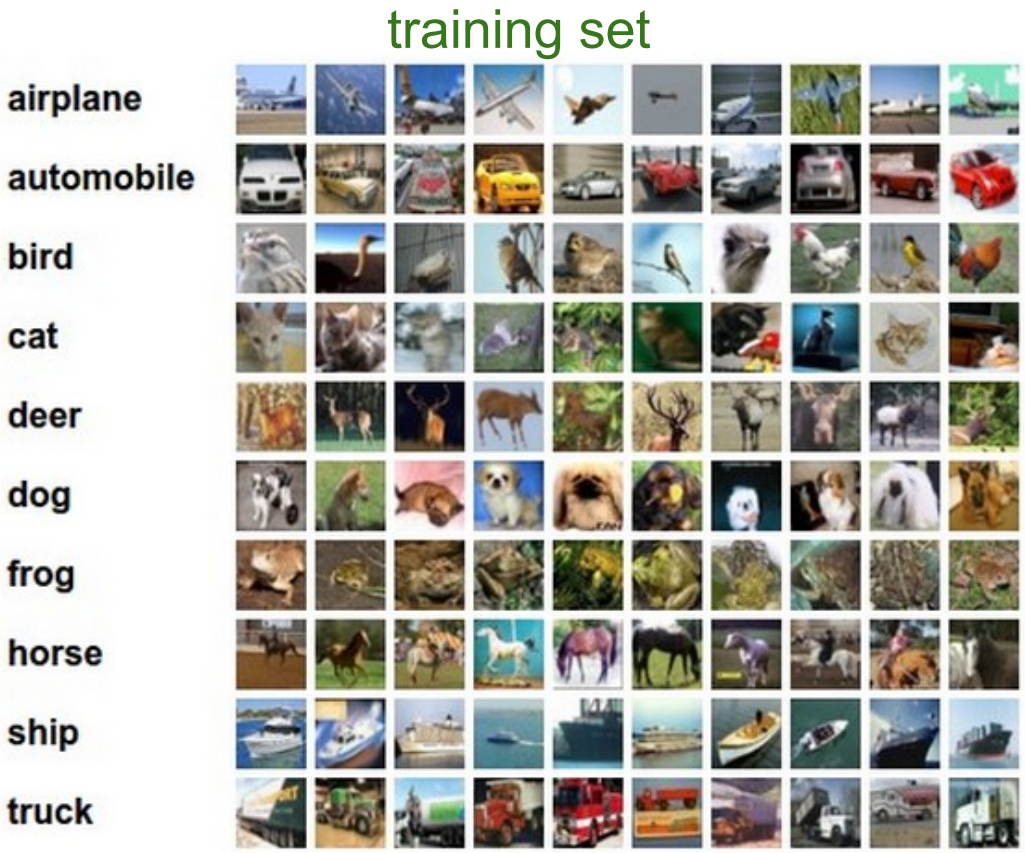
Lecture 3:
Linear Classification

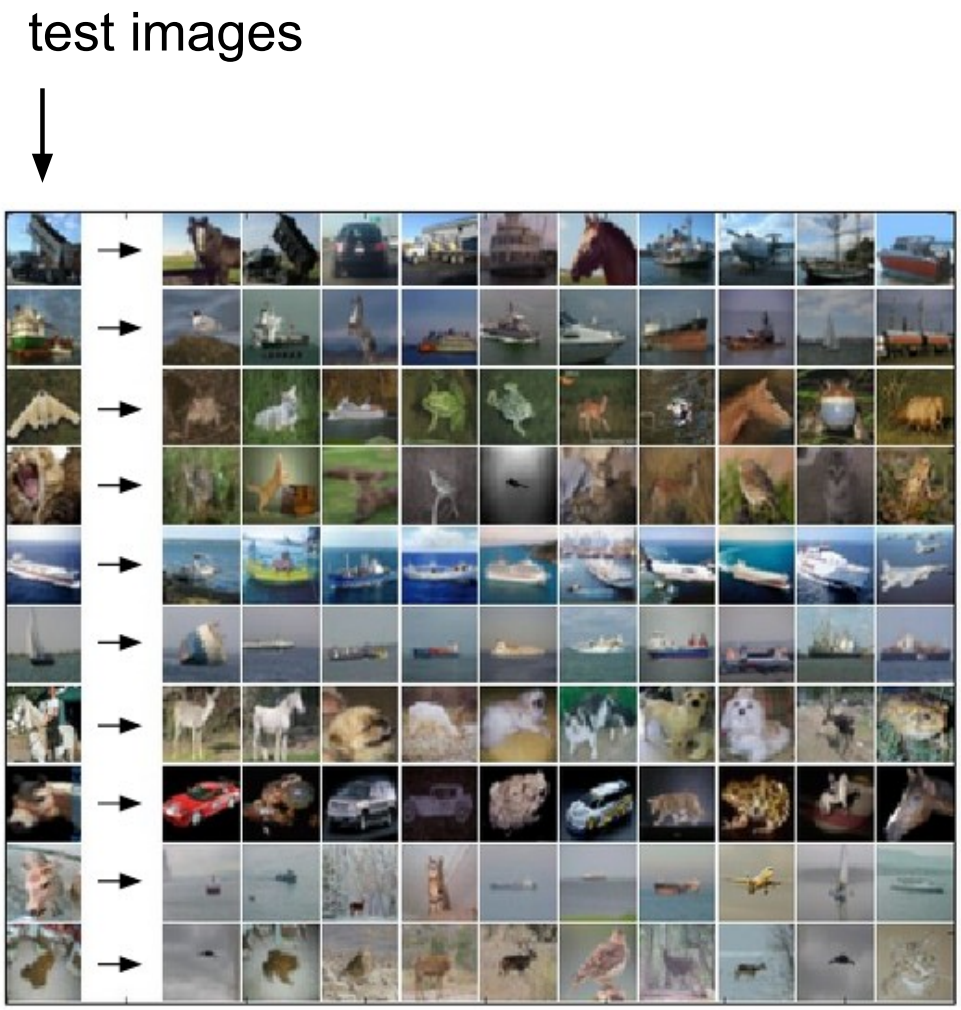
Last time: Image Classification

assume given set of discrete labels
{dog, cat, truck, plane, ...}

