CS5300 Advanced Algorithms   Kishor Kumar Andekar

HW # 7   700744713 (CRN : 12008)

1. Do you think it is possible for a minimum spanning tree to have a cycle ? Justify your Answer.

It is Not possible for a Minimum spanning tree to have a cycle. It is a most economical way of connecting all vertices of a Weighted undirected graph together using the edges of the Graph.

· The Minimum spanning Tree does not have any cycle.
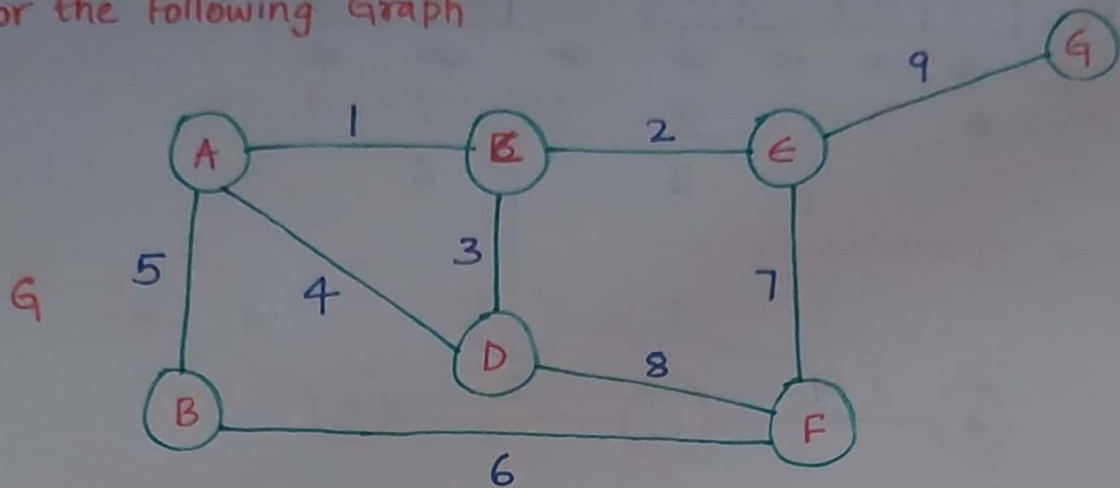
· It has |v| vertices and |v|-1 Edges

Example:

If we have a 4 Vertices and 3 Edges. To get a Loop Number of vertices should be equal to Number of Edges. If Number of vertices and Number of Edges are equal then it is not a Minimum Spanning Tree.
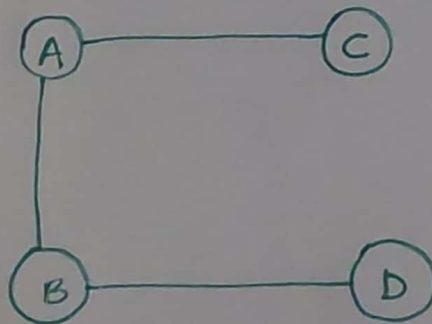
Number of vertices = 4

Number of Edges = 3

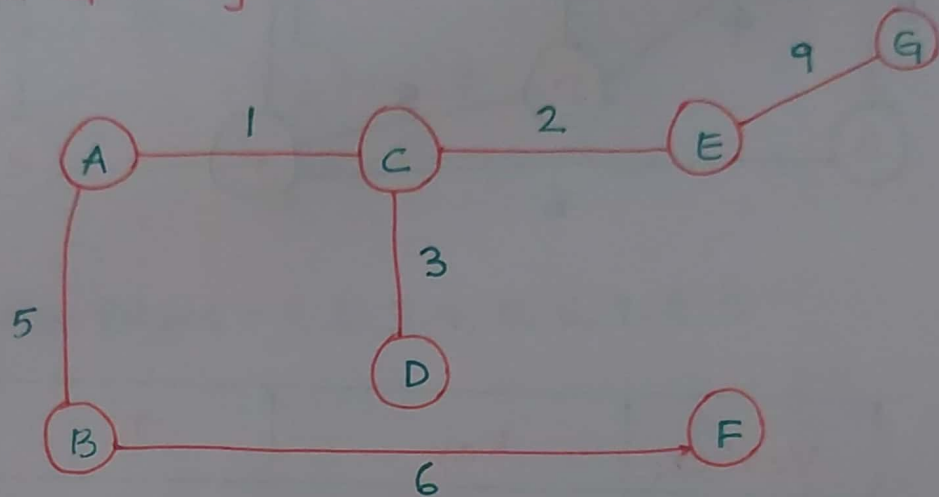## 2. Use Prim's algorithm to find a minimum spanning tree for the following Graph

