HOMEWORK-5

3/12/2023

MANOJ SUVVALA

700744733

1. Given the recurrence relation

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 7T\left(\frac{n}{5}\right)+10n & \text{otherwise} \end{cases}$$

find $T(625)$.

Sol) Given,

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 7T\left(\frac{n}{5}\right)+10n & \text{otherwise} \end{cases}$$

$T(625) = 7T\left(\frac{625}{5}\right) + 10(625)$ — (1) ($\frac{625}{5} = 125$)

$T(125) = 7T\left(\frac{125}{5}\right) + 10(125)$ — (2) ($\frac{125}{5} = 25$)

$T(25) = 7T\left(\frac{25}{5}\right) + 10(25)$ — (3) ($\frac{25}{5} = 5$)

$T(5) = 7T(5/5) + 10(5)$ — (4) ($5/5 = 1$)

$T(1) = 1$, substituting $T(1)=1$ in above equ

(4) $\rightarrow T(5) = 7(1)+50 = 57$

(3) $\rightarrow T(25) = 7(57)+250 = 649$

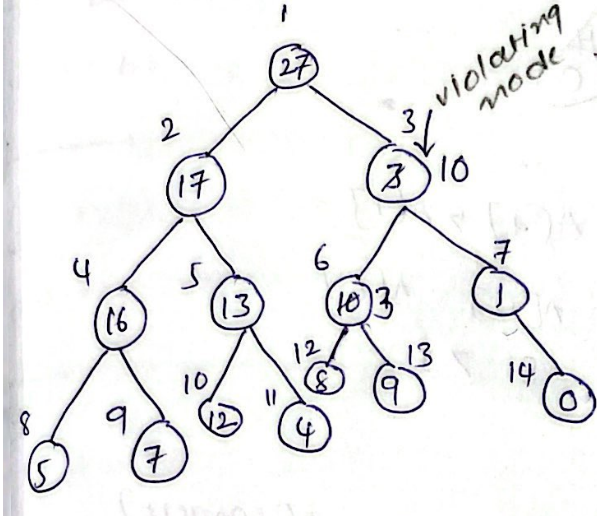
(2) $\rightarrow T(125) = 7(649)+1250 = 5793$

(1) $\rightarrow T(625) = 7(5793)+6250 = 46801$

$\therefore T(625) = 46801.$

2) Illustrate the operation of MAX-HEAPIFY(A,3) on the array A - <27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0>

Sol A = [27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 10]



i	left	right	largest
3	6	7	6
6	12	13	13
13			13

Given MAX-HEAPIFY(A,3) where i-2 and left = 2i, right = 2i+1

heap size = 14

$l \leq A$ heap size $\quad A[l] > A[i]$

$6 < 14 \quad A[6] \quad A[3]$

$10 > 3$

largest = l = 6

$r \leq A \cdot$ heapsize $\quad A[r] > A[\text{largest}]$

$7 < 14 \quad A[7] \quad A[6]$

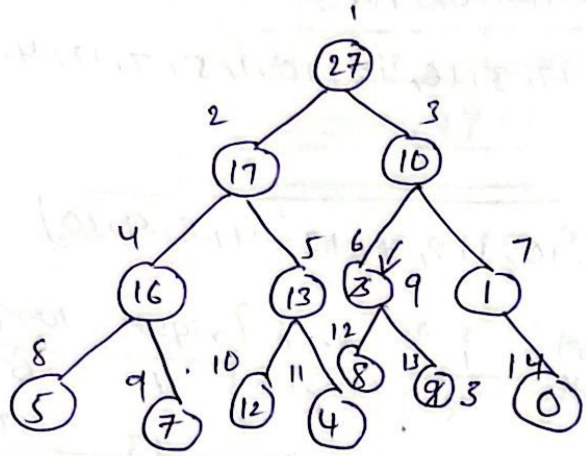
$1 \ngtr 10$

largest $\neq i$

$6 \neq 3$

exchange A[3] with A[6]

MAX-HEAPIFY(A,6)