cat

k-Nearest Neighbor





Linear Classification

1. define a **score function**

class scores

Linear Classification

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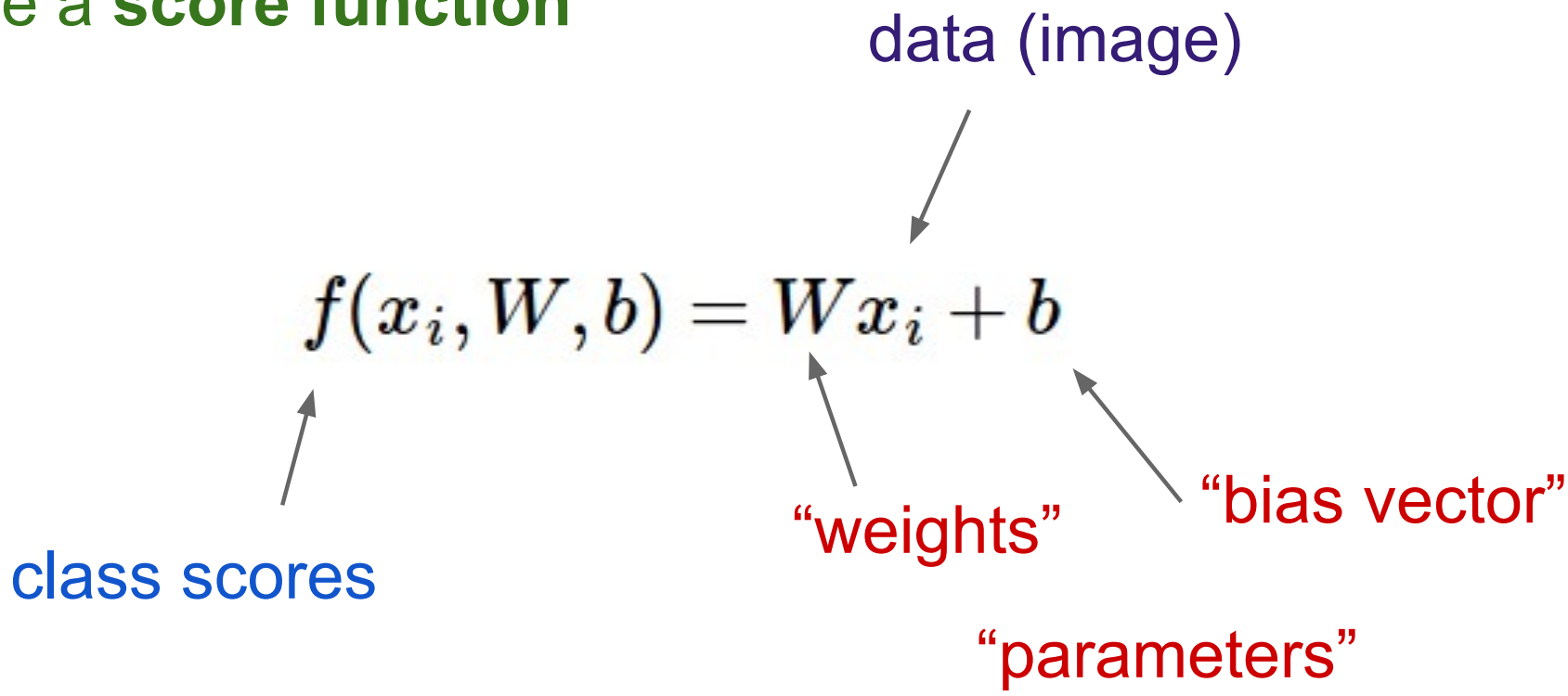


