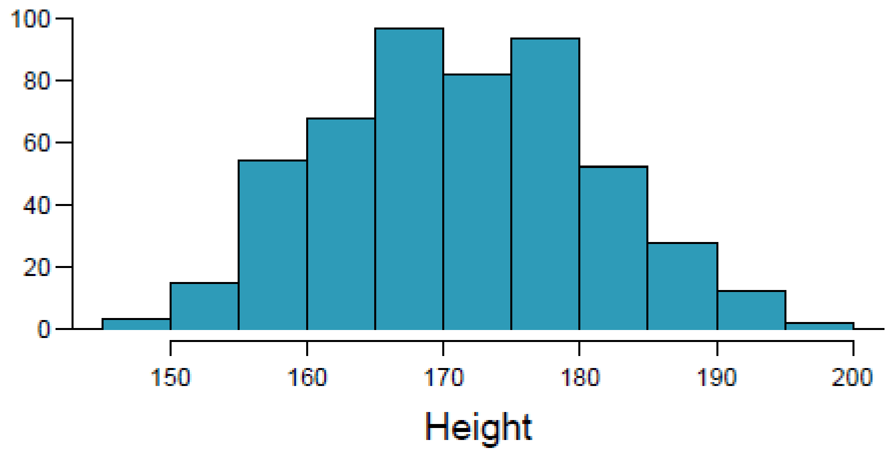
4.4 Heights of adults. Researchers studying anthropometry collected body girth measurements and skeletal diameter measurements, as well as age, weight, height and gender, for 507 physically active individuals. The histogram below shows the sample distribution of heights in centimeters.[38]



Min	147.2
Q1	163.8
Median	170.3
Mean	171.1
SD	9.4
Q3	177.8
Max	198.1

(a) What is the point estimate for the average height of active individuals? What about the median? **(See the next page for parts (b)-(e).)**

(b) What is the point estimate for the standard deviation of the heights of active individuals? What about the IQR?

(c) Is a person who is 1m 80cm (180 cm) tall considered unusually tall? And is a person who is 1m 55cm (155cm) considered unusually short? Explain your reasoning.

(d) The researchers take another random sample of physically active individuals. Would you expect the mean and the standard deviation of this new sample to be the ones given above? Explain your reasoning.

(e) The sample means obtained are point estimates for the mean height of all active individuals, if the sample of individuals is equivalent to a simple random sample. What measure do we use to quantify the variability of such an estimate (Hint: recall that $SD_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$)? Compute this quantity using the data from the original sample under the condition that the data are a simple random sample.

Ans:

a) The point estimate for the average height (mean) is 171.1 and the median is 170.3

b) The point estimate for the standard deviation is 9.4 and IQR is (177.8 – 163.8) = 14

c) The Z score for 180 cms = (180 – 171.1) / 9.4
= 0.9468

This translates to a probability of 82.81%. I would not consider this as unusual.

The Z score for 155 cms = (155 – 171.1) / 9.4

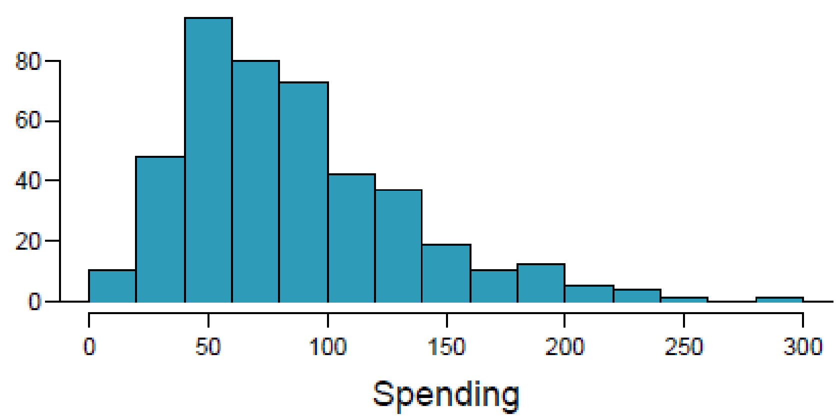
= -1.713

This translates to a probability of 4.34 %. I would consider this as unusual.

