Homework 2: Friday, 13 September, 2019

Use constructions of various Hadamard codes using a normalized Hadamard matrix to prove all of the following results. Assume that Hadamard matrices of appropriate orders exist (*state which orders you would require*). Also assume that d is even.

1. Show that $A(2d, d) = 4d$.

2. Let $2d > n \geq d$ and let d be a positve even integer. Let k denote the number $\left\lfloor \frac{d}{2d-n} \right\rfloor$ and let:

$$a = d(2k+1) - n(k+1) \qquad b = kn - d(2k-1)$$

   Show that a and b are both non-negative integers.

3. Show that:

$$n = (2k-1)a + (2k+1)b \qquad d = ka + (k+1)b$$

4. Show that:

   (i) If n is even then so are both a and b.

   (ii) If n is odd and k is even then b is even.

   (iii) If n is odd and k is also odd then a is even.

5. Let C_i be (n_i, M_i, d_i) code with $i = 1, 2$. Depending on whether $M_1 > M_2$ (or the other way round), we chop off appropriate codewords to construct a new code $C = aC_1 \oplus bC_2$ by taking a copies of C_1 and b copies of C_2 (and concatenate them). Show that C is an (n, M, d) code with $M = \min\{M_1, M_2\}$ and $n = an_1 + bn_2$ and $d \geq ad_1 + bd_2$.

6. Show that if $n \geq 4$ is the order of a Hadamard matrix, then A_n which is an $(n-1, n, \frac{n}{2})$ code exists. Also show that by considering only those codewords of A_n that have a 0 on the first coordinate we get R_n an $(n-2, \frac{n}{2}, \frac{n}{2})$ code.

7. construct C as follows:

(i) If n is even then:

$$C = \frac{a}{2}R_{4k} \oplus \frac{b}{2}R_{4k+4}$$

(ii) If n is odd and k is even then:

$$C = aA_{2k} \oplus \frac{b}{2}R_{4k+4}$$

(iii) If n is odd and k is odd then:

$$C = \frac{a}{2}R_{4k} \oplus bA_{2k+2}$$

In all the cases, show that C meets the Plotkin bound: $M = 2k$.