Topics in Algorithms - End-semester examination

August 6, 2020

- 1. You are given an instance (G = (V, E), s, t) of the max-flow problem, and a non-negative integer B. Let f be a max-flow in G. You are allowed to increase the capacities of some edges, subject to a total increase of B units, so as to achieve a maximum increase in the max-flow value. Design an algorithm for this problem and prove the correctness. If you think the problem is NP-hard, justify.
- 2. There are n items and n sellers. Each seller j charges a cost c_{ij} for item i. The goal is to buy one item from each seller so as to minimize cost. Design an algorithm using flows and prove its correctness.
- 3. Consider the following statements about a stable marriage instance with strict and complete lists, M be the matching obtained by executing the Gale-Shapley algorithm:
 - (a) In M, at least one person gets his/her first choice.
 - (b) In M, at least one person gets a match from the first half of his/her list.

Are these statements true/false? Justify.