d) It is highly unlikely that the new sample will have the same mean and standard deviation as there will always be sampling variations.

e) The Standard Error (SE) quantifies the standard deviation of the sampling distribution.
For our set of samples, SE = 9.4 / sqrt(507)
= 0.4175

4.14 Thanksgiving spending, Part I. The 2009 holiday retail season, which kicked off on November 27, 2009 (the day after Thanksgiving), had been marked by somewhat lower self-reported consumer spending than was seen during the comparable period in 2008. To get an estimate of consumer spending, 436 randomly sampled American adults were surveyed. Daily consumer spending for the six-day period after Thanksgiving, spanning the Black Friday weekend and Cyber Monday, averaged \$84.71. A 95% confidence interval based on this sample is (\$80.31, \$89.11). Determine whether the following statements are true or false, and explain your reasoning.



(a) We are 95% confident that the average spending of these 436 American adults is between \$80.31 and \$89.11.
(b) This confidence interval is not valid since the distribution of spending in the sample is right skewed.
(c) 95% of random samples have a sample mean between \$80.31 and \$89.11.
(d) We are 95% confident that the average spending of all American adults is between \$80.31 and \$89.11.
(e) A 90% confidence interval would be narrower than the 95% confidence interval since we don't need to be as sure about our estimate.
(f) In order to decrease the margin of error of a 95% confidence interval to a third of what it is now, we would need to use a sample 3 times larger.
(g) The margin of error is 4.4.

Ans:

a) TRUE – The confidence interval is the likelihood that the **Population mean** falls within the range and not the just the sample mean (since the sample a subset of the population). It means that we are 95% confident that average spending of the population is between $80.31 and $89.11 and not just the sample's average spend.

b) TRUE – Given that the sample is right skewed and the CI has been built using this sample, I would think that this is invalid.

c) FALSE – The CI mentioned above is only for 1 sample. A 95% confidence means that if we take many samples and constructed CI for these sample means then 95% of those CIs will have the population mean.

d) TRUE. This is the right interpretation.

e) TRUE – A 90% CI is more narrower that a 95% CI.

f) Let's first calculate the current margin of error for a 95% CI.

The standard deviation	= (upperlimit – lowerlimit) / (Z / 2)
	= (89.11 - 80.31) / (1.96 / 2)
	= 2.2449
Current Margin of Error	= Z * SE
	= Z * (sd / SQRT(n))
	= 1.96 * (2.2449 / SQRT(436))
	= 0.2107

To decrease this to a third would be to have the Margin of error to be

(0.2107 / 3) = 0.0702

Using this value in the above formulae, we get the sample size to be

$$n = (Z * sd / ME)^2$$

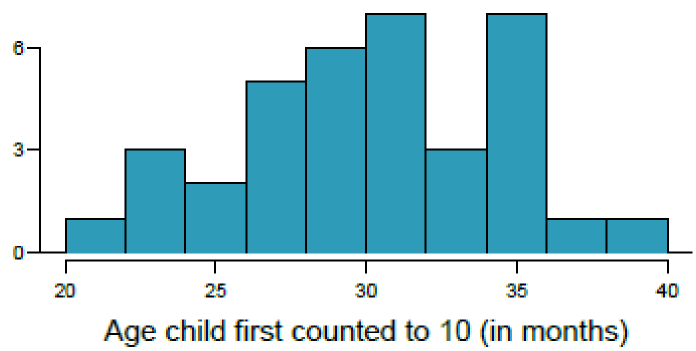
$$= (1.96 * 2.2449 / 0.0702)^2$$

$$= 3929$$

So to get a third of current margin of error, we need 3929 samples which is more than a third of 436.

g) FALSE – As calculated above, the margin of error is 0.2107

4.24 Gifted children, Part I. Researchers investigating characteristics of gifted children collected data from schools in a large city on a random sample of thirty-six children who were identified as gifted children soon after they reached the age of four. The following histogram shows the distribution of the ages (in months) at which these children first counted to 10 successfully. Also provided are some sample statistics.[43]



n	36
min	21
mean	30.69
sd	4.31
max	39

(a) Are conditions for inference satisfied?

