Questionverse – 1, 2, 3, 4, 7, 8, 9, 10, 11, 53.

1) It reminds me of a list of columns in Excel.
It denotes a matrix.
We use matrices for geometrical transformations and coordinate changes.
Matrices are used in cryptography (during key generation), 3D games (to convert into different objects), etc.

2) Function from X to Y is an expression, rule, or law that assigns each element of X exactly one element of Y.

$f: X \to Y$

Injective function is a function in which one to one mapping exists. So, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

Surjective function: A function is surjective if for every $y \in Y$, there exists $x \in X$ such that $f(x) = y$.

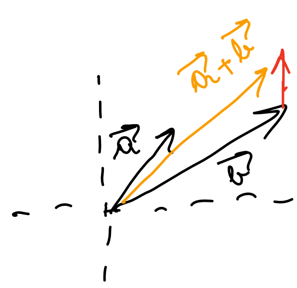
Bijective function is a function such that each $x \in X$ maps to exactly 1 unique $y \in Y$

3) $f: R^2 \longrightarrow R^2$

R^2 means we are dealing with 2 dimensional vector.

Eg vector addition

$\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$



4) According to me, a very nice function would be a function that gives the span of the vectors.

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{aligned} \text{span} &= a\vec{v} + b\vec{w} \\ &= a\begin{bmatrix} 1 \\ 2 \end{bmatrix} + b\begin{bmatrix} 3 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} a + 3b \\ 2a + 4b \end{bmatrix} \end{aligned}$$

One should study this function to get our intuition about the vectors that are a part of the vector space

7) Properties of vector space:

a) The addition operation of a finite list of vectors $v, v_2, \ldots, v_k$ can be calculated in any order. The solution of addition will be same.

b) If $x + y = 0$, then the value should be $y = -x$

c) The negation of 0 is 0.

d) The negation of the negation of a vector is the vector itself.

e) if $x + y = x$ if & only if $y = 0$. Only 0 is the vector that behaves like 0.

f) The product of any vector with 0 gives the zero vector.

g) For every real number c, any scalar times of the zero vector is 0.

h) if $cx = 0$, the either $c = 0$ or $x = 0$.

i) The scalar value -1 times the vector is the negation of the vector.

8) Consider $\begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} = \vec{V}$

Closure property

☆ $c \cdot \begin{bmatrix} x_1 \\ 0 \\ -x_1 \end{bmatrix} = \begin{bmatrix} cx_1 \\ 0 \\ -cx_1 \end{bmatrix}$

☆ $\begin{bmatrix} x_1 \\ 0 \\ -x_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ 0 \\ -x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 0 \\ -(x_1 + x_2) \end{bmatrix}$

Both of them are a part of the vector space $\begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}$ of

Thus, it is a subspace
since
$c\vec{x}, c\vec{x_2} \in \vec{V}$
$\vec{x}_1 + \vec{x}_2 \in \vec{V}$

9)

λ	$(1, 7)$
0	$(0, 0)$
1	$(1, 7)$
2	$(2, 14)$
$\vdots$	

This represents $y = 7x$
if $\vec{v} \Rightarrow$ "$7x$"

scalar multiplication
$= (7 \times x_1) c$
$= (7c) x_1 \in \vec{V}$

For $x = 0$, 0^{th} element exists

Additive property.

$$\begin{bmatrix} 7x_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 7x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 7(x_1+x_2) \\ 0 \end{bmatrix}$$

Since closure property is satisfied, it is a subspace

(0) R^3

consider real nos x_1, x_2, x_3

scalar multiplication:

$$c\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} cx_1 \\ cx_2 \\ cx_3 \end{bmatrix} \in R^3$$

Additive

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{bmatrix} \in R^3$$

closure property is satisfied.

Thus, it is a vector space.

11) It denotes the linear combinations of the vectors which is the span of the vectors. It is always a subspace of the vector space.

12) Yes, it is same as the set of linear equati-ons.

We are trying to find the values of x & y which satisfy the equations through the matrix approach. Intuitively it is a new optimized & more mathematical way of looking at the same equation.

Thanks,
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