1) Given

$$\int_2^3 e^{-2(x-3)^2} dx$$

Soln: Convert the above equation in Gaussian distribution function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{--- (1)}$$

$$\Rightarrow \sigma\sqrt{2\pi} \int_2^3 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-3}{1/2}\right)^2} \quad (-\infty < x < \infty) \quad \text{--- (ii)}$$

$\Rightarrow$ Compare equation (1) & (ii)

$$\sigma = 1/2, \quad \mu = 3$$

⇒ lets apply z distribution.

$$Z = \frac{x-\mu}{\sigma}$$

So, $x = 3$ ⇒ $Z_b = \frac{3-3}{0.5} = 0$

When $x = 2$ ⇒ $Z_a = \frac{2-3}{0.5} = -2$

~~Replace the above~~ z value

From equation II after simplify.

$$\Rightarrow \sigma\sqrt{2\pi}\; p\,(2 < X < 3)$$

Replace X value with above z value.

$$\Rightarrow \sigma\sqrt{2\pi}\; p\left(\frac{2-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{3-\mu}{\sigma}\right)$$

$$\Rightarrow 0.5\sqrt{2\pi}\; p\,(-2 < Z < 0)$$

$$\Rightarrow \cancel{0.5}\sqrt{\frac{\cancel{2}\pi}{2}} \times \left[p(z<0) - p(z<-2)\right]$$

$$\Rightarrow \underset{1.25}{\cancel{0.5 \times 1.4148}}\left[0.5 - 0.0228\right]$$

$$\Rightarrow 1.25 \times 0.4772$$

$$\Rightarrow \boxed{0.5965 \quad \text{Ans}}$$

Q2. Given $X \sim N(\mu, \sigma^2) \Rightarrow \frac{X-\mu}{\sigma} \sim N(0,1)$

$P(X \leq 60) = 0.1$ —— (I)

$P(X \geq 90) = 0.05$ —— (II)

Aim: To find value of μ & σ ?

Soln.

From equation (I)

$$P(X \leq 60) = 0.1$$

$$\Rightarrow P\left(\frac{X-\mu}{\sigma} \leq \frac{60-\mu}{\sigma}\right) = 0.1$$

$$\Rightarrow P\left(Z \leq \frac{60-\mu}{\sigma}\right) = 0.1$$

From negative Z table

$$\frac{60-\mu}{\sigma} = -1.28$$

$$\Rightarrow \mu - 1.28\sigma = 60 \quad \text{—— (III)}$$

From equation (II)

$$P(X \geq 90) = 0.05$$

$$\Rightarrow P\left(Z \geq \frac{90-\mu}{\sigma}\right) = 0.05$$

2

$$\Rightarrow 1 - P\left(z < \frac{90-\mu}{\sigma}\right) = 0.05$$

$$\Rightarrow P\left(z < \frac{90-\mu}{\sigma}\right) = 0.95$$

$$\Rightarrow \frac{90-\mu}{\sigma} = 1.65$$

$$\Rightarrow \mu + 1.65\sigma = 90 \quad \text{--- (IV)}$$

Subtracting equation (III) & (IV)

$$\begin{array}{rl} \mu + 1.65\sigma &= 90 \\ \mu - 1.28\sigma &= 60 \\ - \quad + \quad & \quad - \\ \hline (-) \quad 0 \quad 2.93\sigma &= 30 \end{array}$$

$$\Rightarrow \boxed{\begin{array}{l} \sigma = 10.23 \\ \mu = 73 \end{array}}$$ Ans

③ Given:

$$\{X_1 + \cdots X_n\} \sim N(\mu, \sigma^2)$$

prove that

$$\bar{X}_n = \frac{\{X_1 + \cdots X_n\}}{n}$$

Show that $\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

3, Soln:

prop 1: For any $\alpha \in \mathbb{R}$, $\alpha \neq 0$
if $X \sim N(\mu, \sigma^2)$ than $\alpha X \sim N(\alpha\mu, \alpha^2\sigma^2)$

prop 2: let X_1, X_2 is independent random variable such that $X_1 \sim N(\mu_1, \sigma_1^2)$ &
$X_2 \sim N(\mu_2, \sigma_2^2)$

So if we see $X_1, X_2, \dots X_n$ are independent random variable and

$$X_i \sim (\mu_i, \sigma_i^2)$$

$$X = X_1 + X_2 + X_3 + \dots X_n \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$$

