CS5300 Advanced Algorithms

HW #5

Kishor Kumar Andekar
700744713 (CRN:12008)

1. Illustrate the operation of MAX-HEAP-INSERT (A, 11) on the heap

$A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$

	1	2	3	4	5	6	7	8	9	10	11	12
A	15	13	9	5	12	8	7	4	0	6	2	1

A.heap-size = A.heap-size + 1 = 12 + 1 = 13

A[A.heap-size] = $-\infty$

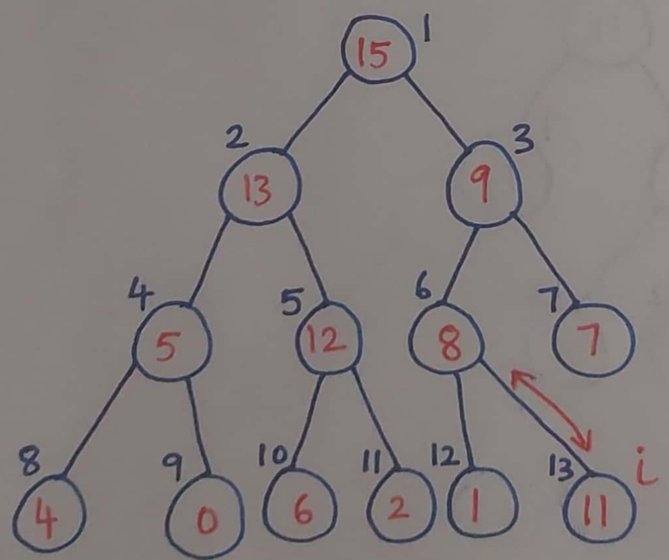
A[13] = 11

HEAP-INCREASE-KEY (A, 13, 11)

Key < A[i] $\Rightarrow$ 11 < A[13]

$\Rightarrow$ 11 < 11 # False

A[i] = Key $\Rightarrow$ A[13] = 11



(a)

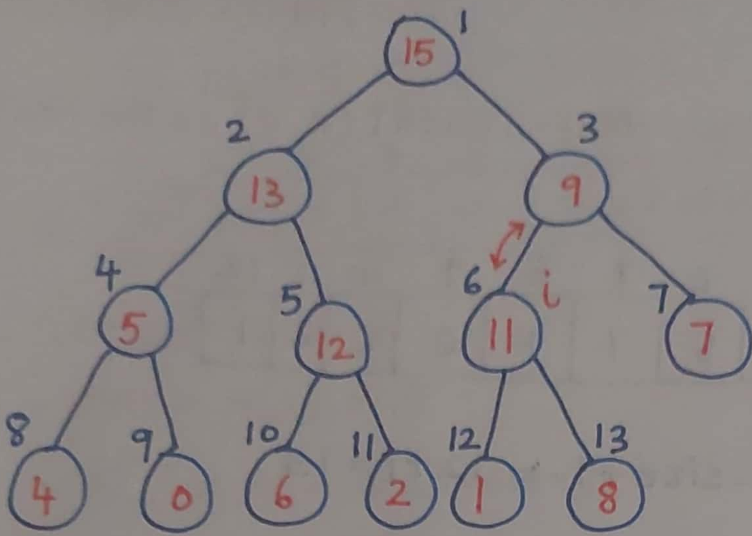
13 > 1 and A[PARENT(i)] < A[i]

13 > 1 and A[6] < A[13]

13 > 1 and 8 < 11 # True

Exchange A[i] with A[PARENT(i)]

Exchange A[13] with A[6]



(b)

$i = \text{PARENT}(i)$

$\text{PARENT}(i) = \left\lfloor \frac{i}{2} \right\rfloor$

$\text{PARENT}(13) = \left\lfloor \frac{13}{2} \right\rfloor = 6$

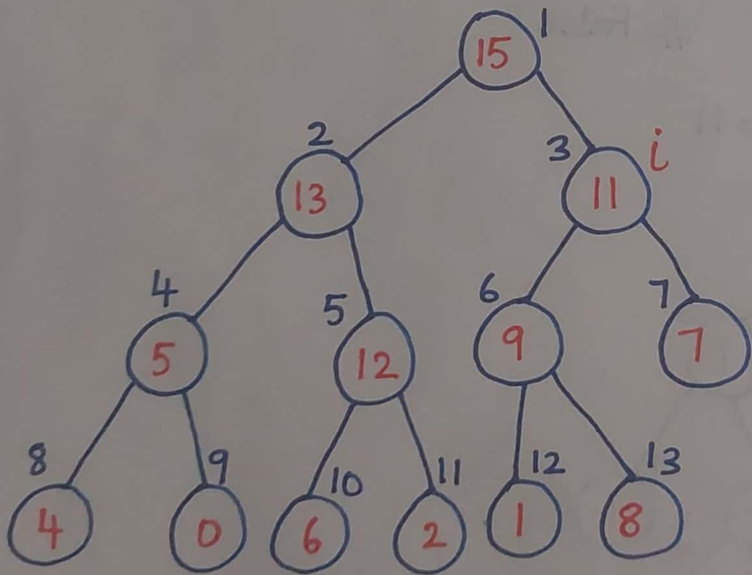
$i = \text{PARENT}(13) = 6$

6>1 and A[3] < A[6]

