Q 1) Given

population

$\mu_0 = 1\,kg$

$\sigma = 0.1$

Sample

$n = 100$

$\bar{X} = 1.03\,kg$

Soln:

Null Hypothesis:

$H_0 : \mu_0 = 1\,kg$ | process mean is 1 kg

$H_a : \mu_0 \neq 1\,kg$ | process mean is not equal to 1 kg.

~~Soln.~~

Now the value of test Statistic is

$$\sqrt{n}\,(\bar{x} - \mu_0)/\sigma = \frac{\sqrt{100}\,(1.03 - 1)}{0.1}$$

$$\Rightarrow \frac{10 \times 0.03}{0.1} = 3$$

So p-value is given by

$$\text{p-value} = p\,(z < 3) = 0.9989$$

Since this value is greater than 0.05, we will reject null hypothesis.

$\Rightarrow$ The process mean is still not 1 kg.

Q8 Given

population:

$\mu_0 = 0.8$ sec

Sample:

$n = 28$

$\bar{x} = 1$ sec

$\bar{s} = 0.3$ sec

Soln:

$H_0 : \mu = 0.8$

$H_1 : \mu \neq 0.8$

Lets apply t test statistic because $n < 30$ & std of population is not given.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{(1 - 0.8) \times \sqrt{28}}{0.3}$$

$$= 3.527$$

The degree of freedom is $n - 1 = 27$

$$\text{p-value} = P(|T_{27}| > 3.53) = 2P(T_{27} > 3.53)$$

$$= 2 \times [1 - 0.9992] = 2 \times 0.0008$$

$$= 0.0016$$

p value is less than 0.05

$\boxed{\text{We Reject the Null Hypothesis}}$

Q3// Given :

population

$\mu_0 = 7.6$ pounds

$\sigma = 1.2$ pounds

Sample

$n = 16$

$\bar{x} = 7.2$ pounds

Soln:

$$\boxed{\begin{array}{l} H_0 : \mu \geq 7.6 \\ H_1 : \mu < 7.6 \end{array}}$$

a) $\alpha = 0.05$

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.2 - 7.6}{1.2/\sqrt{16}} = -1.33$$

$DoF = n-1 = 15$

$$\text{p-value} = P(T_{15} \geq |T|)$$
$$= P(T_{15} > 1.33) = 0.1$$

Since $0.1 > 0.05$

So Accept the null hypothesis.

There is no strong evidence to reject the hatchery's claim.

b) $\alpha = 0.01$

$$\text{p-value} = p\left(T_{15} \geq |T|\right)$$

$$= p\left(T_{15} \geq 1.38\right) = 0.1$$

Since $0.1 > \cancel{0.01} \; 0.01$

So Accept the null hypothesis.

c)

$$\text{p value} = p\left(T_{15} \geq |T|\right)$$

$$= p\left(T_{15} \geq 1.38\right)$$

$$= 0.1$$

Q4 // Given

<u>population:</u>

$\cancel{\mu_0 = 60}$ $\mu_0 = 56.5$

$\cancel{\sigma = 5.6}$

<u>Sample</u>

$\cancel{X_0} = 60$

$s = 5.6$ $\alpha = 0.05$

$n = 456$

Soln:

$H_0 : \mu = 56.5$

$H_1 : \mu > 56.5$

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{60 - 56.5}{5.6/\sqrt{456}} = 13.46$$

$$\text{p value} = P(T_{455} \geq |T|)$$

$$= P(T_{455} \geq 13.46)$$

$= 0$ which is less then 0.05

we have evidence to reject null hypothesis

So the Claim of IITJ is ~~Curse~~ True.

Q.5. Given:

$H_0 : \mu \leq 100$

$H_1 : \mu > 100$

$n = 20$

$\bar{x} = 105$

$\mu_0 = 100, \ s = 5$

$$T = \frac{105-100}{5/\sqrt{20}} = \frac{5 \times \sqrt{20}}{5} = 4.47$$

$$\text{p. value} = P(T_{19} \geq |T|)$$

$$= P(T_{19} \geq T) = P(T_{19} > 4.47)$$

$$= 0.0001$$

Q6, Given:

$H_0 : \mu = 30$ $\quad\quad \alpha = 0.05$

$H_1 : \mu < 30$ $\quad\quad n = 10$

$$\bar{x} = \frac{26+24+20+25+27+25+28+30+26+33}{10}$$

$$= 26.4$$

$s = 3.902$, using $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N-1}}$

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{26.4 - 30}{3.5/\sqrt{10}} = -3.27$$

$$\text{p-value} = p\left(T_{n-1} \geq |T|\right)$$

$$= p\left(T_9 \geq 3.27\right) = 0.0048$$

Since 0.0048 is less then 0.05

So, Reject Null hypothesis.

