OPTIMAL CONTROL OF VARIABLE SPEED WIND TURBINES OPERATING IN BELOW AND ABOVE RATED SPEED

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***Abstract*—Wind power was recognized as a valuable resource thousands of years before internal combustion engines and modern power plants were developed. There are many different types of wind turbines in use around the world, used in diverse applications from powering small pumps on remote ranches to producing bulk electric power for modern cities. And here this project proposes a MIMO linear quadratic regulator (LQR) controller designed for a horizontal variable speed wind turbine by focusing on the operating range referring to the above and below rated wind speeds. This operating condition of wind turbines make them subject to fluctuating loads that create exhaustion and lead to damage. Minimizing the load would reduce the needed materials, and increase the life span and quality of the energy produced. The trade-off between the wind energy conversion maximization and the minimization of the fatigue in the mechanical structure defines the optimality. Therefore, the solution is design of an LQR controller to increase the system performance.**

Keywords*— LQR controller, Horizontal variable wind speed turbine, Electrical power output.

INTRODUCTION

Wind industries are designing challenging wind turbines which supports wind energy and transforms them into electricity. These wind turbines are classified into two different types which are horizontal wind turbine and vertical wind turbine. In this project, we are using horizontal variable speed wind turbine because of its advantages over vertical wind turbine. Wind turbines are large, complex dynamically flexible structures that they are capable of operating in turbulent and unpredictable environmental conditions and also efficiency and reliability are dependent on control design strategies. And the objective of optimal control theory is to determine the control signals that will cause a process to satisfy the physical constraints and at the same time to minimize or maximize some performance criterion.

Moreover, Control system design parameters are obtained after various methods of analysis and acceptable performance are defined in frequency and time domain. For example, rise time, settling time, gain and bandwidth. However, in case of multiple input and multiple output system, they are required to meet the demands of modern technology. But, here the first important challenge is to mostly emphasis on load reduction and grid integration. And the second challenge is to deliver good quality energy from the undesirable source. Therefore, based on the value of wind speed two regions are being analyzed which is shown in Fig.1.

The first one corresponds to low wind operation, where our goal is to increase the output power of the wind turbine and the second one operates in the region greater than the rated value of wind speed i.e., around 14 m/s which is called nominal value and it aims to maintain the power constant throughout.

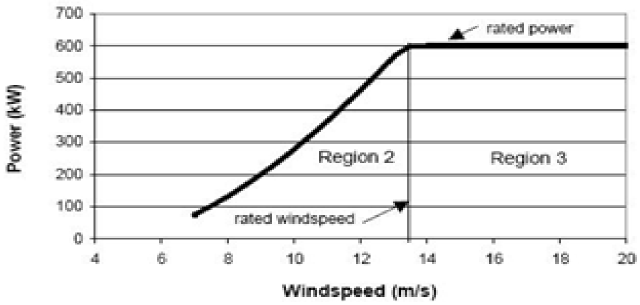

Fig 1. Curve of wind turbine speed versus power

Therefore, LQR Regulator, provides us to be a good solution due to the fact that it facilitates multivariable and multi-objective control design.

MATHEMATICAL MODEL

To compute the model, we have considered a mechanical system of arbitrary complexity which is given by the equation of motion:

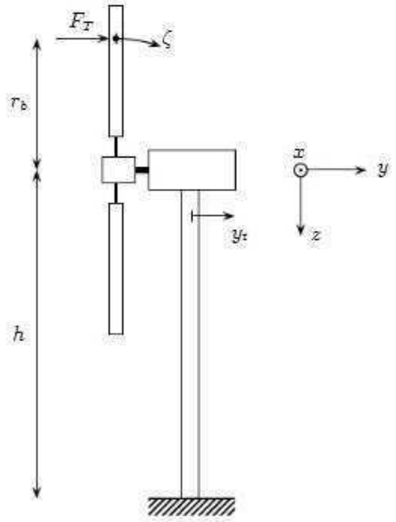

Fig 2. The mechanical structure of the wind turbine

$$M \cdot \ddot{q} + C \cdot \dot{q} + K . q = Q(\dot{q}, q, t, u) \quad (1)$$

Where $M, C, K,$ are the mass, damping and stiffness matrices, Q is the vector of forces acting on the system, and q_i is the generalized coordinate? For our model, the generalized coordinate are $(q = w_T, w_G, \zeta_1, \zeta_2, y_T)$, Where w_T is the angular speed of the rotor, w_G stands for the angular speed of the generator, ζ_1 *and* ζ_2 are the flaps of the blades, and y_T represents the horizontal movement of the tower. And here since the forces acting on the blades are equal, we are considering $\zeta_1 = \zeta_2 = \zeta$ which transforms q into q = (ω_T, ω_G, ζ, y_T) Now, to find