6>1 and 9 < 11 # True

$i = \text{PARENT}(6) = \left\lfloor \frac{6}{2} \right\rfloor = 3$

Exchange A[6] with A[3]



(c)

3>1 and A[1] < A[3]

3>1 and 15 < 11 # False Do Nothing

2. Illustrate the operation of HEAP-EXTRACT-MAX on the heap

$A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$

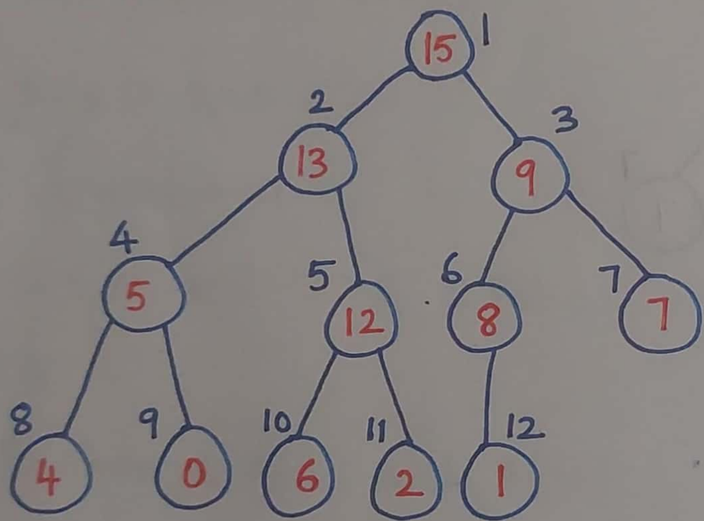
	1	2	3	4	5	6	7	8	9	10	11	12
A	15	13	9	5	12	8	7	4	0	6	2	1

Heap-size = 12

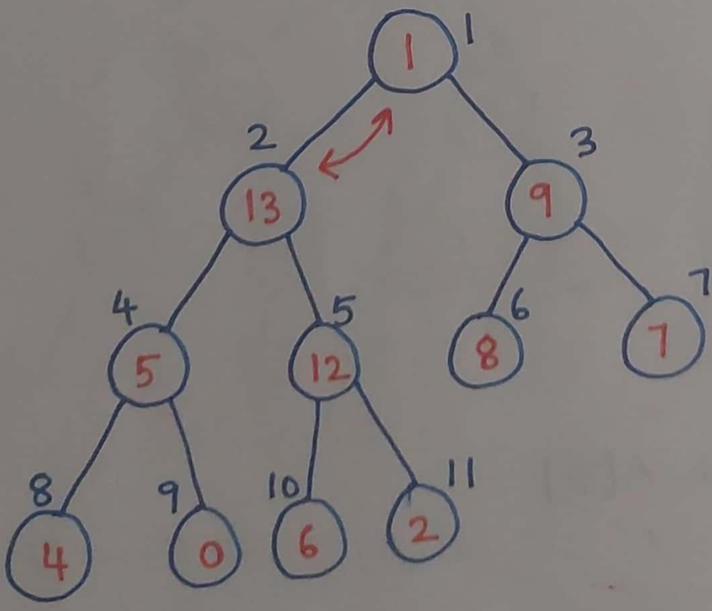
max = A[1] = 15

A[1] = A[A·heap-size] = A[12]

A·heap-size = A·heap-size − 1 = 12 − 1 = 11



(a)



MAX-HEAPIFY (A, 1)

i	l	r	largest
~~1~~	~~2~~	~~3~~	~~2~~
~~2~~	~~4~~	~~5~~	~~4~~
~~5~~	~~10~~	~~11~~	~~5~~
10	20	21	10

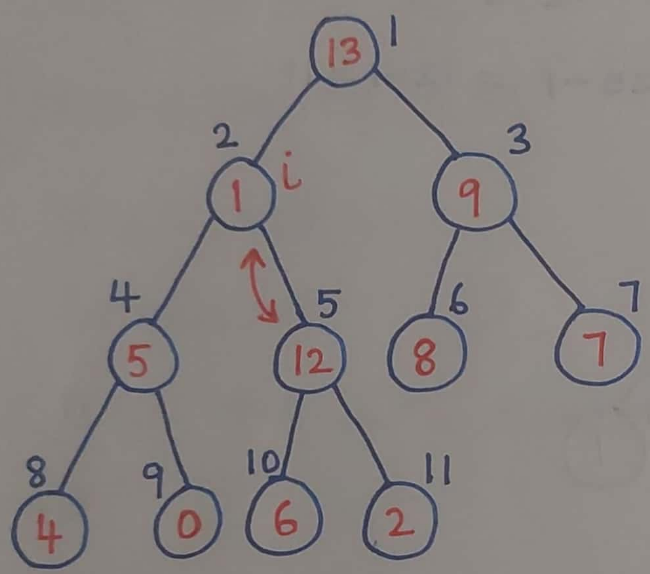
$2 \le 11$ and $13 > 1$

Largest = 2

$3 \le 11$ and $9 > 13$ # False

$2 \ne 1$

Exchange A[1] with A[2]



