

## **Part 1**

# **BuildingNewFunctions**

BNF1.tex

**Exercise 1** Let  $f$  be a function defined by  $f(x) = x^2$  and  $g$  be a function defined by  $g(x) = -5x + 3$ . Use the pair of functions  $f$  and  $g$  to find the following values, if they exist. If the value does not exist, enter DNE.

(a)  $(f + g)(2) = \boxed{-3}$

(b)  $(f - g)(-1) = \boxed{-7}$

(c)  $(g - f)(1) = \boxed{-3}$

(d)  $(f \cdot g)\left(\frac{1}{2}\right) = \boxed{\frac{1}{8}}$

(e)  $\left(\frac{f}{g}\right)(0) = \boxed{0}$

(f)  $\left(\frac{g}{f}\right)(-2) = \boxed{\frac{13}{4}}$

BNF2.tex

**Exercise 2** Let  $f$  be a function defined by  $f(x) = 3x$  and  $g$  be a function defined by  $g(x) = \frac{1}{3x+1}$ . Use the pair of functions  $f$  and  $g$  to find the following values, if they exist. If the value does not exist, enter DNE.

(a)  $(f + g)(2) = \boxed{\frac{43}{7}}$

(b)  $(f - g)(-1) = \boxed{-\frac{5}{2}}$

(c)  $(g - f)(1) = \boxed{-\frac{11}{4}}$

(d)  $(f \cdot g)\left(\frac{1}{2}\right) = \boxed{\frac{3}{5}}$

(e)  $\left(\frac{f}{g}\right)(-2) = \boxed{30}$

(f)  $\left(\frac{g}{f}\right)(0) = \boxed{DNE}$

BNF3.tex

**Exercise 3** Let  $f$  be a function defined by  $f(x) = x^3$  and  $g$  be a function defined by  $g(x) = \frac{1}{x^3}$ . Use the pair of functions  $f$  and  $g$  to find the following values, if they exist. If the value does not exist, enter DNE.

(a)  $(f + g)(2) = \boxed{\frac{65}{8}}$

(b)  $(f - g)(-1) = \boxed{0}$

(c)  $(g - f)(1) = \boxed{0}$

(d)  $(f \cdot g)\left(\frac{1}{2}\right) = \boxed{1}$

(e)  $\left(\frac{f}{g}\right)(0) = \boxed{DNE}$

(f)  $\left(\frac{g}{f}\right)(-2) = \boxed{\frac{1}{64}}$

BNF4.tex

**Exercise 4** Consider the functions  $f$  and  $g$  defined by the table of values below.

$x$	$f(x)$
-3	2
-2	4
-1	0
0	2
1	2
2	3
3	-2

$x$	$g(x)$
-3	-3
-2	-1
-1	-3
0	0
1	3
2	1
3	2

Use the pair of functions  $f$  and  $g$  to find the following values, if they exist. If the value does not exist, enter DNE.

(a)  $(f + g)(-3) = \boxed{-1}$

$$(b) (f - g)(2) = \boxed{2}$$

$$(c) (f \cdot g)(-1) = \boxed{0}$$

$$(d) (g - f)(1) = \boxed{1}$$

$$(e) \left(\frac{f}{g}\right)(0) = \boxed{DNE}$$

$$(f) \left(\frac{f}{g}\right)(3) = \boxed{-1}$$

$$(g) \left(\frac{g}{f}\right)(-1) = \boxed{DNE}$$

$$(h) (g \cdot f)(-2) = \boxed{-4}$$

BNF5.tex

**Exercise 5** Say a store sells various fruits. Let  $B(t)$  represent the number of bananas sold on day  $t$  and  $M(t)$  represent the number of mangos sold on day  $t$ . Say  $P(t)$  represents the price of bananas on day  $t$  and  $Q(t)$  represents the price of mangos on day  $t$ .

- (a) Which of the following represents the total number of bananas and mangos sold on day  $t$ ?

