Q1

```
import numpy as np
def bisection(f,a,b,tol,L,H,C_ocean,C_land):
    if f(a,L,H,C_ocean,C_land)*f(b,L,H,C_ocean,C_land) >= 0:
        print("Bisection method fails.")
        return None
    a_n = a
    b_n = b
    error_metric = f(a,L,H,C_ocean,C_land) - f(b,L,H,C_ocean,C_land)
    while error_metric<=tol:
        m_n = (a_n + b_n)/2
        f_m_n = f(m_n,L,H,C_ocean,C_land)
        if f(a_n,L,H,C_ocean,C_land)*f_m_n < 0:
            a_n = a_n
            b_n = m_n
        elif f(b_n,L,H,C_ocean,C_land)*f_m_n < 0:
            a_n = m_n
            b_n = b_n
        elif f_m_n == 0:
            print("Found exact solution.")
            return m_n
        else:
            print("Bisection method fails.")
            return None
        error_metric = f(a_n,L,H,C_ocean,C_land) -
f(b_n,L,H,C_ocean,C_land)
    return (a_n + b_n)/2
def func(x,L,H,C_ocean,C_land):
  p1  = np.sqrt(H**2+x**2)
  p2  = L-x
  total_cost = p1*C_ocean + p2*C_land

  return total_cost
def my_pipe_builder(C_ocean, C_land, L, H):
  a = 0
  b= L
  tol  = 1e-6
  print("Soltuion ", bisection(func,a,b,tol,L,H,C_ocean,C_land))
# Test case
my_pipe_builder(2,-1.5,1,0.5)
```

```
Found exact solution.
Soltuion  0.2529042306098583
```

Q2

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap



TOL = 1.e-8

def newton(z0, f, fprime, MAX_IT=1000):
    """The Newton-Raphson method applied to f(z).

    Returns the root found, starting with an initial guess, z0, or
False
    if no convergence to tolerance TOL was reached within MAX_IT
iterations.

    """
    z = z0
    for i in range(MAX_IT):
        dz = f(z)/fprime(z)
        if abs(dz) < TOL:
            return z
        z -= dz
    return False
def get_root_index(roots, r):
    """Get the index of r in the list roots.

    If r is not in roots, append it to the list.

    """
    try:
        return np.where(np.isclose(roots, r, atol=TOL))[0][0]
    except IndexError:
        roots.append(r)
        return len(roots) - 1
def newton_fractal(f, fprime, n=200, domain=(-1, 1, -1, 1)):
    """Plot a Newton Fractal by finding the roots of f(z).

    The domain used for the fractal image is the region of the complex
plane
    (xmin, xmax, ymin, ymax) where z = x + iy, discretized into n
values along
    each axis.

    """

    roots = []
    m = np.zeros((n, n))
```

```
    xmin, xmax, ymin, ymax = domain
    for ix, x in enumerate(np.linspace(xmin, xmax, n)):
        for iy, y in enumerate(np.linspace(ymin, ymax, n)):
            z0 = x + y*1j
            r = newton(z0, f, fprime)
            if r is not False:
                ir = get_root_index(roots, r)
                m[iy, ix] = ir
    nroots = len(roots)
    for i in range(len(roots)):
      print("Root_"+str(i+1)+" : ", roots[i])
    # return forth root
    return roots[3]

def f(z):
  return z**4-1
def fprime(z):
  return 4*z**3
iterations = 250
newton_fractal(f, fprime, n=iterations)
```