MAX-HEAPIFY (A, 2)

$4 \le 11$ and $5 > 1$

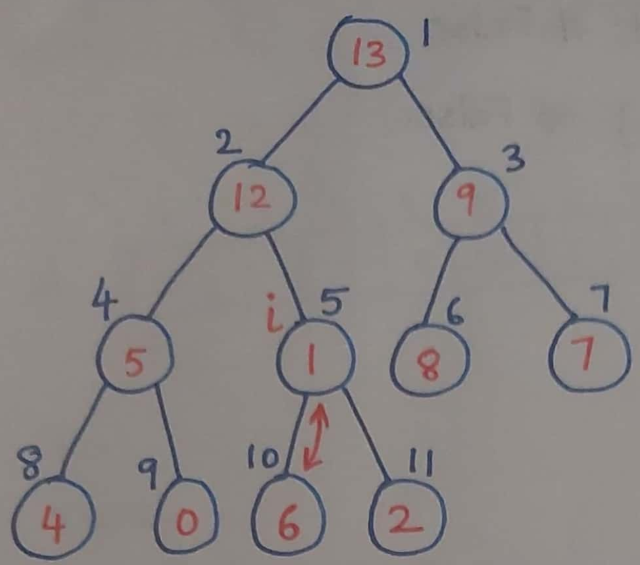
largest = 4

$5 \le 11$ and $12 > 5$

Largest = 5

$5 \ne 2$

Exchange A[2] with A[5]



MAX-HEAPIFY (A, 5)

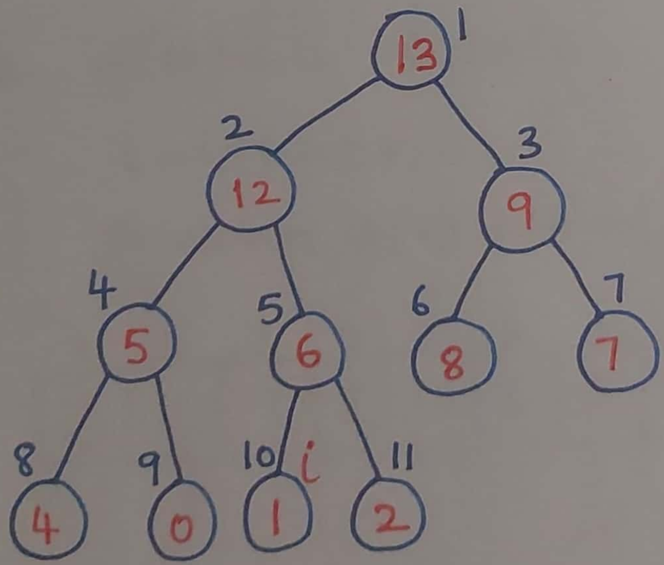
$10 \leq 11$ and $6 > 1$

Largest = 10

$11 \leq 11$ and $2 > 6$ # False

$10 \neq 5$

Exchange A[5] with A[10]



MAX-HEAPIFY (A, 10)

$20 \leq 11$ and $A[20] > A[10]$ # False

$21 \leq 11$ and $A[21] > A[10]$ # False

$10 \neq 10$ # False

return MAX

return 15

3. Illustrate the operation of PARTITION on the array

$A = \langle 15, 1, 9, 5, 13, 6, 8, 4, 21, 2, 6, 12 \rangle$

	1	2	3	4	5	6	7	8	9	10	11	12
A	15	1	9	5	13	6	8	4	21	2	6	12
i	p, j											r

pivot Element

j	i
~~1~~	~~0~~
~~2~~	~~1~~
~~3~~	~~2~~
~~4~~	~~3~~
~~5~~	~~4~~
~~6~~	~~5~~
~~7~~	~~6~~
~~8~~	~~7~~
~~9~~	8
~~10~~	
11	

$p = 1$

$r = 12$

$x = A[r] = A[12] = 12$

$i = p - 1 = 1 - 1 = 0$

$j = 1, \; A[j] \le x \Rightarrow A[1] \le 12$

$\Rightarrow 15 \le 12$ # False, Do Nothing

	1	2	3	4	5	6	7	8	9	10	11	12
	15	1	9	5	13	6	8	4	21	2	6	12
i	p	j										r

$j = 2, \; A[2] \le 12 \Rightarrow 1 \le 12$

$i = i + 1 = 0 + 1 = 1$

Exchange A[1] with A[2]

1	2	3	4	5	6	7	8	9	10	11	12
1	15	9	5	13	6	8	4	21	2	6	12
p, i		j									r

$j = 3, \; A[3] \le 12 \Rightarrow 9 \le 12$

$i = 1 + 1 = 2$

Exchange A[2] with A[3]

1	2	3	4	5	6	7	8	9	10	11	12
1	9	15	5	13	6	8	4	21	2	6	12
p	i		j								r

$j=4, \; A[4] \leq 12 \Rightarrow 5 \leq 12$

$i = 2+1 = 3$

Exchange $A[3]$ with $A[4]$

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	15	13	6	8	4	21	2	6	12
p		i		j							r

