Unit - 1 : Probability and Statistics

1. **Introduction To Probability :-**

The words "chance", and "Probability" are quite familiar to us. statements like, "He or She may pass the examination with good marks", "Probability that it may rain today", Convey the sense of uncertainty about the occurence of some event.

2. **Random Experiment :-**

A random experiment is defined an experiment whose outcome can not be determined in advance is a random experiment.

Examples of random experiments are:

1. Tossing a coin.
2. Throwing a die.
3. Aiming a Target.

In all these Cases there are number of possible results which can occur but the actual result that will occur can not be determined in advance.

3. **Sample space :-**

The set of all possible outcomes in a random experiment is called a sample space.

Each element of a sample space is called a <u>sample point</u>.

Examples of sample space and sample points are:

1. In the throwing of a die, the sample point space $S = \{1, 2, 3, 4, 5, 6\}$ $n(s) = 6$ and typical sample points are 2 and 5.

2. In tossing a coin the sample space $S = \{Head, Tail\}$, $n(s) = 2$.

3. When three coins are thrown simultaneously the sample space $S = \{HHH, HHT, THH, HTH, HTT, THT, TTH, TTT\}$

$n(s) = 8$.

4. <u>Event</u> :- Any subset of a sample space is called an event and denoted by E

Ex. 1. In an experiment of tossing of a coin, getting Head is an event or getting Tail is an event.

$E_1 = \{H\}$ or $E_2 = \{T\}$.

⑤ Exhaustive Events :-

Exhaustive means including or considering all elements or aspects.

" The outcomes of a random experiment are Called exhaustive, if these cover all the possible outcomes of the experiment."

In a throw of a coin, the possible outcomes are head and tail i.e. these are two exhaustive cases. In the experiment of rolling a die, the outcomes 1, 2, 3, 4, 5, 6 (six cases) are exhaustive.

⑥ Favourable Events :-

The events which entail the required happening are said to be favourable events.

For example, in a throw of two die, the number of cases favourable to getting a sum 7 is, i.e. (1,6), (6,1), (2,5), (5,2), (3,4), (4,3).

⑦ Mutually exclusive events :-

" Two events are known as mutually exclusive when the occurence of one of them excludes the occurence of the other, i.e. " Two or more events are mutually exclusive or disjoint events if the events cannot occur simultaneously, in other words, occurence of one of the events prevents the occurence of others."

If A and B are any two events defined on a sample space S and $A \cap B = \phi$, i.e. there is no common sample points between them, then the events A and B are said to be mutually exclusive.

e.g. While tossing a coin we either get a head or tail but not both.

8 Equally likely events

" Events are said to be equally likely if there is no reason to expect any one in preference to any other.

e.g. In tossing a fair Coin, the two possible outcomes. Head and Tail are equally likely. Similarly, in a throw of an unbiased die the coming up of 1, 2, 3, ~~4~~4, 5, 6 is equally likely.

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Aspect	Exclusive Events	Equally likely Events
1. Definition	Events that cannot occur simultaneously.	Events, where each outcome has an equal probability.
2. Example	If one event happens, then other cannot.	Each outcome has an equal chance of occuring.
3. Probability	$P(A \cap B) = P(\phi) = 0$, where A and B are exclusive events.	$P(A) = P(B)$ in equally likely events.
4. Coin Toss Example	Getting a Head and Tail on the same coin toss	Getting a Head or a tail on a fair coin toss.
5. Dice Roll Example	Getting a 2 and a 4 on a single roll of a six sided die.	Rolling any number from 1 to 6 on a fair six-sided die.
6. Sample point	Events cannot have ~~same equal~~ common sample point.	Events may have common sample points.

In a rolling a die, sample space

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{2, 4, 6\} \leftarrow$ getting evens

$E_2 = \{1, 3, 5\} \leftarrow$ getting odds

$E_3 = \{2, 3, 5\} \leftarrow$ getting primes

$E_1 \cap E_2 = \phi \Rightarrow P(E_1 \cap E_2) = 0 \Rightarrow E_1$ & E_2 are mutually inclusive

$P(E_1) = P(E_2) = P(E_3) = \frac{3}{6} = \frac{1}{2} \Rightarrow E_1$ & E_2, and E_3 are equally likely events.

9 ## Independent Events :-

Events are said to be independent if the occurence of one of them does not affect the occurence of any of the other.

For, Examples, $P(A \cap B) = P(A) \cdot P(B)$

(i) In a tossing two coins, the appearance of head on one coin does not affect the appearance of the head on the other coin.

10 ## Dependent Events :-

If the happening of one event influence the probability of the happening of the other (these not being mutually exclusive) then the second event is said to be dependent on the first. For examples,

(i) If A draws a card from a full pack and does not replace it, the draw afterward made by B is dependent on the draw made by A.

