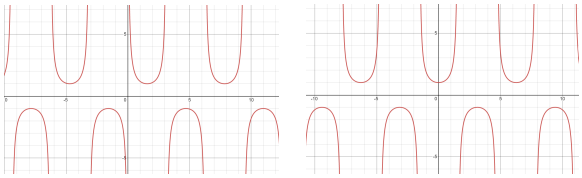

Figure 7: A on the left and B on the right

Multiple Choice:

(i) *A*

(ii) *B* ✓

(iii) *C*

(iv) *D*

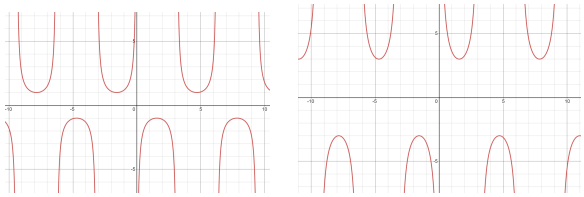

Figure 8: C on the left and D on the right