Linear Classification

1. define a **score function**

(assume CIFAR-10 example so
32 x 32 x 3 images, 10 classes)

data (image)
[3072 x 1]

$$f(x_i, W, b) = Wx_i + b$$

class scores

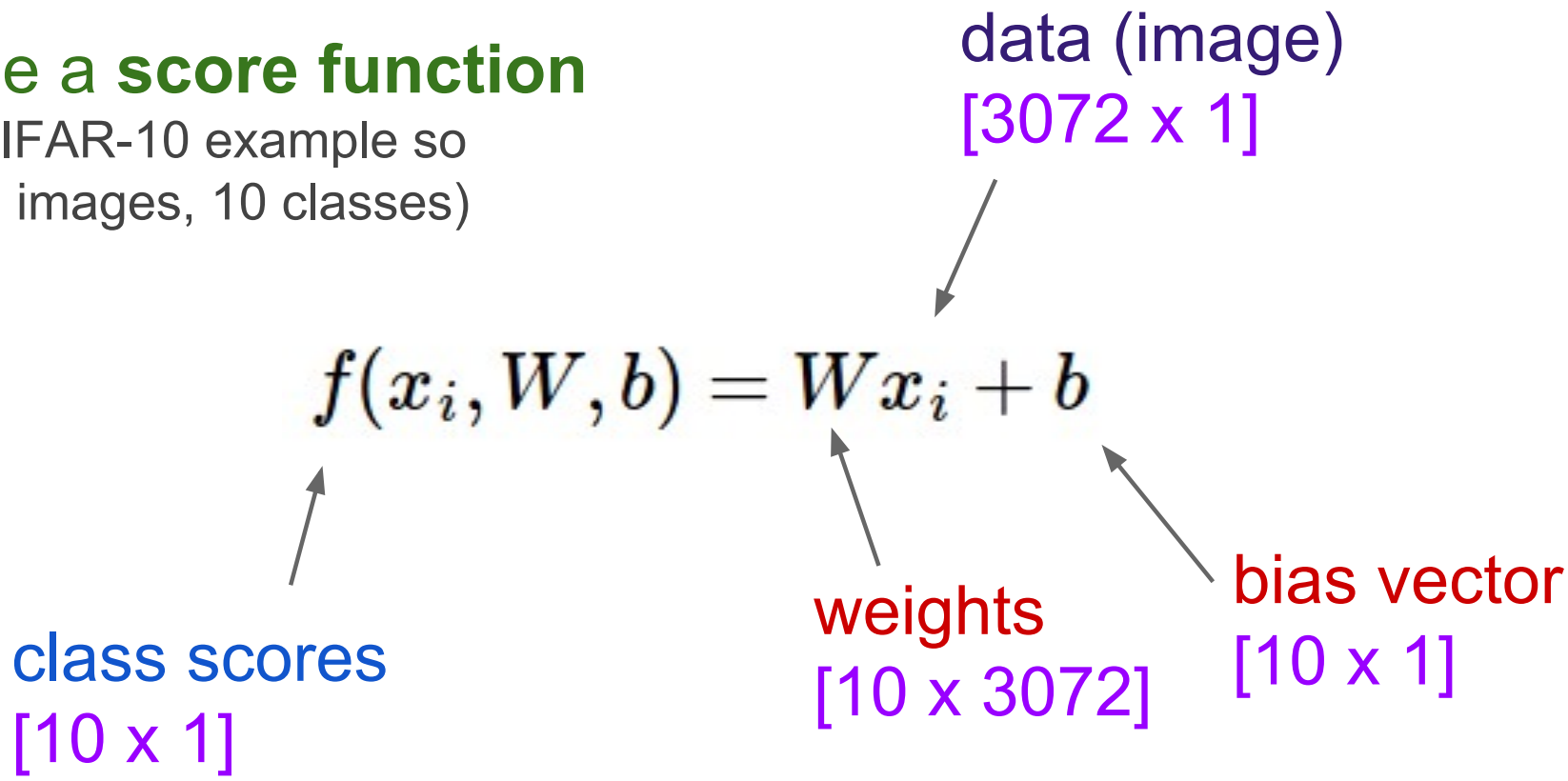
weights

bias vector

Linear Classification

