

## Introduction

An algorithm is a well-defined computational procedure that takes some value, or set of values, as **input** and produces some value, or set of values, as **output**.

An algorithm is a sequence of computational steps that transform the input into the output.

An algorithm is a tool for solving a well-specified computational problem.

**The sorting Problem:** The input is sequence of numbers  $\langle a_1, a_2, \dots, a_n \rangle$ . The output is a permutation (reordering)  $\langle a'_1, a'_2, \dots, a'_n \rangle$  of the input sequence such that  $a'_1 \leq a'_2 \leq \dots \leq a'_n$ .

**Example:** If the input is  $\langle 31, 41, 59, 26, 41, 58 \rangle$  a sorting algorithm should return the sequence  $\langle 26, 31, 41, 41, 58, 59 \rangle$  as output.

The above input sequence is called an **instance** of the sorting problem. In general, an **instance of a problem** consists of the input (satisfying whatever constraints are imposed in the problem statement) needed to compute a solution to the problem.

An algorithm is said to be **correct** if, for every input instance, it halts with the correct output. We say that a correct algorithm **solves** the given computational problem. An incorrect algorithm might not halt at all on some input instances, or it might halt with an answer other than the desired one.

**Analyzing an algorithm:** Predicting the resources required such as memory, communication bandwidth, or computational time. In most cases, the computational time is really what we want to measure.

**Efficiency:** Algorithms devised to solve the same problem often differ dramatically in their efficiency. These differences can be much more significant than differences due to hardware and software.

**Example:** Suppose we have two computers, computer A, a fast computer running insertion sort. Computer B, a slow computer running merge sort. Each must sort an array of ten million numbers. Suppose that computer A executes ten billion instructions per second and computer B executes only ten million instructions per second ( Notice that computer A is 1000 times faster than computer B ). Suppose Computer A requires  $2n^2$  instructions to sort  $n$  numbers and computer B requires  $50n \lg n$  ( $\lg n = \log_2 n$ ) instructions to sort the  $n$  numbers. Then  
Computer A takes

$$\frac{2.(10^7)^2 \text{ instructions}}{10^{10} \text{ instructions / second}} = 20,000 \text{ seconds (more than 5.5 hours)}$$

As for computer B, it takes

$$\frac{50.10^7 . \lg 10^7 \text{ instructions}}{10^7 \text{ instructions / second}} \approx 1163 \text{ seconds (less than 20 minutes)}$$

This means that computer B solves the problem at hand 20 times faster than computer A!