```
Root_1 :  (-3.2665559226941115e-13+1.0000000000003602j)
Root_2 :  (1.0000000000000022+1.955133178273462e-15j)
Root_3 :  (3.256421917228081e-09-0.999999997807702j)
Root_4 :  (-0.9999999999999997-6.198137121753741e-16j)
```

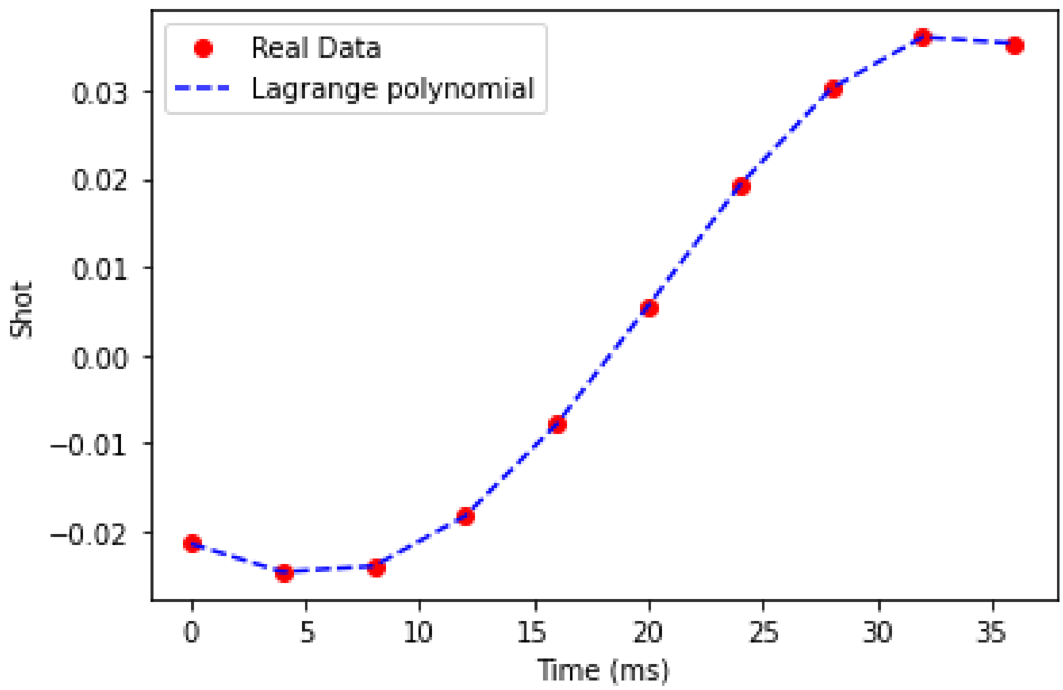
```
(-0.9999999999999997-6.198137121753741e-16j)
```

Q3

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.interpolate import lagrange
def langarange_(x, y, x_p):
  yp = 0
  # Implementing Lagrange Interpolation
  for i in range(len(x)):

      p = 1

      for j in range(len(y)):
          if i != j:
              p = p * (x_p - x[j])/(x[i] - x[j])

      yp = yp + p * y[i]
  return yp
```

```
# def lagrange_main():
x= list(range(0,40,4))
y=[-0.021373, -0.024578,-0.023914, -0.018227,-
0.00781,0.005602,0.019264, 0.030235, 0.036059, 0.035334]
predict = []
for xp in x:
  predict.append(langarange_(x, y, xp))

plt.plot(x,y, 'ro', label='Real Data')
plt.plot(x,predict, '--b', label='Lagrange polynomial')
plt.xlabel("Time (ms)")
plt.ylabel("Shot")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7f18231d6210>
```