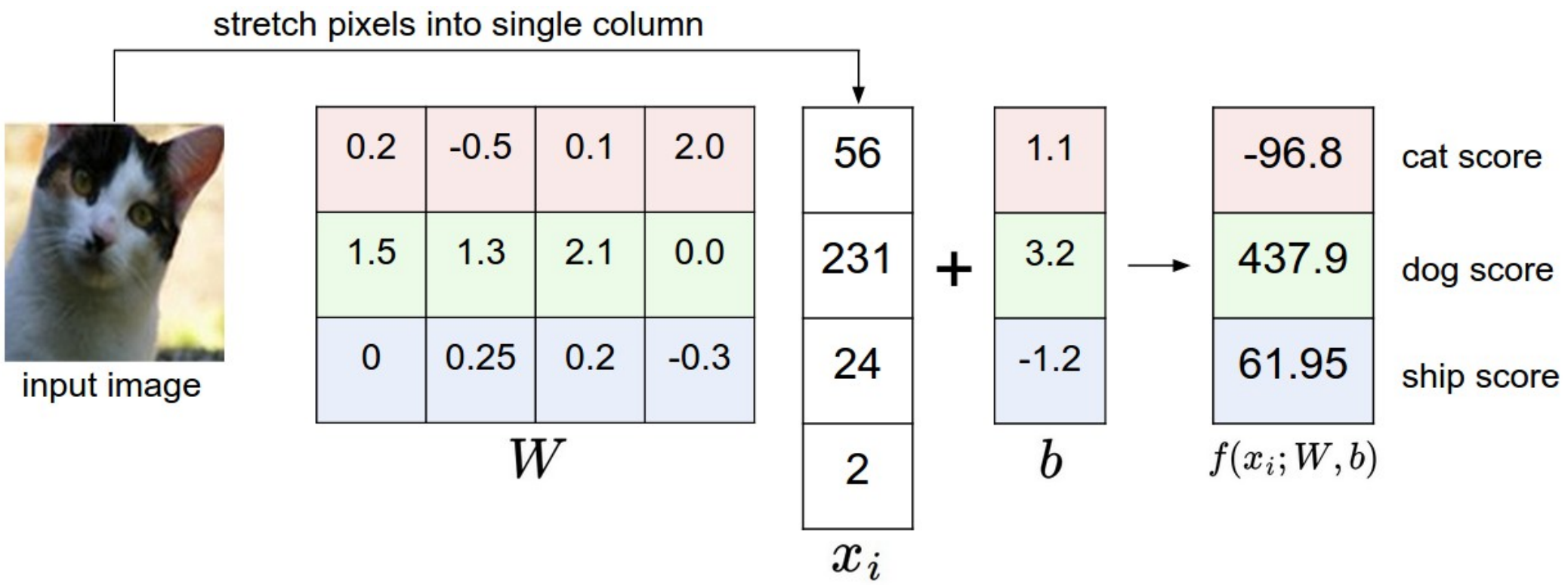
1. define a **score function**

(assume CIFAR-10 example so
32 x 32 x 3 images, 10 classes)



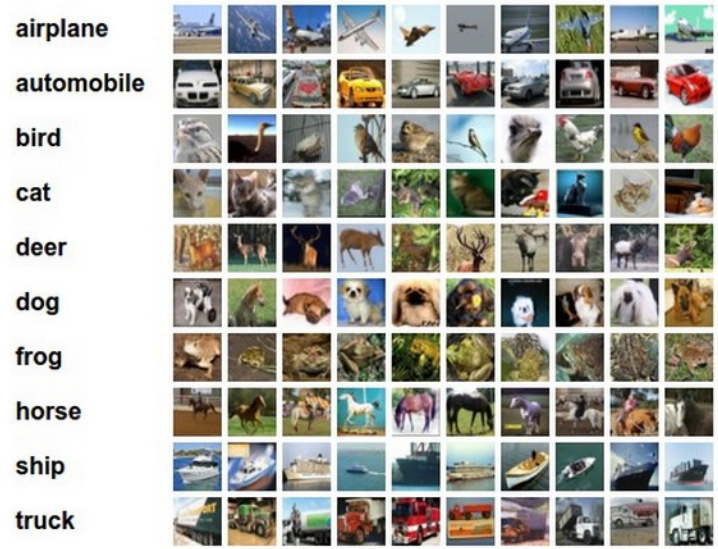
Linear Classification

$$f(x_i, W, b) = Wx_i + b$$



Interpreting a Linear Classifier

Question:
what can a linear classifier do?