$j=5, \; A[5] \leq 12 \Rightarrow 13 \leq 12$ # Do Nothing

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	15	13	6	8	4	21	2	6	12
p		i			j						r

$j=6, \; A[6] \leq 12 \Rightarrow 6 \leq 12$

$i = 3+1 = 4$

Exchange $A[4]$ with $A[6]$

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	13	15	8	4	21	2	6	12
p			i			j					r

$j=7, \; A[7] \leq 12 \Rightarrow 8 \leq 12$

$i = 4+1 = 5$

Exchange $A[5]$ with $A[7]$

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	8	15	13	4	21	2	6	12
p				i			j				r

$j=8$, $A[8] \le 12 \Rightarrow 4 \le 12$

$i = 5+1 = 6$

Exchange $A[6]$ with $A[8]$

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	8	4	13	15	21	2	6	12
p					i			j			r

$j=9$, $A[9] \le 12 \Rightarrow 21 \le 12$ # Do Nothing

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	8	4	13	15	21	2	6	12
p					i				j		r

$j=10$, $A[10] \le 12 \Rightarrow 2 \le 12$

$i = 6+1 = 7$

Exchange $A[7]$ with $A[10]$

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	8	4	2	15	21	13	6	12
p						i				j	r

$j=11$, $A[11] \le 12 \Rightarrow 6 \le 12$

$i = 7+1 = 8$

Exchange $A[8]$ with $A[11]$

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	8	4	2	6	21	13	15	12
							i				j, r

p

Exchange A[i+1] with A[r]

Exchange A[8+1] With A[12]

Exchange A[9] With A[12]

1	2	3	4	5	6	7	8	9	10	11	12
1	9	5	6	8	4	2	6	12	13	15	21
							i				r

p

Return $i+1 = 8+1 = 9$

4. Prove that the solution for the recurrence $T(n) = T(n-1) + \theta(n)$ is $T(n) = \theta(n^2)$

Given Recurrence Equation $T(n) = T(n-1) + \theta(n)$

So, To prove that the solution for above recurrence equation is $T(n) = \theta(n^2)$

We need to prove that the solution for below recurrence equation as,

1. $T(n) = T(n-1) + O(n)$ is $T(n) = O(n^2)$

Proving the above Recurrence Equation

Inductive Hypothesis:

Assume $T(K) \leq C \cdot K^2 \;\; \forall \;\; K \leq n$

In particular, $K = n-1$

i.e., $T(n-1) \leq c(n-1)^2$

Show $T(n) \leq c \cdot n^2$

$$
\begin{aligned}
T(n) &= T(n-1) + C \cdot n \\
&\leq c(n-1)^2 + C \cdot n \\
&\leq cn^2 - 2Cn + C + C \cdot n \\
&\leq cn^2 - c(n-1)
\end{aligned}
$$

$T(n) \leq cn^2$ if $-c(n-1) \leq 0$

Hence $\boxed{T(n) = O(n^2)}$ $\forall \; n \geq 1$

2. $T(n) = T(n-1) + \Omega(n)$ is $T(n) = \Omega(n^2)$

Proving the above Reccurrence Equation

Inductive Hypothesis:

Assume $T(K) \geq C \cdot K^2 \ \forall \ K < n$

In particular, $K = n-1$

i.e., $T(n-1) \geq c(n-1)^2$

To prove, $T(n) \geq cn^2$

$$\begin{aligned} T(n) &\geq T(n-1) + cn \\ &\geq c(n-1)^2 + cn \\ &\geq cn^2 - 2cn + c + c \cdot n \\ &\geq cn^2 - c(n-1) \end{aligned}$$

$T(n) \geq cn^2$ if $-c(n-1) \geq 0$

Hence, $\boxed{T(n) = \Omega(n^2)}$ $\forall \ n \geq 1$

So, From above $T(n) = O(n^2)$ and $T(n) = \Omega(n^2)$

We can Say that $\boxed{T(n) = \Theta(n^2)}$

Hence, Proved

5. Illustrate the operation of COUNTING-SORT on the array $A = \langle 5, 0, 2, 0, 2, 3, 4, 6 \rangle$

	1	2	3	4	5	6	7	8
A	5	0	2	0	2	3	4	6

K = Largest element of A Array = 6

	1	2	3	4	5	6	7	8
B								

$C[i] = 0$

	0	1	2	3	4	5	6
C	0	0	0	0	0	0	0

i	$C[i] = 0$
0	$C[0] = 0$
1	$C[1] = 0$
2	$C[2] = 0$
3	$C[3] = 0$
4	$C[4] = 0$
5	$C[5] = 0$
6	$C[6] = 0$