Q 7. Given population | Sample

$\mu_0 = 210$ | $\bar{X} = 200$

$\bar{s} = 35$

$\alpha = 0.05$

~~a) Sample size = 28~~

$H_0 \Rightarrow \mu \geq 210$

$H_1 \Rightarrow \mu < 210$

a) $n = 25$

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{200 - 210}{35/\sqrt{25}} = -1.428$$

$$\text{p-value} = p\left(T_{24} \geq 1.428\right) = 0.083$$

Since 0.083 is greater then 0.05

So Accept the null hypothesis.

b) $n = 64$

$$T = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{200 - 210}{35/\sqrt{64}} = -2.28$$

$$\text{p-value} = p\left(T_{63} \geq |T|\right)$$

$$= p\left(T_{63} \geq 2.28\right) = 0.013$$

Since p-value is less then 0.05

So Reject the null hypothesis.

Q 8) Given:

population	Sample
$\mu_0 = 9$	$\bar{x} = \frac{9+10+8+12+13+10+9}{7}$
$H_0 \Rightarrow \mu = 9$	$= 10.14$
$H_1 \Rightarrow \mu \neq 9$	$S = \sqrt{\frac{\Sigma(x_i - \mu_0)}{n-1}} = 1.77$
	$n = 7$

$$T = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = 1.70$$

$$\text{p-value} = p\left(T_6 \geq |T|\right) = 2p\left(T_6 > |1.71|\right)$$

$$= 2 \times 0.0748 = 0.1496$$

Since p value is greater then 0.05.

So, Accept the null hypothesis.

$\sigma = 20$

b) $$\chi^2 = \frac{(n-1)\, s^2}{\sigma^2}$$

$$= \frac{5 \times (78.84)^2}{20^2} = 77.618$$

$$\text{p-value} = P\left(\chi^2_{n-1} < c\right)$$

$$= P\left(\chi^2_5 < 77.618\right) = 1$$

Since 1 is greater then 0.05

So Accept the null hypothesis.

Q9.

Given $\alpha = 0.05$

H_0 : range = 2500

H_1 : range < 2,500

a)

$n = 6$

$\mu_0 = 2500$

$$\bar{x} = \frac{2490 + 2510 + 2360 + 2410 + 2300 + 2400}{6}$$

$= 2411.6$

$$S = \cancel{78.84} \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = 78.84$$

$$T = \frac{2411.6 - 25000}{78.84/\sqrt{6}} = -2.73$$

p-value $= P\left(T_5 \geq |-2.73|\right) = 0.0266$

Since 0.0206 is less than 0.05

So Rejecting null hypothesis.

$\sigma = 20$

b)

$$\chi^2 = \frac{(n-1)\, s^2}{\sigma^2}$$

$$= \frac{5 \times (78.84)^2}{20^2} = 77.618$$

$$\text{p-value} = P\left(\chi^2_{n-1} < c\right)$$

$$= P\left(\chi^2_5 < 77.618\right) = 1$$

Since 1 is greater then 0.05

So Accept the null hypothesis.

Q 10,

(iv)

$H_0 : \sigma^2 > (0.1)^2$

$H_1 : \sigma^2 \leq (0.1)^2$

Given

$s = 0.08$ $\alpha = 0.1$

$\sigma = 0.1$

$$\chi_c^2 = \frac{(n-1)\, s^2}{\sigma^2} = 49 \times \frac{0.08^2}{0.1^2}$$

$$= 49 \times \frac{0.08 \times 0.08}{0.1 \times 0.1} = 0.64 \times 49$$

$$= 31.36$$

$$P\text{value} = P\left(\chi_{n-1}^2 < c\right)$$

$$= P\left(\chi_{49}^2 < 31.36\right) = 0.02$$

Since 0.02 is less than 0.1

So, Reject the null hypothesis.

Q 11. $\alpha = 0.05$

$H_0 : \sigma^2 > 5^2$

$n = 20$

$H_1 : \sigma^2 = 5^2$

$s = 8$

$\sigma = 5$

So, $$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2} = \frac{19 \times 64}{25}$$

$$= 48.64$$

p-value $= P\left(\chi^2_{n-1} < c\right)$

$= P\left(\chi^2_{19} < 48.64\right)$

$= 1$

Since 1 is greater than 0.05

Accept null hypothesis.