HOMEWORK#8 MANOJ SUVVALA

ADVANCED ALGORITHMS 700744733

(1) Use algorithm 3.1 discussed in class to complete the following

a) bin(3,2) b) bin(4,3) c) bin(5,1)

Ans) Using algorithm 3.1,

we know that binomial coefficient is,

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The recurrence equation for this binomial coefficient is,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \quad 0 < k < n$$

$$\binom{n}{k} = 1 \quad \text{if } k = 0 \text{ (or) } k = n$$

so using this recurrence equation,

a) $\text{bin}(3,2) = \binom{3}{2}$

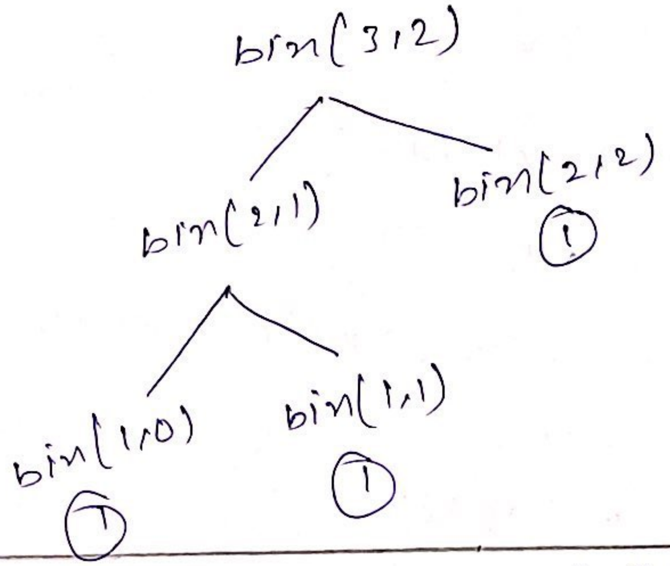
$$= \binom{3-1}{2-1} + \binom{3-1}{2} \Rightarrow \binom{2}{1} + \binom{2}{2}$$

$$\text{bin}(2,1) = \binom{2}{1} \Rightarrow \binom{2-1}{1-1} + \binom{2-1}{1} \Rightarrow \binom{1}{0} + \binom{1}{1}$$

$$\text{bin}(2,2) = \binom{2}{2} = 1 \quad \text{as } k = n$$

bin(1,0) = $\binom{1}{0} = 1$ as k = 0

bin(1,1) = $\binom{1}{1} = 1$ as k = n



so, bin(3,2) = 1 + 1 + 1 = 3

(b) bin(4,3) $= \binom{4}{3} \Rightarrow \binom{4-1}{3-1} + \binom{4-1}{3} \Rightarrow \binom{3}{2} + \binom{3}{3}$

bin(3,2) $\Rightarrow \binom{3}{2} \Rightarrow \binom{3-1}{2-1} + \binom{3-1}{2} \Rightarrow \binom{2}{1} + \binom{2}{2}$

