Random Variable :-

A rule that assigns a real number to each outcome is called random variable. The rule is nothing but a function of the variable, say, X that assigns a unique value to each outcome of the random experiment.

Discrete Random Variable :-

A random variable X, which can take only a finite number of values in an interval of the domain is called discrete random variable.

Ex. 1 Number appearing on top of a dice when it is thrown.

2 The number of telephone calls received per day.

3 Number of mistakes in a page.

4 Number of defective items in a lot.

Discrete Probability Distribution :-

If a random variable x can assume a discrete set of values say $x_1, x_2, \ldots x_n$ with respect to probabilities $p_1, p_2, \ldots p_n$ such that

$$\boxed{p_1 + p_2 + \cdots + p_n = 1 \quad \text{i.e.} \sum_{i=1}^{n} p_i = 1}$$

then occurence of values x_i with respective probability p_i is called the discrete probability distribution of X.

For example, In a throw of a pair of dice the sum X is discrete random variable which is an integer between 2 and 12 with probabilities $P(X)$,

sample space are:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

$P(X)$ given as

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

This constitute a discrete probability distribution

Probability Mass Function (Pmf) :-

Probability function of probability mass function of a random variable X ~~which~~ is ~~an integer~~ mathematical function $p(x)$ which gives the probabilities corresponding to different possible discrete set of values say $x_1, x_2, \ldots x_n$ of variable x. i.e. $p(x_i) = p(x = x_i)$

= probability that and ou variable x assumes values x_i

The function $p(x)$ satisfies the condition :-

(i) $p(x_i) \geq 0$ (ii) $\Sigma p(x_i) = 1$.

Cumulative Distribution function :-

If X is a random variable then $P(X \leq x)$ is called the ~~com~~ cumulative distribution (cdf) denoted by $F(x) = p(X \leq x)$.

Expectation of a discrete Random Variable :-

If x is discrete random variable which assumes the discrete set of values $x_1, x_2, \dots, x_n$ with respective probabilities $p_1, p_2, \dots p_n$ then the expectation or expected value of x is denoted by $E(X)$ and defined as

$$E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \dots + x_n p_n = \sum_{i=1}^{n} x_i p_i$$

similarily the expected value of X^2 is defined as $E(x^2) = \sum_{i=1}^{n} x_i^2 p_i$

Properties :- ① If X is a random variable and 'a' is constant then

(i) $E(a) = a$

(ii) $E(ax) = aE(X)$

(iii) $E(X - \mu) = 0$

② $E(X \pm Y) = E(X) \pm E(Y)$

③ $E(XY) = E(X)\, E(Y)$ if X and Y are independent

④ $E(Y) = aE(X) + b$ [if $y = ax + b$]

Variance and Standard Deviation of Discrete R.V.

The variable of discrete random variable X is expected value of $(X-\mu)^2$ where μ is mean of variable X

$$Var(X) = V(x) = E(X-\mu)^2 = \sum_{i=1}^{n} p_i (x_i - \mu)^2$$

$$= \sum p (x-\mu)^2 = \sum p (x^2 + \mu^2 - 2x\mu)$$

$$= \sum p x^2 + \mu^2 \sum p - 2\mu \sum p x$$

$$= \sum p x^2 + \mu^2 \cdot 1 - 2\mu \cdot \mu \qquad \left[\sum p = 1, \; \mu = E(X) = \sum p x \right]$$

$$= \sum p x^2 + \mu^2 - 2\mu^2$$

$$= \sum p x^2 - \mu^2$$

$$\boxed{Var(X) = E(X^2) - \{E(X)\}^2}$$

The standard deviation (SD) of a random variable x is denoted by s, then

$$\boxed{SD(X) = \sqrt{Var(X)}}$$

Ex. (1) A pair of two coins is tossed, what is the expected value?

