

Natural Numbers : Numbers which are used for counting the objects are called natural numbers. They are denoted by N.

$$N = \{ 1, 2, 3, \dots \}$$

All positive integers are natural numbers.

Whole numbers :- When 'zero' is included in the natural numbers, they are known as whole numbers.

They are denoted by W.

$$W = \{ 0, 1, 2, 3, \dots \}$$

Integers : All natural numbers, zero and negatives of natural numbers are called as integers.

They are denoted by I.

$$I = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$$

Rational numbers : The numbers which can be expressed in the form of $\frac{p}{q}$ where P and Q are integers and $q \neq 0$ are called rational numbers

They are called by Q.

$$\text{E.g.} = \frac{1}{2}, \frac{12}{8}, -6 \text{ (as } -6 = \frac{-6}{1} \text{) etc.}$$

Irrational numbers : The numbers which cannot be written in the form of $\frac{p}{q}$ where P and Q are integers and $q \neq 0$ are called irrational numbers.

$$\text{e.g.- } \sqrt{3}, \sqrt{7}, \frac{2}{17} \text{ etc}$$

When these numbers are expressed in decimal form, they are neither terminating nor repeating.

$$\text{e.g.} = \frac{1}{7}, \frac{2}{17} \text{ etc.}$$

Real numbers : Real numbers include both rational as well as irrational numbers.

Positive or negative, large or small, whole numbers or decimal numbers are all real numbers.

e.g.= 1, 13.79, -0.01, $\frac{2}{3}$ etc.

Imaginary numbers : An imaginary number is a complex number that can be written as a real number multiplied by the imaginary unit 'i' which is defined by its property $i^2 = -1$

Note : Zero (0) is considered to be both real and imaginary number.

Prime number : A prime number is a natural number greater than 1 and is divisible only by 1 and itself.

e.g. 2, 3, 5, 7, 11, 13, 17, 19etc.

Note :- 2 is the only even prime number.

Composite Numbers : A number, other than 1, which is not a prime number is called a composite number .

E.g. 4, 6, 8, 9, 10, 12, 14, 15etc.

Note : 1 is neither a prime number nor a composite number.

2 there are 25 prime numbers between 1 and 100.

To find whether a number is prime or not-

To check whether the number is prime or not,

1 We take an integer larger than the square root of the number. Let the number be 'k'.

2 Test the divisibility of the given number by every prime number less than 'k'.

3 If it is not divisible by any of them, then the given number is prime otherwise it is a composite number.

E.g.= Is 881 a prime number ?

Sol- The appropriate square root of 881 is 30.

Prime number less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

881 is not divisible by any of the above numbers, so it is a prime number.

Co-prime numbers : Two numbers are co-prime if their HCF is 1.

E.g. (2,3), (3,4), (5,7), (3,13) etc.

Even numbers : The number which is divisible by 2 is called even number.

E.g. – 2, 4, 6, 8,

Odd numbers – The number which is not divisible by 2 is called odd number.

e.g. = 3, 5, 7, 9,

Consecutive numbers : A series of numbers in which the succeeding number is greater than the preceding number by 1 is called a series of consecutive numbers.

i.e., Difference between two consecutive numbers is 1.

Some Rules on Counting Numbers

1. Sum of all the first n natural numbers

$$= \frac{n(n+1)}{2}$$

Q. Find the sum of first 20 natural numbers.

Ans- Sum of 1 to 20

$$\begin{aligned} &\text{Sum of 1 to 20} \\ &\frac{20(20+1)}{2} = 210 \end{aligned}$$

Q. Find the sum of numbers from 11 to 20.

$$\text{Ans Sum of 1 to 20} = \frac{20(20+1)}{2} = 210$$

$$\text{Sum of 1 to 10} = \frac{10(10+1)}{2} = 55$$

$$\text{Sum of 11 to 20} = 210 - 55 = 155$$

2. Sum of first n odd numbers =

$$n^2$$

Q. What is the sum of first 10 odd numbers ?

$$\text{Ans- Sum of first 10 odd numbers} = (10)^2 = 100$$

Q. Find the sum of 9+11+13+.....+29

$$\text{Ans} - 1+3+5+\dots\dots\dots+29 = (15)^2 = 225$$

(as there are 15 odd numbers from 1 to 29)

$$1+3+5+7 = (4)^2 = 16$$

$$9+11+13+29 = 225 - 16 = 209$$

3. Sum of first n even numbers

$$n(n+1)$$

Q. What is the sum of even numbers between 1 and 50 ?

$$\text{Ans} - \text{No. of even numbers between 1 and 50} = \frac{50}{2} = 25$$

$$\begin{aligned} \text{Sum of even numbers between 1 and 50} \\ = 25(25+1) = 25 \times 26 = 650 \end{aligned}$$

Q. Find the value of 12+14+.....+30.

Ans- (2+4+6+.....+30) has 15 even numbers

$$2+4+6+\dots+30=15(15+1)=240$$

$$\text{Similarly } 2+4+6+8+10=5(5+1)=30$$

$$12+14+\dots+30=240-30=210$$

4. Sum of squares of first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

Q. what is the value of $1^2 + 2^2 + \dots + 10^2$?

Ans- $1^2 + 2^2 + \dots + 10^2$?

$$\begin{aligned} &= \frac{10(10+1)(2 \times 10+1)}{6} \\ &= \frac{10 \times 11 \times 21}{6} = 385 \end{aligned}$$

5. Sum of cubes of first n natural numbers.

$$= \left[\frac{n(n+1)}{2} \right]^2$$

Q. What is the value of $1^3 + 2^3 + \dots + 5^3$?

Ans- $1^3 + 2^3 + \dots + 5^3$

$$= \left[\frac{5(5+1)}{2} \right]^2 = \left[\frac{5 \times 6}{2} \right]^2 = 225$$

Divisibility Rules

➤ **Divisibility by 2** : Number Whose last digit is either even or zero is divisible by 2.

➤ **Divisibility by 3** : If the sum of the digits of a number is divisible by 3, the number is also divisible by 3.

➤ **Divisibility by 4** : If the last two digits of a Number is divisible by 4 or the number having two or more zeros at the end, the numbers is divisible by 4.

- • Divisibility by 5: If a number is divisible by 5 or 0, the number is divisible by 5.
- • Divisibility by 6 : If a number is divisible by both 2 and 3 the number is also divisible by 6.
- • Divisibility by 8 : If the last three digits of a number is divisible by 8 or the last three digits of a number are zeros, the number is divisible by 8.
 - • Divisibility by 9 : If the sum of all the digits of a number is divisible by 9, the number is also divisible by 9.
 - • Divisibility by 10. The number which ends with zero is divisible by 10.
 - • Divisibility by 11. If the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.
 - • Divisibility by 12. The number which is divisible by both 3 and 4 is also divisible by 12.