import numpy as np

import matplotlib.pyplot as plt

def lda(X, y):

    class\_labels = np.unique(y)

    mean\_vectors = []

    for label in class\_labels:

        mean\_vectors.append(np.mean(X[y == label], axis=0))

    d = X.shape[1]

    SW = np.zeros((d, d))

    SB = np.zeros((d, d))

    overall\_mean = np.mean(X, axis=0)

    for i, label in enumerate(class\_labels):

        class\_scatter = np.zeros((d, d))

        for x in X[y == label]:

            x = x.reshape(d, 1)

            mean\_vec = mean\_vectors[i].reshape(d, 1)

            class\_scatter += (x - mean\_vec).dot((x - mean\_vec).T)

        SW += class\_scatter

        n = X[y == label, :].shape[0]

        mean\_vec = mean\_vectors[i].reshape(d, 1)

        overall\_mean = overall\_mean.reshape(d, 1)

        SB += n \* (mean\_vec - overall\_mean).dot((mean\_vec - overall\_mean).T)

    SW\_inv = np.linalg.inv(SW)

    eig\_vals, eig\_vecs = np.linalg.eig(SW\_inv.dot(SB))

    eig\_pairs = [(np.abs(eig\_vals[i]), eig\_vecs[:, i]) for i in range(len(eig\_vals))]

    eig\_pairs = sorted(eig\_pairs, key=lambda k: k[0], reverse=True)

    W = eig\_pairs[0][1].reshape(d, 1)

    X\_lda = X.dot(W)

    return X\_lda, W