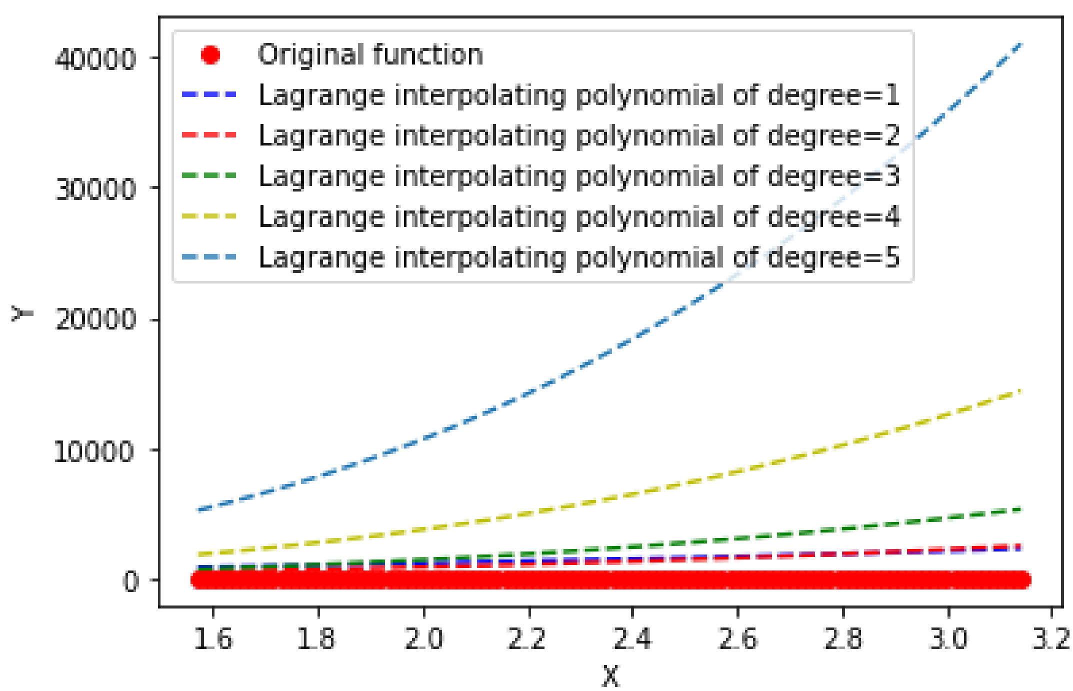


Q4

```
import numpy as np
from scipy import linalg
import matplotlib.pyplot as plt
#pip install PyPolynomial
from pypoly import Polynomial
N=100
X=np.linspace(np.pi/2, np.pi, N)
y = X**2*np.cos(X)

y_out = []
```

```
for order in [1, 2,3,4,5]: # reuired order of equatioin
  equations = np.array([[po ** order] for po in X])
  coefficients =linalg.lstsq(equations, y)
  p = Polynomial(*coefficients)
  out = []
  for x in X:
    out.append(p(x))
  y_out.append(out)
len(y_out)
plt.plot(X,y, 'ro', label='Original function')
plt.plot(X,y_out[0], '--b', label='Lagrange interpolating polynomial
of degree=1')
plt.plot(X,y_out[1], '--r', label='Lagrange interpolating polynomial
of degree=2')
plt.plot(X,y_out[2], '--g', label='Lagrange interpolating polynomial
of degree=3')
plt.plot(X,y_out[3], '--y', label='Lagrange interpolating polynomial
of degree=4')
plt.plot(X,y_out[4], '--', label='Lagrange interpolating polynomial of
degree=5')

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
```

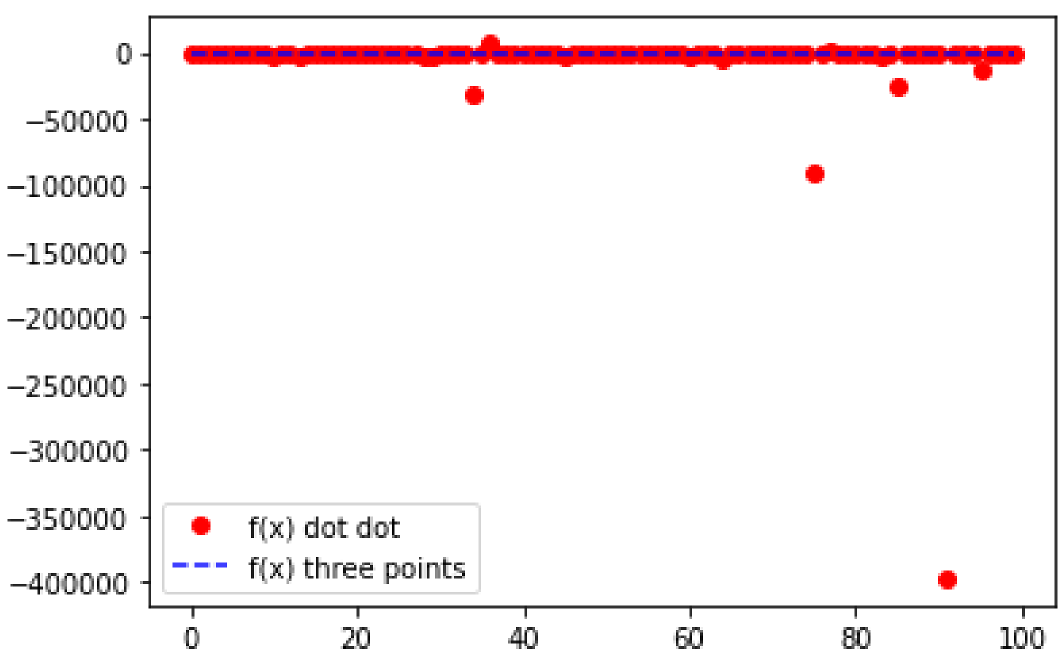
```
<matplotlib.legend.Legend at 0x7f1817a6a6d0>
```