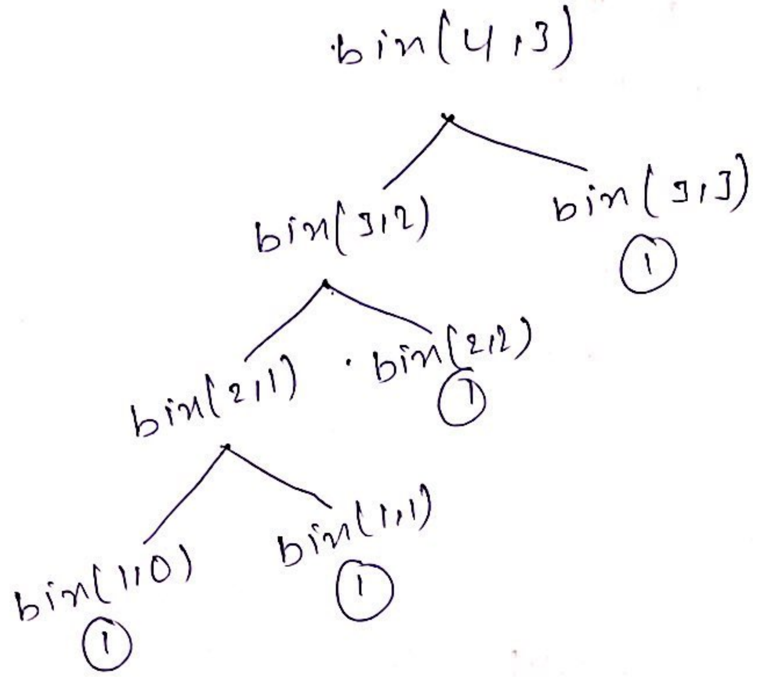
bin(2,1) $= \binom{2}{1} \Rightarrow \binom{2-1}{1-1} + \binom{2-1}{1} \Rightarrow \binom{1}{0} + \binom{1}{1}$

bin(3,3) = $\binom{3}{3} = 1$ as k = n

bin(2,2) = $\binom{2}{2} = 1$ as k = n

bin(1,0) = $\binom{1}{0} = 1$ as k = 0

bin(1,1) = $\binom{1}{1} = 1$ as k = n



so $bin(4,3) = 1+1+1+1 = 4$

(c) $bin(5,1) = \binom{5}{1} = \binom{5-1}{1-1} + \binom{5-1}{1}$

$= \binom{4}{0} + \binom{4}{1}$

$bin(4,1) = \binom{4}{1} = \binom{4-1}{1-1} + \binom{4-1}{1}$

$= \binom{3}{0} + \binom{3}{1}$

$bin(3,1) = \binom{3}{1} \Rightarrow \binom{3-1}{1-1} + \binom{3-1}{1}$

$= \binom{2}{0} + \binom{2}{1}$

$bin(2,1) = \binom{2}{1} \Rightarrow \binom{2-1}{1-1} + \binom{2-1}{1}$

$= \binom{1}{0} + \binom{1}{1}$

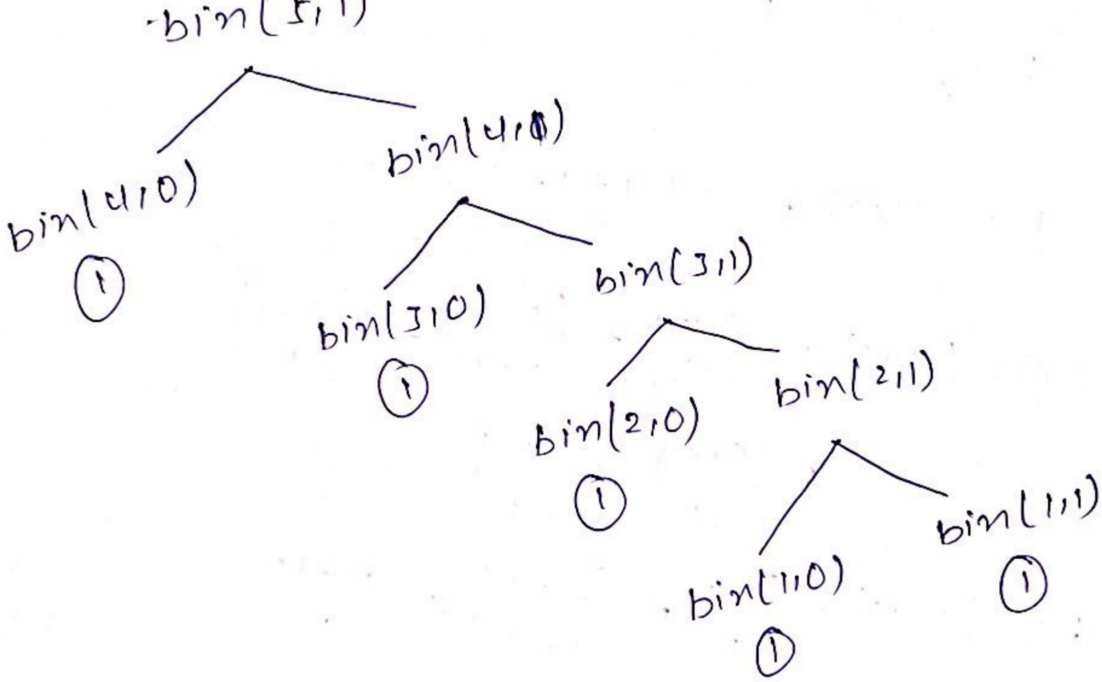
$bin(1,1) = \binom{1}{1} = 1$ as $k = n$

