CHAPTER

NUMBER SYSTEM

3.1 INTRODUCTION

The various number systems are decimal system, binary system, hexadecimal system, etc. We are familiar with decimal number system as this system is used in our day to day work. Digital computers can understand only binary language (0 & 1). To understand the operation of a computer we need binary, octal and hexadecimal number system.

3.1.1 Base or Radix

Total numbers available in a number system is called base or radix. For example, the base of decimal number system is ten because the numbers available in decimal system are 0,1,2,3,4,5,6,7,8,9. The base of binary system is two because the numbers are 0 and 1. The base of octal system is eight because the numbers are 0, 1, 2, 3, 4, 5, 6, 7. The base of the hexadecimal system is 16 because the numbers are 0-9, A, B, C, D, E, F. The first sixteen numbers in all the number systems are as shown in the following table:

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	8	8
9	1001	9	9
10	1010	10	A

(Continued)

Decimal	Binary	Octal	Hexadecimal
11	1011	11	B
12	1100	12	C
13	1101	13	D
14	1110	14	E
15	1111	15	F

3.1.2 Positional Weights

(a) The **decimal number system** has weights in powers of 10.
For example $(375.5)_{10}$ can be represented as follows:

10^2	10^1	10^0	10^{-1}
3	7	5	5

$= 3 * 100 + 7 * 10 + 5 * 1 + 5/10$
$= 300 + 70 + 5 + 0.5$
$= 375.5$

(b) **Binary number system** has weights in powers of two.
For example $(1101.11)_2$

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
1	1	0	1	1	1

$= 1 * 8 + 1 * 4 + 0 * 2 + 1 * 1 + 1/2 + 1/4$
$= 8 + 4 + 0 + 1 + 0.5 + 0.25$
$= (13.75)_{10}$

(c) **Octal number system** has weights in powers of eight.
For example $(134.2)_8$ can be represented as follows:

8^2	8^1	8^0	8^{-1}
1	3	4	2

$= 1 * 64 + 3 * 8 + 4 * 1 + 2/8$
$= 64 + 24 + 4 + 0.25$
$= (92.25)_{10}$

(d) **Hexadecimal number** system has weights in powers of 16.
For example $(2AF.48)_{16}$ can be represented as follows:

16^2	16^1	16^0	16^{-1}	16^{-2}
2	A	F	4	8

$= 2 * 256 + 10 * 16 + 15 * 1 + 4/16 + 8/256$

$= 512 + 160 + 15 + 0.25 + 0.03125$

$= (687.28125)_{10}$

3.2 CONVERSION FROM ONE BASE TO ANOTHER

There are two types of conversion:

1. Successive division and multiplication (Example: decimal to other systems).
2. Putting positional weights (Example: other number systems to decimal).

3.2.1 Successive Division and Multiplication

(a) Decimal to Binary: To convert decimal to binary, successively divide the given decimal by two. Multiply the fraction by 2 (4 times). The decimal to binary conversion is called **double dabble** method.

Example 1: $(35.12)_{10} = (?)_2$

2 | 35		
2 | 17	1	
2 | 8	1	
2 | 4	0	
2 | 2	0	
1	0	

$0.12 * 2 = 0.24$	0
$0.24 * 2 = 0.48$	0
$0.48 * 2 = 0.96$	0
$0.96 * 2 = 1.92$	1

Ans. $(35.12)_{10} = (100011.0001)_2$

Example 2: $(62.8125)_{10} = (?)_2$

2 | 62	
2 | 31	0
2 | 15	1
2 | 7	1
2 | 3	1
1	1

$0.8125 * 2 = 1.6250$	1
$0.6250 * 2 = 1.2500$	1
$0.2500 * 2 = 0.5000$	0
$0.5000 * 2 = 1.0000$	1

