CS 5300
Advanced Algorithms
Midterm Exam Version A

Name:

The point value of each problem is 3 points unless indicated otherwise. You **MUST** show all your work to receive full credit. Work neatly. **GOOD LUCK!**

1. What is the average case running time of an insertion sort algorithm?
   a) O(n)
   b) O(n lg n)
   c) O(lg n)
   d) $O(n^2)$

2. Insertion sort is an example of an incremental algorithm.
   a) True
   b) False

3. How many passes does an insertion sort algorithm consist of?
   a) n
   b) n-1
   c) n+1
   d) n^2

4. (8 pts.) Suppose computer A is running a sorting algorithm and it is supposed to sort an array of one million numbers. Suppose that computer A executes a billion instructions per second, and suppose computer A requires $\mathbf{100}\boldsymbol{n^2}$ instructions to sort n numbers. Find the time it takes computer A to sort the one million numbers?

5. (10 pts.) Use the buttom-up approach to illustrate the operations of merge-sort on the array $A = < 3,1,15,11,2,7,15,8,13 >$. Use the notes discussed in class as a guide

6. (8 pts.) Prove that $2^{n-1} = O(2^n)$?

7. (8 pts.) Show that $5n^2 + 2n + 15 = o(n^3)$

8. (8 pts.) Show that $2n^2 + n$ is $\Omega(n^2)$

9. (12 pts.) Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 7 & if\ n = 1 \\ 2T(n/2) + 5n^2 & otherwise \end{cases}$$

Show, by induction, that $T(n) = 10n^2 - 3n$

10. (12 pts.) Draw the recursion tree for $T(n) = T(\frac{3n}{5}) + T\left(\frac{2n}{5}\right) + \Theta(n)$ and find the height of the tree

11. (18 pts.) Solve the following recurrences

a) $T(n) = 9T(\frac{n}{3}) + n$

b) $T(n) = 3T\left(\frac{n}{4}\right) + nlgn$

c) $T(n) = 8T\left(\frac{n}{2}\right) + n^3 \lg^2 n$

12. (10 pts.) Given the following algorithm:

Algorithm Foo (A):
Input: An array A storing $n \geq 1$ integers.
Output: ?
$k = A[0]$
$for\ i = 1\ to\ n - 1\ do$
$\quad k = k + A[i]$
$return\ k$

a) What is the output?

b) What is the time complexity $T(n)$?