$bin(1,0) = \binom{1}{0} = 1$ as $k = 0$

$bin(2,0) = \binom{2}{0} = 1$ as $k = 0$

$bin(3,0) = \binom{3}{0} = 1$ as $k = 0$

$bin(4,0) = \binom{4}{0} = 1$ as $k = 0$



$\therefore$ $bin(5,1) = 1+1+1+1+1 = 5$

Q2 Use algorithm 3.2 discussed in class to complete the following

a) bin(3,2)

b) bin(4,3)

c) bin(5,1)

Ans) From algorithm 3.2

The recursion equation using dynamic programming is

$$B[i,j] = \binom{i}{j} = B[i-1, j-1] + B[i-1, j] \cdot \text{ if } 0 < j < i$$

$$B[i,j] = 1 \quad \text{if } j = 0 \text{ (or) } j = i$$

Using this equation,

a) bin (3,2)

i \ j	0	1	2	3
0	1			
1	1	1		
2	1	2	1	
3	1	3	3	

ROW 0 → bin(0,0) = B[0,0] = 1 as j = i

bin(0,1) = B[0,1] = X as j > i

ROW 1 → bin(1,0) = B[1,0] = 1 as j = 0

bin(1,1) = B[1,1] = 1 as j = i

ROW 2 bin(2,0) = B[2,0] = 1 as j = 0

bin(2,1) = B[2,1] = B[1,0] + B[1,1]

⇒ 1 + 1 = 2

as j = 0 & j = i

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$bin(2,2) = B[2,2] = 1$ as $j = i$

ROW 3

$bin(3,0) = B[3,0] = 1$ as $j = 0$

$bin(3,1) = B[3,1] = B[2,0] + B[2,1] = 1 + 2 = 3$

so, $bin(3,2) = 3$.

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b) $Bin(4,3)$

	0	1	2	3	4
0	1				
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	6	4	

ROW 0

$Bin(0,0) = 1$ as $j = 1$

ROW 1

$bin(1,0) = B[1,0] = 1$ as $j = 0$

$bin(1,1) = B[1,1] = 1$ as $j = i$

ROW 2

$bin(2,0) = B[2,0] = 1$ as $j = 0$

$bin(2,1) = B[2,1] = B[1,0] + B[1,1] = 1 + 1 = 2$

ROW 3

$bin(3,0) = B[3,0] = 1$ as $j = 0$

$bin(3,1) = B[3,1] = B[2,0] + B[2,1] = 1 + 2 = 3$

$bin(3,2) = B[3,2] = B[2,1] + B[2,2] = 2 + 1 = 3$

$bin(3,3) = 1$ as $j = i$

ROW 4 bin(4,0) = 1 , as j = 0

bin(4,1) = B[4,1] = B[3,0] + B[3,1] = 1+3 = 4

bin(4,2) = B[4,2] = B[3,1] + B[3,2] = 3+3 = 6

bin(4,3) = B[4,3] = B[3,2] + B[3,3] = 3+1 = 4

bin(4,3) = 4

ⓒ bin(5,1)