**Multiple Choice:**

- (i)  $(B + M)(t)$  ✓
- (ii)  $(B - M)(t)$
- (iii)  $(B \cdot M)(t)$
- (iv)  $\left(\frac{B}{M}\right)(t)$

- (b) Which of the following represents the total money  $R(t)$  made by selling bananas on day  $t$ ?

**Multiple Choice:**

- (i)  $(B + P)(t)$
- (ii)  $(B - P)(t)$
- (iii)  $(B \cdot P)(t)$  ✓

(iv)  $\left(\frac{B}{P}\right)(t)$

- (c) Which of the following represents the total money  $S(t)$  made by selling mangos on day  $t$ ?

**Multiple Choice:**

- (i)  $(M + Q)(t)$   
 (ii)  $(Q - M)(t)$   
 (iii)  $(M \cdot Q)(t)$  ✓  
 (iv)  $\left(\frac{M}{Q}\right)(t)$

Which of the following represents the total money made by selling bananas and mangos on day  $t$ ?

**Multiple Choice:**

- (a)  $(R + S)(t)$  ✓  
 (b)  $(R - S)(t)$   
 (c)  $(R \cdot S)(t)$   
 (d)  $\left(\frac{R}{S}\right)(t)$

BNF6.tex

## Exercise 6

Let  $h$  be a function defined by  $h(x) = \frac{2 \sin(3x)}{5\sqrt{3x^2}}$ . Which of the following definitions of  $f$  and  $g$  satisfy  $(f \cdot g)(x) = h(x)$ ?

**Multiple Choice:**

- (a)  $f(x) = 2 \sin(3x)$  and  $g(x) = 5\sqrt{3x^2}$   
 (b)  $f(x) = \frac{1}{2 \sin(3x)}$  and  $g(x) = 5\sqrt{3x^2}$   
 (c)  $f(x) = \sin(3x)$  and  $g(x) = \frac{10}{\sqrt{3x^2}}$

(d)  $f(x) = \frac{2\sin(3x)}{5}$  and  $g(x) = \frac{1}{\sqrt{3x^2}}$  ✓

Let  $h$  be a function defined by  $h(x) = 2x^2 + 2x - 2$ . Which of the following definitions of  $f$  and  $g$  satisfy  $(f + g)(x) = h(x)$ ?

**Multiple Choice:**

- (a)  $f(x) = 2x^2$  and  $g(x) = 2x + 2$
- (b)  $f(x) = x^2$  and  $g(x) = x^2 + 2x$
- (c)  $f(x) = x^2 + x - 1$  and  $g(x) = x^2 + x - 1$  ✓
- (d)  $f(x) = x^2$  and  $g(x) = x^2 + 2x - 1$

Let  $h$  be a function defined by  $h(x) = \sin(x) - \cos(x)$ . Which of the following definitions of  $f$  and  $g$  satisfy  $(f - g)(x) = h(x)$ ?

**Multiple Choice:**

- (a)  $f(x) = \sin(x) + \cos(x)$  and  $g(x) = 2\cos(x)$  ✓
- (b)  $f(x) = 2\sin(x)$  and  $g(x) = \cos(x)$
- (c)  $f(x) = \sin(x)$  and  $g(x) = -\cos(x)$
- (d)  $f(x) = -\cos(x)$  and  $g(x) = \sin(x)$

BNF7.tex

## Exercise 7

Let  $h$  be a function defined by  $h(x) = 5\tan(x)$ . Which of the following definitions of  $f$  and  $g$  satisfy  $\left(\frac{f}{g}\right)(x) = h(x)$ ?

**Multiple Choice:**

- (a)  $f(x) = 5\sin(x)$  and  $g(x) = 5\cos(x)$
- (b)  $f(x) = \sin(x)$  and  $g(x) = \cos(x)$
- (c)  $f(x) = \sin(x)$  and  $g(x) = \frac{\cos(x)}{5}$  ✓

(d)  $f(x) = 5 \sin(x)$  and  $g(x) = \frac{1}{5 \cos(x)}$

Let  $h$  be a function defined by  $h(x) = 2 \tan(x)$ . Which of the following definitions of  $f$  and  $g$  satisfy  $(f \cdot g)(x) = h(x)$ ?

