Laplace Transforms:

Laplace Transform of Standard Functions

$L\{1\} = \frac{1}{s}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$, $n = 1, 2, 3.......$ Where n is a positive integer
$L\{e^{at}\} = \frac{1}{s-a}$	$L\{e^{-at}\} = \frac{1}{s+a}$
$L\{\sin at\} = \frac{a}{s^2+a^2}$	$L\{\cos at\} = \frac{s}{s^2+a^2}$
$L\{\sinh at\} = \frac{a}{s^2-a^2}$	$L\{\cosh at\} = \frac{s}{s^2-a^2}$
$L\{u(t)\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

Properties of the Laplace transform:

$f(t)$	$L\{f(t)\} = F(s)$
Translation (first Shifting Theorem)	$L\{e^{at} f(t)\} = F(s-a)$
Multiplication by t	$L\{t^n\ f(t)\} = (-1)^n \frac{d^n}{ds^n}\{F(s)\}$
Time scale	$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$
Integration	$L\left\{\int_0^t f(t)\,dt\right\} = \frac{1}{s} L\{f(t)\}$
Division by t	$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\ ds.$

Transform of a Periodic Function: If $f(t)$ is periodic with a period T, then

$$L\{f(t)\} = \frac{1}{\left(1-e^{-sT}\right)}\int_0^T e^{-st} f(t)\,dt \,.$$

Second Shifting Theorem:

If $F(s) = L\{f(t)\}$ **and** $a > 0$, **then** $L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}$.

Transforms of the derivatives:

$L\{f'(t)\} = sL\{f(t)\} - f(0)$

$L\{f''(t)\} = s^2L\{f(t)\} - s\,f(0) - f'(0)$

$L\{f^n(t)\} = s^nL\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-2}f''(0) - \cdots\cdots\cdots - f^{n-1}(0)$

Inverse Laplace Transform:

Transform $F(s) = L\{f(t)\}$	**Inverse Transform** $f(t) = L^{-1}\{F(s)\}$
$L\{t^n\} = \frac{n!}{s^{n+1}}$ Where n is a positive integer	$\frac{t^{n-1}}{(n-1)!} = L^{-1}\left\{\frac{1}{s^n}\right\}$ Where n is a positive integer
$L\{e^{at}\} = \frac{1}{s-a}$	$e^{at} = L^{-1}\left\{\frac{1}{s-a}\right\}$
$L\{\sinh at\} = \frac{a}{s^2-a^2}$	$\sinh at = L^{-1}\left\{\frac{a}{s^2-a^2}\right\}$
$L\{\cosh at\} = \frac{s}{s^2-a^2}$	$\cosh at = L^{-1}\left\{\frac{s}{s^2-a^2}\right\}$
$L\{\sin at\} = \frac{a}{s^2+a^2}$	$\sin at = L^{-1}\left\{\frac{a}{s^2+a^2}\right\}$
$L\{\cos at\} = \frac{s}{s^2+a^2}$	$\cos at = L^{-1}\left\{\frac{s}{s^2+a^2}\right\}$

Formulae on Shifting rule:

$$L[e^{at}\cos bt] = \frac{s-a}{(s-a)^2+b^2} \qquad L^{-1}\left[\frac{s-a}{(s-a)^2+b^2}\right] = e^{at}\cos bt$$

$$L[e^{at}\sin bt] = \frac{b}{(s-a)^2+b^2} \qquad L^{-1}\left[\frac{b}{(s-a)^2+b^2}\right] = e^{at}\sin bt$$

$$L[e^{-at}\cos bt] = \frac{s+a}{(s+a)^2+b^2} \qquad L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right] = e^{-at}\cos bt$$

$$L[e^{-at}\sin bt] = \frac{b}{(s+a)^2+b^2} \qquad L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right] = e^{-at}\sin bt$$

$$L[e^{at}\sinh bt] = \frac{b}{(s-a)^2-b^2} \qquad L^{-1}\left[\frac{b}{(s-a)^2-b^2}\right] = e^{at}\sinh bt$$

$$L[e^{at}\cosh bt] = \frac{s-a}{(s-a)^2-b^2} \qquad L^{-1}\left[\frac{s-a}{(s-a)^2-b^2}\right] = e^{at}\cosh bt$$

$$L[e^{-at}\cosh bt] = \frac{s+a}{(s+a)^2-b^2} \qquad L^{-1}\left[\frac{s+a}{(s+a)^2-b^2}\right] = e^{-at}\cosh bt$$

$$L[e^{-at}\sinh bt] = \frac{b}{(s+a)^2-b^2} \qquad L^{-1}\left[\frac{b}{(s+a)^2-b^2}\right] = e^{-at}\sinh bt$$

Convolution Theorem:

Let $f(t)$ and $g(t)$ be piecewise continuous on $[0,\infty)$ and $F(s)=L\{f(t)\}$ & $G(s)=L\{g(t)\}$.

Then, $L^{-1}\{F(s)G(s)\} = \int_0^t f(u)g(t-u)\,du.$