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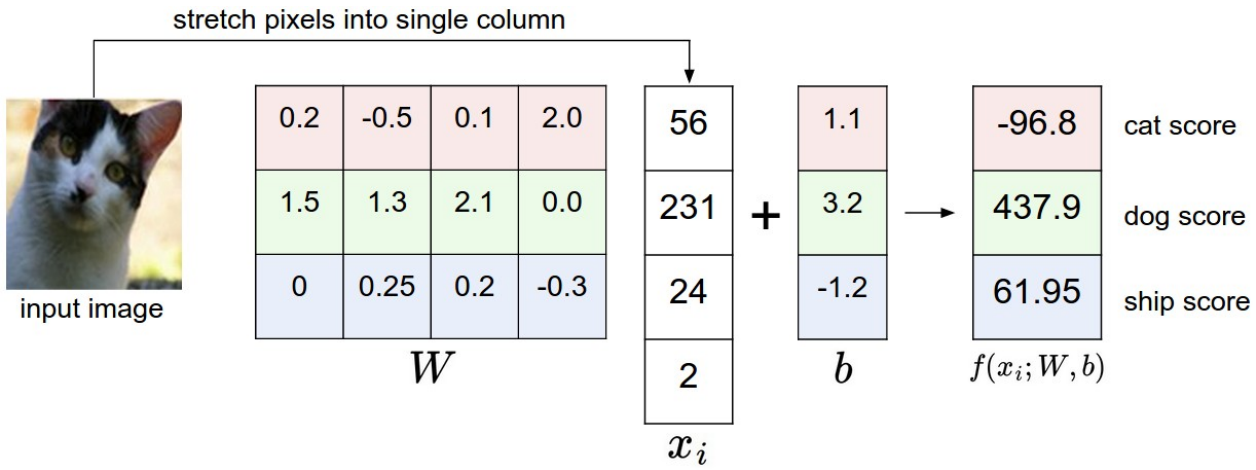
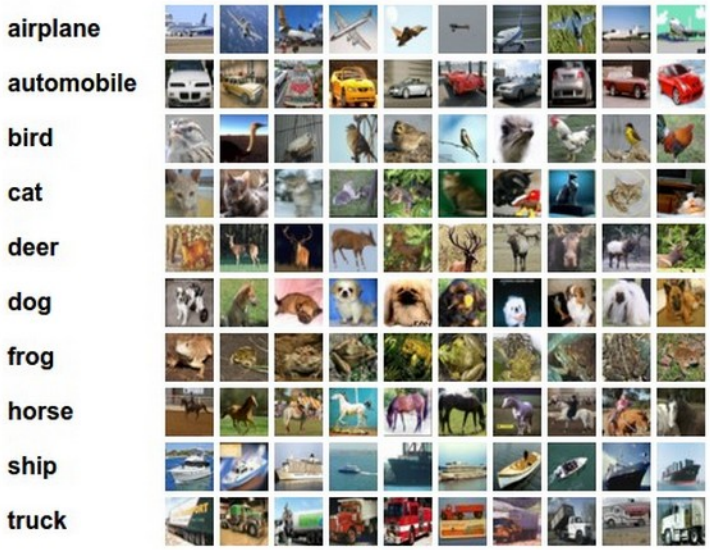


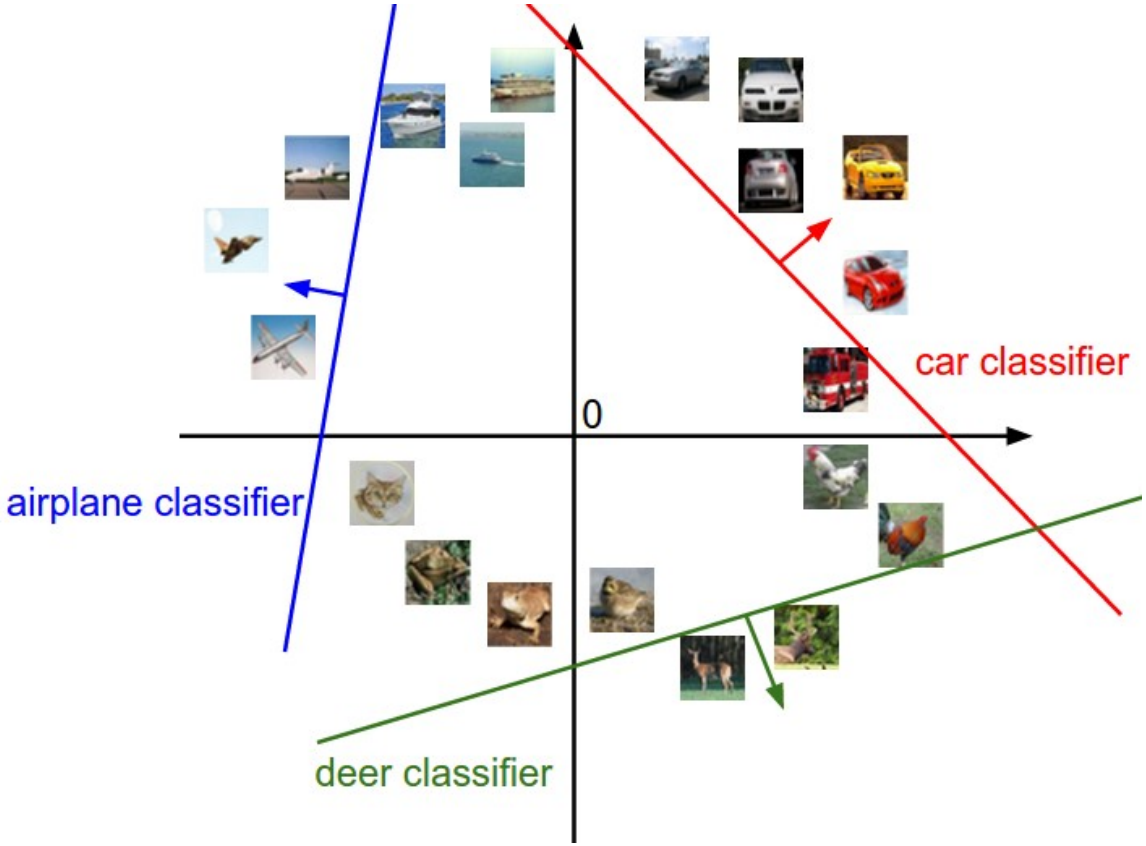
Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

