CS 5300
Advanced Algorithms
Midterm Exam

Name:

The point value of each problem is 8 points unless indicated otherwise. You **MUST** show all your work to receive full credit. Work neatly. **GOOD LUCK!**

1. (6 pts.) Find a theta notation for the number of times the statement $x = x + 1$ is executed

```
for i = 1 to n
    for j = 1 to i
        x = x +1
```

2. What is the output of the following algorithm? Analyze its worst-case running time, and express it using "Big-Oh" notation.

Algorithm Foo (A):
Input: An array A storing $n \geq 1$ integers.
Output: ?

```
k = A[0]
for i = 1 to n − 1 do
    k = k + A[i]
return k
```

3. (4 pts.) Which of the following is correct with regard to insertion sort?

   a) insertion sort is stable and it sorts In-place

   b) insertion sort is unstable and it sorts In-place

   c) insertion sort is stable and it does not sort In-place

   d) insertion sort is unstable and it does not sort In-place

4. (4 pts.) Which of the following sorting algorithm is best suited if the elements are already sorted?

   a) Heap Sort

   b) Quick Sort

   c) Insertion Sort

   d) Merge Sort

5. (4 pts.) Insertion sort is an example of an incremental algorithm.

   a) True

   b) False

6. (4 pts.) Consider the code given below, which runs insertion sort:

```
void insertionSort(int arr[], int array_size)
  {
    int i, j, value;
    for (i = 1; i < array_size; i++)
    {
            value = arr[i];
            j = i;
            while (________ )
            {
                    arr[j] = arr[j - 1];
                    j = j - 1;
            }
            arr[j] = value;
    }
}
```

Which condition will correctly implement the while loop?

a) (j > 0) || (arr[j − 1] > value)

b) (j > 0) && (arr[j − 1] > value)

c) (j > 0) && (arr[j + 1] > value)

d) (j > 0) && (arr[j + 1] < value)

7. Use the bottom-up approach to illustrate the operations of merge-sort on the array

$$A = < 11, 10, 5, 2, 3, 1, 15, 14, 8, 1 >$$

8. Is $2^{n+2} = O(2^n)$? If true, prove it; otherwise, show why it is false

9. Show that $5n^2 + 2n + 15 = o(n^3)$

10. Show that $2n^2 + n$ is $\Omega(n^2)$

11. (10 pts.) Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 7 & if\ n = 1 \\ 2T(n/2) + 5n^2 & otherwise \end{cases}$$

Show, by induction, that $T(n) = 10n^2 - 3n$

12. (10 pts.) Draw the recursion tree for $T(n) = T\left(\frac{n}{5}\right) + T(\frac{4n}{5}) + O(n)$ and find the height of the tree

13. (18 pts.) Solve the following recurrences

a) $T(n) = 9T(\frac{n}{3}) + n$

b) $T(n) = 7T\left(\frac{n}{4}\right) + n^2 lgn$

c) $T(n) = 3T\left(\frac{n}{3}\right) + \Theta(nlgn)$