Complexity Theory: Assignment 1

asifkhan@cmi.ac.in, arsricharan@cmi.ac.in

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1. If $EXP \neq NEXP$, then $P \neq NP$.

2. Let $HALT$ be the algorithm which on input (α, x) runs the machine M_α encoded by α on x and accepts if M_α halts on x. Show that $HALT$ is NP-hard. Is it NP-complete?

3. If L_1, L_2 are in NP, then are $L_1 \cup L_2$ and $L_1 \cap L_2$ in NP?

4. Show that $TAUTOLOGY$ is $coNP$-complete. Show that $NP = coNP$ if and only if $3SAT$ and $TAUTOLOGY$ are polynomial-time reducible to one another.

5. Prove that if every unary NP language is in P, then $EXP = NEXP$. A unary language is a subset of $\{1\}^n$.

6. Let $\Sigma_2 SAT$ denote the following problem: Given a formula ψ of the form

$$\psi = \exists x \forall y (\phi(x, y) = 1).$$

where ϕ is a CNF formula, need to decide if ψ is true. Prove that if $P = NP$, then $\Sigma_2 SAT$ is in P.