Example training classifiers on CIFAR-10:

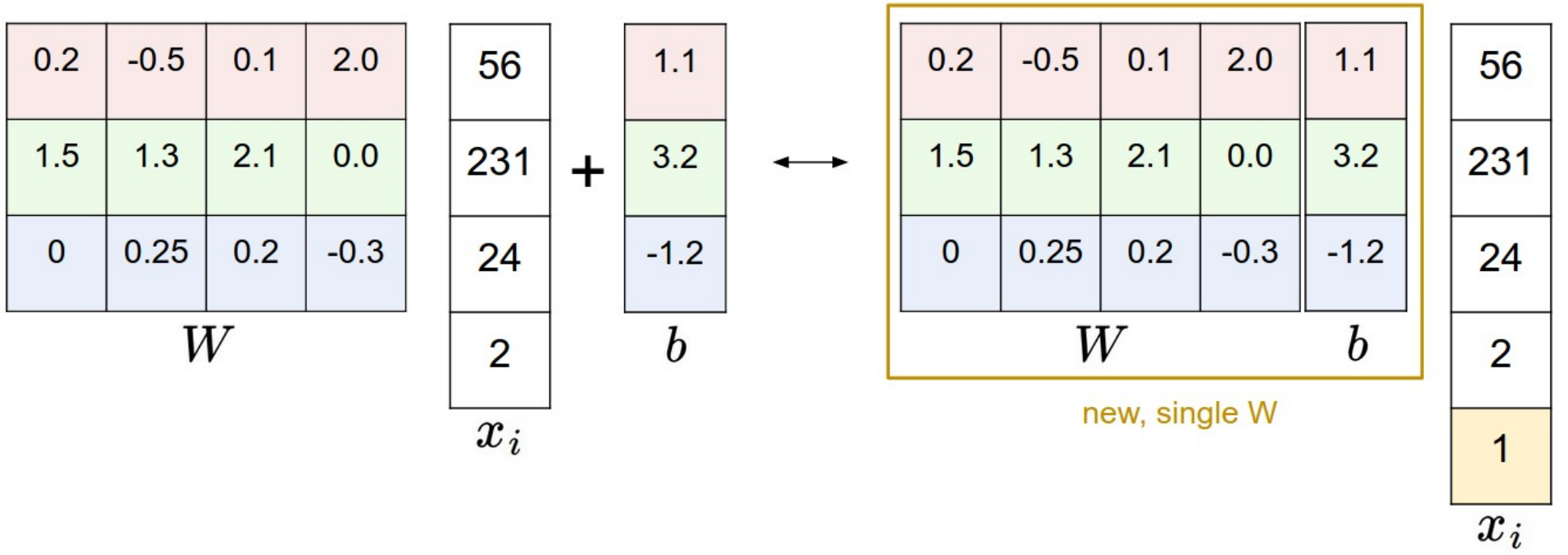
Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

