Part 1

Composition of Functions

CoF3.tex

Exercise 1 *Suppose that $r = f(t)$ is the radius, in centimeters, of a circle at time t minutes, and $A(r)$ is the area, in square centimeters, of a circle of radius r centimeters.*

Which of the following statements best explains the meaning of the composite function $(A(f(t))$?

Multiple Choice:

(a) *The area of a circle, in square centimeters, of radius r centimeters.*

(b) *The area of a circle, in square centimeters, at time t minutes.* ✓

(c) *The radius of a circle, in centimeters, at timet minutes.*

(d) *The function f of the minutes and the area.*

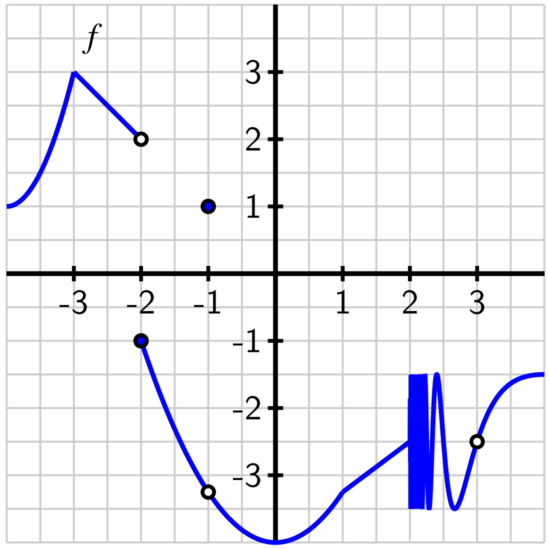
(e) *None of these choices.*

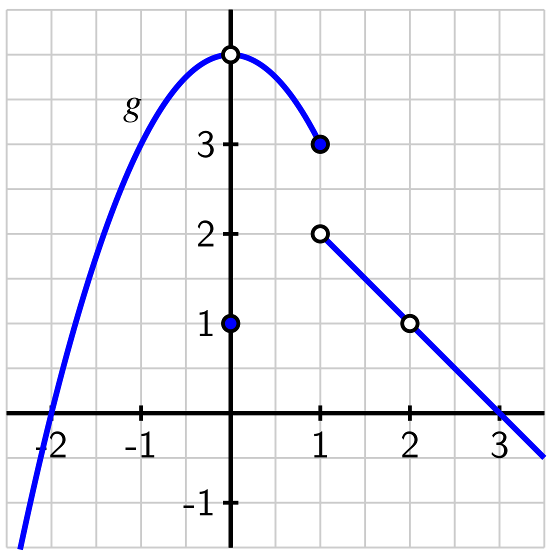
Suppose that $r = f(t) = t^3$. Recall that $A(r) = \pi r^2$. Find $A(f(t) = \boxed{\pi r^6}$.

CoF4.tex

Exercise 2 *Let functions f and g be given by the graphs below.*

An open circle means there is not a point at that location on the graph. For instance, $f(-1) = 1$, but $f(3)$ is not defined. If any answers below are not defined, write"undefined".





Determine:

- $f(f(-2)) = \boxed{1}$
- $f(g(1)) = \boxed{undefined}$
- $g(f(-2)) = \boxed{3}$

- $g(g(0)) = \boxed{3}$
- $g(f(-3)) = \boxed{0}$
- $f(g(2)) = \boxed{undefined}$

CoF5.tex

Exercise 3 *Let functions r and s be defined by the table below.*

t	-4	-3	-2	-1	0	1	2	3	4
$r(t)$	4	1	2	3	0	-3	2	-1	-4
$s(t)$	-5	-6	-7	-8	0	8	7	6	5

Problem 3.1 *Determine:*

- $(s \circ r)(3) = \boxed{-8}$
- $(s \circ r)(-4) = \boxed{5}$
- $(s \circ r)(0) = \boxed{0}$

Problem 3.2 *Select all the values that are in the domain of r.*

Select All Correct Answers:

(a) -8

(b) -7

(c) -6

(d) -5

(e) -4 ✓

(f) -3 ✓

(g) -2 ✓

(h) -1 ✓

(i) 0 ✓

(j) 1 ✓

(k) 2 ✓

(l) 3 ✓

(m) 4 ✓

(n) 5

(o) 6

(p) 7

(q) 8

Problem 3.3 *Select all the values that are in the domain of s.*

Select All Correct Answers:

(a) -8

(b) -7

(c) -6

(d) -5

(e) -4 ✓

(f) -3 ✓

(g) -2 ✓

(h) -1 ✓

(i) 0 ✓

(j) 1 ✓

(k) 2 ✓

(l) 3 ✓

(m) 4 ✓

(n) 5

(o) 6

(p) 7

(q) 8

Problem 3.4 *Select all the values that are in the range of r.*

Select All Correct Answers:

(a) -8

(b) -7

(c) -6

(d) -5

(e) -4 ✓

(f) -3 ✓

(g) -2 ✓

(h) -1 ✓

(i) 0 ✓

(j) 1 ✓

(k) 2 ✓

(l) 3 ✓

(m) 4 ✓

(n) 5

(o) 6

(p) 7

(q) 8

Problem 3.5 *Select all the values that are in the range of s.*

Select All Correct Answers:

(a) -8 ✓

(b) -7 ✓

(c) -6 ✓

(d) -5 ✓

(e) -4

(f) -3

(g) -2

(h) -1

(i) 0 ✓

(j) 1

(k) 2

(l) 3

(m) 4

(n) 5 ✓

(o) 6 ✓

(p) 7 ✓

(q) 8 ✓

Problem 3.6 *Select all the values that are in the domain of $s \circ r$.*

Select All Correct Answers:

(a) -8

(b) -7

(c) -6

(d) -5

(e) -4 ✓

(f) -3 ✓

(g) -2 ✓

(h) -1 ✓

(i) 0 ✓

(j) 1 ✓

(k) 2 ✓

(l) 3 ✓

(m) 4 ✓

(n) 5

(o) 6

(p) 7

(q) 8

Problem 3.7 *Select all the values that are in the domain of $r \circ s$.*

Select All Correct Answers:

(a) -8

(b) -7

(c) -6

(d) -5

(e) -4

(f) -3

(g) -2

(h) -1

(i) 0 ✓

(j) 1

(k) 2

(l) 3

(m) 4

(n) 5

(o) 6

(p) 7

(q) 8

CoF6.tex

Exercise 4 *For each of the following functions, find two simpler functions f and g such that the given function can be written as a composite function $g \circ f$. The functions f and g should each be a famous function or a polynomial.*

- *If $g(f(x)) = \sin(x^2)$, then we could decompose this function into $g(x) = \boxed{\sin(x)}$ and $f(x) = \boxed{x^2}$.*
- *If $g(f(x)) = \sqrt{2x^5 - 7}$, then we could decompose this function into $g(x) = \boxed{\sqrt{x}}$ and $f(x) = \boxed{2x^5 - 7}$.*
- *If $g(f(x)) = e^{3x-x^2}$, then we could decompose this function into $g(x) = \boxed{e^x}$ and $f(x) = \boxed{3x - x^2}$.*
- *If $g(f(x)) = |\ln(x)|$, then we could decompose this function into $g(x) = \boxed{|x|}$ and $f(x) = \boxed{\ln(x)}$.*
- *If $g(f(x)) = 5e^{4x} + 7e^{3x} - 11e^x + 4$, then we could decompose this function into $g(x) = \boxed{5x^4 + 7x^3 - 11x + 4}$ and $f(x) = \boxed{e^x}$.*

CoF7.tex

Exercise 5 *Use the given pair of functions to find the following values if they exist. If the value is not defined, write "undefined".*

Problem 5.1 $f(x) = x^2$, $g(x) = 2x + 1$

- $(g \circ f)(0) = \boxed{1}$
- $(f \circ g)(-1) = \boxed{1}$
- $(f \circ f)(2) = \boxed{16}$
- $(g \circ f)(-3) = \boxed{19}$

- $(f \circ g)\left(\frac{1}{2}\right) = \boxed{4}$
- $(f \circ f)(-2) = \boxed{16}$

Problem 5.2 $f(x) = |x-1|$, $g(x) = x^2 - 5$

- $(g \circ f)(0) = \boxed{-4}$
- $(f \circ g)(-1) = \boxed{5}$
- $(f \circ f)(2) = \boxed{0}$
- $(g \circ f)(-3) = \boxed{11}$
- $(f \circ g)\left(\frac{1}{2}\right) = \boxed{\frac{23}{4}}$
- $(f \circ f)(-2) = \boxed{2}$

Problem 5.3

$f(x) = \sqrt{3-x}$, $g(x) = x^2 + 1$

- $(g \circ f)(0) = \boxed{4}$
- $(f \circ g)(-1) = \boxed{1}$
- $(f \circ f)(2) = \boxed{\sqrt{2}}$
- $(g \circ f)(-3) = \boxed{7}$
- $(f \circ g)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{7}}{2}}$
- $(f \circ f)(-2) = \boxed{\sqrt{3-\sqrt{5}}}$

Problem 5.4 $f(x) = \sqrt[3]{x+1}$, $g(x) = 4x^2 - x$

- $(g \circ f)(0) = \boxed{3}$

- $(f \circ g)(-1) = \boxed{\sqrt[3]{6}}$
- $(f \circ f)(2) = \boxed{\sqrt[3]{\sqrt[3]{3}+1}}$
- $(g \circ f)(-3) = \boxed{4\sqrt[3]{4} + \sqrt[3]{2}}$
- $(f \circ g)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt[3]{12}}{2}}$
- $(f \circ f)(-2) = \boxed{0}$