Ans. $(62.8125)_{10} = (111110.1101)_2$

(b) Decimal to Octal

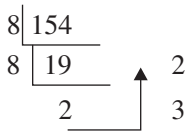
Example 1: $(245.13)_{10} = (\ ?\ )_8$

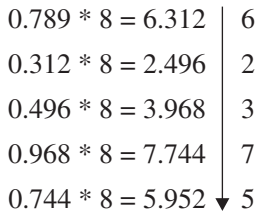
8 | 245	
8 | 30	5
3	6

$0.13 * 8 = 1.04$	1
$0.04 * 8 = 0.32$	0
$0.32 * 8 = 2.56$	2
$0.56 * 8 = 4.48$	4

Ans. $(245.13)_{10} = (356.1024)_8$

Example 2: $(154.789)_{10} = (\ ?\ )_8$

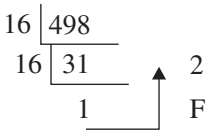


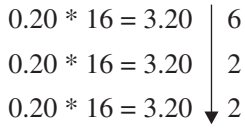


Ans. $(154.789)_{10} = (232.62375)_8$

(c) Decimal to Hexadecimal

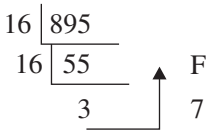
Example 1: $(498.20)_{10} = (\ ?\ )_{16}$

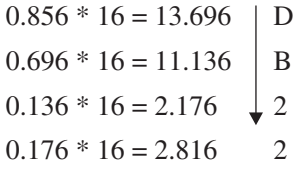




Ans. $(498.20)_{10} = (1F2.333)_{16}$

Example 2: $(895.856)_{10} = (\ ?\ )_{16}$





Ans. $(895.856)_{10} = (37F.DB22)_{16}$

3.2.2 Putting Positional Weights

(a) Binary to Decimal

Example 1: $(10110.101)_2 = (?)_{10}$

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	0	1	1	0	1	0	1

$= 1 * 16 + 0 * 8 + 1 * 4 + 1 * 2 + 0 * 1 + 1/2 + 0/4 + 1/8$

$= 16 + 0 + 4 + 2 + 0 + 0.5 + 0 + 0.125 = 22.625$

Ans. $(10110.101)_2 = (22.625)_{10}$

Example 2: $(11110.110)_2 = (\ ?\ )_{10}$

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	1	1	1	0	1	1	0

$= 1 * 16 + 1 * 8 + 1 * 4 + 1 * 2 + 0 * 1 + 1/2 + 1/4 + 0/8$

$= 16 + 8 + 4 + 2 + 0 + 0.5 + 0.25 + 0 = 30.75$

Ans. $(11110.110)_2 = (30.75)_{10}$

(b) Octal to Decimal

Example 1: $(127.46)_8 = (\ ?\ )_{10}$

8^2	8^1	8^0	8^{-1}	8^{-2}
1	2	7	4	6

= 1 * 64 + 2 * 8 + 7 * 1 + 4/8 + 6/64
= 64 + 16 + 7 + 4/8 + 6/64
= 87.5938

Ans. $(127.46)_8 = (87.5938\ )_{10}$

Example 2: $(540.657)_8 = (\ ?\ )_{10}$

8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}
5	4	0	6	5	7

= 5 * 64 + 4 * 8 + 0 * 1 + 6/8 + 5/64 + 7/512
= 320 + 32 + 0 + 0.75 + 0.0781 + 0.01367
= 352.84177

Ans. $(540.657)_8 = (352.84177)_{10}$

(c) Hexadecimal to Decimal

Example 1: $(A35F.BD)_{16} = (?)_{10}$

16^3	16^2	16^1	16^0	16^{-1}	16^{-2}
A	3	5	F	B	D

= 10 * 4096 + 3 * 256 + 5 * 16 + 15 * 1 + 11/16 + 13/256
= 40960 + 768 + 80 + 15 + 0.6875 + 0.05078
= 41823.7383

Ans. $(A35F.BD)_{16} = (41823.7383)_{10}$

Example 2: $(6BB.418)_{16} = (\ ?\ )_{10}$

16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}
6	B	B	4	1	8

= 6 * 256 + 11 * 16 + 11 * 1 + 4 /16 + 1 /256+ 8 /4096
= 1536 + 176 + 11 + 0.25000 + 0.00390 + 0.00195
= 1723.25585

Ans. $(6BB.418)_{16} = (1723.25585)_{10}$

3.2.3 Other Conversions

(a) Binary to Octal: To convert binary to octal, split the given binary in groups of three from right to left for integers and for left to right for fractions. For less number of bits in the split group, add zeroes. Write the corresponding octal digits.