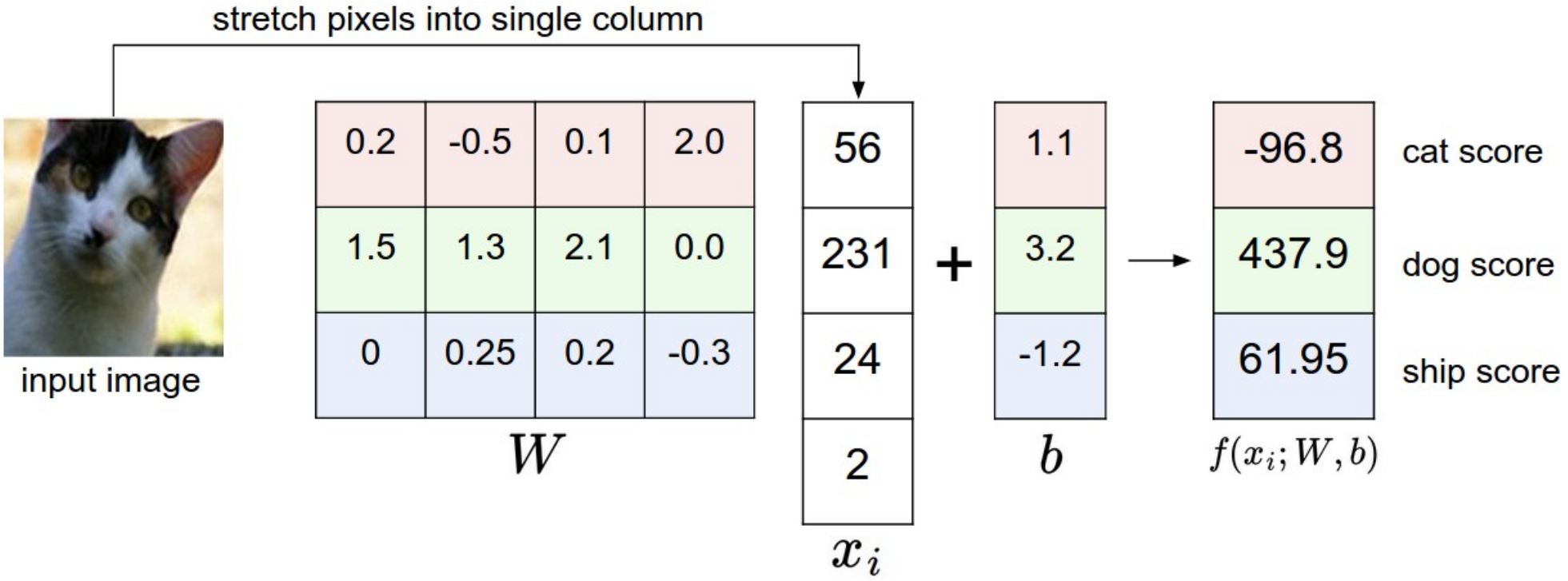
Bias trick

$$f(x_i, W, b) = Wx_i + b \longrightarrow f(x_i, W) = Wx_i$$



So far:

We defined a (linear) **score function**: $f(x_i, W, b) = Wx_i + b$



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Example:

$$f(x_i, W) = [13, -7, 11]$$

$$y_i = 0$$

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- scores, label ⟶ *loss*.

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Example:

$$f(x_i, W) = [13, -7, 11]$$

$$y_i = 0$$

Question: if you were to assign a single number to how "unhappy" you are with these scores, what would you do?

2. Define a **loss function** (or cost function, or objective)
One (of many ways) to do it: **Multiclass SVM Loss**

$$L_i = \sum_{j \neq y_i} \max(0, f(x_i, W)_j - f(x_i, W)_{y_i} + \Delta)$$

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(One possible generalization of Binary Support Vector Machine to multiple classes)

$$L_i = C \max(0, 1 - y_i w^T x_i) + R(W)$$

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One (of many ways) to do it: **Multiclass SVM Loss**

$$L_i = \sum_{j \neq y_i} \max(0, f(x_i, W)_j - f(x_i, W)_{y_i} + \Delta)$$

loss due to example i

sum over all incorrect labels

difference between the correct class score and incorrect class score

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e.g. 10

$$f(x_i, W) = [13, -7, 11]$$

$$y_i = 0$$

---

loss = ?

Example: $$L_i = \sum_{j \neq y_i} \max(0, f(x_i, W)_j - f(x_i, W)_{y_i} + \Delta)$$

e.g. 10

$$f(x_i, W) = [13, -7, 11]$$

$$y_i = 0$$

$$L_i = \max(0, -7 - 13 + 10) + \max(0, 11 - 13 + 10)$$

$$L_i = \sum_{j \neq y_i} \max(0, f(x_i, W)_j - f(x_i, W)_{y_i} + \Delta)$$

```
def L_i_vectorized(x, y, W):
  scores = W.dot(x)
  margins = np.maximum(0, scores - scores[y] + 1)
  margins[y] = 0
  loss_i = np.sum(margins)
  return loss_i
```

There is a bug with the objective…

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right]$$

L2 Regularization

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$

Regularization strength

L2 regularization: motivation

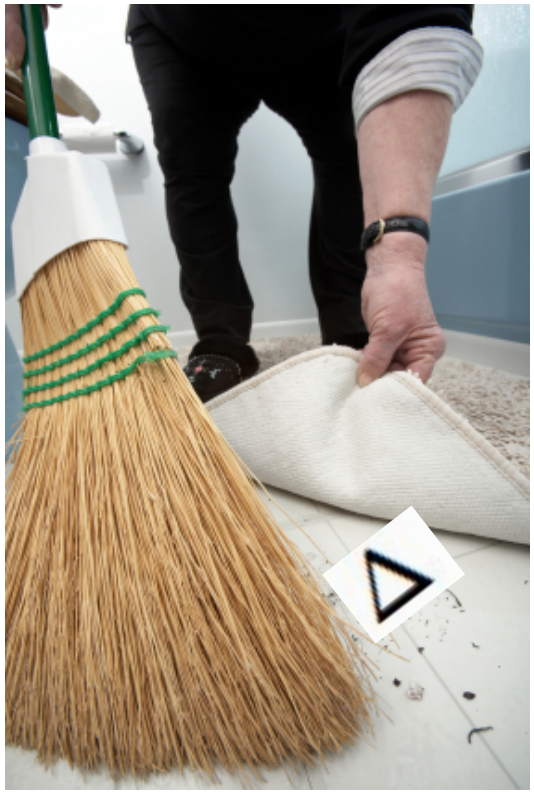
$$x = [1, 1, 1, 1]$$

$$w_1 = [1, 0, 0, 0]$$

$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

$$w_1^T x = w_2^T x = 1$$

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$



$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$

Do we have to cross-validate both Δ and λ ?

