6. Bayesian Calculus

Course: Introduction to AI
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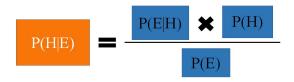
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Bayes Theorem

- Harking back to **Thomas Bayes**

- Q: How can we update our belief in a hypothesis/claim, given evidence related to the claim.

- A: Comparative rarity of the evidence w.r.t claim/hypothesis (presence or absence of hypothesis)

Formula for Bayes' Theorem

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$$

where,

- $P(H|E)$ – the probability of hypothesis (H) given evidence (E)
- $P(E|H)$ – the probability of observing evidence (E) given hypothesis (H)
- $P(H)$ – the probability of hypothesis (H)
- $P(E)$ – the total probability of observing evidence (E)

Bayes Theorem - Historical Review

- The theorem is named after English statistician, **Thomas Bayes**.

- Considered the foundation of the special statistical inference approach called the *Bayesian inference* or *Bayesian calculus*.

- Long ignored in favor of Boolean calculations, Bayes' Theorem has recently become more popular due to increased calculation capacity for performing its complex calculations.

- Bayesian inference is fundamental to Bayesian statistics, being considered "*to the theory of probability what Pythagoras's theorem is to geometry.*"

Holy Trinity: Prior, Likelihood, and Posterior

Bayes theorem states the following:

- *Posterior = Prior * Likelihood* (needs normalising)

- **Prior:** Probability of hypothesis claim prior to observing related evidence
- **Posterior:** Probability of hypothesis post assimilating the evidence
- **Likelihood:** The likelihood of evidence under the assumption that hypothesis holds

Reasoning in Odds*

Odds for multiple hypotheses*

Exhaustive and Mutually-exclusive

Sample Problem - 1

- Imagine you are a financial analyst at an investment bank. According to your research of publicly-traded companies, 60% of the companies that increased their share price by more than 5% in the last three years replaced their CEOs during the period.

- At the same time, only 35% of the companies that did not increase their share price by more than 5% in the same period replaced their CEOs. Knowing that the probability that the stock prices grow by more than 5% is 4%, find the probability that the shares of a company that fires its CEO will increase by more than 5%.

Solution

Before finding the probabilities, we first define the notation of the probabilities.

- $P(H) = 0.04$

- $P(E) = \frac{P(E|H) \times P(H)}{P(E|H) \times P(H) + P(E|\bar{H}) \times P(\bar{H})} = \frac{0.6 \times 0.04}{0.6 \times 0.04 + 0.35 \times 0.96} = 0.36$

- $P(E|H) = 0.6$

- $P(H|E) = \frac{P(E|H) \times P(H)}{P(E)} = \frac{0.6 \times 0.04}{0.36} = 0.066$

Sample Problem - 2

- Imagine there is a drug test that is 98% accurate, meaning that 98% of the time, it shows a true positive result for someone using the drug, and 98% of the time, it shows a true negative result for nonusers of the drug.

- Assume 0.5% of people use the drug. If a person selected at random tests positive for the drug, determine the probability the person is actually a user of the drug.

Solution

Overview