CS 5300 Advanced Algorithms
HW # 3

1. Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 2 & if\ n = 1 \\ T(n-1) + 2 & otherwise \end{cases}$$

   Show, by induction, that $T(n) = 2n$

2. Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 1 & if\ n = 1 \\ T(n-1) + n^2 & otherwise \end{cases}$$

   Show, by induction, that $T(n) = \frac{n(n+1)(2n+1)}{6}$

3. Show that the solution for $T(n) = T(\sqrt{n}) + \Theta(1)$ is $T(n) = \Theta(lglg(n))$
4. Draw the recursion tree for $T(n) = T\left(\frac{2n}{5}\right) + T\left(\frac{3n}{5}\right) + \Theta(n)$ and find the height of the tree, then generate the guess for the solution.
5. Draw the recursion tree for $T(n) = 2T\left(\frac{n}{2}\right) + \Theta(n^2)$ and find the height of the tree, then generate the guess for the solution.
6. Solve the following recurrence: $T(n) = 16T\left(\frac{n}{4}\right) + \Theta(n^2 lgn)$
7. Solve the following recurrence equation: $T(n) = 4T(\frac{n}{2}) + \Theta(n^2)$
8. Solve the following recurrence: $T(n) = 7T\left(\frac{n}{2}\right) + \Theta(n^3)$
9. Solve the following recurrence: $T(n) = 3T\left(\frac{n}{2}\right) + \Theta(n^2 lgn)$
10. Solve the following recurrence: $T(n) = 5T\left(\frac{n}{2}\right) + \Theta(n^2)$
11. Solve the following recurrence: $T(n) = 11T\left(\frac{n}{3}\right) + \Theta(n^2)$