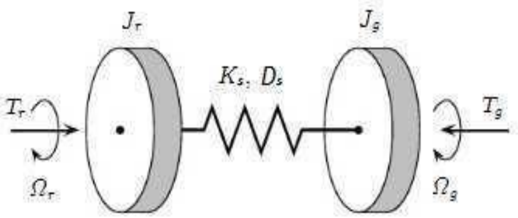

Fig. 3 The two-mass model representation of the drive train

Therefore, the forces acting on the system are: C_{aero}, the aerodynamic torque, C_{em}, the electromagnetic torque, and F_{aero}, representing the thrust. The aerodynamic torque and the force acting on the entire rotor are expressed in terms of non-dimensional power coefficient C_P and the thrust coefficient C_T respectively as follows,

$$C_P = \frac{P}{0.5 . \rho . A . v^3 . N_g . N_b}, N = \frac{120}{f} \quad (2)$$

$$\omega_T = \frac{60 . v . TSR}{\pi . D}$$

$$\text{TSR} = \frac{Blade\ tip\ speed}{Wind\ speed\ in\ mph}$$

$$\omega_a = \frac{D}{2} . \left(\frac{C_P . \rho . \frac{D^2}{2} . v^3}{41}\right)^{0.5} \quad (3)$$

$$\omega = \frac{v}{R}, C_{em} = \frac{P}{\omega},$$

Where P represents wind power, ρ represents the air density, N_g is the generator efficiency, f represents the frequency and N_b is the gear box efficiency, v is the average wind speed, ω is the angular velocity, R is the radius of the rotor, D is the diameter of the rotor and TSR is the tip speed ratio. The power coefficient is significant because it offers information about the efficiency of the turbine and it also helps in defining the control objectives Moreover, it also defines the aerodynamic torque that helps in moving the turbine's rotor. Here power and thrust coefficient depends upon tip speed ratio and pitch angle of the blades.

To derive the mathematical models, let us consider the LaGrange equation,

$$\frac{d}{dt}\left(\frac{\delta E_c}{\delta \dot{q}_i}\right) - \frac{\delta E_c}{\delta q_i} + \frac{\delta E_d}{\delta \dot{q}_i} + \frac{\delta E_P}{\delta q_i} = Q \quad (4)$$

Here E_c, E_d, E_P denotes the kinetic, dissipated and potential energies. Therefore, we now obtain

$$E_c = \frac{J_t}{2} . {\omega_t}^2 + \frac{J_G}{2} . {\omega_G}^2 + \frac{M_T}{2} . \left({\dot{y}_T}^2 + r_P . \dot{\zeta}\right)^2$$

$$E_D = \frac{d_A}{2} . (\omega_T - \omega_G)^2 + d_P . (r_P . \dot{\zeta})^2 \quad (5)$$

$$E_P = \frac{k_A}{2} . (\theta_T - \theta_G)^2 + k_P . (r_P . \zeta)^2 + \frac{k_T}{2} . {y_T}^2$$

These energies were calculated under the supposition that the generalized force that acts on the rotor is applied on a point situated at the distance r_P on each blade from the hub of the rotor (Fig. 2). In the above equations, J_T and J_G represent the rotor and the generator moments of inertia, M_T and M_P are the masses of the tower and of the blade, d_P, d_A and d_T represent the damping coefficients for the blade, drive shaft and tower. Similarly, k_P, k_A and k_T stand for the spring coefficients of the blade, drive shaft and tower. Θ_T and Θ_G are the angular positions of the rotor and generator. The interconnection of the models of different plant subsystems, leads to a highly nonlinear system. For this study, we have supposed that the saturation values in position are -45˚ and 45˚, and that the servomotor does not exceed the speed of 10˚ /s.