Q5

```
import numpy as np
import matplotlib.pyplot as plt
def Gaussian_Func_dot_dot(x,std, mean):
  fx = 1/(np.sqrt(2*np.pi)*std)
  fx = fx * (x-mean)**2/(std**2) - 1/(std**2)
  fx = fx * np.exp(-(x-mean)**2/x*std)
  return fx
def Gaussian_Func(x,std, mean):
  fx = 1/(np.sqrt(2*np.pi)*std)
  fx = fx * np.exp(-(x-mean)**2/x*std)
  return fx
def Gaussain_three_points(x,std, mean,dx):
  # f(x+dx) - 2f(x) + f(x-dx) / dx2
  return (Gaussian_Func(x+dx,std, mean)-2*Gaussian_Func(x,std, mean)
+Gaussian_Func(x-dx,std, mean))/ dx**2

# intialization
mu, sigma = 0, 0.1 # mean and standard deviation
x = np.random.normal(mu, sigma, 100)
dx=1e-6
y_dot = Gaussian_Func_dot_dot(x,s, mu)
y_3P = Gaussain_three_points(x,s, mu,dx)
plt.plot(list(range(len(x))),y_dot, 'ro', label='f(x) dot dot ')
plt.plot(list(range(len(x))),y_3P, '--b', label='f(x) three points')
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7f4fcb9e9cd0>
```



#Q6

```
def f(x):
  return 1/(1+(x-np.pi)**2)
def midpoint(f, a, b, n):
    h = float(b-a)/n
    result = 0
    for i in range(n):
        result += f((a + h/2.0) + i*h)
    result *= h
    return result

def trapz(f,a,b,N=50):

    x = np.linspace(a,b,N+1) # N+1 points make N subintervals
    y = f(x)
    y_right = y[1:] # right endpoints
    y_left = y[:-1] # left endpoints
    dx = (b - a)/N
    T = (dx/2) * np.sum(y_right + y_left)
    return T
def simps(f,a,b,N=50):

    if N % 2 == 1:
        raise ValueError("N must be an even integer.")
    dx = (b-a)/N
    x = np.linspace(a,b,N+1)
    y = f(x)
    S = dx/3 * np.sum(y[0:-1:2] + 4*y[1::2] + y[2::2])
    return S
def Composite_simps(f,a,b,N, tol):
  exactValue = np.arctan(5-np.pi)+np.arctan(np.pi)
  absError = 0
  for n in range(2, N, 2):
      h = (b-a)/n
      xI0 = f(a) + f(b)
      xI1 = 0
      xI2 = 0
      # Use simpsons composite ruls to eventually approximate the
integral
      for i in range(1,n):
          x = a + i*h
          if i%2 == 0: #even
              xI2 = xI2 + f(x)
          else:
              xI1 = xI1 + f(x)
      xI = h*(xI0 + 2*xI2 + 4*xI1)/3 # our approximation
      absError = abs(xI - exactValue)
      if absError < tol:
          break
```