	0	1	2	3	4	5	6
C	~~0~~	0	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~
	~~1~~		~~1~~	1	1	1	1
	2		2				

j	
	$c[A[j]] = c[A[j]] + 1$
1	$c[A[1]] = c[A[1]] + 1$
	$c[5] = c[5] + 1 \Rightarrow 0 + 1 = 1$
2	$c[A[2]] = c[A[2]] + 1$
	$c[0] = c[0] + 1 \Rightarrow 0 + 1 = 1$
3	$c[A[3]] = c[A[3]] + 1$
	$c[2] = c[2] + 1 = 0 + 1 = 1$
4	$c[A[4]] = c[A[4]] + 1$
	$c[0] = c[0] + 1 \Rightarrow 1 + 1 = 2$
5	$c[A[5]] = c[A[5]] + 1$
	$c[2] = c[2] + 1 \Rightarrow 1 + 1 = 2$
6	$c[A[6]] = c[A[6]] + 1$
	$c[3] = c[3] + 1 \Rightarrow 0 + 1 = 1$
7	$c[A[7]] = c[A[7]] + 1$
	$c[4] = c[4] + 1 \Rightarrow 0 + 1 = 1$

8 $\quad C[A[8]] = C[A[8]]+1$

$C[6] = C[6]+1 \Rightarrow 0+1 = 1$

	0	1	2	3	4	5	6
C	2	0	2	1	1	1	1

	0	1	2	3	4	5	6
C	2	$\cancel{0}$	$\cancel{2}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$
		2	4	5	6	7	8

$i \quad C[i] = C[i] + C[i-1]$

1 $\quad C[1] = C[1] + C[1-1]$

$C[1] = C[1] + C[0]$

$C[1] = 0+2 = 2$

2 $\quad C[2] = C[2] + C[2-1]$

$C[2] = C[2] + C[1]$

$C[2] = 2+2 = 4$

3 $\quad C[3] = C[3] + C[3-1]$

$C[3] = C[3] + C[2]$

$C[3] = 1+4 = 5$

4 $\quad C[4] = C[4] + C[4-1]$

$C[4] = C[4] + C[3]$

$C[4] = 1+5 = 6$

5 $C[5] = C[5] + C[5-1]$

$C[5] = C[5] + C[4]$

$C[5] = 1 + 6 = 7$

6 $C[6] = C[6] + C[6-1]$

$C[6] = C[6] + C[5]$

$C[6] = 1 + 7 = 8$

C

0	1	2	3	4	5	6
2	2	4	5	6	7	8

B

1	2	3	4	5	6	7	8
0	0	2	2	3	4	5	6

C

0	1	2	3	4	5	6
2	2	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~
~~1~~		~~3~~	4	5	6	7
0		2				

j 8 to 1

j $B[C[A[j]]] = A[j]$ $C[A[j]] = C[A[j]] - 1$

8 $B[C[A[8]]] = A[8]$ $C[A[8]] = C[A[8]] - 1$

$B[C[6]] = 6$ $C[6] = C[6] - 1$

$B[8] = 6$ $C[6] = 8 - 1 = 7$

7 $B[C[A[7]]] = A[7]$ $C[A[7]] = C[A[7]]-1$

$B[C[4]] = 4$ $C[4] = C[4]-1$

$B[6] = 4$ $C[4] = 6-1 = 5$

6 $B[C[A[6]]] = A[6]$ $C[A[6]] = C[A[6]]-1$

$B[C[3]] = 3$ $C[3] = C[3]-1$

$B[5] = 3$ $C[3] = 5-1 = 4$

5 $B[C[A[5]]] = A[5]$ $C[A[5]] = C[A[5]]-1$

$B[C[2]] = 2$ $C[2] = C[2]-1$

$B[4] = 2$ $C[2] = 4-1 = 3$

4 $B[C[A[4]]] = A[4]$ $C[A[4]] = C[A[4]]-1$

$B[C[0]] = 0$ $C[0] = C[0]-1$

$B[2] = 0$ $C[0] = 2-1 = 1$

3 $B[C[A[3]]] = A[3]$ $C[A[3]] = C[A[3]]-1$

$B[C[2]] = 2$ $C[2] = C[2]-1$

$B[3] = 2$ $C[2] = 3-1 = 2$

2 $B[C[A[2]]] = A[2]$ $C[A[2]] = C[A[2]]-1$

$B[C[0]] = 0$ $C[0] = C[0]-1$

$B[1] = 0$ $C[0] = 1-1 = 0$

1 $B[C[A[1]]] = A[1]$ $\quad$ $C[A[1]] = C[A[1]]-1$

$B[C[5]] = 5$ $\quad$ $C[5] = C[5]-1$

$B[7] = 5$ $\quad$ $C[5] = 7-1 = 6$

B

1	2	3	4	5	6	7	8
0	0	2	2	3	4	5	6

6. Illustrate the Operation of RADIX-SORT on the following list of English words : COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX

i	COW	SEA	TAB	BAR
~~1~~	DOG	TEA	BAR	BIG
~~2~~	SEA	MOB	EAR	BOX
3	RUG	TAB	TAR	COW
	ROW	DOG	SEA	DIG
	MOB	RUG	TEA	DOG
	BOX	DIG	DIG	EAR
	TAB →	BIG →	BIG →	FOX
	BAR	BAR	MOB	MOB
	EAR	EAR	DOG	NOW
	TAR	TAR	COW	ROW
	DIG	COW	ROW	RUG
	BIG	ROW	NOW	SEA
	TEA	NOW	BOX	TAB
	NOW	BOX	FOX	TAR
	FOX	FOX	RUG	TEA

