hw02_q9

We have:

$$precision = \frac{TP}{TP + FP} \qquad q.1$$

$$sensitivity = \frac{TP}{TP + FN} \qquad Eq.2$$

From Eq.1, let's find the bounds of the precision relative to TP:

$$\lim_{TP \to 0} \frac{TP}{TP + FP} = \frac{0}{0 + FP}$$

$$\lim_{TP \to 0} \frac{TP}{TP + FP} = 0$$

$$\lim_{TP \to \infty} \frac{TP}{TP + FP} = \frac{\frac{TP}{TP}}{\frac{TP}{TP} + \frac{FP}{TP}}$$

$$\lim_{TP \to \infty} \frac{TP}{TP + FP} = \frac{1}{1 + \frac{FP}{TP}}$$

$$\lim_{TP \to \infty} \frac{TP}{TP + FP} = \frac{1}{1 + 0}$$

$$\lim_{TP \to \infty} \frac{TP}{TP + FP} = 1$$

from Eq.1, let's find the bounds of the precision relative to FP:

$$\lim_{FP \to 0} \frac{TP}{TP + FP} = \frac{TP}{0 + TP}$$

$$\lim_{FP \to 0} \frac{TP}{TP + FP} = 1$$

$$\lim_{FP \to \infty} \frac{TP}{TP + FP} = \frac{\frac{TP}{FP}}{\frac{TP}{FP} + \frac{FP}{FP}}$$

$$\lim_{FP \to \infty} \frac{TP}{TP + FP} = \frac{0}{0 + \frac{FP}{FP}}$$

$$\lim_{FP \to \infty} \frac{TP}{TP + FP} = \frac{0}{0 + 1}$$

$$\lim_{FP \to \infty} \frac{TP}{TP + FP} = 0$$

Hence, the precision $\frac{TP}{TP+FP}$ has bounds of [0,1]

Hence, also following the same steps above, the sensitivity $\frac{TP}{TP+FN}$ has bounds of [0,1]

Therefore, we can state the following:

$$0 \leq precision \leq 1$$

$$0 \leq sensitivity \leq 1$$

$$0 \leq precision * sensitivity \leq 1 * sensitivity$$

$$0 \leq 2 * precision * sensitivity \leq 2 * sensitivity \qquad Eq.3$$

Also, since the max sensitivity is 1 and max precision is 1 as shown above, we can state:

$$0 \leq precision + sensitivity \leq 2 \quad Eq.4$$

Now divide Eq.3 by Eq.4 we get:

$$0 \leq (2 * precision * sensitivity)/(precision + sensitivity) \leq 2 * sensitivity/2$$

$$0 \leq (2 * precision * sensitivity)/(precision + sensitivity) \leq sensitivity qquad Eq.5$$

We know that the sensitivity max value 1. Hence, Eq.5 has a lower bound of 0 and upper bound of 1.

Therefore, F1 score will always be between 0 and 1.