Topic:OpenQASM
Subtopic:Quantum Teleportation

Reddi Kishore

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Introduction

Quantum teleportation is a technique used to transfer quantum information from source to the destination.Suppose Alice and Bob are two friends, Alice wants to transfer the qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. But according to no-cloning theorem in quantum mechanics we cannot copy a unknown quantum state,hence Alice cannot simply copy the quantum state. we can only copy the classical states.
For teleportation to happen it needs an entangled pair of qubits along with two classical bits.To transfer a qubit Alice and bob should need a third party to send them an entangled pair. Alice do some operations on his qubit, then sent it to the Bobs end, and then Bob performs some operations to get the qubit that Alice sent.
All these operations are done by using a OpenQASM(Open Quantum Assembly Language) programming language, which is used to design the quantum circuits in a easy and simpler manner.Building of quantum circuits will be done in IBM Quantum Composer using 'Open Quantum Assembly language'(OpenQASM). Step by step explanation to the circuit will be explained in this report.

1.Quantum Teleportation

Step-1:

Let's setup the circuit by using three qubits and two classical bits. '//' symbols are used to denote a comment which explains the corresponding code.

Code:

```
OPENQASM 2.0;             // Denotes a file in OpenQASM format
include"qelib1.inc";      //Open and parse the source file 'qelib1.inc'
qreg q[3];         //a quantum register of name 'q' will be created with 3 qubits
creg c1[1];         //a classical register named c1 of 1 bit will be created
creg c2[2];        //anther classical register with 1 bit
```

Figure 1: quantum circuit

Step-2:

Here q[0] i.e $|\psi\rangle$is a qubit that needs to be transported from Alice to Bob. While q[1] and q[2] are the entangle pair qubits that are used to transport the quantum state $|\psi\rangle$ between Alice and Bob. q1 is held by Alice while q2 is held by bob.

let us define the state q[0] i.e $|\psi\rangle$ as

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$

To create an entangled pair between q[1] and q[2], initially we need to apply Hadamard(H) gate on q[1]. let's take q[1] and q[2] are in state $|0\rangle$ initially, now apply Hadamard(H) gate.

$$H\,|q1\rangle = H\,|0\rangle = |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Now apply a CNOT gate with q[1] as control and q[2] as target.It creates an Bell state $|\beta_{00}\rangle$

$$\begin{aligned}|q1\rangle\,|q2\rangle &= \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]|0\rangle \\ &= \frac{|00\rangle + |10\rangle}{\sqrt{2}} \\ CNOT\,|q1q2\rangle &= CNOT\left[\frac{|00\rangle + |10\rangle}{\sqrt{2}}\right] \\ |\beta_{00}\rangle &= \frac{|00\rangle + |11\rangle}{\sqrt{2}}\end{aligned}$$

Let's see the OpenQASM code corresponding to the above mathematical equations

Code

```
OPENQASM 2.0;
include "qelib1.inc";
qreg q[3];
creg c1[1];
creg c2[1];

h q[1];                // H-gate applied on q1
cx q[1], q[2];        // CNOT gate with q1 as a control and q2 as target
barrier q;            // Barrier is used to separate the steps
```

Circuit

Let's the see the circuit build by using the above code in IBM Quantum composer software.

Figure 2: Quantum Circuit

Step-3:

Alice holds two qubits q0 i.e $|\psi\rangle$ and q1. Now Alice should apply CNOT gate with q0 as a control and q1 as a target i.e $(CNOT \otimes I \otimes I)$ on a combined state of three qubits. and then Hadamard gate is applied on the combined state i.e $(H \otimes I \otimes I)$. the combined state of three qubits will be

$$|\psi\rangle \otimes |\beta_{00}\rangle = (a|0\rangle + b|1\rangle)\left[\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}}\left[a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle\right]$$

Apply $(H \otimes I \otimes I)(CNOT \otimes I)$ on the combined state.

$$(H\otimes I\otimes I)(CNOT\otimes I)\big(|\psi\rangle\otimes|\beta_{00}\rangle\big) = (H\otimes I\otimes I)(CNOT\otimes I)\frac{1}{\sqrt{2}}\left[a|000\rangle+a|011\rangle+b|100\rangle+b|111\rangle\right]$$

$$= (H \otimes I \otimes I)\frac{1}{\sqrt{2}}\left[a|000\rangle + a|011\rangle + b|110\rangle + b|101\rangle\right]$$

$$= \frac{1}{2}\left[|00\rangle\left[a|0\rangle+b|1\rangle\right]+|01\rangle\left[a|1\rangle+b|0\rangle\right]+|10\rangle\left[a|0\rangle-b|1\rangle\right]+|11\rangle\left[a|1\rangle-b|0\rangle\right]\right]$$

let's see the OpenQASM code corresponding to the above mathematical equations

Code

```
OPENQASM 2.0;
include "qelib1.inc";
qreg q[3];
creg c1[1];
creg c2[1];

h q[1];
cx q[1], q[2];
barrier q;

cx q[0], q[1];   // CNOT gate with q0 as control and q1 as target
h q[0];          // H-gate applied on q0
barrier q;       // barrier to separate the steps
```

Circuit

By implementing the above code in IBM Quantum composer we can get the circuit given in figure-3.