7. Illustrate the operation of BUCKET-SORT on the array A = <0.72, 0.17, 0.19, 0.62, 0.49, 0.20, 0.81, 0.52, 0.61, 0.47, 0.39, 0.59, 0.68, 0.99, 0.16>

$n = A.length = 15$

i	Insert $A[i] \rightarrow B[\lfloor n * A[i] \rfloor]$
1	$A[1] \rightarrow B[\lfloor 15 * A[1] \rfloor]$
	$A[1] \rightarrow B[\lfloor 15 * 0.72 \rfloor]$
	$A[1] \rightarrow B[10]$
2	$A[2] \rightarrow B[\lfloor 15 * A[2] \rfloor]$
	$A[2] \rightarrow B[\lfloor 15 * 0.17 \rfloor]$
	$A[2] \rightarrow B[2]$
3	$A[3] \rightarrow B[\lfloor 15 * A[3] \rfloor]$
	$A[3] \rightarrow B[\lfloor 15 * 0.19 \rfloor]$
	$A[3] \rightarrow B[2]$
4	$A[4] \rightarrow B[\lfloor 15 * A[4] \rfloor]$
	$A[4] \rightarrow B[\lfloor 15 * 0.62 \rfloor]$
	$A[4] \rightarrow B[9]$
5	$A[5] \rightarrow B[\lfloor 15 * A[5] \rfloor]$
	$A[5] \rightarrow B[\lfloor 15 * 0.49 \rfloor]$
	$A[5] \rightarrow B[7]$