Example 1: $(10110011.011101)_2 = (\ ?\ )_8$

$= 010\ 110\ 011\ .\ 011\ 101$
$= 2\quad 6\quad 3\quad 3\quad 5$

Ans. $(10110011.011101)_2 = (263.35)_8$

Example 2: $(11100101.1111)_2 = (\ ?\ )_8$

$= 011\ 100\ 101\ .\ 111\ 100$
$= 3\quad 4\quad 5\quad .\quad 7\quad 4$

Ans. $(11100101)_2 = (345.74)_8$

(b) Binary to Hexadecimal: To convert binary to hexadecimal, split the binary groups of four from right to left for integers and left to right for fractions. For less number of bits in the split group, add zeroes. Write the corresponding hexadecimal digits.

Example 1: $(100111111010.11010011)_2 = (\ ?\ )_{16}$

1001 1111 1010 . 1101 0011
9 F A . D 3

Ans. $(100111111010.11010011)_2 = (9FA.D3)_{16}$

Example 2: $(1111100101.101010)_2 = (\ ?\ )_{16}$

$= 0011\quad 1110\quad 0101\quad .\quad 1010\quad 1000$
$= 3\quad E\quad 5\quad .\quad A\quad 8$

Ans. $(1111100101.101010)_2 = (3E5.A8)_{16}$

(c) Octal to Binary: To convert octal to binary, write the octal equivalent of binary as shown in the table.

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Example 1: $(6054.1327)_8 = (?)_2$

= 110 000 101 100 . 001 011 010 111

Ans. $(6054.1327)_8 = (110000101100.001011010111)_2$

Example 2: $(7204.7005)_8 = (?)_2$

= 111 010 000 100 . 111 000 000 101

Ans. $(7204.7005)_8 = (111010000100.111000000101)_2$

(d) Octal to Hexadecimal: To convert octal to hexadecimal, first convert the given octal into binary and split the binary in groups of four. Write the corresponding hexadecimal as shown in the table.

Example 1: $(4056.1327)_8 = (?)_{16}$

Step 1: (Octal to Binary)

= 100 000 101 110 . 001 011 010 111

Step 2: (Binary to Hexadecimal)

= 1000 0010 1110 . 0010 1101 0111

= 8 2 E . 2 D 7

Ans. $(4056.1327)_8 = (42E.2D7)_{16}$

Example 2: $(513.001)_8 = (?)_{16}$

Step 1: (Octal to Binary)

= 101 001 011 . 000 000 001

Step 2: (Binary to Hexadecimal)

= 0001 0100 1011 . 0000 0000 1000

= 1 4 B . 0 0 8

Ans. $(513.001)_8 = (14B.008)_{16}$

(e) Hexadecimal to Binary: To convert hexadecimal to binary, write the four bit binary for the corresponding hexadecimal digit as shown in the table. For example:

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010

(Continued)

Hexadecimal	Binary
B	1011
C	1100
D	1101
E	1110
F	1111

Example 1: $(CF6.AB2)_{16} = (?)_2$

= 1100 1111 0110 . 1010 1011 0010

Ans. = $(CF6.AB2)_{16} = (110011110110.101010110010)_2$

Example 2: $(CAE.F0F)_{16} = (?)_2$

= 1100 1010 1110 . 1111 0000 1111

Ans. = $(CAE.FOF)_{16} = (110010101110 .111100001111)_2$

(f) Hexadecimal to Octal: To convert hexadecimal to octal, first convert the given hexadecimal into binary and split the binary groups of three and write the corresponding octal digit.

Example 1: $(810.751)_{16} = (?)_8$

Step 1: (Hexadecimal to Binary)

= 1000 0001 0000 . 0111 0101 0001

Step 2: (Binary to Octal)

= 100 000 010 000 . 011 101 010 001

= 4 0 2 0 . 3 5 2 1

Ans. $(810.751)_{16} = (4020.3521)_8$

Example 2: $(FACE.1234)_{16} = (?)_8$

Step 1: (Hexadecimal to Binary)

= 1111 1010 1100 1110 . 0001 0010 0011 0100

Step 2: (Binary to Octal)

= 001 111 101 011 001 110 . 000 100 100 011 010

= 1 7 5 3 1 6 . 0 4 4 3 2

Ans. $(FACE.1234)_{16} = (175316.04432)_8$