Math Assignment - 2

Questionverse Questions - 1, 2, 3, 4, ~~7~~8, 9, 10, 11, 53

1) Matrices remind me of numerical columns of an SQL table (and also of my report card in school).

~~Matrices are used to perform~~

Matrices denote and are used to perform linear transformations from one set of coordinates to another.

Matrices are used to - solve simultaneous linear equations, in graphics processing to perform image transformatn in 3-D video games to render frames & reflections, etc.

2) A functn with input X & output Y is an expression or rule such that for every X there is only one (unique) output of Y

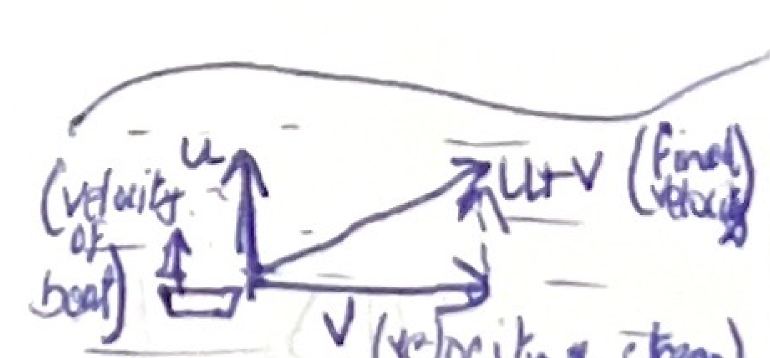
Injective ~~Surjective~~ functn : Functn where one to one mapping exists. i.e., if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

Surjective functn : Functn where onto mapping exists: i.e., for every $y \in Y$ there exists an $x \in X$.

Bijective functn : Both one-to one & onto mapping exists i.e., for every $y \in Y$ there exists only 1 $x \in X$.

3) An example of $f: R^2 \longrightarrow R^2$ function would be vector addition or summation.

Lets say we want to calculate the velocity of a boat moving across the stream



$f(\vec{u}, \vec{v}) = \vec{u} + \vec{v}$

4) A good example of a very nice funct^n ~~could~~ ~~be a~~ of $f: R^2 \rightarrow R^2$ would be a linear transformat^n that shows the reflect^n of a vector on the Y-axis.

i.e $f(x, y) = (-x, y)$

Represented by a linear transformat^n matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

This ~~matrix~~ funct^n can be further enhanced to give a reflect^n along a point (say origin $(0,0)$)

$f(x, y) = (-x, -y)$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Study of such matrices is very useful in photography & graphic processing applications where we need image reflections from mirrors, water bodies, etc.

7) R^2 is a vector space in 2 dimensions & its ~~properties include~~ contains all real valued points in a 2-dimensional coordinate system.

Properties of vector spaces

i) The addit^n operat^n of a finite set of vectors $(v_1, v_2 \cdots v_k)$ can be calculated in any order (Commutative)
i.e., it is commutative

$$\vec{V_1} + \vec{V_2} + \vec{V_3} = \vec{V_3} + \vec{V_1} + \vec{V_2}$$

ii) If $\vec{x} - \vec{y} = 0$ then $\vec{y} = -\vec{x}$

iii) Negation of negation of a vector is the vector i.e., $-(-\vec{x}) = \vec{x}$

iv) Negation of 0 is 0.

v) If $\vec{x}+\vec{y}=\vec{x}$, then $\vec{y}=0$ & 0 is unique

vi) Product of 0 with any vector is $\vec{0}$ vector.

vii) For any real number k, $k \times \vec{0} = \vec{0}$

viii) if $c\vec{x}=0$, then $c=0$ or $\vec{x}=0$.

ix) Scalar -1 multiplied by vector $\vec{v}$ is the negation of the vector

8) Let us consider $x-y=0$ or $y=x$ as a subspace of R^2

To prove this, we need to prove 2 conditions

i) Closure under addition

For x_1 & x_2 we have

(x_1, x_1), (x_2, x_2)

$f(x_1) + f(x_2)$

$= (x_1+x_2, x_1+x_2)$ → Same form as before. Hence, closed under additⁿ

ii) Closure under scaling

for x_1

$c \times f(x_1) = c(x_1, x_1)$

$= 6(cx_1, cx_1)$ → Same form as before. Hence, closed under scaling

Since, both properties are satisfied, so $x-y=0$ is a subspace of R^2

9) set $\{\alpha(1,7) \mid \alpha \in R\}$

$\alpha = -1 \Rightarrow (-1,-7)$

$\alpha = 1 \Rightarrow (1,7)$

$\alpha = 2 \Rightarrow (2,14)$

⋮

so on

This represents the line $y = 7x$ or $y - 7x = 0$

For Subspace,

i) $f(x_1) \Rightarrow (x_1, 7x_1)$

$f(x_2) \Rightarrow (x_2, 7x_2)$

$f(x_1) + f(x_2) = (x_1 + x_2, 7(x_1 + x_2))$

↳ Closed under additⁿ

ii) $f(cx_1) = (cx_1, (7c)x_1)$

↳ Closed under scalar multiplicatⁿ

$f(0) = (0,0)$

$\vec{0}$ is a subset of the space

Hence, the given set is a subspace of R^2.

10)

R^3

To check vector space,

Lets say $P_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \in R$ and $P_2 \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \in R$

$P_1 + P_2 = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix} \in R$ ($\because$ real no + real no = real no)

$$CP_1 = C\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} Cx_1 \\ Cy_1 \\ Cz_1 \end{bmatrix} \in R$$ ($\because$ real no $\times$ real no = real no)

Closed under addition & scalar multiplication
Hence, R^3 is a vector space.

11) For $(1,2,3)$ and $(4,5,7) \in R$

set $\{\alpha(1,2,3) + \beta(4,5,7) \mid \alpha, \beta \in R\}$
denotes the linear span of the set of vectors.

~~It is a s~~ Linear span is always a subspace of the original vector space.

53) $x - 2y = 15$ - (1)
$x + 4y = 19$ - (2)

(2) - (1)

$= 6y = 4$
$y = 2/3$

$x = 2y + 15 = \frac{4}{3} + 15 = \frac{49}{3}$

This is the same as

$$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 19 \end{bmatrix}$$

In this case, we can perform row transformatn or find the inverse of the matrix & multiply on both sides to get the value of x.