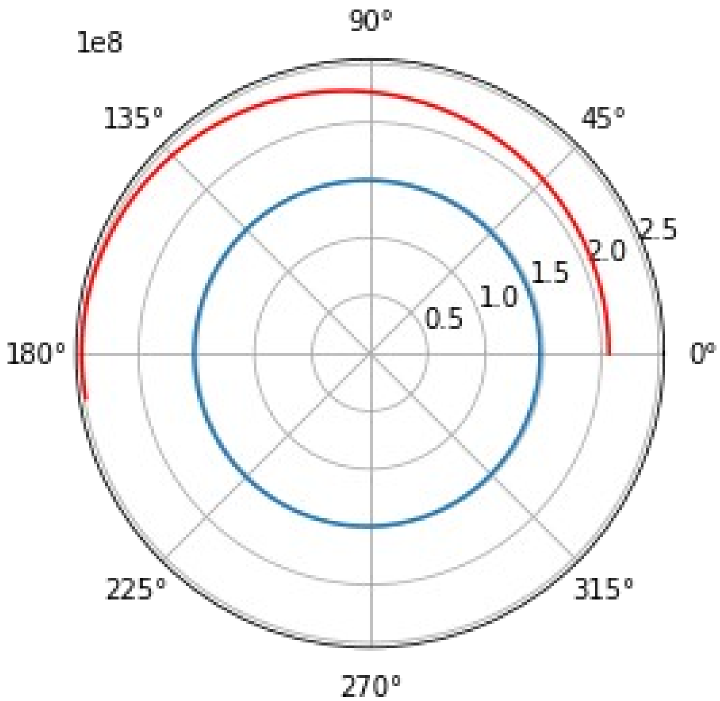
```
  return absError
n=50
a = 0
b = 5
exactValue = np.arctan(5-np.pi)+np.arctan(np.pi)
f_midpoint= midpoint(f, a, b, n)
f_trapz = trapz(f,a,b,n)
f_simps = simps(f,a,b,n)
tol =  1e-4
f_c_simp_error =Composite_simps(f,a,b,n, tol)
print("Mid Point Rule error         :  ", abs(exactValue-f_midpoint) )
print("Trapzoid rule error          :  ", abs(exactValue-f_trapz))
print("Simpson rule error           :  ", abs(exactValue-f_simps))
print("Composite simpson rule error :  ",f_c_simp_error )
```

```
Mid Point Rule error         :   0.0001001957355359906
Trapzoid rule error          :   0.00020042542611919956
Simpson rule error           :   1.809228722393641e-07
Composite simpson rule error :   3.7766006237838212e-06
```

Q7

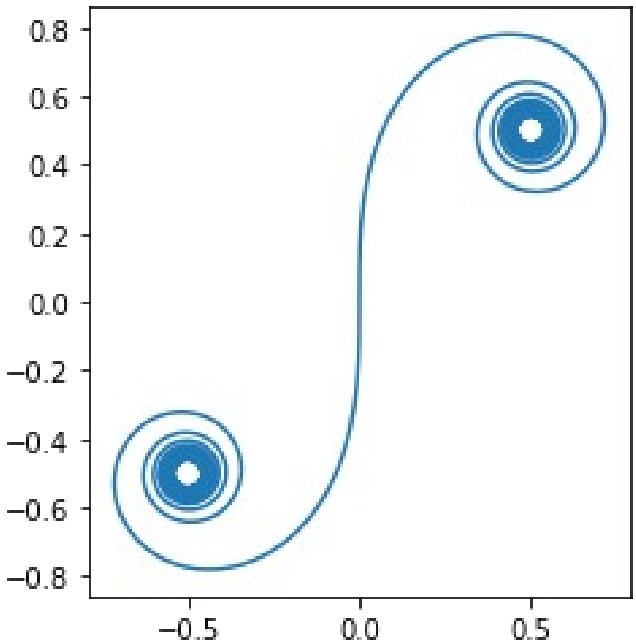
```
import math
from matplotlib import pyplot as plt

def step_one(t, p):
     """
     M = mean anomaly
     M = 2pi * t
         -------
            P
     """
     return (2 * math.pi * t) / p

def step_two(m, e):
     """
     M = mean anomaly
     E = eccentric anomaly
     e = eccentricity

     M = E - esinE
     """
     def M(E): return E - (e * math.sin(E))
     E = 0
     while m > M(E):
          E += 1
     while M(E) > m:
          E -= 0.00001
     return E
```

```python
def step_three(e, E):
    """
    (1 - e)tan^2(theta/2) = (1 + e)tan^2(E/2)
    e = eccentricity
    theta = true anomaly
    E = eccentric anomaly
    """
    def l(theta): return (1-e)*(math.tan(theta/2))**2
    r = (1+e)*(math.tan(E/2))**2
    theta = 0
    while l(theta) < r:
        theta += 0.1
    while r < l(theta):
        theta -= 0.00001
    return [theta, 2*(math.pi - theta) + theta]

def step_four(a, e, E):
    """
    a = semi-major axis
    e = eccentricity
    E = eccentric anomaly

    r = a(1 - ecosE)
    """
    return a * (1 - (e * math.cos(E)))

def calculate(e, t, p, a):
    M = step_one(t, p)
    E = step_two(M, e)
    if list(math.modf(float(t) / p))[0] > 0.5:
        theta = step_three(e, E)[1]
    if list(math.modf(float(t) / p))[0] < 0.5:
        theta = step_three(e, E)[0]
    r = step_four(a, e, E)
    return [theta, r]

e_theta, e_r = [], []
m_theta, m_r = [], []
for x in range(0, 365):
    e_coords = calculate(0.0167, x, 365, 1.496E8)
    e_theta.append(e_coords[0])
    e_r.append(e_coords[1])
    m_coords = calculate(0.0935, x, 687, 2.2792E8)
    m_theta.append(m_coords[0])
    m_r.append(m_coords[1])
plt.polar(e_theta, e_r)
plt.polar(m_theta, m_r, 'r')
plt.show()
```