Vertex	Key	Π
A	~~∞~~ 0	Nil
B	~~∞~~ 5	~~Nil~~ A
C	~~∞~~ 1	~~Nil~~ A
D	~~∞~~ ~~4~~ 3	~~Nil~~ ~~A~~ C
E	~~∞~~ 2	~~Nil~~ C
F	~~∞~~ ~~7~~ 6	~~Nil~~ ~~E~~ B
G	~~∞~~ 9	~~Nil~~ E

Q

A	B	C	D	E	F	G

$\cancel{\infty}$ $\cancel{\infty}$ $\cancel{\infty}$ $\cancel{\infty}$ $\cancel{\infty}$ $\cancel{\infty}$ $\cancel{\infty}$

0   5   1 $\cancel{4}$ 2 $\cancel{7}$ 9
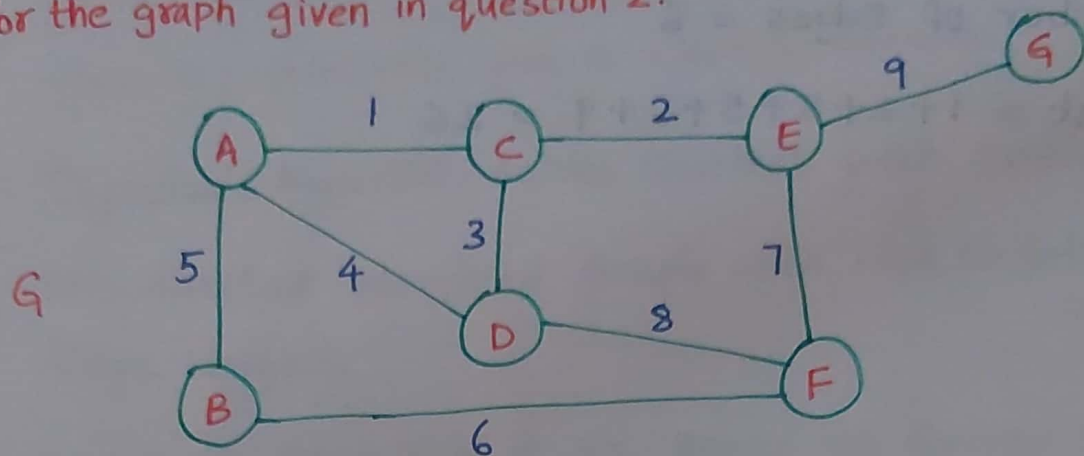
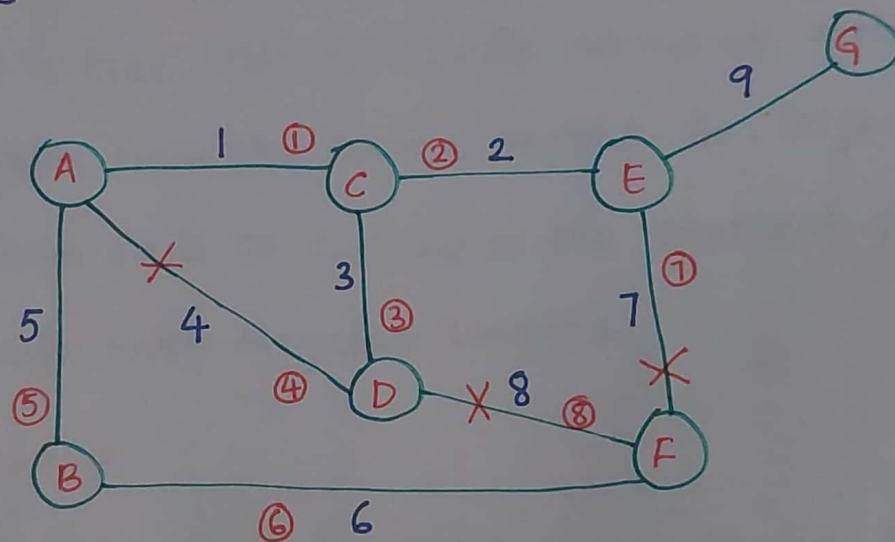   3   6