By combining all the equations, we get

$$\dot{x}(t) = A . x(t) + B . u(t) \quad (6)$$
$$y(t) = C . x(t) + D . u(t)$$

Where m_v represents a perturbation acting on the system and A, B, C, D, are as follows.

$$A=\begin{bmatrix} -1.1908*10^{-08} & 1.4770*10^{-05} & 8.6167*10^{-06} & -1.1308*10^{-06} & -2.1809*10^{-07} & -3.1325*10^{-06} & -1.4353*10^{-06} & 4.723*10^{-07} & 2.9313*10^{-06} \\ 2.17*10^{-08} & -1.13*10^{-06} & -5.0266*10^{-70} & 3.1841*10^{07} & 6.1057*10^{-08} & -3.6456*10^{-08} & -1.593*10^{-08} & -1.2360*10^{-07} & 1.262*10^{-05} \\ -3.2249*10^{-09} & -8.4253*10^{-06} & -3.7209*10^{-06} & -1.4056*10^{-07} & -2.6316*10^{-08} & -3.8155*10^{-07} & -1.8789*10^{-07} & 4.7862*10^{-08} & 3.263*10^{-05} \\ 5.6009*10^{-05} & 1.5156*10^{-04} & 8.1582*10^{-05} & 8.5439*10^{-04} & 1.6360*10^{-04} & 2.2662*10^{-05} & 1.6955*10^{-05} & -3.2910*10^{-04} & 0.0100 \\ 5.6832*10^{-05} & -8.3673*10^{-04} & -3.573*10^{-04} & 8.4709*10^{-04} & 1.6227*10^{-04} & -1.6843*10^{-05} & -2.5621*10^{-06} & -3.2706*10^{-04} & 0.0116 \\ 1.4477*10^{-08} & 6.9508*10^{-06} & 3.8520*10^{-06} & -1.1091*10^{-07} & -2.1765*10^{-08} & -1.0704*10^{-06} & -4.8623*10^{-07} & 5.725*10^{-08} & -1.7575*10^{-06} \\ 1.6427*10^{-07} & 6.3092*10^{-05} & 3.3888*10^{-05} & 3.7562*10^{-08} & 2.2905*10^{-09} & -7.7959*10^{-06} & -3.5189*10^{-06} & 1.0454*10^{-07} & -4.6028*10^{-06} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.2132*10^{-49} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0608 \end{bmatrix}$$

$$[B=]\begin{bmatrix} 0.0010 & 8.9947*10^{-08} & -2.622*10^{-04} \\ 0.0012 & -4.1593*10^{-09} & -0.0023 \\ 0.0037 & 1.8583*10^{-09} & -9.3880*10^{-04} \\ 0.6248 & -1.1203*10^{-05} & -4.8728 \\ 0.8086 & -1.1110*10^{-05} & -4.9197 \\ 1.2626*10^{-05} & -1.4654*10^{-12} & -6.8600*10^{-07} \\ 3.263*10^{-05} & 6.3159*10^{-13} & 2.6564*10^{-07} \\ 0 & 0 & 1.0 \\ 6.7085 & 0 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad D=\begin{bmatrix} 0 & 0 & 10^{-5} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here the system consists of three inputs and four outputs where the input consist of v_m the average value of the wind speed, and the two control variables: the pitch angle β, and the electromagnetic torque C_{em}. The state vectors as $x^T = \left(\theta_T - \theta_G, \zeta, y_T, \omega_T, \omega_G, \dot{\zeta}, y_T, \beta, v\right)^T$ the output of the system is given by $y = (P_{el}, \omega_T, \zeta, y_T)$ and the command signal is given by $u = (\beta, C_{em})$ where$P_{_el}$ represents electric power generated by the turbine and here we use the normalized value where it can be computed as $P_{\_el} = \omega_G \, . C_{em}$.

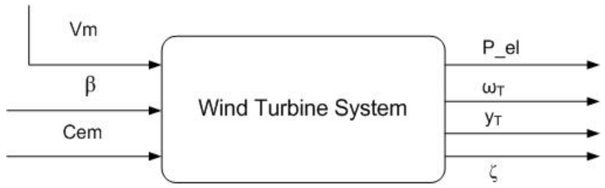

Fig.4 The block scheme of the controlled system

The output variable ω_T is maintained constant and therefore, value of the flap mode of the blades and tower of the blades are varied as much as possible. The pitch angle and the electromagnetic torque are fixed here.