Thus if $\{X_1, \dots X_n\} \sim N(\mu, \sigma^2)$

$$X = X_1 + \dots + X_n \sim N\left(\sum_{i=1}^{n} \mu, \sum_{i=1}^{n} \sigma_i^2\right)$$

$$X_1 + \dots + X_n \sim N(n\mu, n\sigma^2)$$

Now $\bar{X} = \frac{X}{n}$ where $X \sim N(\mu, n\sigma^2)$

From prop 1: $\bar{X} \sim N\left(\frac{1}{n} n\mu, \left(\frac{1}{n}\right)^2 n\sigma^2\right)$

$$\Rightarrow \boxed{\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{Ans}}$$

4) Given :

$N(1, 4)$ $\qquad \mu = 1$

$\sigma = 2$

and $p(x^2 > 4) = ?$

Soln :

$p(x^2 > 4)$ $\qquad \left\{ \begin{array}{l} x^2 > 4 \\ \Rightarrow x < -2, x > 2, \end{array} \right.$

$\Rightarrow p(x < -2 \text{ and } x > 2)$

$\Rightarrow p(x < -2) + p(x > 2)$

$\Rightarrow \left(p\left(z < \frac{-2-1}{2}\right) + 1 - p\left(z < \frac{2-1}{2}\right) \right.$

$\Rightarrow p\left(z < -\frac{3}{2}\right) + 1 - p\left(z < \frac{1}{2}\right)$

$\Rightarrow 0.06681 + 1 - 0.69146$

$\Rightarrow \boxed{0.375 \text{ Ans}}$

5/ Given : X has normal distribution with parameters a & b.

$$\left(\frac{X-a}{b}\right)^2$$ is a chi square distribution with def 1

6) Given

$\mu = \text{mean} = m$

$\sigma = \text{std} = 3$

$p(X > 15) = p(Z < 1)$

$\Rightarrow p\left(\frac{X-\mu}{\sigma} > \frac{15-\mu}{\sigma}\right) = p(Z < 1)$

$\Rightarrow p\left(Z > \frac{15-m}{3}\right) = p(Z < 1)$

$\Rightarrow 1 - p\left(Z < \frac{15-m}{3}\right) = p(Z < 1)$

Total prob is equal to 1

From z +ve table

$\Rightarrow 1 - p\left(Z < \frac{15-m}{3}\right) = 0.84134$

$\Rightarrow p\left(Z < \frac{15-m}{3}\right) = 0.1587$

$\Rightarrow$ From −ve z table

$p\left(Z < \frac{15-m}{3}\right) = p(Z < -1)$

So $\frac{15-m}{3} = -1$

$\boxed{m = 18}$

Ans

Q 7, Given : x follows normal distribution

$\mu = 0$, $\sigma = 1$. mean of $3x^2 = ?$

Soln :

$$X \sim N(0, 1)$$

~~as per χ^2 definition~~

$$\Rightarrow X^2 \sim \chi^2(1)$$

As per chi square definition we can say that if x is standard normal variable, then x^2 is distributed chi-square with one degree free also $3x^2 \Rightarrow x^2 + x^2 + x^2$ so degree of freedom will be 3.

$\Rightarrow$ The mean of x^2 is degree of freedoms and its variance twice of degree of freedom.

So $\boxed{\text{mean of } 3x^2 = 3}$ Ans.

9) Given

$\alpha = 0.05$,
$df = 15$

From χ^2 table

$$\chi^2_{\alpha, df} = \chi^2_{0.05, 15} = \boxed{24.996}$$ Ans.

8) Given

$df = 26$

$$P\left(\chi^2_{26} \leq 30\right)$$

$$\Rightarrow 1 - P\left(\chi^2_{26} > 30\right)$$

From Chi Square Calculator

Assume

Significance level $= 0.05$

$$\Rightarrow 1 - 0.267611$$

$$\Rightarrow \boxed{0.7325}$$ Ans

Q 10/ Given:
Three coordinate error are in normal distribution

$\mu = 0, \quad \sigma = 2$

Soln:
If D is the distance, then

$$D^2 = X_1^2 + X_2^2 + X_3^2$$

where X_i is the error in the i^{th} coordinate.
Since
$Z_i = X_i/2, \quad i = 1, 2, 3$ are all
standard normal random variables, it follows that

$$P(D^2 > 9) = P\{Z_1^2 + Z_2^2 + Z_3^2 > 9/4\}$$

$$= P\{\chi_3^2 > 9/4\}$$

$$\boxed{= 0.5222}$$ Ans

11) Given

a) $p\{T_{12} \le 1.4\} = ?$

Soln:

$$p(T_{12} \le 1.4)$$

$$\Rightarrow 1 - p(T_{12} > 1.4)$$

$$\Rightarrow 1 - \frac{0.1 + 0.05}{2}$$

From t table

$$\Rightarrow 1 - 0.075$$

$$\Rightarrow \boxed{0.925 \text{ Ans}}$$

b) $t_{0.025, 9} = ?$

$\alpha = 0.025$

$df = 9$

From t one tail t table.

$$\boxed{t_{0.025, 9} = 2.2625 \quad \text{Ans}}$$

12/ Given

$n = 25000$

$\mu = 320$

$std = \sigma = 540$

Soln: let X denote the total yearly claim number of policy holder.

let X_i denote the yearly claim of policy holder i, with $n = 25000$.