Problem 5.5 $f(x) = \dfrac{3}{1-x}$, $g(x) = \dfrac{4x}{x^2+1}$

- $(g \circ f)(0) = \boxed{\frac{6}{5}}$
- $(f \circ g)(-1) = \boxed{1}$
- $(f \circ f)(2) = \boxed{\frac{3}{4}}$
- $(g \circ f)(-3) = \boxed{\frac{48}{25}}$
- $(f \circ g)\left(\frac{1}{2}\right) = \boxed{-5}$
- $(f \circ f)(-2) = \boxed{undefined}$

`CoF8.tex`

Exercise 6 *Use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.*

Problem 6.1 *For* $f(x) = x^2 - x + 1$ *and* $g(x) = 3x - 5$

- $(g \circ f)(x) = \boxed{3x^2 - 3x - 2}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$

- $(f \circ g)(x) = \boxed{9x^2 - 33x + 31}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{x^4 - 2x^3 + 2x^2 - x + 1}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$

Problem 6.2 *For* $f(x) = x^2 - 4$ *and* $g(x) = |x|$

- $(g \circ f)(x) = \boxed{|x^2 - 4|}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{x^2 - 4}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{x^4 - 8x^2 + 12}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$

Problem 6.3 *For* $f(x) = 3x - 5$ *and* $g(x) = \sqrt{x}$

- $(g \circ f)(x) = \boxed{\sqrt{3x - 5}}$ *with domain* $\left[\boxed{\frac{5}{3}}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{3\sqrt{x} - 5}$ *with domain* $\left[\boxed{0}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{9x - 20}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$

Problem 6.4 *For* $f(x) = \dfrac{x}{2x + 1}$ *and* $g(x) = \dfrac{2x + 1}{x}$

- $(g \circ f)(x) = \boxed{\dfrac{4x + 1}{x}}$ *with domain* $\left(\boxed{-\infty}, \boxed{-\frac{1}{2}}\right) \cup \left(\boxed{-\frac{1}{2}}, \boxed{0}\right), \cup \left(\boxed{0}, \boxed{\infty}\right)$
- $(f \circ g)(x) = \boxed{\dfrac{2x + 1}{5x + 2}}$ *with domain* $\left(\boxed{-\infty}, \boxed{-\frac{2}{5}}\right) \cup \left(-\boxed{\frac{2}{5}}, \boxed{0}\right) \cup \left(\boxed{0}, \boxed{\infty}\right)$
- $(f \circ f)(x) = \boxed{\dfrac{x}{4x + 1}}$ *with domain* $\left(\boxed{-\infty}, \boxed{-\frac{1}{2}}\right) \cup \left(\boxed{-\frac{1}{2}}, \boxed{-\frac{1}{4}}\right) \cup \left(\boxed{-\frac{1}{4}}, \boxed{\infty}\right)$

Problem 6.5 *For* $f(x) = |x|$ *and* $g(x) = \sqrt{4-x}$

- $(g \circ f)(x) = \boxed{\sqrt{4-|x|}}$ *with domain* $\left[\boxed{-4}, \boxed{4}\right]$
- $(f \circ g)(x) = \boxed{|\sqrt{4-x}|}$ *with domain* $\left(\boxed{-\infty}, \boxed{4}\right]$
- $(f \circ f)(x) = \boxed{|x|}$ *with domain* $\left(\boxed{-\infty}, \boxed{\infty}\right)$

CoF1.tex

Exercise 7 *Let* $f(x) = \dfrac{1}{x}$.

(a) *Compute* $AV_{[x,x+1]}$. *Assume* $[x, x+1]$ *is in the domain of* f. *Your answer will involve the variable* x.

$AV_{[x,x+1]} = \boxed{-\dfrac{1}{x^2+x}}$.

(b) *Compute* $AV_{[x,x+h]}$. *Assume* $[x, x+h]$ *is in the domain of* f. *Your answer will involve the variables* x *and* h.

$AV_{[x,x+h]} = \boxed{-\dfrac{1}{x^2+xh}}$.

CoF2.tex

Exercise 8 *Let* $f(x) = x^3$.

(a) *Compute* $AV_{[2,2+h]}$. *Your answer will involve the variable* h.

$AV_{[2,2+h]} = \boxed{12+6h+h^2}$.

(b) *Compute* $AV_{[x,x+2]}$. *Your answer will involve the variable* x.

$AV_{[x,x+2]} = \boxed{3x^2+6x+4}$.

(c) *Compute* $AV_{[x,x+h]}$. *Your answer will involve the variables* x *and* h.

$AV_{[x,x+h]} = \boxed{3x^2+3xh+h^2}$.