Tutorial Sheet-02 (Unit-01)

IS41-Statistics, Probability and Linear Programming												
NAME:											Marks:	
USN:												

Multiple Regression and Random variables

1. **Write the normal equations for $Y = a + bX_1 + cX_2$**

2. **Random Variables**

A real number associated with outcome of an experiment is known as a random variable.

Example(1)

Suppose two fair coins are tossed. Then S={HH, HT, TH, TT}

Let X be the random variable corresponding to number of heads. Then X takes values 0,1,2.

Outcome	HH	HT	TH	TT
X	2	1	1	0

Range of random variable

The set of all real numbers of a random variate X is called range of X.

In example(1), Range of X={0,1,2}

Discrete Probability distribution

Let X be a discrete random variable assuming the values $x_1, x_2, x_3\ x_4, \ldots, x_n$.
With each possible outcome x_i we associate a number $p_i = P(X=x_i)=P(x_i)$ called the probability of x_i.

Then $P(x_i)$ is called the probability mass function (PMF) of the random variable X if the following conditions are satisfied.

(i) $P(x_i) \geq 0 \quad \forall i$

(ii) $\sum P(x_i) = 1$

The set $\{P(xi)\}$ is called the probability distribution of the random variable.

Continuous Probability distribution

Let X be a continuous random variable assuming values x over an interval.

We assign a real number $f(x)$ satisfying the conditions

(i) $f(x) \geq 0 \quad \forall i$

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

Then $f(x)$ is called the probability density function (PDF) of the continuous variable X.

If (a,b) is a subinterval of the range space of X then the probability of x which lies in (a,b) is denoted by $P(a \leq X \leq b)$ and is defined as

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

3.	**Cumulative distribution function**
	Let X be a random variable (discrete or continuous). We define F(x) to be the cumulative distribution function (CDF) or simply distribution function if
	$$\mathrm{F(x) = P(X \leq x)}$$
	If X is a discrete random variable, then
	$$\mathrm{F(x_i) = P(X \leq x_i) = P(x_1) + P(x_2) + \cdots P(x_i)}$$
	If X is a continuous variable with PDF $f(x)$ then
	$$\mathrm{F(x) = P(X \leq x)} = \int_{-\infty}^{\mathrm{x}} f(x)dx$$
	Note: $f(x) = \frac{d}{dx}\big(\mathrm{F(x)}\big)$

4.	**Mean and variance:** (For both DRV & CRV)
5.	**Prove that** σ^2 **= E(X²) – [E(X)]²**

Tutorial-02

1.	Estimate the value of y when $x_1 = 2.5$ & $x_2 = 3.5$ by finding the relation of the form $y = a + bx_1 + cx_2$ by the method of least squares for the following data

x_1	2	4	6	8	10	12
x_2	-2	0	1	2	3	4
y	18	15	14	11	11	9

Solution:

x_1	x_2						
2	-2						
4	0						
6	1						
8	2						
10	3						
12	4						

2.	The probability distribution of a random variable X is given by the following table. Find k and determine mean & variance.

X	0	1	2	3	4	5
P(X= x)	k	5k	10k	10k	5k	k

3. The probability mass function of a variate X is

X	0	1	2	3	4	5	6
P(X)	K	3k	5k	7k	9k	11k	13k

Find
(i) k, $P(X < 4)$, $P(X \geq 5)$, $P(3 < X < 6)$
(ii) Mean and variance

4. A random variable X has the following probability function

X	1	2	3	4	5	6	7
P(X)	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) $P(X \geq 6)$ (iii) $P(X < 6)$ (iv) $P(1 \leq X < 5)$ (v) E(X) (vi) variance

5.	Find which of the following is a probability density function.

function	PDF (Yes/No)	Reason
$f_1(x) = \begin{cases} 2x, 0 < x < 1 \\ 0. \ otherwise \end{cases}$		
$f_2(x) = \begin{cases} 2x, -1 < x, 1 \\ 0, otherwise \end{cases}$		
$f_3(x) = \begin{cases} \lvert x \rvert, & \lvert x \rvert \leq 1 \\ 0, otherwise \end{cases}$		
$f_4(x) = \begin{cases} 2x, & 0 < x \leq 1 \\ 4 - 4x, & 1 < x < 2 \\ 0, & otherwise \end{cases}$		

6.	A random variable X has the density function $f(x) = \frac{k}{1+x^2}$,$-\infty < x < \infty$, determine k and evaluate P(X ≥ 0). Also find mean and variance.

7.	If a continuous random variable X has pdf $f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0, & elsewhere \end{cases}$ Obtain (i) P[X < 1] (ii) $P[\lvert X \rvert > 1]$ (iii) P[2X + 3 > 5]
8.	Is the function $f(x) = \begin{cases} e^{-x}, & 0 \le x < \infty \\ 0, & elsewhere \end{cases}$ a density function of the continuous random variable X? i). If so, determine $P(1 \le X \le 2)$. ii). Also, find the cumultive distribution function $F(x)$

9.	The pdf of a random variable X is given by $P(X = x) = \begin{cases} x & , 0 \leq x \leq 1 \\ 2 - x & , 1 < x \leq 2 \\ 0 & , elsewhere \end{cases}$ Find (i) cumulative distribution function F(X) and (ii) $P(X \geq 1.5)$.
10.	The diameter of an electric cable is assumed to be a continuous random variable with pdf $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$, 0 elsewhere. Verify that the above is a pdf. Also find its mean and variance.