Based on central Limit theorem.

that $X = \sum_{i=1}^{n} X_i$ will be approximately a normal distribution with

$$\text{mean} = n\mu = 25000 \times 320 = 8 \times 10^6$$

$$std = \sqrt{n}\,\sigma = \sqrt{25000} \times 540 = 8.5 \times 10^4$$

Now

$$p(X > 8.3 \times 10^6)$$

$$\Rightarrow p\left(\frac{X - \mu n}{\sigma\sqrt{n}} > \frac{8.3 \times 10^6 - \mu n}{\sigma\sqrt{n}}\right)$$

$$\Rightarrow p\left(Z > \frac{8.3 \times 10^6 - 8 \times 10^6}{8.5 \times 10^4}\right)$$

$$p\left(Z > \frac{0.3 \times 10^{\cancel{6}\,2}}{8.5 \times \cancel{10^4}}\right) \Rightarrow p\left(Z > \frac{30}{8.5}\right)$$

$$1 - p\left(Z < \frac{30}{8.5}\right) = 1 - 0.99977$$

$$\boxed{= 0.00023}$$

13

Given

population

$\mu = 167$

$\sigma = 27$

a)

Sample Size

$n = 36$

b)

Sample Size

$n = 144$

$P(163 < x < 170) = ?$

Soln :

a) Let z be the standard normal random variable.

It follows Central Limit theorem that $\bar{x}$ is approximately normal with $\mu = 167$ and

$$\sigma = 27/\sqrt{n} = \frac{27}{\sqrt{36}} = 27/6 = 4.5$$

$n = 36$

$\Rightarrow$ Therfore $p(163 < x < 170)$

$$\Rightarrow p\left(\frac{163-167}{4.5} < Z < \frac{170-167}{4.5}\right)$$

$$\Rightarrow p\left(\frac{-4}{4.5} < Z < \frac{3}{4.5}\right)$$

$$\Rightarrow p\left(z < \frac{3}{4.5}\right) - p\left(z < \frac{-4}{4.5}\right)$$

$$\Rightarrow p(z < 0.667) - P(z < -0.889)$$

$$\Rightarrow 0.7486 - 0.1867$$

$$\Rightarrow \boxed{0.5619 \quad \text{Ans}}$$

Soln

$n = 144$ $\quad \sigma = {}^{27}/_{\sqrt{n}} = \frac{27}{\sqrt{144}} = \frac{\cancel{27}^{9}}{\cancel{12}_{4}}$

$\mu = 167$

$= 2.25$

$\Rightarrow P(163 < X < 170)$

$\Rightarrow P\left(\frac{163-167}{2.25} < Z < \frac{170-167}{2.25}\right)$

$\Rightarrow \cancel{P\left(\frac{3}{2.25} < Z\right.}$

$\Rightarrow P\left(Z < \frac{3}{2.25}\right) - P\left(Z < \frac{-4}{2.25}\right)$

$\Rightarrow P(Z < 1.33) - P(Z < -1.778)$

$\Rightarrow 0.9082 - 0.0375$

$\Rightarrow \boxed{0.8707 \quad \text{Ans.}}$

14) Given :

$\mu = d$

$\sigma = 2$

$\varepsilon = \pm 0.5$

Soln

$\Rightarrow$ If the astronomer makes n measurements then $\bar{x}$, the sample mean of these measurements will be approximately a normal random variable with mean d and std $2/\sqrt{n}$

The probability will lie between $d \pm 0.5$ is obtained as follows.

$$p(-0.5 < \bar{x} < 0.5) = p\left(\frac{-0.5}{2/\sqrt{n}} < \frac{\bar{x}-d}{2/\sqrt{n}} < \frac{0.5}{2/\sqrt{n}}\right)$$

$$\approx p\left(-\sqrt{n}/4 < z < \sqrt{n}/4\right)$$

$$= 2p\left(z < \sqrt{n}/4\right) - 1$$

where z is a standard normal random variable.