Minimum Spanning Tree :

Number of Vertices = 7

Number of Edges = 6
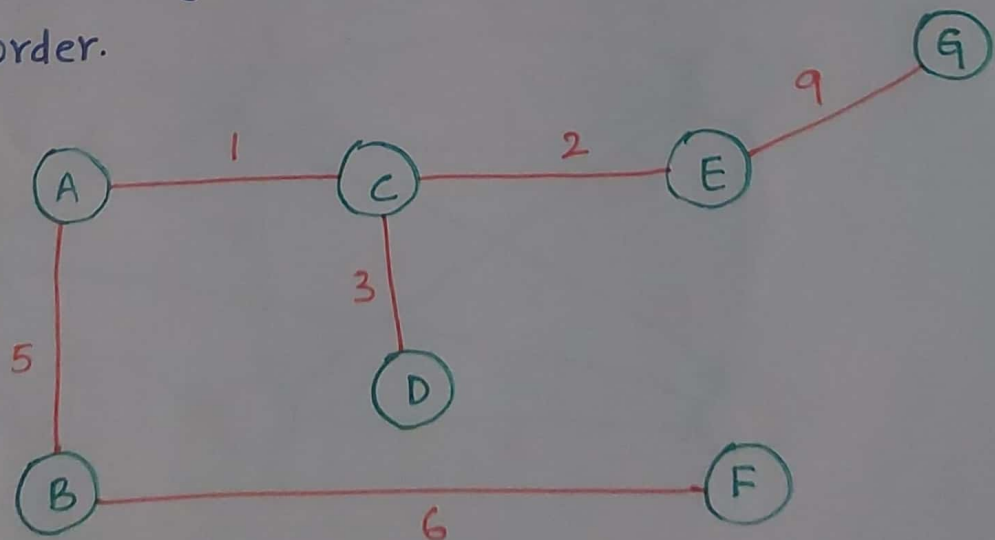
Cost = 1 + 2 + 3 + 5 + 6 + 9 = 26

3. Use Kruskal's algorithm to find a minimum spanning tree for the graph given in question 2.

Sorting the Edges : 1, 2, 3, 4, 5, 6, 7, 8, 9

Minimum Spanning Tree after considering the edges in sorted order.

Number of Vertices = 7

Number of Edges = 6

Cost = 1 + 2 + 3 + 5 + 6 + 9 = 26

Number of Vertices = 7

Number of Edges = 6
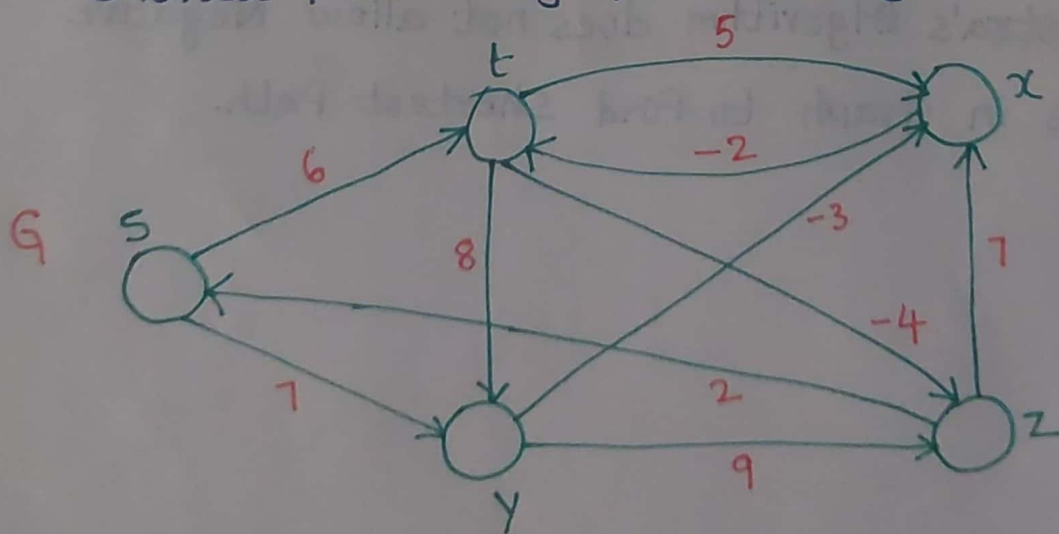
Cost = 1 + 2 + 3 + 5 + 6 + 9 = 26