Probability :-

If there are n exhaustive, mutually exclusive and equally likely outcomes in performing an experiment, of which m are favourable to an event A then the probability p of the event A to occur, is given by

$$p = P(A) = \frac{\text{Number of favourable Cases}}{\text{Number of mutually exclusive and equally likely events, exhaustive events}}$$

$$= \frac{m}{n}$$

Ex. (1) A and B throw alternaterely with a pair of dice. One who first throws a total of nine wins. What are their respective chances of winning if A starts the game?

Solution :- Given that A and B throw alternatively with a pair of dice.

The probability of getting total of nine is

$P\{(6,3),(5,4),(4,5),(3,6)\} = \frac{4}{36} = \frac{1}{9}$

Given that A starts the same

The probability of winning the A is $\frac{1}{9} = \frac{1}{9}$.

Suppose he can't. Then B will throw

The probability of getting total of nine by B is

$$= (\text{not getting A})(\text{getting B})$$

$$= \left(1-\frac{1}{9}\right)\left(\frac{1}{9}\right) = \left(\frac{8}{9}\right)\left(\frac{1}{9}\right)$$

If not the chance will go to given A. If he can get total of nine the probability will be $= (\text{not winning A})(\text{not winning B})(\text{winning A})$

$$= \left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right) = \left(\frac{8}{9}\right)^2\left(\frac{1}{9}\right)$$

The probability of the winning B in the second trial $= (\text{not winning A})(\text{not winning B})(\text{not winning A})(\text{winning B})$

$$= \left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right) = \left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)$$

The probability of A winning

$$= (\text{wining A}) + (\text{not winning A})(\text{not winning B})(\text{winning A})$$

$$= \frac{1}{9} + \left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right) + \left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right) + \cdots$$

$$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \cdots\right]$$

$$= \frac{1}{9}\left[\frac{1}{1-\left(\frac{8}{9}\right)^2}\right] = \frac{1}{9}\,\frac{1}{\frac{81-64}{81}}$$

$$= \frac{1}{\cancel{9}} \times \frac{\cancel{81}\,9}{17} = \frac{9}{17} = 0.529$$

Ex. 2 Find the probability of drawing a card from a well ~~defined~~ shuffled pack such that the drawn card is either a king or queen.

Solution :- A pack containing 52 ~~pack~~ cards of a pack will have 4 kings and 4 queens.

Hence $P(A) = \frac{4}{52}$, $P(B) = \frac{4}{52}$

Hence $P(A \cup B) = P(A) + P(B) = \frac{4}{52} + \frac{4}{52} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$.

is the required result.

Ex. 3 If the probability of the horse A winning the race is $\frac{1}{5}$ and the probability of the horse B winning the same race is $\frac{1}{6}$, what is the probability that one of the horses will win the race?

Solution Probability of winning of the horse A $= \frac{1}{5}$

Probability of winning of the horse B $= \frac{1}{6}$

$$P(A \cup B) = P(A) + P(B) = \frac{1}{5} + \frac{1}{6} = \frac{6+5}{5 \times 6} = \frac{11}{30}.$$

Ex. 4. Two dice are thrown together. Find the probability that the sum of divisible by 2 or 3.

Soln :- Let the event "that the sum is divisible by 2" be denoted by A and the event "that the sum is divisible by 3" be denoted by B

Total number of pairs $= 6 \times 6 = 36$ which are:

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(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

A : (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1) (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6) = 18 pairs.

B : (1,2), (1,5), (2,1), (2,4), (3,3), (3,6), (4,2), (4,5), (5,1), (5,4), (6,3), (6,6) = 12 pairs

$A \cap B$:= (1,5), (2,4), (3,3), (4,2), (5,1), (6,6) = 6 pairs

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{12}{36} - \frac{6}{36}$$

$$= \frac{18+12-6}{36} = \frac{18+6}{36} = \frac{24}{36} = \frac{2}{3}.$$

= 0.667 is the required result.

Examples for Probability

① Three coins are tossed, find the probability of getting at least 2 heads.

Soln. sample space $S = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$

$$p = \frac{\text{no. of the favourable cases}}{\text{Total number of cases}}$$

$$p = \frac{4}{8} = \frac{1}{2}$$

② What is the probability that if a card is drawn at random from an ordinary pack of cards, it is : (i) a red card, (ii) a club (iii) one of the ~~count~~ face cards (jack or queen or king).

Soln:- Number of exhaustive cases = 52

(i) There are 26 red cards & 26 black cards in an ordinary pack.

∴ Favourable Cases = 26 (number of red cards)

∴ Probability of getting a red card $= \frac{26}{52} = \frac{1}{2}$.

(ii) Number of clubs in a pack = 13

∴ Favourable Cases = 13

$\therefore$ Probability of getting a club $= \frac{13}{52} = \frac{1}{4}$.

(iii) There are $4 \times 3 = 12$ court cards in a pack of cards.

$\therefore$ Number of favourable cases $= 12$

Number of exhaustive cases $= 52$

$\therefore$ Probability of getting a ~~court~~ face cards $= \frac{12}{52} = \frac{3}{13}$.