(b) Suppose you read online that children first count to 10 successfully when they are 32 months old, on average. Perform a hypothesis test to evaluate if these data provide convincing evidence that the average age at which gifted children fist count to 10 successfully is less than the general average of 32 months. Use a significance level of 0.10.

(c) Interpret the p-value in context of the hypothesis test and the data.

(d) Calculate a 90% confidence interval for the average age at which gifted children first count to 10 successfully.

(e) Do your results from the hypothesis test and the confidence interval agree? Explain.

Ans:

a) The sample size is 36 which is more than the 30 stipulated. The distribution appears to be roughly normal. I would say that the conditions for inference are satisfied.

b) H_0 = mean = 32

H_A = mean < 32

Significance Level = 0.1

SE = sd / sqrt(n)

$= 4.32 / sqrt(36)$

$= 0.72$

$Z = (30.69 – 32) / 0.72$

$= -1.819444$

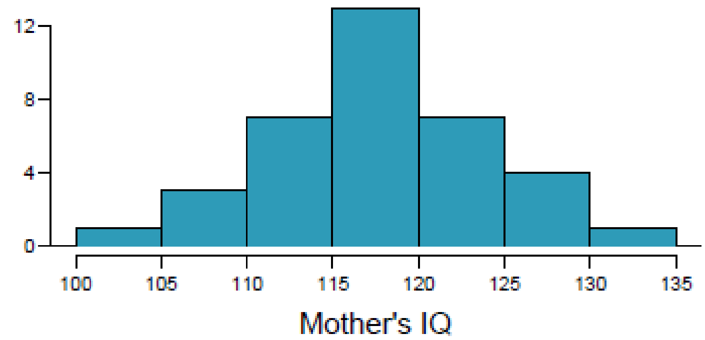
Calculating the probability for this Z, we get 0.0341.

c) The p-value of 0.0341 is much lower than 0.1. This suggests that the mean of 30.69 is not close to the average of 32 months. Therefore we can reject H_0 in favor of H_A

d) Z is calculated as 1.2816 for 90% CI. Now to calculate the CI:
Lower = mean – (Z * SE)
= 30.69 – (1.2816 * 0.72)
= 29.76725
Upper = mean + (Z * SE)
= 30.69 + (1.2816 * 0.72)
= 31.61275
Hence the CI is (29.76725, 31.61275)

e) The results agree because 32 is not within the range of the CI and hence the rejection of the null hypothesis.

4.26 Gifted children, Part II. Exercise 4.24 describes a study on gifted children. In this study, along with variables on the children, the researchers also collected data on the mother's and father's IQ of the 36 randomly sampled gifted children. The histogram below shows the distribution of mother's IQ. Also provided are some sample statistics.



n	36
min	101
mean	118.2
sd	6.5
max	131

(a) Perform a hypothesis test to evaluate if these data provide convincing evidence that the average IQ of mothers of gifted children is different than the average IQ for the population at large, which is 100. Use a significance level of 0.10.

(b) Calculate a 90% confidence interval for the average IQ of mothers of gifted children.

(c) Do your results from the hypothesis test and the confidence interval agree? Explain.