GENERAL PROCEDURE OF THE LINEAR QUADRATIC CONTROLLER DESIGN

In general, simulation results of a wind turbine can be obtained with three different types of controllers such as classical PID regulator, a full state feedback and a fuzzy regulator. Since, PID and Full state feedback regulators do not allow a rigorous control design to perform a fine tuning of the trade-off between the energy performance and the reliability demands we use Linear Quadratic Regulator as a state feedback in this project. For this design, the cost function is defined as

$$J = \int_0^{\infty} (y^T . Q . y + u^T . R . u) . dt \qquad (7)$$

The feedback control which minimizes the value of cost is given by: $u = -K . x$

Where K is given by $K = R^{-1} . B^T . P_C$, P_C is given by solution to the equation:

$$P_c . A + A^T . P_C - P_C . B . R^{-1} . B^T . P_C + Q = 0 \qquad (8)$$

This matrix ensures that the reference input matrix is scaled in order to equate the feedback signal provided by the LQR regulator. This algorithm guaranties that no matter, any two symmetric and positive definite matrixes Q and R that we chose in order to minimize the quadratic criteria, there is always a matrix P_c, which is also symmetric and positive definite, that represents the riccati equation(10).Therefore, to minimize the flap modes of the blades, the tower oscillation, to maintain the electrical power level and the angular speed of the rotor at desired levels we can replace the variables y and u by the corresponding vectors discussed before in section two. The weighting matrixes R and Q are selected based on the Bryson's rule which states that it contains matrixes such that all diagonals with the non-zero elements are scaled and the variables that are present in the optimization criteria have a maximum value of one. And the state vectors and units are numerically different as per the different components of the command such as pitch angle and electromagnetic torque. Hence, if we choose large values of Q compared to values in R, then the value of mechanical weights get minimized.

RESULTS

The operating point for the linearization of the system corresponds to the average value of the wind speed of 15m/s. Here the normalized electrical power and the angular rotor speed was chosen as constants with the appropriate values in order to minimize the variations of electrical power and to keep the rotor speed constant. R and Q values are chosen such that they provide very good performance of the system.

Where Q and R are defined as,

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.03 & 0 & 0 & 0 \\ 0 & 10^{-5} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The cost function is written as

$$J = \int_0^{\infty} (x^T . Q_1 . x + u^T . R_1 . u + 2 . x^T . S . u),$$

Where $Q_1 = C_1^T . Q . C_1$, $R_1 = R + D_1^T . Q . D_1$,

$S = C_1^T . Q . D_1$,

and the matrices C_1 and D_1 being truncated blocks from the system matrices C and D. These matrices contain the lines and columns from C and D corresponding to the control variables C_{em} and β. Also here the LQR guarantee nominally stable closed loop systems and the electrical power output and the angular speed of the rotor manage to follow the reference and maintain the nominal imposed values. Moreover, the first flap mode of the blades and bending of the towers which were meant to be minimized have extremely small values. And the blades have a deviation of about 5mm while the tower has an insignificant movement on the horizontal direction.

