Final Problem Set

1. Suppose $\rho = p|0\rangle\langle 0| + (1-p)\dfrac{(|0\rangle + |1\rangle)(\langle 0| + \langle 1|)}{2}$

   Evaluate $S(\rho)$ and Compare the value with $H(p, 1-p)$.

2. Suppose $|AB\rangle$ is a pure state shared between Alice and Bob. Show that $|AB\rangle$ is entangled iff $S(B|A) < 0$.

3. i) Show that $H(X, Y|Z) \geq H(X|Z)$

   ii) Show that it is not always the case that
   $$S(A,B|C) \geq S(A|C)$$

   iii) $\quad S(A, B|C) \geq S(A|C) - S(B|C)$

4. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a convex function. For Hermitian operators A, B, Show that
   $$\text{tr}(f(A) - f(B)) \geq \text{tr}((A-B)f'(B))$$

5. Give a detailed proof that "Quantum operations never increase mutual Information".
   Hint: Nielsen-Chuang (Thm 11.15 (3)). Your proof should be self-contained.