Thus $$2p\left(z < \sqrt{n}/4\right) - 1 \geq 0.95$$

or $$p\left(z < \sqrt{n}/4\right) \geq 0.975$$

Since $p(z < 1.96) = 0.975$

$\Rightarrow$ $\sqrt{n}/4 \geq 1.96$

That is, at least 62 observations are necessary.

15) Given:

$$T \sim N(20, 3)$$

Sample Size $N = 15$

Aim To find $P(s^2 > 12) = ?$

we know that $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)}$

therefore $P(s^2 > 12) = P\left(\frac{14 s^2}{9} > \frac{12 \times 14}{9}\right)$

$\Rightarrow P\left(\chi^2_{14} > \frac{\cancel{12}^{4} \times 14}{\cancel{9}_{3}}\right)$

$\Rightarrow P\left(\chi^2_{14} > 18.67\right)$

$\Rightarrow \boxed{0.18 \quad \text{Ans}}$

16//

Given

mean = 147 pounds

std = 62 pounds

$n = 25$

let x_i be the amount consumed by the ith member of the sample.

$i = 1, \ldots 25$, x_i is independent random variable.

Required probability

$$p\left(\frac{x_1 + \cdots x_{25}}{25} > 150\right) = p(\bar{x} > 150)$$

$\bar{x}$ is the sample mean of the 25 sample values.

x_i is the independent random variables with mean 147 and std 62

It follows from the Central Limit theorem that their sample mean will be approx normal with mean 147 and standard deviation

$$\frac{\sigma}{\sqrt{n}} = \frac{62}{5}$$

16/

Thus with Z being a standard normal random variable we have

$$p(\bar{x} > 150) = p\left\{\frac{\bar{x}-147}{12.4} > \frac{150-147}{12.4}\right\}$$

$$\approx p(z > 0.242)$$

$$\boxed{\approx 0.404 \quad \text{Ans}}$$

17//

Given Z_1, Z_2 are iid standard normal variable.

Find $P(Z_1^2 < 2Z_2^2)$

properties : If χ_n^2 & χ_m^2 are independent Chi square Random variable with n & m degree of freedom, then $F_{n,m}$ defined by

F-distribution $$F_{n,m} = \frac{\chi_n^2/n}{\chi_m^2/m}$$

As per question $Z_1^2 = \chi_1^2$ with def = 1
$Z_2^2 = \chi_2^2$ $\quad n = m = 1$

$$\Rightarrow P\left(\frac{Z_1^2}{Z_2^2} < 2\right)$$

$\because$ division of 2 chi square is F dist.

$$P(F_{1,1} < 2) \Rightarrow P(1 - F_{1,1} > 2)$$

$$\Rightarrow 1 - 0.39$$

$$\Rightarrow \boxed{0.61 \text{ Ans}}$$

Q.18 Given:

X_1, X_2, X_3, X_4 are from Standard normal population

Q. probability distribution of

$13X_1^2 + 17X_2^2 + 19X_3^2 + 41X_4^2$

Ans

According to the question X_1, X_2, X_3 and X_4 are from Standard normal population

So As per chi square definition we can say that if X is a Standard normal distribution, than X^2 is distributed with chi square with 1 degree of freedom.

So

$$13X_1^2 + 17X_2^2 + 19X_3^2 + 41X_4^2$$

we can say with reference to question 7's soln It is chi square distribution with $13+17+19+41 = 90$ degree of freedom.

19) Given

$$X \sim \chi^2_{(5)} \qquad df = 5$$

$$P(1.145 < X < 12.83)$$

$$\Rightarrow P(X < 12.83) - P(X < 1.145)$$

$$\Rightarrow \cancel{1} - P(X > 12.83) - \cancel{1} + P(X > 1.145)$$

$$\Rightarrow P(X > 1.145) - P(X > 12.83)$$

From chi square table

$$\Rightarrow$$

$$0.950 - 0.025$$

$$\Rightarrow \boxed{0.925 \quad \text{Ans}}$$

8) Given

$$P(\chi^2_{26} \leq 30)$$

$$\Rightarrow 1 - P(\chi^2_{26} > 30)$$

From chi Square Calculator

$df = 26$, Chi-Square Score $= 30$, $\alpha = 0.05$

So $1 - 0.267611$

$$\Rightarrow \boxed{0.7324 \quad \text{Ans}}$$

2) Given

$X \sim N(10, 25) \Rightarrow n = 501$

By Chi square Theorem.