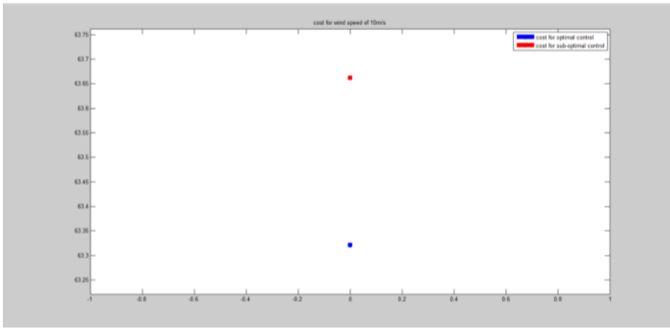

Fig. 8 Optimal cost and Suboptimal cost wind speed of 10 m/s

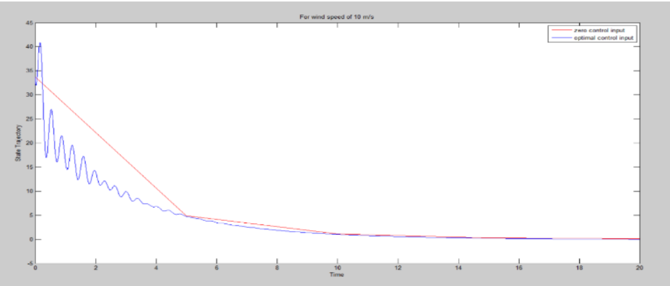

Fig. 5 State trajectories when wind speed is 10 m/s

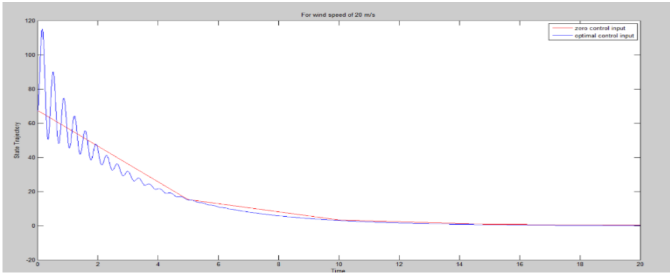

Fig. 9 state trajectories when wind speed is 20 m/s

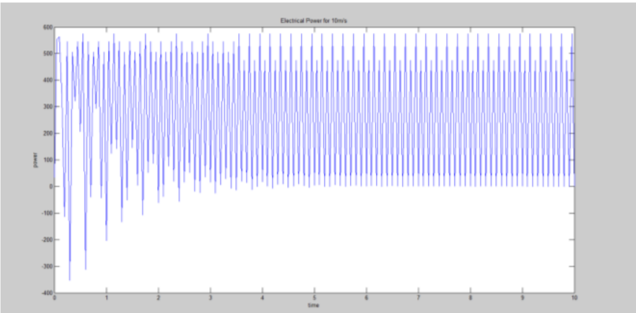

Fig. 6 Power vs Time output for wind speed of 10 m/s

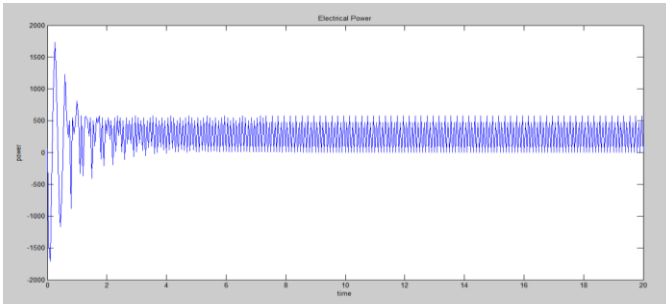

Fig. 10 Power vs Time output for wind speed of 20 m/s

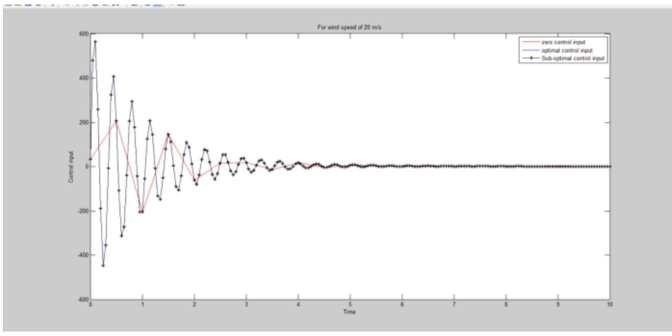

Fig. 7 Control input for wind speed of 10 m/s

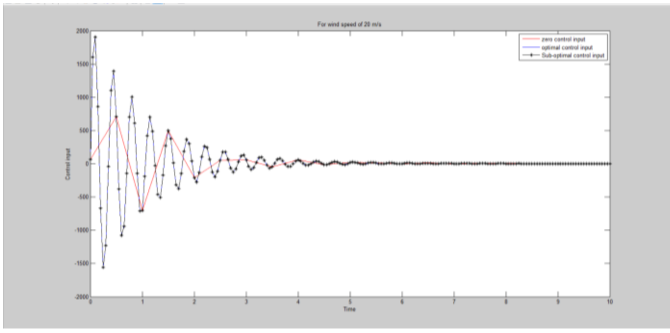

Fig. 11 Control input for wind speed of 20 m/s

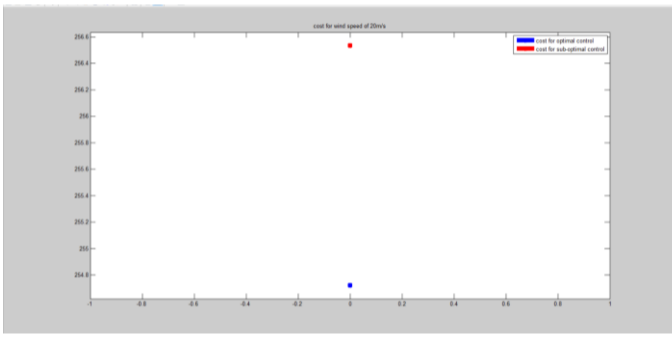

Fig. 12 Optimal and Suboptimal cost for wind speed of 20 m/s

APPENDIX

Physical Measure	Value
Nominal Power	15Kw
Blade deviation	5mm
Wind speed	20m/s
Wind power	4900 Kw
Wind speed	10m/s
Wind power	612.5 Kw
Angular displacement between rotor and generator	720 (20 m/s) 210 (10 m/s)
Air Density	1184kg/m^3
TSR	6
Diameter of the rotor	34m

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