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$

$$\max(0, \delta f + \Delta)$$

$$\rightarrow \max(0, \delta f + \Delta_2) \qquad \text{if } \Delta \rightarrow \Delta_2$$

$$\rightarrow \max(0, \frac{\Delta_2}{\Delta} \delta f + \Delta_2) \qquad \text{then scale the weights by } \frac{\Delta_2}{\Delta}$$

$$= \max(0, \frac{\Delta_2}{\Delta} (\delta f + \Delta))$$

$$= \frac{\Delta_2}{\Delta} \max(0, \delta f + \Delta)$$

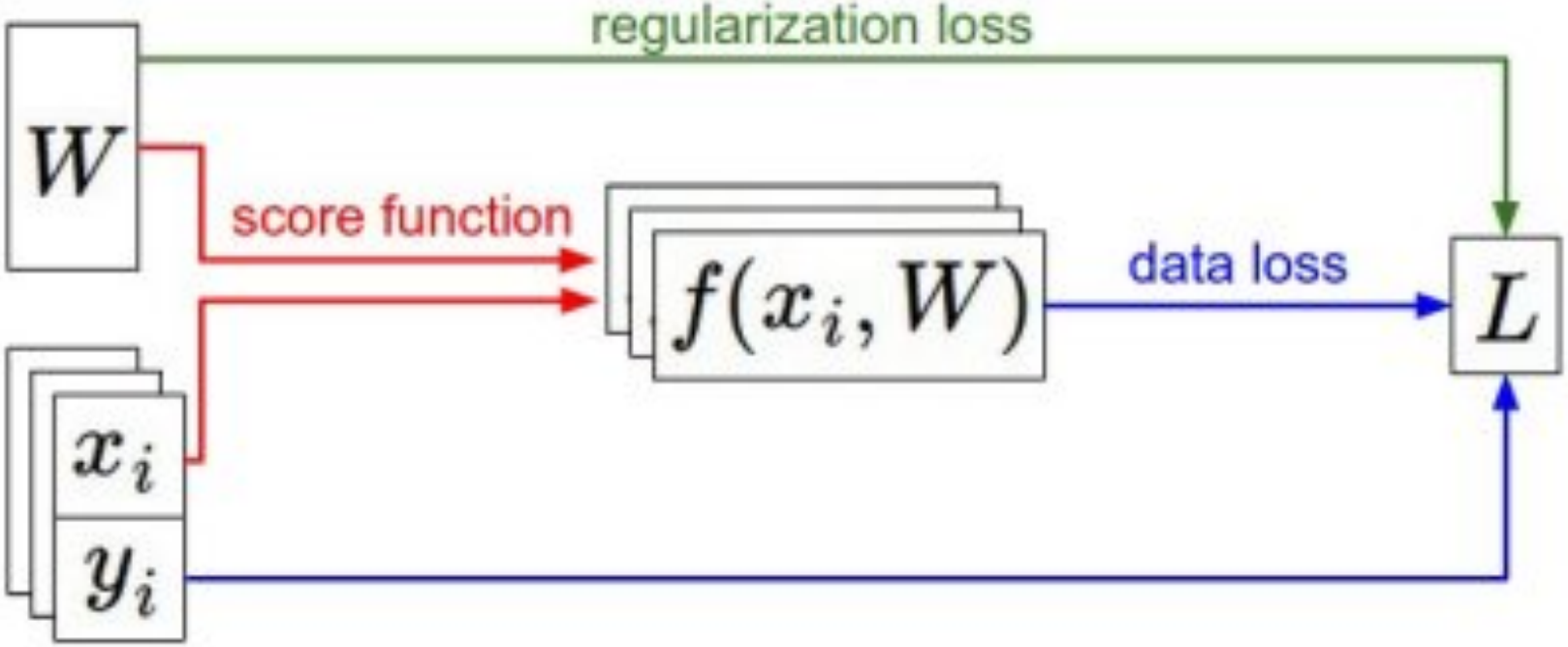
So far…

1. Score function

$$f(x_i, W, b) = Wx_i + b$$

2. Loss function

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$


regularization loss
W
score function
$f(x_i, W)$
data loss
L
x_i
y_i