4. Can Dijkstra's algorithm be used to find the shortest paths in a graph with some Negative Weights? If so, prove it; otherwise give a Counter Example.

Dijkstra's Algorithm solves shortest path problem for directed Weighted Graph with Non Negative Edge Weights.
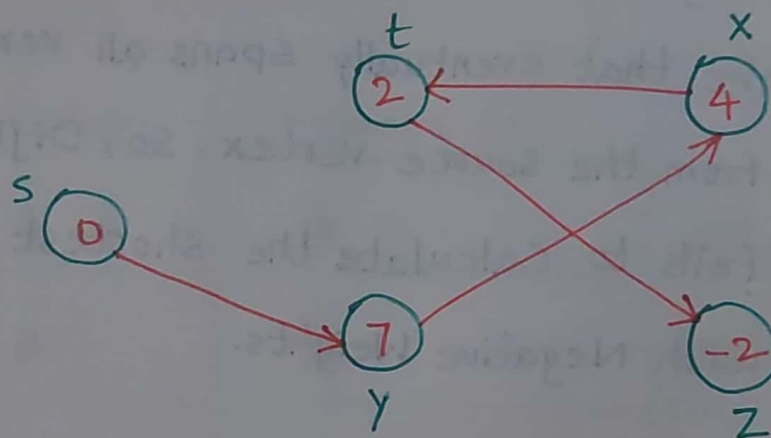
Dijkstra's Algorithm is also Known as Greedy Algorithm. Dijkstra's Algorithm starts at Source Vertex and it grows a tree, that eventually spans all vertices reachable from the source Vertex. So, Dijkstra's algorithm fails to Calculate the shortest path Correctly with Negative Weights.

Counter Example:

Only BellMan-Ford Algorithm allows to find the Shortest Path in a graph with Negative Weights.
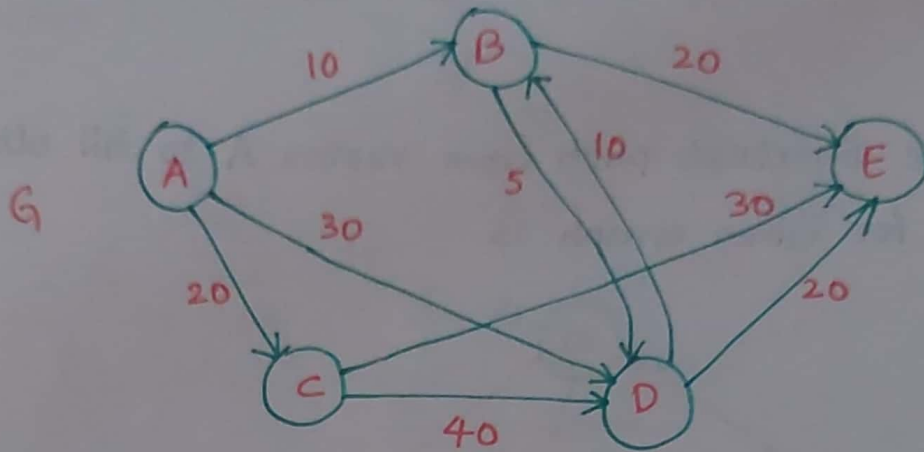
i	Vertex	d	π
~~1~~	s	~~∞~~ 0	Nil
~~2~~	t	~~∞~~ ~~8~~ 2	~~Nil~~ ~~s~~ x
~~3~~	x	~~∞~~ ~~11~~ 4	~~Nil~~ ~~t~~ y
~~4~~	y	~~∞~~ 7	~~Nil~~ s
5 STOP	z	~~∞~~ ~~7~~ −2	~~Nil~~ ~~y~~ t

Hence, Only Bellman – Ford Algorithm find the Shortest path With Negative Weights in the Graph

So, Dijkstra's Algorithm does not allow Negative Weights in Graph to find Shortest Path.