Soln. Expected value or mean value $= E(x) = \mu = \sum_{i=1}^{n} p_i x_i$

In tossing of two coins, probability distribution is represented in tabular form as follows

X	0	1	2	Total
P(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1

HH	HT	TH	TT
X = 2	X = 1	X = 1	X = 0

$$\therefore \quad E(X) = \frac{1}{4} \times 0 + \frac{1}{2} \times 1 + \frac{1}{4} \times 2 = 1$$

$$\boxed{E(x) = 1}$$

As the probability of getting no head, one head and two heads is respectively $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

In the tossing of two coins, we have

$$\Sigma p x^2 = \frac{1}{4} \cdot (0)^2 + \frac{1}{2} \cdot (1)^2 + \frac{1}{4} \cdot 2^2 = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\therefore \text{ Variance} = \sigma^2 = \Sigma p x^2 - \mu^2 = \frac{3}{2} - 1^2$$

$$= \frac{1}{2}.$$

Ex. (2)

Ex. (2) A pair of dice is thrown together, find the expected value.

Solⁿ. In a throw of pair of dice the sum (X) is a discrete random variable which is an integer between 2 and 12 with the probabilities as given below:

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\therefore$ Expected value $= E(X) = \mu$

$$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36}$$

$$+ \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$= \frac{252}{36} = 7.$$

In case of a pair of dices we have

$$\Sigma px^2 = \frac{1}{36}\cdot 4 + \frac{2}{36}\cdot 9 + \frac{3}{36}\cdot 16 + \frac{4}{36}\cdot 25 + \frac{5}{36}\cdot 36 + \frac{6}{36}\cdot 49$$

$$+ \frac{5}{36}\cdot 64 + \frac{4}{36}\cdot 81 + \frac{3}{36}\cdot 100 + \frac{2}{36}\cdot 121 + \frac{1}{36}\cdot 144$$

$$= \frac{1}{36}\left[4+18+48+100+180+294+320+324+300 + 242 + 144\right]$$

$$= \frac{1}{36}\left[1974\right] = \frac{329}{6}$$

$$\text{Variance} = \sigma^2 = \frac{329}{6} - 7^2$$

$$= \frac{329 - 294}{6}$$

$$= \frac{35}{6}$$

$$\text{stadard deviation} = \sigma = \sqrt{\frac{35}{6}}.$$

Ex. 3 A random variable X has the following distribution

X	−2	−1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

Determine : (i) k, (ii) Mean, (iii) Variance.

Solⁿ :- (i) since $\Sigma p(x) = 1$

$\Rightarrow 0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$\Rightarrow 0.6 + 4K = 1$

$\Rightarrow 4K = 0.4$

$\Rightarrow \boxed{K = 0.1}$

Now the probability distribution of the random variable X is

X	−2	−1	0	1	2	3
P(X)	0.1	0.1	0.2	0.2	0.3	0.1

4. Find the mean and variance of uniform probability distribution $f(x) = \frac{1}{n}$ for $x = 1, 2, \ldots n$.

Soln.

X	1	2	3	...	n
P(X)	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$

$$\text{Mean} = E(X) = \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \cdots + \frac{n}{n}$$

$$= \frac{1+2+3+\cdots+n}{n}$$

$$= \frac{1}{n}\left[\Sigma n\right]$$

$$= \frac{1}{n}\left[\frac{n(n+1)}{2}\right]$$

$$= \frac{n+1}{2}$$

$$\text{Variance} = E(X^2) - \{E(X)\}^2 = \frac{1^2+2^2+\cdots+n^2}{n} - \left[\frac{n+1}{2}\right]^2$$

$$= \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{(n+1)}{2}\right]$$

$$= \left(\frac{n+1}{2}\right)\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{(n+1)}{2}\left[\frac{(n-1)}{6}\right] = \frac{n^2-1}{12}.$$

(5) For discrete probability distribution

x	0	1	2	3	4	5	6	7
f	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine (i) k (ii) mean (iii) variance (iv) smallest value of x such that $P(X \le x) > \frac{1}{2}$.

