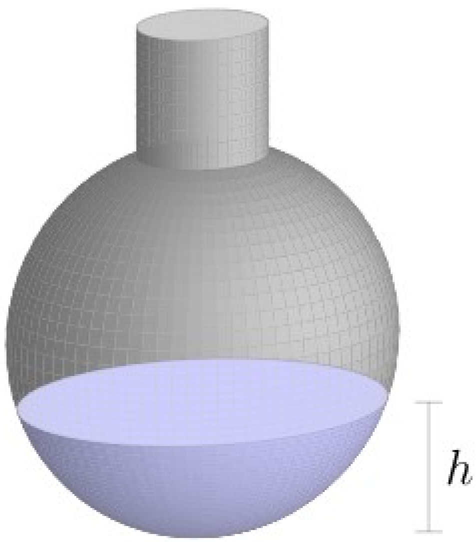
Changing in Tandem - Written Exercises

Problem 1 *Suppose we have an unusual tank whose base is a perfect sphere with radius 3 feet, and then atop the spherical base is a cylindrical "chimney" that is a circular cylinder of radius 1 foot and height 2 feet. The tank is initially empty, but then a spigot is turned on that pumps water into the tank at a constant rate of 1.25 cubic feet per minute.*



Let V denote the total volume of water (in cubic feet) in the tank at any time t (in minutes), and h the depth of the water (in feet) at given time t.

a. *It is possible to use calculus to show that the total volume this tank can hold is $V_{\mathit{full}} = \pi(20 + \frac{38}{3}\sqrt{2}) \approx 119.12$ cubic feet. In addition, the actual height of the tank (from the bottom of the spherical base to the top of the chimney) is $h_{\mathit{full}} = \sqrt{8} + 2 \approx 4.83$ feet. How long does it take the tank to fill? Why?*

b. *On the blank axes provided below, sketch (by hand) possible graphs of how V and t change in tandem and how h and t change in tandem.*

Learning outcomes:
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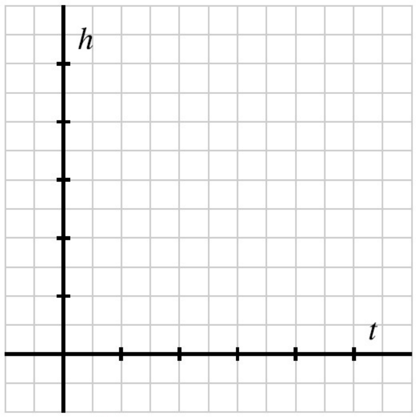
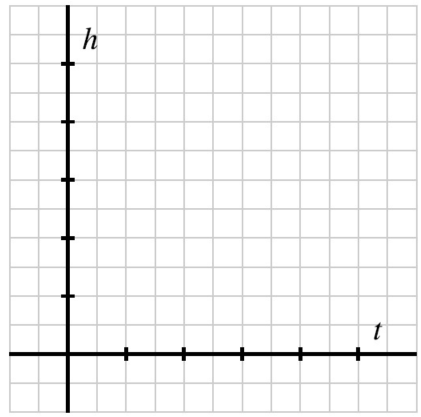




For each graph, label any ordered pairs on the graph that you know for certain, and write at least one sentence that explains why your graphs have the shape they do.

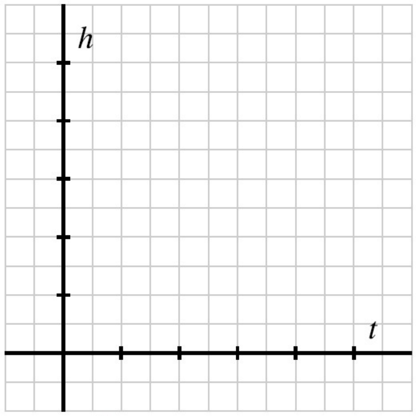
c. *How would your graph(s) change (if at all) if the chimney was shaped like an inverted cone instead of a cylinder? Explain and discuss.*

