Exponential Modeling: Early Exponentials

We use exponents when exploring real world situations.

Early Exponentials

Example 1. *An athlete signs a contract saying that they will earn \$8.3 million with an increase of 4.8% each year of the 5 year contract. Use this scenario to fill in the following table:*

Year of : Contract	*Athlete's Salary*	*Calculate the next year's salary using the previous year's salary*
1	\$8.3 *million*	*N/A*
2	\$ 8.7 *million*	$8.3 + 8.3 \times$ .048
3	\$ 9.1 *million*	8.7 + 8.7 × .048
4	\$ 9.5 *million*	9.1 + 9.1 × .048
5	\$ 10.0 *million*	9.5 + 9.5 × .048

You may have calculated the salaries for each year by first finding 4.8% of the previous year's salary, and then adding that value to the previous year's salary. Doing it this way, you have to type two calculations into a calculator (calculate 4.8%, then record that value, then enter the addition of that value to the salary). If you did it this way, think about how to write (and do) the computation with just one calculation entry into a calculator.

$$8.3 + 8.3 \times \boxed{.048} = 8.3 \times \boxed{1.048}$$

What we have developed above is an exponential function to describe this athlete's salary. Let's look at each value in our function and identify what each piece represents.

$$y = 8.3(1.048)^x$$

8.3 1.048 x y What we have outlined above is an understanding of what each piece of an exponential function in the form y=a*Rx means in terms of a given context. So we can now use this understanding to create an exponential function for exponential scenarios without having to go through the "step by step" process as we did in the original table for the athlete salary problem.

Learning outcomes:
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Example 2. *Some CDs (a banking investment option) are offering rates that give about 3.10% yield per year as long as a minimum amount, such as $5,000, is invested. "3.10% yield" simply means the investment increases (earns) about 3.10%. CDs are only given for a certain amount of time (usually a certain number of years). When a CD "matures" (ends), you usually have the option to renew the CD.*

(a) *Use this scenario to fill out the table below*

After Year:	*Value of investment*	*Calculate the value at the end of the next year using the previous year's v*
0	$\$ \boxed{5,000}$	$\$ \boxed{5000}(\boxed{1.031})$
1	$\$ \boxed{5,155}$	$\$ \boxed{5000}(\boxed{1.031})$
2	$\$5,314.81$	$\$ \boxed{5000}(\boxed{1.031})$
3	$\$5,479.57$	$\$ \boxed{5000}(\boxed{1.031})$

(b) *Suppose you invest the minimum amount of $5,000 in order to get this 3.10% yield. Write a function to describe how much the investment will be worth after x years. Write the units of each value and identify what each represents.*

(c) *Fill in the first column of the table below with the values you calculated in your pre class work. Then, enter the appropriate values into your function from above to fill in the last column of the table.*

After Year:	*Value of investment from previous table*	*Value of investment from function*
0	$\$5,000$	
1	$\$5,155$	
2	$\$5,314.81$	
3	$\$5,479.57$	

How do these values compare? Are they exactly the same? Should they be? Explain.

(d) *Suppose this is a 7 year CD. How much will your investment be worth at the end of the CD?*

(e) *Suppose you keep renewing this CD with this rate every time it "matures" (comes to the end). About how many years will it take to double the initial investment? Use "Guess and Check" to answer this question. We want our function to output: Because*

(i) *Find the closest whole number that gives us less than we want:*

(ii) *Find the closest whole number that gives us more than we want:*

(iii) *Which of these values is closer to what we want? Use whichever value is closer as your estimated value for the answer.*

Exploration 4. Suppose there is a new virus that reportedly doubles infection cases in about 20 days. What would be this virus' infection rate (as a percent)? Hint: Set up an exponential function, assuming that there were initially 80 recorded infections. After you have figured out the infection rate, think about/explain why an initial number of infections was not needed in order to answer this question.