Overview of the reading course - Spectral Graph Theory

Kishlaya Jaiswal

Week 1

- Spectral theorem for symmetric operators
- Rayleigh quotient
- Adjacency matrix as an operator and quadratic form
- Connectivity and bipartite-ness from adjacency spectrum
- Wilf's theorem on upper bound for chromatic number
- Hoffman's bound

Week 2

- Introduce Laplacian/Normalized laplacian
- Basic properties of Normalized Laplacian spectrum
- Isopertimetric ratio

Week 3

27/01

- Laplacian eigenvalues for various graphs - complete, cycles, hypercubes, paths
- Theory of characters when considered as eigenvectors for Cayley graphs

31/01

- Sparsest Cut problem
- Cheeger's inequality and proof

Week 4

03/02

- Finish proof of Cheeger's inequality
- Outline characterization of cheeger constant
- Sobolev inequalities

Week 5

10/02

- Vertex expansion
- Cheeger constant characterization

Week 6

17/02

- Spring networks and energy of the system
- Resistor networks

21/02

- Effective Resistance
- Work out proof sketch for effective resistance of an edge = probability of that edge in a spanning tree

Mid-sem

Week 8

- Planar embeddings
- Barycentric co-ordinates and Tutte's theorem

Week 9

- Pseudo-random generators
- 9 random bits suffice for a random walk

COVID-19 outbreak

Week 10

16/03

- Zigzag product of graphs
- $USTCON \in L$

01/08

- Submitted first report on a rigoruous proof for effective resistance of an edge = probability of that edge in a random spanning tree

14/08

- Submitted second report on a detailed discussion of the USTCON problem and Omer Reingold's proof

Sources covered

- Chapter $1, 2, 3$ of Spectral Graph Theory by Fan R.K. Chung
- Part 1 and 2 of $Lx = b$ by Nisheeth K. Vishnoi
- Lectures $7, 8, 9, 11$ from Daniel Spielman's lecture notes
- Chapter 5 of Luca Trevisan's lecture notes