Soln. (i) $\Sigma f(x) = 1$

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0$

$\Rightarrow 10k(k+1)-1(k+1)=0$

$\Rightarrow (k+1)(10k-1)=0$

$\Rightarrow k=-1,\ k=1/10$

$\therefore$ $k=\frac{1}{10}$ since probability can never be negative.

x	0	1	2	3	4	5	6	7
f	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100}+\frac{1}{10}=\frac{17}{100}$

(ii) Mean $= \Sigma x f(x) = 0\times0 + 1\times\frac{1}{10} + 2\times\frac{2}{10} + 3\times\frac{2}{10} + 4\times\frac{3}{10} + 5\times\frac{1}{100}$
$+ 6\times\frac{2}{100} + 7\times\frac{17}{100}$

$= \frac{10+40+60+120+5+12+119}{100} = \frac{366}{100} = 3.66$

(iii) Variance $= E(x^2) - \{E(x)\}^2$

$$\sigma^2 = \left[0 + 1^2 \times \frac{1}{10} + 2^2 \times \frac{2}{10} + 3^2 \times \frac{2}{10} + 4^2 \times \frac{3}{10} + 5^2 \times \frac{1}{100} + 6^2 \times \frac{2}{100} + 7^2 \times \frac{17}{100}\right] - \{3.66\}^2$$

$$= \frac{10 + 80 + 180 + 480 + 25 + 72 + 833}{100} - 13.3956$$

$$= \frac{1680}{100} - 13.3956$$

$$= 16.8 - 13.3956 = 3.40$$

(iv) $P(x \le 0) = f(0) = 0$

$P(x \le 1) = f(0) + f(1) = 0 + 0.1 = 0.1$

$P(x \le 2) = 0 + 0.1 + 0.2 = 0.3$

$P(x \le 3) = 0.3 + 0.2 = 0.5 = \frac{1}{2}$

$P(x \le 4) = 0.5 + 0.3 = 0.8$

$\therefore$ Smallest value of X such that $P(X \le x) \ge \frac{1}{2}$

is 4.

Moment of Random Variable :-

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Moment of Random Variable

The "moments" of a random variable (or its distribution) are expected values of powers or related functions of the random variable.

The rth moment of x is $\mu'_r = E(x^r) = \sum x^r P(X=x)$

In particular, the first moment is mean

$$\mu_x = E(x) = \mu'_1$$

The rth central moment of x is $\mu_r = E(X-\mu_x)^r$

In particular, the second central moment is variance $\sigma^2_x = Var X = E(X-\mu_x)^2$.

Relation Between rth moment of random variable and rth central moment of random variable

$$\mu_1 = E(X-\mu_x) = E(X) - E(\mu_x) = E(X) - E(X) = 0$$

$$\mu_2 = E(X-\mu_x)^2 = E(x^2 + \mu_x^2 - 2x\mu_x)$$

$$= E(x^2) + \mu_x^2 - 2\mu_x E(x)$$

$$= E(x^2) + \mu_x^2 - 2\mu_x^2$$

$$= E(x^2) - \mu_x^2$$

$$\mu_2 = \mu'_2 - \mu_1'^2 \quad [\text{first central moment is mean}]$$

$$\mu_3 = E(X-\mu_x)^3 = E(x^3 + \mu_x^3 - 3x^2\mu_x + 3\mu_x^2 x)$$

$$= E(x^3) + E(\mu_x^3) - 3E(x^2)\mu_x + 3\mu_x^2 E(X)$$

$$\mu_3 = E(X^3) - \mu_1'^3 - 3\mu_1'\mu_2' + 3\mu_1'^2\mu_1'$$

$$= \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' + 6\mu_1'^2\mu_2' - 4\mu_1'\mu_2' - 3\mu_1'^4$$

Let X be a discrete random variable having probability mass function

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{if } x = 1 \\ 1/3 & \text{if } x = 2 \\ 1/6 & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

find the third moment of X.

Third moment $= E(x^3)$

$$= \sum x^3 P_X(x)$$

$$= \frac{1}{2}\cdot 1^3 + \frac{1}{3}\cdot 2^3 + \frac{1}{6}\cdot 3^3 + 0$$

$$= \frac{1}{2} + \frac{8}{3} + \frac{27}{6}$$

$$= \frac{3+16+27}{6}$$

$$= \frac{46}{6}$$

$$= \frac{23}{3}.$$

Probability Density Function :-

The probability density function (p.d.f.) of a random Variable X usually denoted by $f_x(x)$ or simply $f(x)$ has following properties :-

$f(x) \geq 0 , \quad -\infty < x < \infty$

$\int_{-\infty}^{\infty} f(x)\, dx = 1.$

A continuous random variables X has a probability density function defined by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & \text{if } -3 \leq x \leq -1 \\ \frac{1}{16}(6-2x^2) & \text{if } -1 \leq x < 1 \\ \frac{1}{16}(3-x)^2 & \text{if } 1 < x \leq 3 \\ 0 & \text{elsewhere.} \end{cases}$$

verify f(x) is a density function. and also find the mean of the random variable X.

