

6. Bayesian Calculus

Course: Introduction to AI **Instructor: Saumya Jetley**

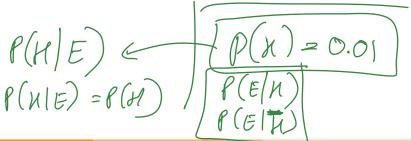
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Bayes Theorem



- Harking back to **Thomas Bayes**
- Q: How can we update our belief in a hypothesis/claim, given evidence related to the claim.



Bayes Theorem



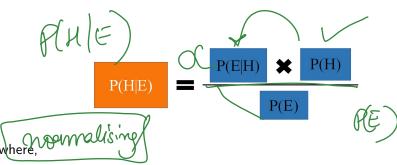
- Harking back to Thomas Bayes
- Q: How can we update our belief in a hypothesis/claim, given evidence related to the claim.

A: Comparative rarity of the evidence w.r.t claim/hypothesis (presence or absence of hypothesis)



Formula for Bayes' Theorem





- P(H|E) the probability of hypothesis (H) given evidence (E)
- P(E|H) the probability of observing evidence (E) given hypothesis (H)
- P(H) the probability of hypothesis (H)
- \blacksquare P(E) the total probability of observing evidence (E)

Bayes Theorem - Historical Review



- The theorem is named after English statistician, **Thomas Bayes**.
- Considered the foundation of the special statistical inference approach called the Bayesian inference or Bayesian calculus.
- Long ignored in favor of Boolean calculations, Bayes' Theorem has recently become more popular due to increased calculation capacity for performing its complex calculations.

Bayesian inference is fundamental to Bayesian statistics, being considered "to the theory of probability what Pythagoras's theorem is to geometry."

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Holy Trinity: Prior, Likelihood, and Posterior

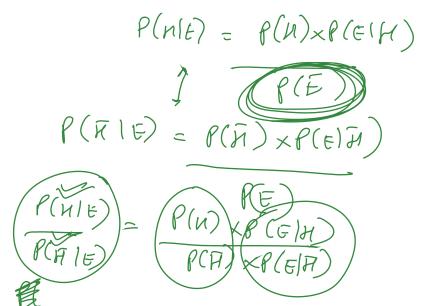


Bayes theorem states the following:

- Posterior = Prior * Likelihood (needs normalising)
- Prior: Probability of hypothesis claim prior to observing related evidence
- **Posterior:** Probability of hypothesis post assimilating the evidence
- Likelihood: The likelihood of evidence under the assumption that hypothesis holds

Reasoning in Odds*

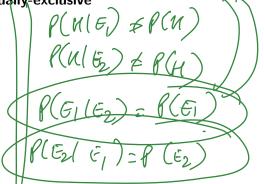




Odds for multiple hypotheses*



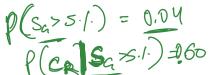
Exhaustive and Mutually-exclusive



Sample Problem - 1



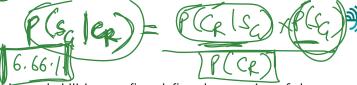
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- Imagine you are a financial analyst at an investment bank. According to your research of publicly-traded companies, 60% of the companies that increased their share price by more than 5% in the last three years replaced their CEOs during the period.
- At the same time, only 35% of the companies that did not increase their share price by more than 5% in the same period replaced their CEOs Knowing that the probability that the stock prices grow by more than 5% is 4%, find the probability that the shares of a company that fires its CEO will increase by more than 5%.



Solution



Before finding the probabilities, we first define the notation of the probabilities.

- **■** *P*(*H*)
- *P*(*E*)
- P(H|E)
- P(E|H)

Sample Problem - 2



- Imagine there is a drug test that is 98% accurate, meaning that 98% of the time, it shows a true positive result for someone using the drug, and 98% of the time, it shows a true negative result for nonusers of the drug.
- Assume 0.5% of people use the drug. If a person selected at random tests positive for the drug, determine the probability the person is actually a user of the drug.

Solution



Overview



- Bayes theorem
- 2 Historical review

- Holy Trinity Prior, Likelihood, Posterior
- 4 Sample problems