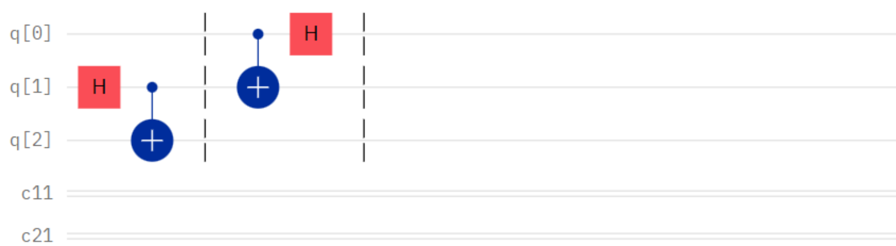

Figure 3: Quantum circuit

Step-4:

From the final equation we got Alice measures the two qubits i.e q0 and q1 by applying measurement gates on both qubits and stores that information into the two classical bits.Then Alice sends these two bits to the Bob.

Code

```
OPENQASM 2.0;
include "qelib1.inc";
qreg q[3];
creg c1[1];
creg c2[1];
```

```
h q[1];
cx q[1], q[2];
barrier q[0], q[1], q[2];

cx q[0], q[1];
h q[0];
barrier q[0], q[1], q[2];

measure q[0] -> c1[0];      //measurement done on q0 and value stores in c1
measure q[1] -> c2[0];      //measurement done on q1 and value stores in c2
barrier q;                  // barrier separates the steps
```

Circuit

By implementing the above code we will get the circuit like this.

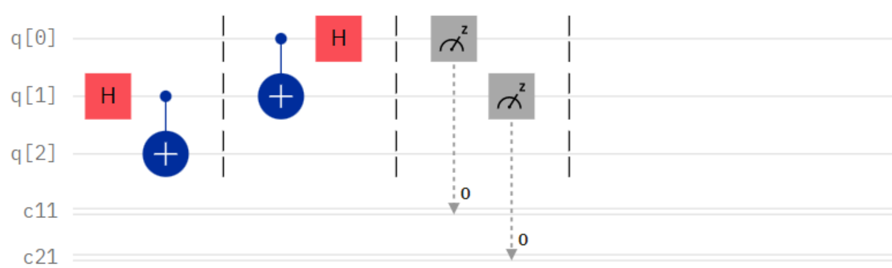

Figure 4: Quantum circuit

Step-5:

Bob receives the information that Alice sent in the form classical bits.
If Alice gets $|00\rangle$ then Bob's receives a state $(a|0\rangle + b|1\rangle)$
If Alice gets $|01\rangle$ then Bob's receives a state $(a|1\rangle + b|0\rangle)$
If Alice gets $|10\rangle$ then Bob's receives a state $(a|0\rangle - b|1\rangle)$
If Alice gets $|11\rangle$ then Bob's receives a state $(a|1\rangle - b|0\rangle)$

Using the bits received over the classical channel ,Bob reconstructs the state on his side by applying the particular gate on q2.
If state is
$(a|0\rangle + b|1\rangle) \rightarrow DoNothing$
(a $|1\rangle + b|0\rangle) \rightarrow Apply \quad X - gate$
(a $|0\rangle - b|1\rangle) \rightarrow Apply \quad Z - gate$
(a $|1\rangle - b|0\rangle) \rightarrow Apply \quad ZX - gate$

Code

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[3];
creg c1[1];
creg c2[1];

h q[1];
cx q[1], q[2];
barrier q[0], q[1], q[2];

cx q[0], q[1];
h q[0];
barrier q[0], q[1], q[2];

measure q[0] -> c1[0];
measure q[1] -> c2[0];
barrier q[0], q[1], q[2];

if (c1 == 1) x q[2];    // X-gate will be applied only if c1 is in 1 state
if (c2 == 1) z q[2];    // Z-gate will be applied only if c2 is in 1 state
```

Circuit

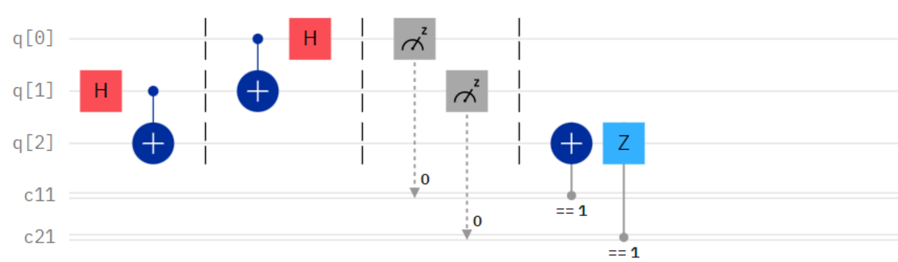

Figure 5: Quantum circuit

** At the end of this protocol Alice will teleport the qubit to Bob.