Q8:

```
from scipy.special import fresnel
from scipy import linspace
import matplotlib.pyplot as plt

t = linspace(-8, 8, 1000)
s, c = fresnel(t)

plt.plot(s, c)
plt.axes().set_aspect("equal")
plt.show()
#  S is smoother

/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5:
DeprecationWarning: scipy.linspace is deprecated and will be removed
in SciPy 2.0.0, use numpy.linspace instead
  """
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:9:
MatplotlibDeprecationWarning: Adding an axes using the same arguments
as a previous axes currently reuses the earlier instance.  In a future
version, a new instance will always be created and returned.
Meanwhile, this warning can be suppressed, and the future behavior
ensured, by passing a unique label to each axes instance.
  if __name__ == '__main__':
```

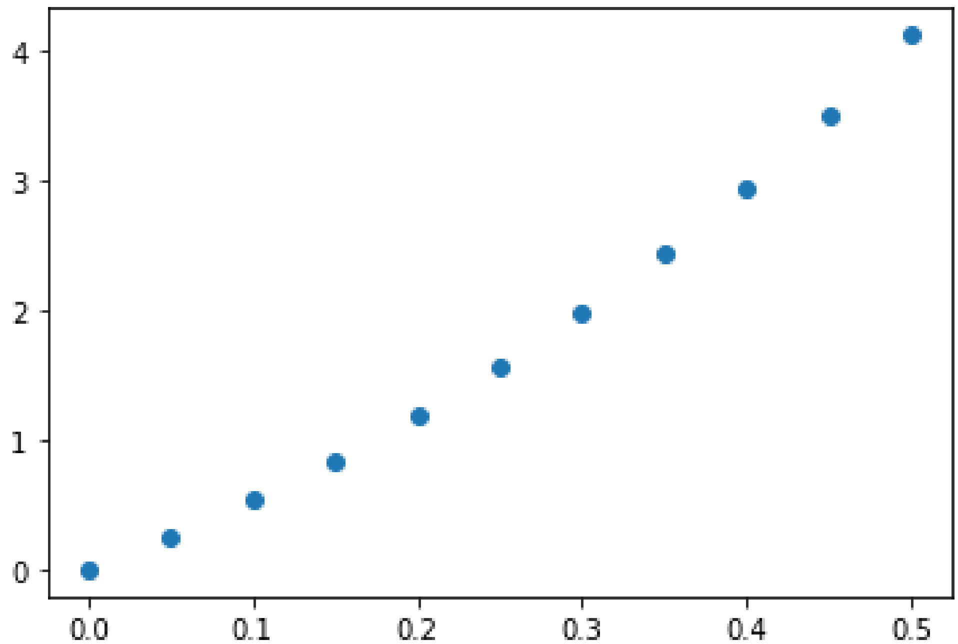


Q9

```
from scipy.optimize import curve_fit
from matplotlib import pyplot
from numpy import arange
def objective(strain, a,b):
  # integral of given form: stress = (a*strain)/(1-b*strain)
  return a*strain/(1-b*strain)
x = [0,
0.05 ,
0.10 ,
0.15 ,
0.20 ,
0.25 ,
0.30 ,
0.35 ,
0.40 ,
0.45 ,
0.50 ]
y= [0, 0.252,0.531,0.840,1.184,1.558,1.975,2.444,2.943, 3.500,4.115]