**Multiple Choice:**

(a)  $f(x) = \tan(x)$  and  $g(x) = 2 \cos(x)$

(b)  $f(x) = 2 \sin(x)$  and  $g(x) = \frac{1}{\cos(x)}$  ✓

(c)  $f(x) = \cos(x)$  and  $g(x) = 2 \sin(x)$

(d)  $f(x) = \frac{2}{\sin(x)}$  and  $g(x) = \cos(x)$

Let  $h$  be a function defined by  $h(x) = \sin(x) \tan(x)$ . Which of the following definitions of  $f$  and  $g$  satisfy  $(f \cdot g)(x) = h(x)$ ?

**Multiple Choice:**

(a)  $f(x) = (\sin(x))^2$  and  $g(x) = \cos(x)$

(b)  $f(x) = \sin(x)$  and  $g(x) = \frac{1}{\cos(x)}$

(c)  $f(x) = (\sin(x))^2$  and  $g(x) = \frac{1}{\cos(x)}$  ✓

(d)  $f(x) = \cos(x)$  and  $g(x) = \left(\frac{1}{\sin(x)}\right)^2$

BNF8.tex

**Exercise 8** Let's discuss some of the properties of the tangent function.

(a) Recall that the sine function is odd and the cosine function is even. Using the fact that  $\tan(x) = \frac{\sin(x)}{\cos(x)}$ , we can deduce that tangent is

**Multiple Choice:**

(i) odd. ✓

(ii) even.

(iii) *neither odd nor even.*

(b) *Recall that the tangent function has period  $\pi$ . Using the fact that  $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$ , answer the following questions.*

*What is  $\tan\left(\frac{2\pi}{3}\right)$ ?*

**Multiple Choice:**

- (i)  $\sqrt{3}$
- (ii)  $-\sqrt{3}$  ✓
- (iii) 3
- (iv) -3
- (v) -1.72

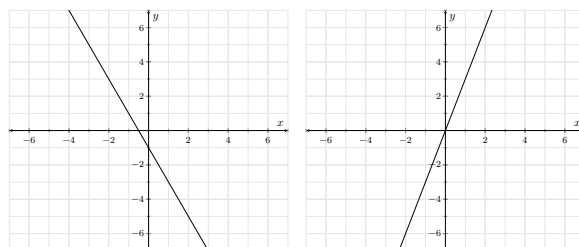
*What is  $\tan\left(\frac{\pi}{3}\right)$ ?*

**Multiple Choice:**

- (i)  $\sqrt{3}$  ✓
- (ii)  $-\sqrt{3}$
- (iii) 3
- (iv) -3
- (v) -1.72

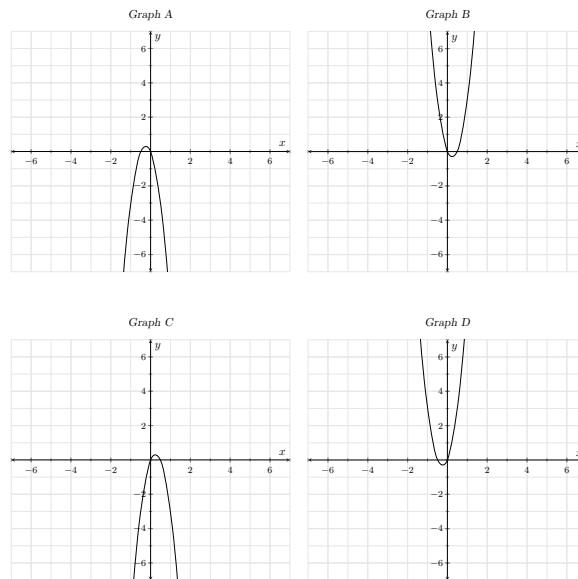
BNF9.tex

**Exercise 9** *Look at the following graphs of the functions  $f$  (on the left) and  $g$  (on the right).*



*Which of the following is the graph of  $f \cdot g$ ?*





**Multiple Choice:**

- (a) *Graph A* ✓
- (b) *Graph B*
- (c) *Graph C*
- (d) *Graph D*