$l \le A$.heapsize $\qquad$ $A[l] > A[i]$

$12 < 14$ $\qquad$ $A[12] \cdot A[6]$

8 7 3

largest = l = 12

$r \le A$.heapsize $\qquad$ $A[r] > A[\text{largest}]$

$13 < 14$ $\qquad$ $A[13] \cdot A[12]$

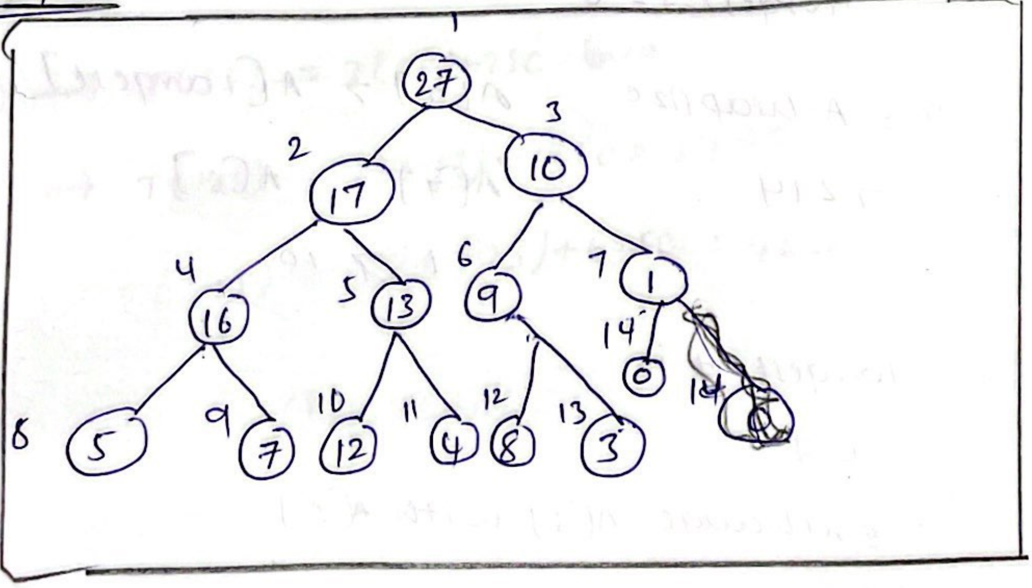
9 7 8

largest = r = 13

exchange A[6] with A[13]

MAXHEAPIFY (A, 13)

<u>output</u>



if we perform MAX-HEAPIFY (A, 13) then 'l' will be 26 'r' will be 27

which are greater than heap size = 14 & largest because the i=13 so we stop performing MAX-HEAPIFY

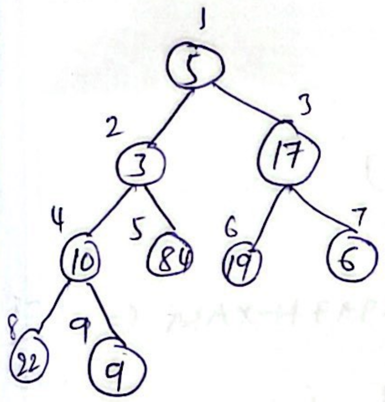
→ The above tree is the output & final array will be A = [27, 17, 10, 16, 13, 9, 1, 5, 7, 12, 4, 8, 3, 0]

---

(3) Illustrate the operation of BUILD-MAX-HEAP on the array A = [5, 3, 17, 10, 84, 19, 6, 22, 9]

Sol

A = < 5, 3, 17, 10, 84, 19, 6, 22, 9 >



BUILD-MAX-HEAP(A)

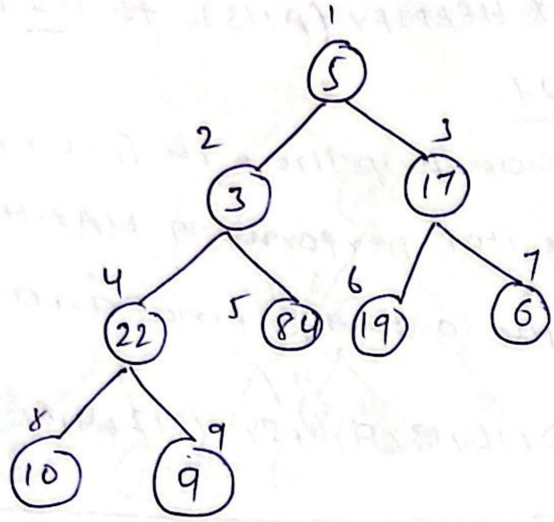
so A.heap size = 9

i = 4 ⇒ MAX-HEAPIFY(A, 4)

i	l	r	largest
4	8	9	8

so exchange A[8] with A[4]