Since f(x) is density function $\int_{-\infty}^{\infty} f(x) dx = 1.$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{-3} f(x)dx + \int_{-3}^{-1} f(x)dx + \int_{-1}^{1} f(x)dx + \int_{1}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx$$

$$= \int_{-\infty}^{-3} 0\, dx + \int_{-3}^{-1} \frac{1}{16}(3+x)^2 dx + \int_{-1}^{1} \frac{1}{16}(6-2x^2)\, dx$$

$$+ \int_{1}^{3} \frac{1}{16}(3-x)^2\, dx + \int_{3}^{\infty} 0 . dx$$

$$= \frac{1}{16}\left[\left\{\frac{(3+x)^3}{3}\right\}_{-3}^{-1} + \left\{6x - \frac{2.x^3}{3}\right\}_{-1}^{1} + \left\{\frac{(3-x)^3}{-3}\right\}_{1}^{3}\right]$$

$$= \frac{1}{16}\left[\left(\frac{8}{3}-0\right)+\left(6-\frac{2}{3}\right)-\left(-6+\frac{2}{3}\right)-\left(0-\frac{8}{3}\right)\right]$$

$$= \frac{1}{16}\left[\frac{8}{3}+6-\frac{2}{3}+6-\frac{2}{3}+\frac{8}{3}\right]$$

$$= \frac{1}{16}\left[\frac{16-4+36}{3}\right] = \frac{1}{16}\times\frac{48}{3} = \frac{48}{48} = 1$$

$$\therefore \boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

Hence $f(x)$ is a density function.

Mean of the random variable X is

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \frac{1}{16}\int_{-3}^{-1} x(3+x)^2\,dx + \frac{1}{16}\int_{-1}^{1} x(6-2x^2)\,dx + \frac{1}{16}\int_{1}^{3} x(3-x)^2\,dx$$

$$= \frac{1}{16}\left[\int_{-3}^{-1} x(9+6x+x^2)\,dx + 0 + \int_{1}^{3} x(9+x^2-6x)\,dx\right]$$

$$= \frac{1}{16}\left[\int_{-3}^{-1}(9x+6x^2+x^3)\,dx + \int_{1}^{3}(9x+x^3-6x^2)\,dx\right]$$

$$= \frac{1}{16}\left[\left(\frac{9x^2}{2}+\frac{6x^3}{3}+\frac{x^4}{4}\right)_{-3}^{-1} + \left(\frac{9x^2}{2}+\frac{x^4}{4}-\frac{6x^3}{3}\right)_{1}^{3}\right]$$

$$= \frac{1}{16}\left[\left(\frac{9}{2}-2+\frac{1}{4}-\frac{81}{2}+54-\frac{81}{4}\right)+\left(\frac{81}{2}+\frac{81}{4}-54-\frac{9}{2}-\frac{1}{4}+2\right)\right] = 0$$

Ex. ② A continuous random variable x has

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & \text{for } -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

represents the density, find the mean and standard deviation of x.

Sol^n :- If $f(x)$ is density function, then it satisfies

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{-1} 0.dx + \int_{-1}^{1} \frac{1}{2}(x+1)\,dx + \int_{1}^{\infty} 0.dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right)\right]$$

$$= \frac{1}{2}[2]$$

$$= 1.$$

Hence $f(x)$ is a density function.

Mean of the random variable X is

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{-1} 0.\,dx + \int_{-1}^{1} x.\frac{1}{2}(x+1)\,dx + \int_{1}^{\infty} 0.\,dx$$

$$= \frac{1}{2}\int_{-1}^{1}(x^2+x)\,dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1}{3}+\frac{1}{2}\right)-\left(\frac{-1}{3}+\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}+\frac{1}{2}+\frac{1}{3}-\frac{1}{2}\right]$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} = \frac{1}{3}.$$

Therefore the mean of the random variable X is $\frac{1}{3}$.