popt, _ = curve_fit(objective, x, y)
# summarize the parameter values

a, b= popt
print('y = %.5f * x + %.5f' % (a, b))
# plot input vs output
pyplot.scatter(x, y)
# define a sequence of inputs between the smallest and largest known
```

```
inputs
x_line = arange(min(x), max(x), 1)
# calculate the output for the range
y_line = objective(x_line, a, b)
# create a line plot for the mapping function
pyplot.plot(x_line, y_line, '--', color='red')
pyplot.show()
```

```
y = 5.03745 * x + 0.78037
```

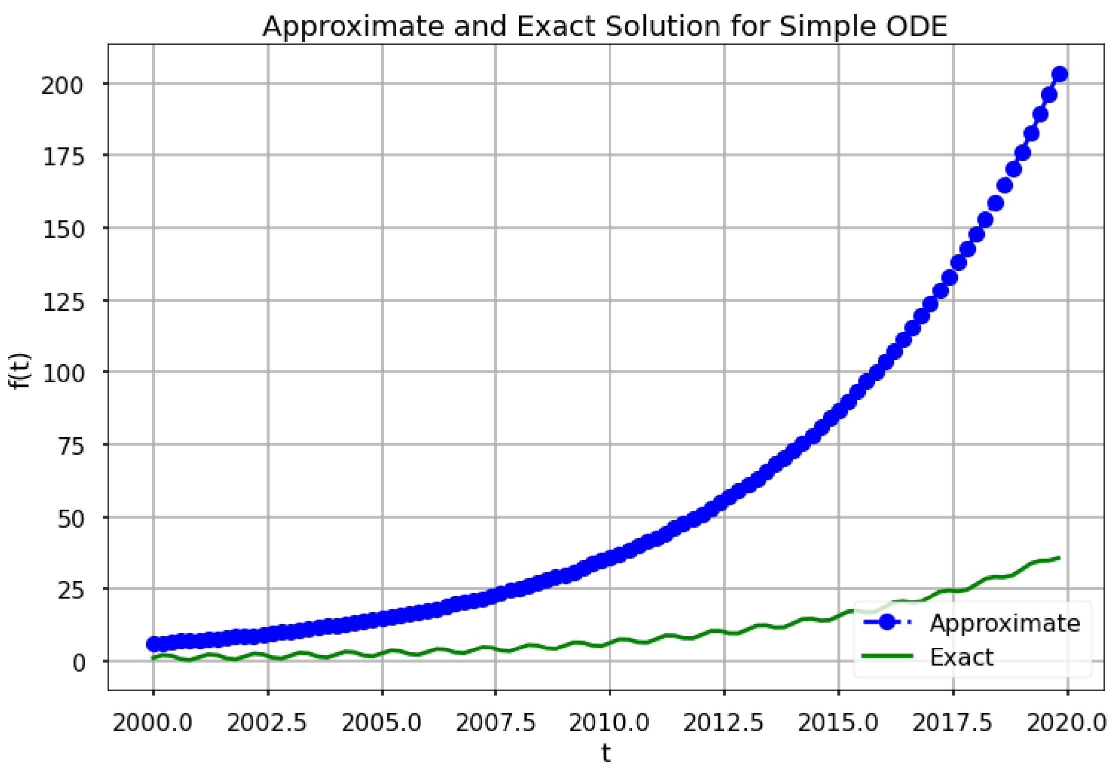


Q10

Eurler forward method

```
import numpy as np
import matplotlib.pyplot as plt

plt.style.use('seaborn-poster')
%matplotlib inline

# Define parameters
f = lambda y, t: 0.2*y- 0.01*y*2 + np.sin(2*np.pi*t) # ODE
h = 0.2 # Step size
t = np.arange(2000, 2020, h) # Numerical grid
s0 = 6 # Initial Condition

# Explicit Euler Method
s = np.zeros(len(t))
s[0] = s0
```

```
for i in range(0, len(t) - 1):
    s[i + 1] = s[i] + h*f(s[i], t[i])

plt.figure(figsize = (12, 8))
plt.plot(t, s, 'bo--', label='Approximate')
plt.plot(t, f(s,t), 'g', label='Exact')
plt.title('Approximate and Exact Solution \
for Simple ODE')
plt.xlabel('t')
plt.ylabel('f(t)')
plt.grid()
plt.legend(loc='lower right')
plt.show()
```