so the tree will be



$\frac{i}{8}$ $\frac{l}{16}$ $\frac{r}{17}$ so stop Max-Heapify as

16 > A.heap size

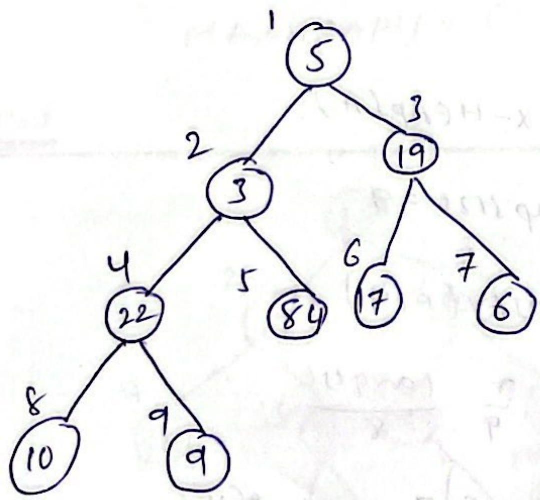
17 > A.heap size

$\underline{i=3}$ ⇒ MAX-HEAPIFY (A, 3)

$\frac{i}{3}$ $\frac{l}{6}$ $\frac{r}{7}$ $\frac{\text{largest}}{6}$

so exchange A[6] with A[3]

so tree will be



$\frac{i}{6}$ $\frac{l}{12}$ $\frac{r}{13}$ so stopping MAX-HEAPIFY as

12 > A.heap size

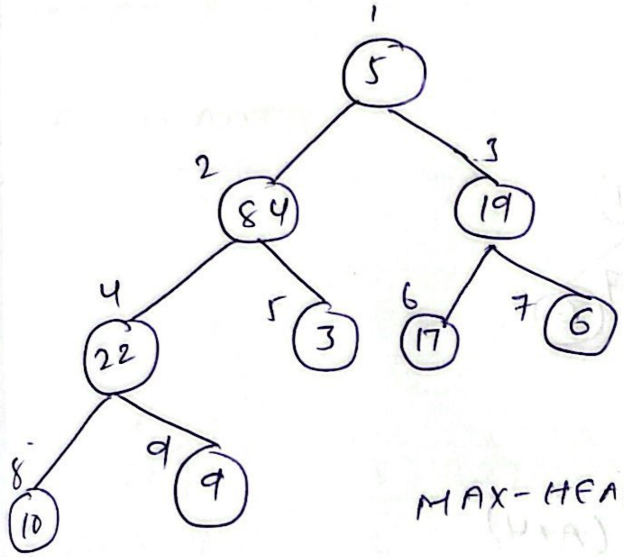
13 > A heap size

i=2 ⇒ MAX-HEAPIFY(A,2)

i	l	r	largest
2	4	5	5

so exchange A[5] with A[2]

so tree will be



MAX-HEAPIFY (A,5)

i	l	r
5	10	11

so stopping MAX-HEAPIFY as

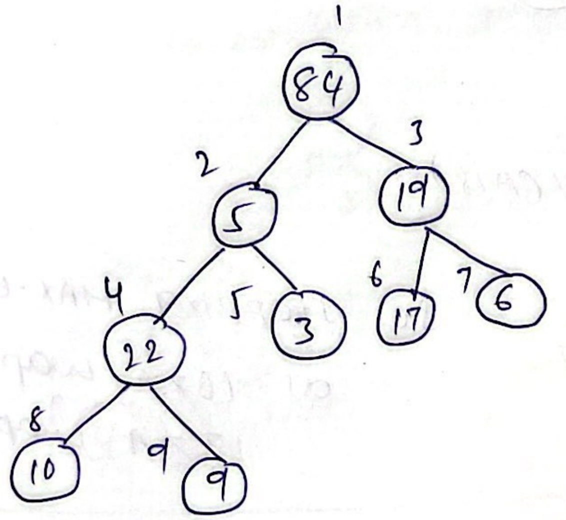
10 > A.heapsize

11 > A heapsize

i=1 ⇒ MAX-HEAPIFY (A,1)

i	l	r	largest
1	2	3	2

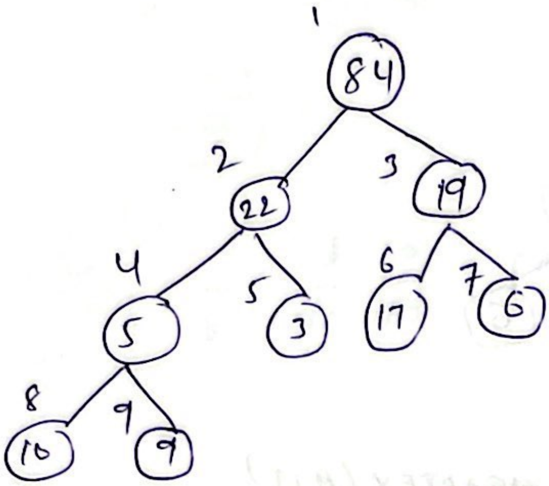
so exchange A[2] with A[1]