$$\frac{(n-1)s^2}{\sigma^2} = \chi^2_{(n-1)}$$

$$\Rightarrow \frac{(501-1)s^2}{25^2} = \chi^2_{(501-1)}.$$

Hence Expected value of s^2 is given by

$$E[s^2] = E\left[\left(\frac{25}{500}\right)\left(\left(\frac{500}{25}\right)s^2\right)\right]$$

$$= \frac{25}{500} E\left[\left(\frac{500}{25}\right) s^2\right]$$

$$= \frac{1}{20} E\left[\chi^2(500)\right]$$

$$= \frac{1}{20} \times 500$$

$$\boxed{= 25}$$

Ans

22 Given $T \sim t(19)$

$$P(|T| \leq c) = 0.95$$

$$c = ?$$

Soln:

$$P(|T| \leq c) = 0.95$$

$$\Rightarrow P(-c \leq T \leq c) = 0.95$$

$$\Rightarrow P(T \leq c) - P(T \leq -c) = 0.95$$

$$\Rightarrow P(T \leq -c) = P(T \geq c)$$ / Area wise

So $P(T \leq c) - P(T > c) = 0.95$

$$\Rightarrow P(T \leq c) - 1 + P(T \leq c) = 0.95$$

$$\Rightarrow \cancel{2} P(T \leq c) = \frac{1.95}{\cancel{2}} = 0.975$$

From T table with df = 19.

$$P\left(T \leq \frac{0.2+.15}{2}\right) = 0.975$$

So $\boxed{c = 0.175}$ Ans

8

24. Given:

X_1, X_2, X_3 & X_4 are Standard normal distribution.

$$W = \frac{X_1 - X_2 + X_3}{\sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2}}$$

$W = ?$

Ans. As per question

$$X_1, X_2, X_3, X_4 \sim N(0,1)$$

As per properties of $E(X)$

$$E(W) = E(X_1 + X_2 + X_3) \cdot E \frac{1}{\sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2}}$$

$$= E(X_1) + E(X_2) + E(X_3) \cdot E\left(\frac{1}{\sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2}}\right)$$

From central limit theorem.

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

than,

$$E(X_1) = \mu = 0$$
$$E(X_2) = \mu = 0$$
$$E(X_3) = \mu = 0$$

$$E(W) = (0 - 0 + 0) \cdot E\left[\frac{1}{\sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2}}\right]$$

$$\boxed{E(W) = 0}$$

Ans.

25//

Given:

mean = μ

$\sigma^2 = 1$

(1) $n = 25$

let x_i ounces fill to be observed for i^{th} bottles.

$$x_i = \{1, 2, \ldots 25\}.$$

$$X \sim N(\mu, \sigma^2)$$

The sample mean of these observation will be approximately a normal random variable with mean μ and std $= 1/\sqrt{n} = \frac{1}{5}$

The probability lies between $\mu - 0.3$ & $\mu + 0.3$.

$$P(\mu - 0.3 < X < \mu + 0.3)$$

$$\Rightarrow P\left(\frac{\mu - 0.3 - \mu}{1/5} < Z < \frac{\mu + 0.3 - \mu}{1/5}\right)$$

$$\Rightarrow P(-0.3 \times 5 < Z < 0.3 \times 5)$$

$$\Rightarrow P(-1.5 < Z < 1.5)$$

$$\Rightarrow P(Z < 1.5) - P(Z < -1.5)$$

$$\Rightarrow 0.93319 \; \cancel{0.4332} - 0.0668$$

$$\Rightarrow \boxed{\cancel{0.3664} \; \text{~~Ans~~}}$$

$\boxed{0.86638 \text{ Ans}}$ "0.86638

Q5

11 $\sigma = \frac{1}{\sqrt{n}}$

$$p(\mu - 0.3 < x < \mu + 0.3) = 0.95$$

$$\Rightarrow p\left(\frac{\mu - 0.3 - \mu}{1/\sqrt{n}} < Z < \frac{\mu + 0.3 - \mu}{1/\sqrt{n}}\right) = 0.95$$

$$\Rightarrow p\left(-0.3\sqrt{n} < Z < 0.3\sqrt{n}\right) = 0.95$$

$$\Rightarrow p\left(Z < 0.3\sqrt{n}\right) - p\left(Z < -0.3\sqrt{n}\right) = 0.95$$

$$\Rightarrow p\left(Z < 0.3\sqrt{n}\right) - P\left(Z > 0.3\sqrt{n}\right) = 0.95$$

$$\Rightarrow p\left(Z < 0.3\sqrt{n}\right) - 1 + p\left(Z < 0.3\sqrt{n}\right) = 0.95$$

$$\cancel{2}\, p\left(Z < 0.3\sqrt{n}\right) = \cancel{1.95}\ 0.975$$

Since $p(Z < 1.96) = 0.975$
From Z table

$$\Rightarrow 0.3\sqrt{n} = 1.96$$

$$\Rightarrow \boxed{n = 42}$$

Ans,