

Exercise 1 Use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.

Problem 1.1 For $f(x) = x^2 - x + 1$ and $g(x) = 3x - 5$

- $(g \circ f)(x) = \boxed{3x^2 - 3x - 2}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{9x^2 - 33x + 31}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{x^4 - 2x^3 + 2x^2 - x + 1}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$

Problem 1.2 For $f(x) = x^2 - 4$ and $g(x) = |x|$

- $(g \circ f)(x) = \boxed{|x^2 - 4|}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{x^2 - 4}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{x^4 - 8x^2 + 12}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$

Problem 1.3 For $f(x) = 3x - 5$ and $g(x) = \sqrt{x}$

- $(g \circ f)(x) = \boxed{\sqrt{3x - 5}}$ with domain $\left[\boxed{\frac{5}{3}}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{3\sqrt{x} - 5}$ with domain $\left[\boxed{0}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{9x - 20}$ with domain $\left(\boxed{-\infty}, \boxed{\infty}\right)$

Problem 1.4 For $f(x) = \frac{x}{2x + 1}$ and $g(x) = \frac{2x + 1}{x}$

- $(g \circ f)(x) = \boxed{\frac{4x + 1}{x}}$ with domain $\left(\boxed{-\infty}, \boxed{-\frac{1}{2}}\right) \cup \left(\boxed{-\frac{1}{2}}, \boxed{0}\right) \cup \left(\boxed{0}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{\frac{2x + 1}{5x + 2}}$ with domain $\left(\boxed{-\infty}, \boxed{-\frac{2}{5}}\right) \cup \left(\boxed{-\frac{2}{5}}, \boxed{0}\right) \cup \left(\boxed{0}, \boxed{\infty}\right)$

- $(f \circ f)(x) = \frac{x}{4x+1}$ with domain $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

Problem 1.5 For $f(x) = |x|$ and $g(x) = \sqrt{4-x}$

- $(g \circ f)(x) = \sqrt{4-|x|}$ with domain $[-4, 4]$
 - $(f \circ g)(x) = |\sqrt{4-x}|$ with domain $[-\infty, 4]$
 - $(f \circ f)(x) = |x|$ with domain $[-\infty, \infty]$
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