Ex. ③ A bag contains 4 white, 5 red and 7 black balls.

(i) What is the probability that three balls drawn at random are all red?

(ii) What is the probability that one is white, one is red and one is black?

Solⁿ: (i) There are 16 balls in all. Three balls can be drawn in $^{16}C_3$ ways (exhaustive number of ways).

Now, three red balls out 5 can be drawn in $^5C_3$ ways. Hence, the required probability is equal to

$$\frac{^5C_3}{^{16}C_3} = \frac{5 \times 4 \times 3!}{3!\, 2 \times 1} \Big/ \frac{16 \times 15 \times 14 \times 13!}{3 \times 2 \times 1 \times 13!}$$

$$= 10 \times \frac{1}{16 \times 5 \times 7} = \frac{1}{56}.$$

(ii) Next, One white ball out of 4 can be drawn in $^4C_1$ ways, one red ball can be drawn in $^5C_1$ ways and one black ball in $^7C_1$ ways

Hence, the total number of favourable ways

$$= {}^4C_1 \times {}^5C_1 \times {}^7C_1 = 4 \times 5 \times 7 = 140$$

Hence, the required probability $= \frac{140}{{}^{16}C_3}$

$$= \frac{140}{\frac{16 \times 15 \times 14 \times 13!}{3!\,13!}}$$

$$= \frac{1}{4}.$$

Ex. (4) If 3 of 20 tyres in storage are defective and 4 of them are randomly choosen for inspection (i.e., each tyre has the same chance of being selected), what is the probability that only one of the defective tyres will be included?

Solⁿ:- Given that defective tyres are 3

Then non-defective tyres are $= 20 - 3 = 17$.

Selecting of 4 tyres from 20, which contains one defective is ${}^3C_1 \times {}^{17}C_3$ ways

Also the number of possible cases are ${}^{20}C_4$

∴ The required probability $= \frac{{}^3C_1 \times {}^{17}C_3}{{}^{20}C_4}$

The required probability $= \frac{{}^{3}C_{1} \times {}^{17}C_{3}}{{}^{20}C_{4}}$

(Note: Here we are selecting 1 defective tyre from 3 tyres that can be done in ${}^{3}C_{1}$ ways and remaining 3 tyres can be taken from 17 non-defective tyres in ${}^{17}C_{3}$ ways.)

$$= \frac{3 \times \frac{17!}{3!\,14!}}{\frac{20 \times 19 \times 18 \times 17 \times 16!}{4!\,16!}}$$

$$= \frac{\cancel{3} \times 17 \times \cancel{16} \times 15 \times \cancel{14!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{14!}} \Bigg/ \frac{\cancel{20} \times \cancel{19} \times \cancel{18} \times 17}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= \frac{\cancel{17} \times 8 \times \cancel{15}}{\cancel{5} \times 19 \times \cancel{3} \times \cancel{17}}$$

$$= \frac{8}{19}.$$

Ex. 5 What is the probability that a non-leap year contains 53 sundays?

Soln:- A non-leap year consists of 365 days of these, there are 52 complete weeks and 1 extra day. That day may be any one of the 7 days. So already we have 52 sundays. For one more Sunday, the probability that getting a one more sunday is $1/7$.

Hence, the probability that a non-leap year contains 53 sundays is $1/7$.

Ex. 6 Find the probability of drawing a card from a well shuffled pack such that the drawn card is either a kind or queen.

Sol^n:- The events ~~that~~ the card drawn is either a king or queen are mutually exclusive events.

Let the event of the kind card drawn be denoted by A and the queen card drawn be denoted by B. A pack containing 52 cards will have 4 kings and 4 queens.

Hence $P(A) = \frac{4}{52}$, $P(B) = \frac{4}{52}$, $A \cap B = \phi$

$$P(A \cup B) = P(A) + P(B) = \frac{4}{52} + \frac{4}{52} = \frac{1}{13} + \frac{1}{13}$$

$$= \frac{2}{13}$$ is the required result.

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Probability of winning of the horse B $= \frac{1}{6}$

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Ex. 8. Two dice are thrown together. Find the probability that the sum of divisible by 2 or 3.

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Total number of pairs = 6×6 = 36 which are:

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(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

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B: (1,2), (1,5), (2,1), (2,4), (3,3), (3,6), (4,2), (4,5), (5,1), (5,4), (6,3), (6,6). = 12 pairs

The points common to the events A and B are (1,5), (5,1), (2,4), (4,2), (3,3), (6,6) = 6 pairs

Obviously the events A and B are not mutually exclusive. Hence, the probability of the event A or B, i.e. " sum divisible by 2 or 3 "

is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{12}{36} - \frac{6}{36}$

$= \frac{18+12-6}{36} = \frac{30-6}{36} = \frac{24}{36} = \frac{2}{3}.$

$= 0.667$ is the required result.