	0	1	2	3	4	5
0	1					
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4	6	4	1	
5	1	5				

ROW 0 bin(0,0) = B[0,0] = 1 as j = i

ROW 1 bin(1,0) = B[1,0] = 1 as j = 0

bin(1,1) = B[1,1] = 1 as j = i

ROW 2 bin(2,0) = B[2,0] = 1 as j = 0

bin(2,1) = B[2,1] = B[1,0] + B[1,1] = 1+1 = 2

ROW 3 bin(3,0) = B[3,0] = 1 as j = 0

$bin(3,1) = B[3,1] = B[2,0] + B[2,1] = 1+2 = 3$

$bin(3,2) = B[3,2] = B[2,1] + B[2,2] = 2+1 = 3$

$bin(3,3) = 1$ as $j = i$

Row 4

$bin(4,0) = Bin[4,0] = 1$ as $j = 0$

$bin(4,1) = B[4,1] = B[3,0] + B[3,1] = 1+3 = 4.$

$bin(4,2) = B[4,2] = B[3,1] + B[3,2] = 3+3 = 6$

$bin(4,3) = B[4,3] = B[3,2] + B[3,3] = 3+1 = 4$

$bin(4,4) = B[4,4] = 1$ as $j = i$

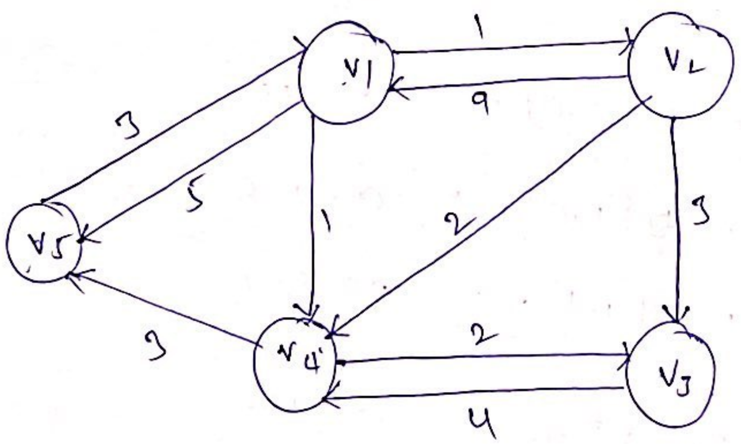
Row 5

$bin(5,0) = B[5,0] = 1$ as $j = 0$

$bin(5,1) = B[5,1] = B[4,0] + B[4,1] = 1+4 = 5$

So, $bin(5,1) = 5$

(3) Given the graph below,



complete the following.

a)

Ans)

So, using dynamic programming method of solving the all-pairs shortest path for the above graph.

$$D^k[i,j] = \min(\text{case }1, \text{case }2)$$
$$= \min(D^{k-1}[i,j], D^{k-1}[i,k] + D^{k-1}[k,j]).$$

a) $D^{(0)}[1][3] = D^0[1,3] = \text{length}(v_1, v_3) = \infty$

b) $D^{(1)}[1][3] = D^1[1,3]$
$= \min(D^0[1,3], D^0[1,1] + D^0(1,3])$
$= \min(\infty, \infty + \infty)$
$= \infty$

c) $D^{(2)}[1][3] = D^2[1,3]$
$= \min(D^1[1,3], D^1[1,2] + D^1[2,3])$

$D^1[1][2] = \cancel{D^2[1,2]} \min(D^0[1,2], D^0[1,1] + D^0[1,2].)$
$= \min(1, \infty + 1) = 1$

$D^1[2,3] = \min(D^0[2,3], D^0(2,1) + D^0(1,3])$
$= \min(3, 9 + \infty.) = 3$

so, $D^2[1,3] = \min(\infty, 1+3) = 4$

d) $D^3[1][3] = D^3(1,3] = \min(D^2[1,3], D^2(1,3] + D^2(3,3])$

$D^2[3,3] = \min(D^1[3,3], D^1[3,2] + D^1[2,3])$

$D^1[3,2] = \min(D^0[3,2], D^0(3,1] + D^0(1,2])$
$= \min(\infty, \infty) = \infty$