$\therefore$ The variance of the random variable X is $Var(X) = E(X^2) - \{E(X)\}^2$

Now $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$= \int_{-1}^{1} x^2.\frac{1}{2}(x+1)\,dx$$

$$E(x^2) = \frac{1}{2}\left\{\int_{-1}^{1} (x^3 + x^2)\,dx\right\}$$

$$= \frac{1}{2}\left[0 + 2\int_0^1 x^2\,dx\right]$$

x^3 is odd function

x^2 is even function

$$= \left(\frac{x^3}{3}\right)_0^1 = \frac{1}{3}.$$

Now $\text{var}(X) = E(x^2) - \{E(x)\}^2$

$$= \frac{1}{3} - \left(\frac{1}{3}\right)^2 = \frac{1}{3} - \frac{1}{9}$$

$$\text{var}(X) = \frac{3-1}{9} = \frac{2}{9}$$

$\therefore$ standard deviation of $X = \frac{\sqrt{2}}{3}$.

If the probability density function

$$f(x) = \begin{cases} kx^2 & \text{if } 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find the ~~efe~~ value 'k' and find the probability between $x = \frac{1}{2}$ and $x = 3/2$.

From the given data, $f(x) = \begin{cases} kx^2, & \text{if } 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$

If $f(x)$ is a density function, then it satisfies $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1.$$

$$\Rightarrow k \cdot \int_{0}^{3} x^3\,dx = 1$$

$$\Rightarrow k \left[\frac{x^4}{4}\right]_0^3 = 1$$

$$\Rightarrow k \left[\frac{81}{4}\right] = 1$$

$$\Rightarrow \boxed{k = \frac{4}{81}}$$

Now, $f(x) = \begin{cases} \frac{4}{81}x^3 & \text{if } 0 \le x \le 3 \\ 0 & \text{, elsewhere} \end{cases}$

(i) $$P\left(\frac{1}{2} \le x \le \frac{3}{2}\right) = \int_{1/2}^{3/2} \frac{4}{81} x^3\,dx$$

$$= \frac{4}{81}\left[\frac{x^4}{4}\right]_{1/2}^{3/2} = \frac{4 \times \frac{1}{4}}{81}\left[x^4\right]_{1/2}^{3/2}$$

$$= \frac{1}{81}\left[\frac{81}{16} - \frac{1}{16}\right]$$

$$= \frac{1}{81} \times \frac{80}{16} = \frac{5}{81} = 0.0617.$$

Let X be a discrete random variable having probability mass function

$$P_X(X) = \begin{cases} 1/2 & \text{if } x = 1 \\ 1/3 & \text{if } x = 2 \\ 1/6 & \text{if } x = 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the third moment of X

Third moment $= E(X^3) = \Sigma x^3 P_X(x)$

$$= \frac{1}{2} \cdot 1^3 + \frac{1}{3} \cdot 2^3 + \frac{1}{6} \cdot 3^3$$

$$= \frac{1}{2} + \frac{8}{3} + \frac{27}{6}$$

$$= \frac{3 + 16 + 27}{6} = \frac{46}{6} = \frac{23}{3}$$

Let X be a discrete random variable with Probability mass function

$$P_X(x) = \begin{cases} 3/4 & \text{if } x = 1 \\ 1/4 & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the third ~~mo~~ central moment of X.

$E(X) = \mu_X = \Sigma x P_X(x) = 1 \times \frac{3}{4} + 2 \times \frac{1}{4} = \frac{5}{4}$

The third central moment of X can be Computed as follows :-

$$E(X - \mu_X)^3 = \Sigma \left(x - \frac{5}{4}\right)^3 P_X(x)$$

$$= \left(1 - \frac{5}{4}\right)^3 \times \frac{3}{4} + \left(2 - \frac{5}{4}\right)^3 \times \frac{1}{4}$$

$$= \left(-1/4\right)^3 \times \frac{3}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4}$$

$$= \left[-\frac{3}{64} + \frac{27}{64}\right]\frac{1}{4} = \frac{\overset{3\,\cancel{6}}{\cancel{24}}}{\underset{32}{\cancel{64}}} \times \frac{1}{\cancel{4}} = \frac{3}{32}$$