5. Use Dijkstra's to find the shortest paths from vertex A to all the other vertices for the graph given below.
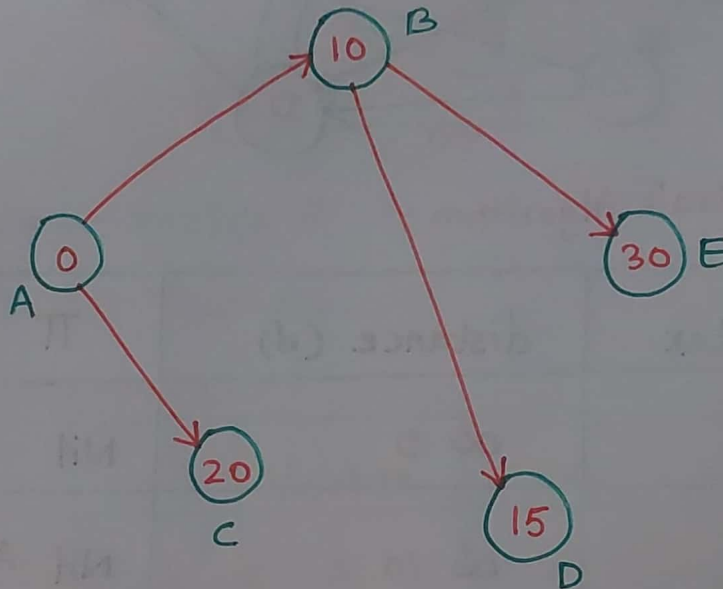

Dijkstra's Algorithm :

Vertex	distance (d)	π
A	~~∞~~ 0	Nil
B	~~∞~~ 10	~~Nil~~ A
C	~~∞~~ 20	~~Nil~~ A
D	~~∞~~ ~~30~~ 15	~~Nil~~ ~~A~~ B
E	~~∞~~ 30	~~Nil~~ B

Q

A	B	C	D	E
∞	∞	∞	∞	∞
0	10	20	3̶0̶	30
			15	

Hence, the Shortest path from vertex A to all other vertices for Given Graph is

B 10

A 0

30 E

20 C

15 D

6. Use Bellman-Ford Algorithm to find the shortest paths from vertex A to all the other vertices for the graph given in question 5.
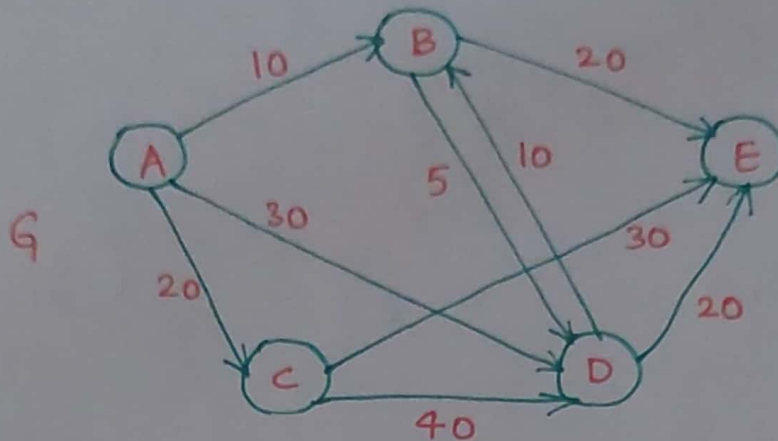
Given source vertex A

Bellman-Ford Algorithm

i = set of Vertices - 1

i		Vertex	d	π
1	Vertex 'B', 'C', 'D' update	A	~~∞~~ 0	Nil
2	Vertex 'D', 'E' update	B	~~∞~~ 10	~~Nil~~ A
3	No update	C	~~∞~~ 20	~~Nil~~ A
4	No update	D	~~∞~~ ~~30~~ 15	~~Nil~~ ~~A~~ B
5	STOP	E	~~∞~~ 30	~~Nil~~ B

Hence, The shortest path from Vertex A to all other
Vertices for given Graph is: