Assignment 3

1. Prove that the Schmidt decomposition gives a way to identity if a pure state is entangled or product. In particular, prove that a pure bipartite state is entangled if and only if it has more than one Schmidt coefficent.

2. Prove that Trace is cyclic. That is

$$Tr(ABC) = Tr(CAB) = Tr(BCA)$$

Prove that:

$$Tr_B \{ |x_1\rangle\langle x_2|_A \otimes |y_1\rangle\langle y_2|_B \}$$

$$= \sum_i \langle i|_B (|x_1\rangle\langle x_2|_A \otimes |y_1\rangle\langle y_2|_B) |i\rangle_B$$

$$= |x_1\rangle\langle x_2|_A \langle y_2|y_1\rangle$$

Where $\{ |i\rangle_B \}$ is a system of orthonormal basis for Bob's side.

3. Dephasing channel is the following:

$$\rho \longrightarrow (1-p)\,\rho + \cancel{p}\; p\, Z \rho Z$$ where

$0 < p < 1$ and ρ is a density operator. Prove that it is indeed a channel. Verify that its action on the Bloch vector is the following:

$$\frac{1}{2}\left(I + r_x X + r_y Y + r_z Z\right)$$
$$\longrightarrow \frac{1}{2}\left(I + (1-2p) r_x X + (1-2p) r_y Y + r_z Z\right)$$

4. Prove that a positive operator is necessarily Hermitian.