$D^1[3,3] = \min(D^0[3,3], D^0(3,1] + D^0[1,3])$

$= \min(\infty, \infty) = \infty$

$\Rightarrow D^2[3,3] = \min(\infty, \infty) = \infty$

$\Rightarrow D^3[1,3] = \min(4, \infty) = 4$

(e) $D^4[1][3] = D^4(1,3] = \min(D^3[1,3], D^3(4,4], + D^3[4,3])$

$D^3[1,4] = \min(D^2[1,4], D^2[1,3] + D^2(3,4])$

$D^2[1,4] = \min(D^0(2,4], D^0[\cancel{1},2] + D^2(2,4])$

$D^1(1,4) = \min(D^0[1,4], D^0(1,1] + D^0[1,4])$

$= \min(1, \infty) = 1$

$D^1[2,4] = \min(D^0(2,4], D^0[2,1] + D^0(1,4])$

$= \min(2, 9+1) = 2$

$\Rightarrow D^2[1,4] = \min(1, 1+2) = 1$

$D^2[3,4] = \min(D^1[3,4], D^1[3,2] + D^1(2,4])$

$D^1[3,4] = \min(D^0(3,4], D^0[3,1] + D^0(1+4])$

$= \min(4, \infty) = 4$

$D^1[2,4] = \min(D^0(2,4], D^0[3,1] + D^0(1,4])$

$= \min \cancel{(4, \infty)} = 2$

$(2, 9+1)$

$D^2[3,4] = \min(4, \infty) = 4$

$D^3(1,4] = \min(1, 4+4) = 1$

so, $D^4[1,3] = \min(4, 1 + D^3[4,3])$

$D^3[4,3] = \min[D^2[4,3], D^2[4,3] + D^2[3,3])$

$D^2(4,3) = \min[D'(4,3], D'[4,2] + D'[2,3])$

$D'[4,3] = \min(D^0(4,3], D^0(4,1] + D^0(1,3] = \min(2,\infty) = 2$

$D'[4,2] = \min(D^0(4,2], D^0(4,1] + D^0(1,2] = \min(\infty,\infty) = \infty$

$D^3(4,3] = \min(2, 2+\infty) = 2$

Hence, $D^4[1,3] = \min[4,3] = 3$

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(4) Use the graph from solution (Question-3) to compute the following

Ans) Using the below equation and the data from Question 3

$D^k[i,j] = \min(\text{case1}, \text{case2})$

$= \min(D^{k-1}[i,j], D^{k-1}[i,k] + D^{k-1}[k,j])$

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a) $D^{(0)}[2][3] = D^0(2,3] = 3$