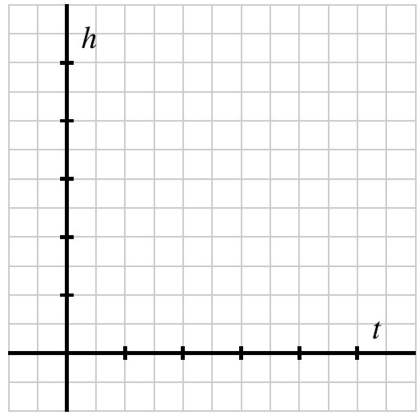
Problem 2 *Suppose we have a tank that is a perfect sphere with radius* 6 *feet. The tank is initially empty, but then a spigot is turned on that is pumping water into the tank in a very special way: the faucet is regulated so that the depth of water in the tank is increasing at a constant rate of* 0.4 *feet per minute.*

Let V denote the total volume of water (in cubic feet) in the tank at any time t (in minutes), and h the depth of the water (in feet) at given time t.

a. *How long does it take the tank to fill? What will the values of V and h be at the moment the tank is full? Why?*

b. *On the blank axes provided below, sketch (by hand) possible graphs of how V and t change in tandem and how h and t change in tandem.*



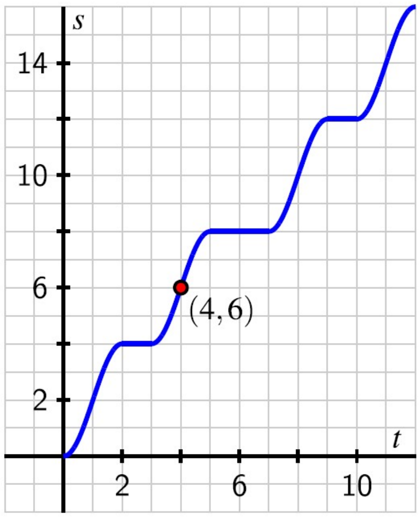


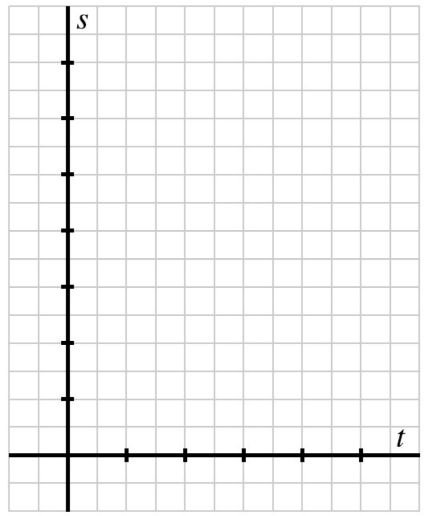
For each graph, label any ordered pairs on the graph that you know for certain, and write at least one sentence that explains why your graphs have the shape they do.

c. *How do your responses change if the tank stays the same but instead the tank is initially full and the tank drains in such a way that the height of the water is always decreasing at a constant rate of* 0.25 *feet per minute?*

Problem 3 *The relationship between the position, s, of a car driving on a straight road at time t is given by the graph pictured at left below. The car's position has units measured in thousands of feet while time is measured in minutes. For instance, the point $(4, 6)$ on the graph indicates that after* 4 *minutes, the car has traveled* 6000 *feet from its starting location.*

You can think of the car's position like mile-markers on a highway. Saying that $s = 500$ means that the car is located 500 feet from "marker zero" on the road.





a. *Write several sentences that explain the how the car is being driven and how you make these conclusions from the graph.*

b. *How far did the car travel between* $t = 2$ *and* $t = 10$*?*

c. *Does the car ever travel in reverse? Why or why not? If not, how would the graph have to look to indicate such motion?*

d. *On the blank axes above, plot points or sketch a curve to describe the behavior of a car that is driven in the following way: from* $t = 0$ *to* $t = 5$ *the car travels straight down the road at a constant rate of* 1000 *feet per minute. At* $t = 5$*, the car pulls over and parks for* 2 *full minutes. Then, at* $t = 7$*, the car does an abrupt U-turn and returns in the opposite direction at a constant rate of* 800 *feet per minute for* 5 *additional minutes. As part of your work, determine (and label) the car's location at several additional points in time beyond* $t = 0, 5, 7, 12$.