Ans:

a) H_0 = mean = 100

H_A = mean > 100

Significance Level = 0.1

SE = sd / sqrt(n)

= 6.5 / sqrt(36)

= 1.0833

Z = (118.2 – 100) / 1.0833

= 16.80

Calculating the probability for this Z, we get a value so small that it is practically 0. This is much lower than the significance level of 0.1. This suggests the IQs of mothers of gifted children mean 118.2 not even close to 100. Therefore, we reject the null hypothesis in favor of the alternative.

b) Z is calculated as 1.2816 for 90% CI. Now to calculate the CI:

Lower = mean – (Z * SE)

= 118.2 – (1.2816 * 1.0833)

= 116.8116

Upper = mean + (Z * SE)

= 118.2 + (1.2816 * 1.0833)

= 119.5884

Hence the CI is (116.8116, 119.5884)

c) The results agree because 100 is not within the range of the CI and hence the rejection of the null hypothesis.

4.34 CLT. Define the term "sampling distribution" of the mean, and describe how the shape, center, and spread of the sampling distribution of the mean change as sample size increases.

Ans:

"Sampling distribution" of the mean is the distribution of mean values calculated from repeated sampling of a population. As the sample size increases, the shape of the distribution tends to be more normal. The spread of the distribution gets smaller as the sample size increases.

4.40 CFLBs. A manufacturer of compact fluorescent light bulbs advertises that the distribution of the lifespans of these light bulbs is nearly normal with a mean of 9,000 hours and a standard deviation of 1,000 hours.

(a) What is the probability that a randomly chosen light bulb lasts more than 10,500 hours?

(b) Describe the distribution of the mean lifespan of 15 light bulbs.

(c) What is the probability that the mean lifespan of 15 randomly chosen light bulbs is more than 10,500 hours?

(d) Sketch the two distributions (population and sampling) on the same scale.

(e) Could you estimate the probabilities from parts (a) and (c) if the lifespans of light bulbs had a skewed distribution?

Ans:

a) The Z score = (10500 – 9000) / 1000

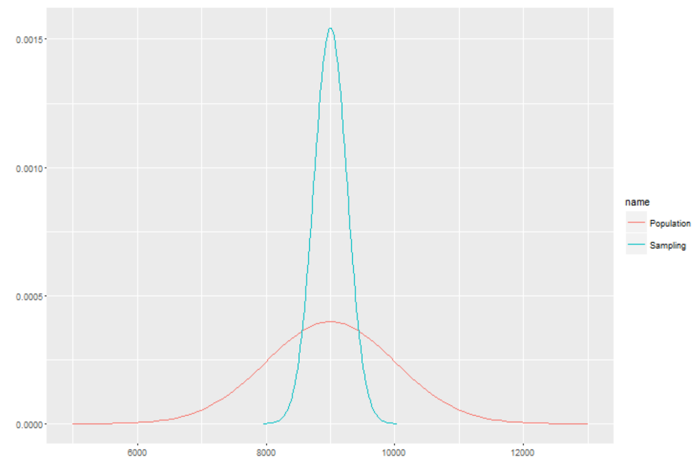
= 1.5

The probability associated with this Z value is 0.933. Since we want the upper tail, we get the probability of lasting more than 10,500 hours = (1 - 0.933) = **0.0668 = 6.68%**

b) Since the lifespans are supposed to be nearly normal, we can assume that a random sampling would also yield a distribution that is nearly normal and is centered around the mean of 9000 hours.

c) The probability of 1 light bulb lasting more than 10,500 hours is 6.68%. Since the probabilities are independent of each other, the probability of 15 light bulbs lasting more than 10,500 hours is 6.68% ^ 15 = 2.35E^{-18} which is practically 0.

d) Sketch of the Population and Sampling distribution



e) No, this approach would not be useful if the distribution was skewed.

4.48 Same observation, different sample size. Suppose you conduct a hypothesis test based on a sample where the sample size is $n = 50$, and arrive at a p-value of 0.08. You then refer back to your notes and discover that you made a careless mistake, the sample size should have been $n = 500$. Will your p-value increase, decrease, or stay the same? Explain.

Ans: As the sample size increases, the p-value decreases. This is because of the fact that as the sample size increases the SE decreases which in turn affects the calculation of the p-value.