

CS 5300
Advanced Algorithms
Midterm Exam Version A

Name:

The point value of each problem is 3 points unless indicated otherwise. You **MUST** show all your work to receive full credit. Work neatly. **GOOD LUCK!**

1. What is the average case running time of an insertion sort algorithm?
 - a) $O(n)$
 - b) $O(n \lg n)$
 - c) $O(\lg n)$
 - d) $O(n^2)$
2. Insertion sort is an example of an incremental algorithm.
 - a) True
 - b) False
3. How many passes does an insertion sort algorithm consist of?
 - a) n
 - b) $n-1$
 - c) $n+1$
 - d) n^2
4. (8 pts.) Suppose computer A is running a sorting algorithm and it is supposed to sort an array of one million numbers. Suppose that computer A executes a billion instructions per second, and suppose computer A requires $100n^2$ instructions to sort n numbers. Find the time it takes computer A to sort the one million numbers?

5. (10 pts.) Use the bottom-up approach to illustrate the operations of merge-sort on the array $A = \langle 3, 1, 15, 11, 2, 7, 15, 8, 13 \rangle$. Use the notes discussed in class as a guide

6. (8 pts.) Prove that $2^{n-1} = O(2^n)$?

7. (8 pts.) Show that $5n^2 + 2n + 15 = o(n^3)$

8. (8 pts.) Show that $2n^2 + n$ is $\Omega(n^2)$

9. (12 pts.) Consider the following recurrence equation, defining $T(n)$, as

$$T(n) = \begin{cases} 7 & \text{if } n = 1 \\ 2T(n/2) + 5n^2 & \text{otherwise} \end{cases}$$

Show, by induction, that $T(n) = 10n^2 - 3n$

10. (12 pts.) Draw the recursion tree for $T(n) = T\left(\frac{3n}{5}\right) + T\left(\frac{2n}{5}\right) + \Theta(n)$ and find the height of the tree

11. (18 pts.) Solve the following recurrences

a) $T(n) = 9T\left(\frac{n}{3}\right) + n$

b) $T(n) = 3T\left(\frac{n}{4}\right) + n \lg n$

c) $T(n) = 8T\left(\frac{n}{2}\right) + n^3 \lg^2 n$

12. (10 pts.) Given the following algorithm:

Algorithm Foo (A):

Input: An array A storing $n \geq 1$ integers.

Output: ?

$k = A[0]$

for $i = 1$ *to* $n - 1$ *do*

$k = k + A[i]$

return k

a) What is the output?

b) What is the time complexity $T(n)$?