

## Homework 1: Tuesday, 27 August, 2019

1. Let  $q \geq 2$  be a natural number and let  $\Gamma$  be a set of  $q$  alphabets. A  $q$ -ary code  $C$  over  $\Gamma$  is a subset of  $\Gamma^n$ . Define the Hamming distance between two words  $x = (x_1x_2 \cdots x_n)$  and  $y = (y_1y_2 \cdots y_n)$  by  $d(x, y) = |\{i : x_i \neq y_i\}|$ .
  - (a) Show that  $d$  is a metric on  $\Gamma^n$ .
  - (b) Show that in this general set up,  $C$  can correct up to  $d$  errors if the minimum distance of  $C$  is  $2d + 1$ .
  - (c) State and prove the sphere packing bound in this general set up of a  $q$ -ary code with minimum distance  $2d + 1$ .
2. Let  $q$  be a prime power and let  $H$  be a matrix with  $m$  rows whose columns are representatives of (all the) 1-dimensional subspaces of  $V(m, q)$ .
  - (a) How many columns does  $H$  have?
  - (b) Show that with  $H$  as a parity check matrix we have a perfect single error correcting  $q$ -ary code **H**.