6 $A[6] \rightarrow B[\lfloor 15 * A[6] \rfloor]$

$A[6] \rightarrow B[\lfloor 15 * 0.20 \rfloor]$

$A[6] \rightarrow B[3]$

7 $A[7] \rightarrow B[\lfloor 15 * A[7] \rfloor]$

$A[7] \rightarrow B[\lfloor 15 * 0.81 \rfloor]$

$A[7] \rightarrow B[12]$

8 $A[8] \rightarrow B[\lfloor 15 * A[8] \rfloor]$

$A[8] \rightarrow B[\lfloor 15 * 0.52 \rfloor]$

$A[8] \rightarrow B[7]$

9 $A[9] \rightarrow B[\lfloor 15 * A[9] \rfloor]$

$A[9] \rightarrow B[\lfloor 15 * 0.61 \rfloor]$

$A[9] \rightarrow B[9]$

10 $A[10] \rightarrow B[\lfloor 15 * A[10] \rfloor]$

$A[10] \rightarrow B[\lfloor 15 * 0.47 \rfloor]$

$A[10] \rightarrow B[7]$

11 $A[11] \rightarrow B[\lfloor 15 * A[11] \rfloor]$

$A[11] \rightarrow B[\lfloor 15 * 0.39 \rfloor]$

$A[11] \rightarrow B[5]$

12 $A[12] \rightarrow B[\lfloor 15 * A[12] \rfloor]$

$A[12] \rightarrow B[\lfloor 15 * 0.59 \rfloor]$

$A[12] \rightarrow B[8]$

13 $A[13] \rightarrow B[\lfloor 15 * A[13] \rfloor]$

$A[13] \rightarrow B[\lfloor 15 * 0.68 \rfloor]$

$A[13] \rightarrow B[10]$

14 $A[14] \rightarrow B[\lfloor 15 * A[14] \rfloor]$

$A[14] \rightarrow B[\lfloor 15 * 0.99 \rfloor]$

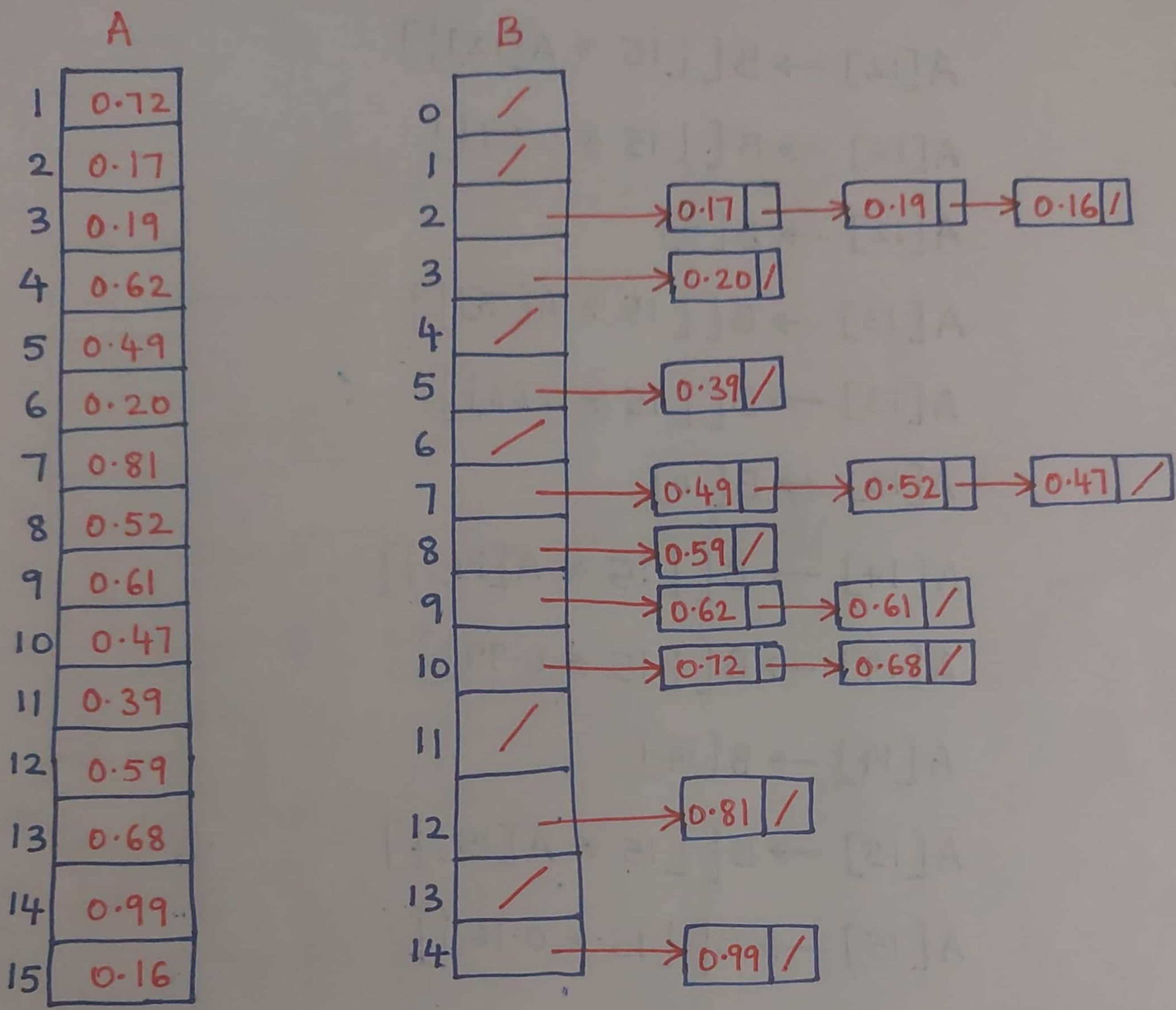
$A[14] \rightarrow B[14]$

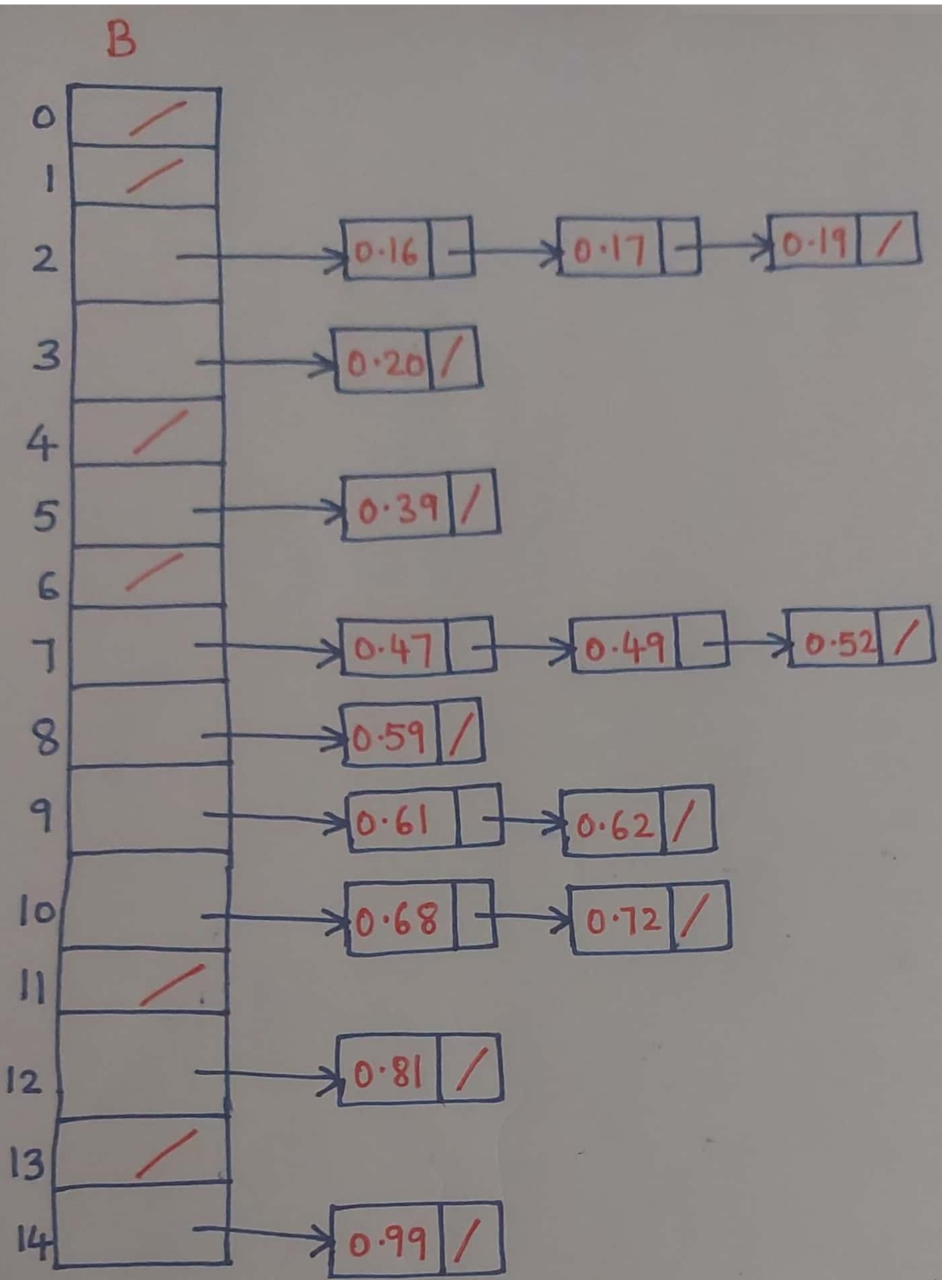
15 $A[15] \rightarrow B[\lfloor 15 * A[15] \rfloor]$