=> MAX HEAPIFY (A,2)

i	l	r	largest
2	4	5	4

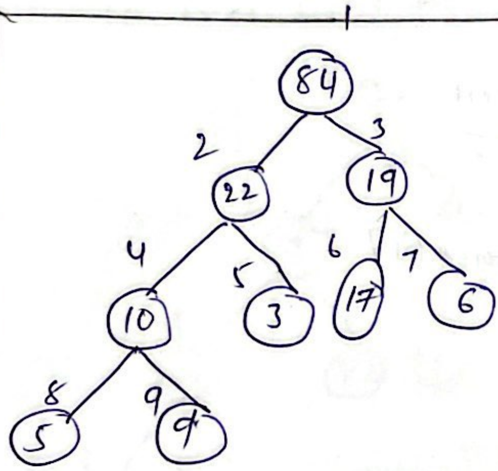
so exchange A[4] with A[2]



=> MAX-HEAPIFY (A,4)

i	l	r	largest
4	8	9	8

so exchange A[8] with A[4]



=> MAX HEAPIFY (A,8)

i	l	r
8	16	17

so stopping MAX-HEAPIFY as 16 > A heapsize

17 > A heapsize

Scanned with CamScanner

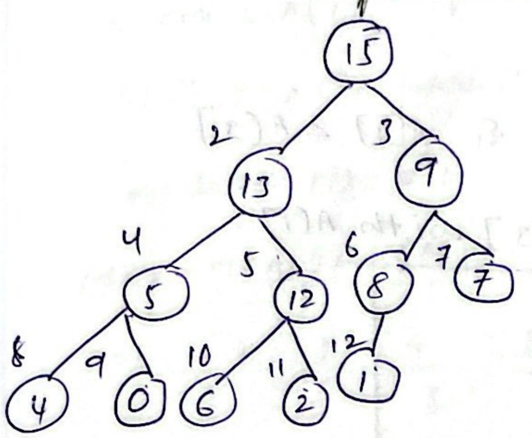
i=0 so STOP the loop & above tree is the final tree & output & final array is

A = [84, 22, 19, 10, 3, 17, 6, 5, 9]

---

(4) Illustrate the operation of MAX-HEAP-INSERT (A, 17) on the heap A = ⟨15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1⟩.

sol Given Array A = [15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1]



MAX-HEAP-INSERT (A, 17)

so, A.heap size = (13)

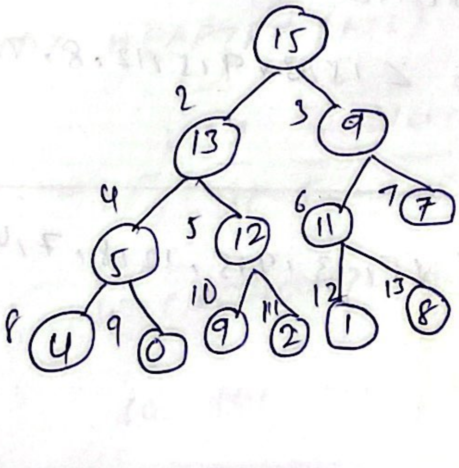
A[13] = $-\infty$

HEAP-INCREASE-KEY (A, 13, 17)

so i=13 key = 17, 17 > & A[6] < A[13]

=) A[13] = 17

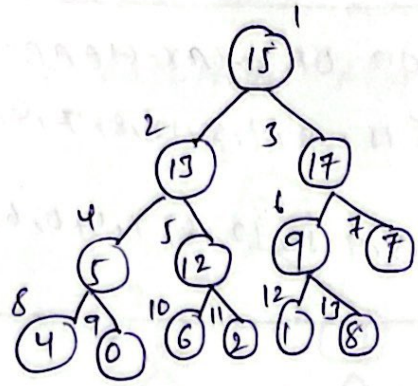
so exchange A[13] with A[6]



so, i=6

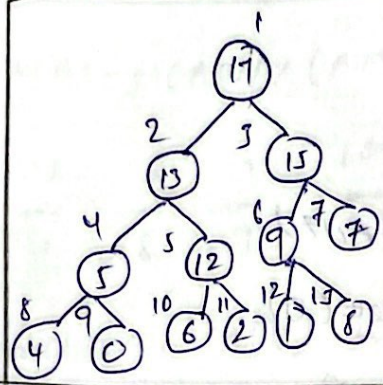
key = 17 ⇒ A[6] = 17 & A[3] < A[6]

exchange A[6] with A[3]



so i=3   key = 17, A[3] = 17 & A[1] < A[3]

so exchange A[3] with A[1]



