6. Bayesian Calculus

Course: Introduction to AI
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October 21, 2022

Bayes Theorem

- Harking back to **Thomas Bayes**
- Q: How can we update our belief in a hypothesis/claim, given evidence related to the claim.

$P(H|E)$

$P(H|E) = P(H)$

$P(H) = 0.01$

$P(E|H)$

$P(E|\bar{H})$

Bayes Theorem

- Harking back to **Thomas Bayes**

- Q: How can we update our belief in a hypothesis/claim, given evidence related to the claim.
- A: Comparative rarity of the evidence w.r.t claim/hypothesis (presence or absence of hypothesis)

$$P(E|H) \longleftrightarrow P(E|\bar{H})$$

Formula for Bayes' Theorem

$P(H|E)$

$\propto$

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$$

$P(E)$

normalising

where,

- $P(H|E)$ – the probability of hypothesis (H) given evidence (E)
- $P(E|H)$ – the probability of observing evidence (E) given hypothesis (H)
- $P(H)$ – the probability of hypothesis (H)
- $P(E)$ – the total probability of observing evidence (E)

Bayes Theorem - Historical Review

theologist

- The theorem is named after English statistician, **Thomas Bayes**.
- Considered the foundation of the special statistical inference approach called the *Bayesian inference* or *Bayesian calculus*.
- Long ignored in favor of Boolean calculations, Bayes' Theorem has recently become more popular due to increased calculation capacity for performing its complex calculations.
- Bayesian inference is fundamental to Bayesian statistics, being considered "*to the theory of probability what Pythagoras's theorem is to geometry.*"

uncertainty.

computational

Holy Trinity: Prior, Likelihood, and Posterior

Bayes theorem states the following:

- *Posterior* = *Prior* * *Likelihood* (needs normalising)

normalising

- **Prior:** Probability of hypothesis claim prior to observing related evidence
- **Posterior:** Probability of hypothesis post assimilating the evidence
- **Likelihood:** The likelihood of evidence under the assumption that hypothesis holds

Reasoning in Odds*

$$P(H|E) = \frac{P(H) \times P(E|H)}{P(E)}$$

$$\updownarrow$$

$$P(\bar{H}|E) = \frac{P(\bar{H}) \times P(E|\bar{H})}{P(E)}$$

$$\frac{P(H|E)}{P(\bar{H}|E)} = \frac{P(H)}{P(\bar{H})} \times \frac{P(E|H)}{P(E|\bar{H})}$$

Odds for multiple hypotheses*

$E_1 \; E_2$

$P(A|B) = P(A)$

$P(H|E) = P(H)$

$P(A \cap B) = P(A) \times P(B)$

Exhaustive and Mutually-exclusive

$P(H|E_1) \neq P(H)$

$P(H|E_2) \neq P(H)$

$P(E_1|E_2) = P(E_1)$

$P(E_2|E_1) = P(E_2)$

Sample Problem - 1

$P(S_G > 5\%) = 0.04$

$P(C_R | S_G > 5\%) = 0.60$

- Imagine you are a financial analyst at an investment bank. According to your research of publicly-traded companies, 60% of the companies that increased their share price by more than 5% in the last three years replaced their CEOs during the period.
- At the same time, only 35% of the companies that did not increase their share price by more than 5% in the same period replaced their CEOs. Knowing that the probability that the stock prices grow by more than 5% is 4%, find the probability that the shares of a company that fires its CEO will increase by more than 5%.

$\boxed{P(S_G) = 0.04}$

$P(C_R | \bar{S}_G) = 0.35$

$P(C_R | S_G) = 0.60$

$P(C_R | \bar{S}_G) = 0.35$

$P(S_G | C_R)$

$\bar{S}_G$

Solution

$$P(S_G | e_R) = \frac{P(C_R | S_G) \times P(S_G)}{P(C_R)}$$

$\boxed{6.66\%}$

Before finding the probabilities, we first define the notation of the probabilities.

- $P(H)$
- $P(E)$
- $P(H|E)$
- $P(E|H)$

Sample Problem - 2

- Imagine there is a drug test that is 98% accurate, meaning that 98% of the time, it shows a true positive result for someone using the drug, and 98% of the time, it shows a true negative result for nonusers of the drug.
- Assume 0.5% of people use the drug. If a person selected at random tests positive for the drug, determine the probability the person is actually a user of the drug.

Solution

Overview

1. Bayes theorem
2. Historical review
3. Holy Trinity - Prior, Likelihood, Posterior
4. Sample problems