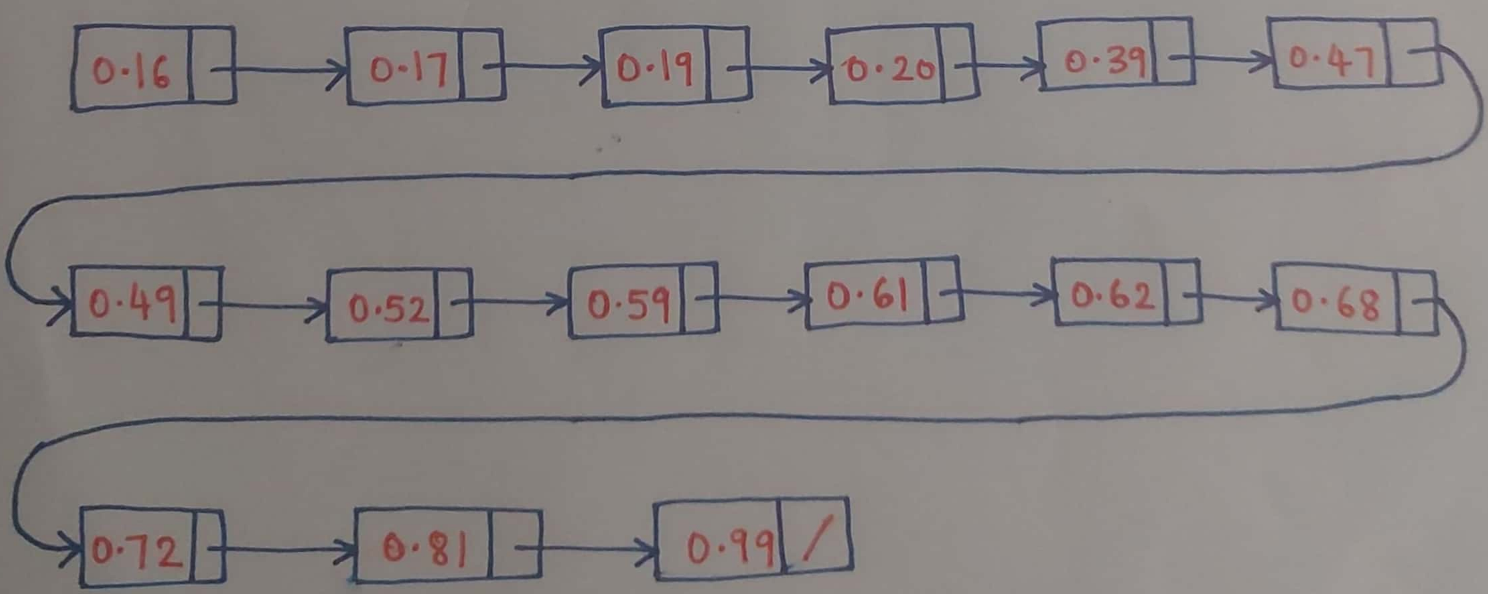
$A[15] \rightarrow B[\lfloor 15 * 0.16 \rfloor]$

$A[15] \rightarrow B[2]$

16 STOP


A
1 0·72
2 0·17
3 0·19
4 0·62
5 0·49
6 0·20
7 0·81
8 0·52
9 0·61
10 0·47
11 0·39
12 0·59
13 0·68
14 0·99
15 0·16
B
0
1
2 0·17 0·19 0·16
3 0·20
4
5 0·39
6
7 0·49 0·52 0·47
8 0·59
9 0·62 0·61
10 0·72 0·68
11
12 0·81
13
14 0·99


B
0
1
2
0·16
0·17
0·19
3
0·20
4
5
0·39
6
7
0·47
0·49
0·52
8
0·59
9
0·61
0·62
10
0·68
0·72
11
12
0·81
13
14
0·99


0·16
0·17
0·19
0·20
0·39
0·47
0·49
0·52
0·59
0·61
0·62
0·68
0·72
0·81
0·99