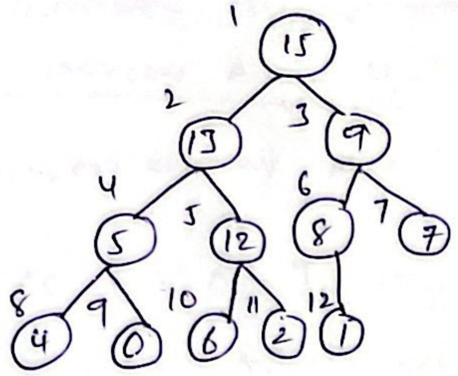
so i=1, so stop the loop. The above will be the final tree & output & final array will be

A = [17, 13, 15, 5, 12, 9, 7, 4, 0, 6, 2, 1, 8]

(5) Illustrate the operation of HEAP-EXTRACT-MAX on the heap A = <15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1

17

sol   Given Array A = [15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1]



HEAP-EXTRACT-MAX (A)

max = A[1] = 15

A[1] = A[A.heap size] = A[12] = 1

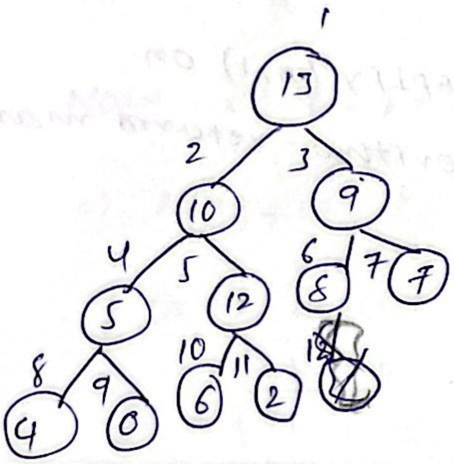
A.heap size = 11

MAX-HEAPIFY (A,1)

=)

i	l	r	largest
1	2	3	3

so exchange A[1] with A[2]

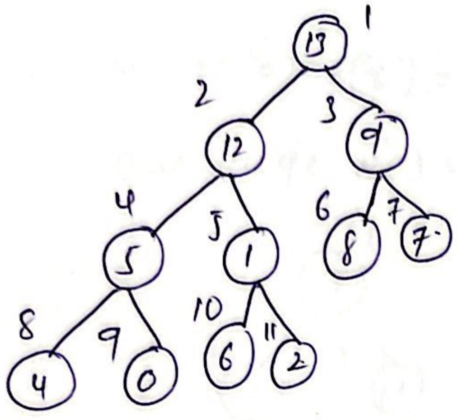


=) MAX HEAPIFY (A,2)

i	l	r	largest
2	4	5	5

so exchange A[2] with A[5]

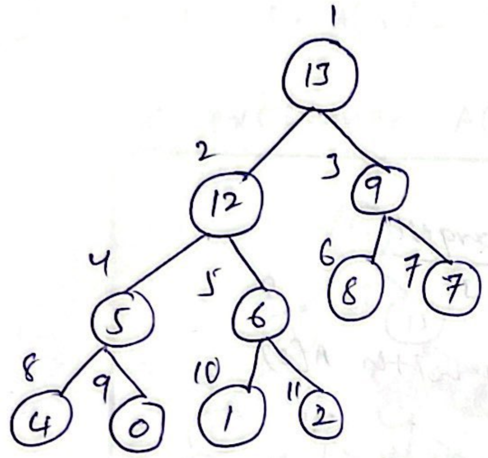
so the tree will be



=) MAX-HEAPIFY (A 1 5) =)

i	l	r	largest
5	10	2	10

So exchange A(5) with A(10)



So, After applying MAX-HEAPIFY (A,1) on HEAP-EXTRACT-MAX, the algorithm return max which is 15

The output will be 15

(6) Illustrate the operation of PARTITION on the array A = < 15, 9, 5, 23, 6, 18, 4, 17 >

