① (a) $f(x) = \begin{cases} cx & \text{if } 0 \le x \le 4 \\ 0 & \text{otherwise.} \end{cases}$

$\cancel{\int_0^4 f(x)\,dx = 1}$ $\cancel{\int_0^4 cx}$ $\int_0^4 f(x)\,dx = \underline{1}$

$\int_0^4 cx\,dx = 1. \quad \left( = \frac{cx^2}{2} \right)$

$8c = 1.$

$\therefore c = 1/8.$

(b) Find $P(-1 \le x \le 1)$.

$= \int_{-1}^{1} \frac{x}{8}\,dx$

$= 0$

(c) $P(X \geq 2)$

$\int_2^4 cx\,dx = \frac{x^2}{16}$

$\frac{4^2}{16} - \frac{2^2}{16}. \quad 1 - \frac{4}{16}$

$1 - \frac{1}{4} = 3/4.$

(d) $P(X < 3 \mid X > 1)$

$\int_1^3 cx\,dx = \frac{3^2}{16} - \frac{1^2}{16} = \frac{8}{16} = 1/2 = 0.5$

(e) $E|X| = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^4 x\,cx\,dx$

$= \frac{x^3}{16} = \frac{4^3}{16} = 4$

(f) $Var(x) = E[X^2] - (E|X|)^2$

$= \int_0^4 x f(x)\,dx - (E|X|)^2$

$= \frac{4^6}{16} - 4^2 = \frac{4^6}{4^2} - 4^2$

$= \frac{4^6 - 4^4}{4^2} = \frac{4^2}{4^2} = 1.$

$\sigma(x) = \sqrt{1} = 1$

g) CDF $F(x)$

~~Calculating the Probability Distribution table~~

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X=x)$ | | | | | |

$F(x) = \int_{-\infty}^{4} f(t)\,dt$

$= \begin{cases} 0 \\ \int_0^x \frac{1}{4-0}\,dt = \frac{x}{4-0} - \frac{0}{4-0} = \frac{x}{4} \\ 1 \end{cases}$

(2) a) $P(-2 < x \leq 0) = \int_{-2}^{0} \frac{3}{2} x^2 \, dx = \frac{3}{2} \int_{-2}^{0} x^2 \, dx = \frac{3}{2} \left[ \frac{0^3}{3} - \frac{(-2)^3}{3} \right]$

$= 4$

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b) $P(x > -0.5) = \int_{-0.5}^{1} \frac{3}{2} x^2 dx = \frac{3}{2} \left[ \frac{1^3}{3} - \frac{(-0.5)^3}{3} \right] = \frac{3}{2} \left[ \frac{1}{3} + \frac{0.125}{3} \right]$

$= \cancel{\frac{3}{2} \times \frac{0.875}{3} = \frac{0.875}{2} = 0.4375} = \frac{3}{2} \times \frac{1.125}{3} = 0.5625$

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c) $P(x > 0.5 \mid x > -0.5) = \frac{P(x > 0.5 \text{ and } x > -0.5)}{P(x > -0.5)} = \frac{P(x > 0.5)}{P(x > -0.5)}$.

$= \cancel{\frac{0.5625}{0.4375} = 1.286} = \frac{0.4375}{0.5625} = 0.778$.

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d). $E[x] = \int_{-1}^{1} x f(x) dx = \frac{3}{2} \int_{-1}^{1} x^3 dx = \frac{3}{2} \left[ \frac{x^4}{4} \Big|_{-1}^{1} \right] = \frac{3}{2} \left[ \frac{1}{4} - \frac{1}{4} \right]$

$= 0$

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e). $Var(x) = E[x^2] - (E|x|)^2 = \int_{-1}^{1} x f(x) dx - (E|x|)^2 = 0$

~~a~~ $\sigma_x = 0$.

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f). CDF $F(x)$