

①

A factory has two plants. Records show that the plant I produces 30% of the items of the output whereas plant II produces 70% of the items. Further, 5% of the items produced by plant I are defective while 1% produced by plant II are defective. If a defective item is drawn at random, find the probability that the defective item was produced by (i) plant I (ii) by plant II.

Solⁿ:-

Let A_1 denote the event of drawing an item produced by ~~pt~~ plant I, A_2 denote the event of drawing an item produced by plant II and B denote the event of drawing a defective item by plant I or by plant II. We are given that

$$P(A_1) = 0.30, \quad P(A_2) = 0.70,$$

$$P(B|A_1) = 0.05, \quad P(B|A_2) = 0.01$$

The required probability are $P(A_1|B)$ and $P(A_2|B)$.

By Bayes's theorem,

$$P(A_1|B) = \frac{P(A_1) P(B|A_1)}{P(A_1) P(B|A_1) + P(A_2) P(B|A_2)}$$

$$= \frac{0.30 \times 0.05}{0.30 \times 0.05 + 0.70 \times 0.01}$$

$$= \frac{15}{22} = 0.682$$

Similarly, $P(A_2|B) = \frac{P(A_2) P(B|A_2)}{P(A_1) P(B|A_1) + P(A_2) P(B|A_2)}$

$$= \frac{0.70 \times 0.01}{0.30 \times 0.05 + 0.70 \times 0.01}$$

$$= \frac{7}{22} = 0.318$$

(3)

② A survey was conducted to find the supplies of the consumer durables for the market. It was found that the three major companies A, B and C have market share of 35%, 25% and 40% respectively out of which 2%, 1% and 3% are not upto the satisfaction.

A consumer buys a product and is dissatisfied with it. What is the probability that it is from the company

Solⁿ: Let X be the event of drawing a dissatisfied item from any company and the event that an item drawn was produced by companies A, B or C, be Y_1, Y_2, Y_3 respectively.

$$\therefore P(Y_1) = 0.35, P(Y_2) = 0.25, P(Y_3) = 0.40$$

$$\text{and } P(X|Y_1) = 0.02, P(X|Y_2) = 0.01$$

$$P(X|Y_3) = 0.03$$

$$P(Y_1)P(X|Y_1) = 0.007, P(Y_2)P(X|Y_2) = 0.0025$$

$$P(Y_3)P(X|Y_3) = 0.012$$

(4)

∴ Required probability

$$= \frac{P(Y_3) P(X|Y_3)}{P(Y_1) P(X|Y_1) + P(Y_2) P(X|Y_2) + P(Y_3) P(X|Y_3)}$$

$$= \frac{0.012}{0.007 + 0.0025 + 0.012}$$

$$= \frac{0.012}{0.0215}$$

$$= 0.558 \text{ Ans.}$$

- ③ An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents is 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

(2)

(5)

Solⁿ let E_1, E_2, E_3 denote the events that a driver selected at random is a scooter, car or truck driver respectively and let E denote the event of a driver meeting with accident. Then

$$P(E_1) = \frac{2000}{12000} = \frac{1}{6}, \quad P(E_2) = \frac{4000}{12000} = \frac{1}{3},$$

$$P(E_3) = \frac{6000}{12000} = \frac{1}{2}$$

$$\text{and } P(E|E_1) = 0.01, \quad P(E|E_2) = 0.03,$$

$$P(E|E_3) = 0.15$$

$$\text{Required probability} = \frac{P(E_1) P(E|E_1)}{P(E_1)P(E|E_1) + P(E_2)P(E|E_2) + P(E_3)P(E|E_3)}$$

$$= \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15}$$

$$= \frac{0.01}{0.01 + 0.06 + 0.45} = \frac{1}{52} \text{ Ans}$$