Sol Given array A =

1	2	3	4	5	6	7	8
15	9	5	23	6	18	4	17

so $x = A[r] = 17$, $i = 0$, $j = 1$

$\Rightarrow A[1] \le 17$ $i = 1$ & exchange A[1] with A[1]

so, A will be

	1	2	3	4	5	6	7	8
A	15	9	5	23	6	18	4	17
	p, i, j							r

Now $x = 17$, $i = 1$, $j = 2$

$\Rightarrow A[2] \le 17$ so, $i = 2$ & exchange A[2] with A[2]

so A will be

	1	2	3	4	5	6	7	8
A	15	9	5	23	6	18	4	17
	p	i, j						r

Now $x = 17$, $i = 2$, $j = 3$

$\Rightarrow A[3] \le 17$, so $i = 3$ & exchange A[3] with A[3]

so A will be

	1	2	3	4	5	6	7	8
A	15	9	5	23	6	18	4	17
	p		i, j					r

Now, $x = 17$, $i = 3$, $j = 4$

$\Rightarrow A[4] \nleq 17$ so A will be

	1	2	3	4	5	6	7	8
A	15	9	5	23	6	18	4	17
	p		i	j				r

Now $x = 17$, $i = 3$, $j = 5$

=) $A[5] \leq 17$, so $i = 4$ & exchange $A[4]$ with $A[5]$

	1	2	3	4	5	6	7	8
A	15	9	5	6	23	18	4	17
	p			i	j			r

Now $x = 17$, $i = 4$, $j = 6$

=) $A[6] \nleq 17$, so A will be

1	2	3	4	5	6	7	8
15	9	5	6	23	18	4	17
p			i		j		r

Now $x = 17$, $i = 4$, $j = 7$

=) $A[7] \leq 17$ so, $i = 5$ & exchange $A[5]$ with $A[7]$.

	1	2	3	4	5	6	7	8
A	15	9	5	6	4	18	23	17
	p				i		j	r

$x = 17$, $i = 5$, $j = 8$ STOP!!

Now exchange $A[6]$ with $A[8]$, so A will be

	1	2	3	4	5	6	7	8
A	15	9	5	6	4	17	23	18
	p					q		r

as $q = i + 1 = 6$

so, if we check we can see that left side elem of q is less than $A[q]$ & right side elements of q is greater than $A[q]$. so it returns <u>6</u>

(1) prove that the solution for the reccurence $T(n) = T(n-1) + O(n)$ is $T(n) = O(n^2)$

---

sol) Given recurrence equation

$$T(n) = T(n-1) + O(n)$$

so to prove $T(n) = O(n^2)$ is solution for the given equation

we need to prove that solution for recurrence equation

(1) $T(n) = T(n-1) + O(n)$ is $T(n) = O(n^2)$

(2) $T(n) = T(n-1) + \Omega(n)$ is $T(n) = \Omega(n^2)$

(1) to prove $T(n) = T(n-1) + O(n)$ is $T(n) = O(n^2)$

Inductive hypothesis:

Assume $T(k) \leq c k^2 \quad \forall k \leq n$

In particular let $k = n-1$

i.e $T(n-1) \leq c(n-1)^2$

show that $T(n) \leq c \cdot n^2$

$$\begin{aligned} T(n) &\leq T(n-1) + O(n) \\ &\leq c(n-1)^2 + cn \\ &\leq cn^2 - 2cn + c + cn \\ &\leq cn^2 - c(n-1) \end{aligned}$$

$\Rightarrow T(n) \leq cn^2$ if $-c(n-1) \leq 0$

Hence, $T(n) = O(n^2) \quad \forall n \geq 1$

(2) To prove $T(n) = T(n-1) + \Omega(n)$ is $T(n) = \Omega(n^2)$

Inductive hypothesis

Assume $T(k) \geq ck^2 \quad \forall k < n$

In particular let $k = n-1$

i.e $T(n-1) \geq c(n-1)^2$

show that $T(n) \geq cn^2$

$$T(n) \geq T(n-1) + cn$$
$$\geq c(n-1)^2 + cn$$
$$\geq cn^2 - c(n-)$$

$\Rightarrow T(n) \geq cn^2$ if $-c(n-1) \leq 0$

Hence $T(n) = \Omega(n^2) \quad \forall n \geq 1$

· Hence $T(n) = O(n^2)$ & $T(n) = \Omega(n^2)$

$\therefore \boxed{T(n) = \Theta(n^2)}$

Hence proved.