b) $D^{(1)}[2][3] = D'[2,3] = \min(D^0[2,3], D^0(2,1] + D^0(1,3])$

c) $D^{(2)}[2][3] = D^2[2,3] = \min(D'[2,3], D'[2,2] + D'[2,3])$

$D'[2,2] = \min(D^0[2,2], D^0[2,1] + D^0(1,2])$

$= \min(\infty, 9+1) = 10$

$\Rightarrow D^2[2,3] = \min(3, 10+3) = 3$

d) $D^{(3)}[2][3] = D^3[2,3] = \min(D^3[2,3], D^3[2,4] + D^3[3,3])$

$= \min(3, \cancel{\infty}\, 3+\infty) = 3$

e) $D^{(4)}[2][3] = D^4[2,3] = \min(D^3[2,3], D^3[2,4] + D^3[4,3])$

$D^2[2,4] = \min(D^1[2,4], D^1[2,2] + D^1[2,4])$

$= \min(2, 0+2) = 2$

$\Rightarrow D^3[2,4] = \min(2,7) = 2$

$\Rightarrow D^4[2,3] = \min(3, 2 + D^3[4,3])$

$D^3[4,3] = \min(D^2[4,3], D^2[4,3] + D^2[3,3])$

$D^2[4,3] = \min(D^1[4,3], D^1[4,2] + D^1[2,3])$

$D^1[4,3] = \min(2, \infty) = 2$

$D^1(4,3) = \min(D^0[4,2], D^0[4,1] + D^0[1,2])$

$= \min(\infty, \infty) = \infty$

$\Rightarrow D^2[4,3] = \min(2, \infty) = 2$

$D^3(4,3) = \min(2, 2+\infty) = 2$

$\Rightarrow D^4[2,3] = \min(3,4) = 3$

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(5) Use the graph from solution 4 to compute the following.

Ans) so using the dynamic programming method of solving the all pairs shortest path for the graph in question 4 & the data from question-4

$D^k(i,j) = \min(\text{case1}, \text{case2})$

$= \min\{D^{k-1}(i,j), D^{k-1}[i,k] + D^{k-1}[k,j]\}$

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a) $D^{(0)}[3][5] = D^0[3,5] = \infty$

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b) $D^{(1)}[3][5] = D^1[3,5] = \min(D^0[3,5], D^0[3,1] + D^0[1,5])$

$= \min(\infty, \infty) = \infty$

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c) $D^{(2)}[3][5] = D^2[3,5] = \min(D^1[3,5], D^1[3,2] + D^1[2,5])$

$D^1[2][5] = \min(D^0[2,5], D^0[2,1] + D^0[1,5])$

$\Rightarrow \min(\infty, 9+5) = 14$

$\Rightarrow D^2[3,5] = \min(\infty, \infty) = \infty$

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d) $D^{(3)}[3][5] = \min(D^3[3,5], D^2[3,3] + D^2[3,5])$

$= \min(\infty, \infty) = \infty$

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e) $D^{(4)}[3][5] = \min(D^3[3,5], D^2[3,4] + D^2[4,5]$.

$D^{(3)}[3,4] = \min(D^2[3,4], D^2[4,3] + D^2[3,5])$

$D^{(2)}[4,5] = \min(D^1[4,5], D^1[4,2] + D^1[2,5])$

$D^{(1)}[4,5] = \min(D^0[4,5], D^0[4,1] + D^0[1,5])$

$= \min(3, \infty) = 3$

$\Rightarrow D^2[4,5] = \min(3, \infty) = 3$

$D^3[4,5] = \min(3, 2+\infty) = 3$

$D^4[3,5] = \min(\infty, 4+3) = 7$

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⑥ List three problems that have polynomial time algorithm. Justify your answer.

Ans) problems that can be solved by a polynomial time algorithm are called tractable problems.

Three problems which have polynomial time algorithm are

1. Binary search
2. Bubble sort
3. Sequential search.

Justification:

From the above three problems, the best & worst case time complexity are $O(1)$, $O(\log n)$, $O(n)$, $O(n^2)$. These time complexities are polynomial functions of the size of the input. Hence, these algorithm are considered polynomial time algorithms.

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⑦ Suppose that problem A & problem B are two different decision problems. Also, it is assumed that problems. A is polynomial-time many-one reducible to problem B. If problem A is NP-complete, is problem B NP-complete? Justify your Answer.

Ans) Given that problem A & problem B are two different decision problems. Furthermore, assume that problem A is polynomial-time many-one reducible to problem B.

Given problem A is NP-complete, then we have to show that problem B also NP-complete.

Since it is given that problem A is reducible to problem B & also since a polynomial time reduction preserves the complexity class, this implies that problem B is also NP & any problem in NP can be reduced to problem in polynomial time.

Therefore, problem B is also NP-complete. So, if problem A is NP-complete & is polynomial time many one reducible to problem B, then problem B is also NP-complete.