Euler Backward

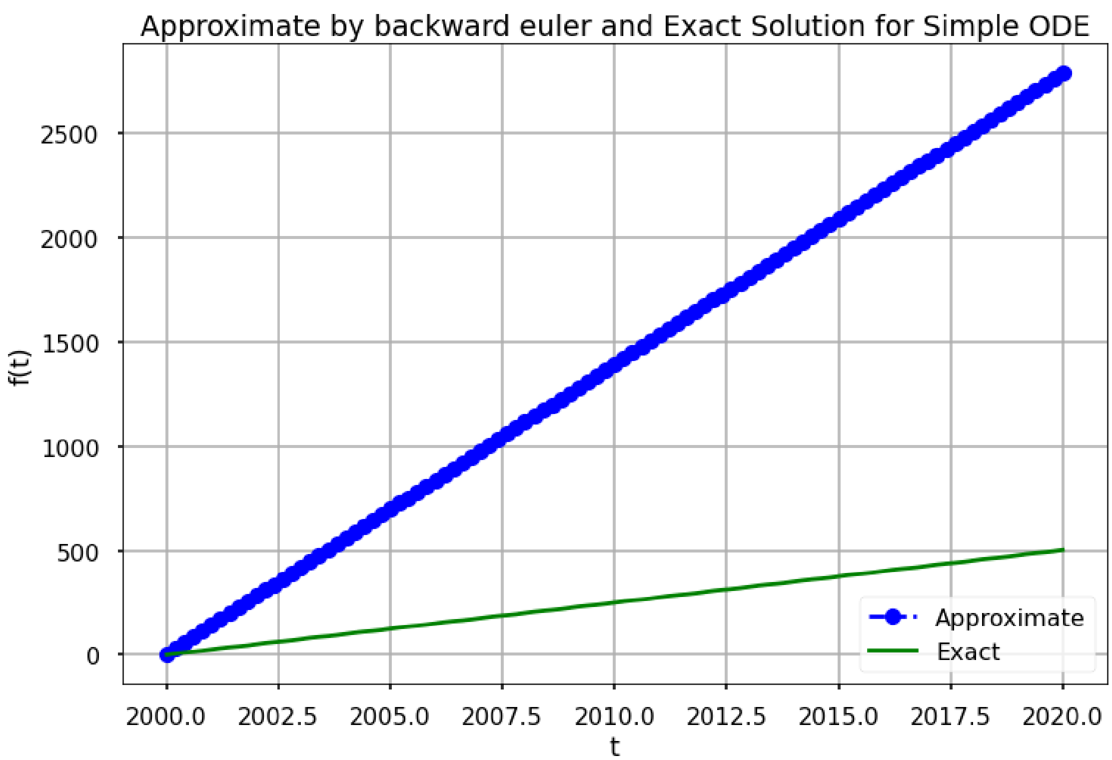
```
def backgward_euler(f_prime, y_0, endpoints, h):
    n = int((endpoints[-1] - endpoints[0])/h)
    x = endpoints[0]
    y = y_0
    x_out, y_out = np.array([x]), np.array([y])
    for i in range(n):
        y_prime = f_prime(x, y)/(1 + h * 8)
        y += h * y_prime
        x += h
        x_out = np.append(x_out, x)
```

```python
        y_out = np.append(y_out, y)

    return x_out, y_out
f = lambda y, t: 0.2*y- 0.01*y*2 + np.sin(2*np.pi*t) # ODE
h = 0.2 # Step size
t = np.arange(2000, 2020, h) # Numerical grid
s0 = 6 # Initial Condition

t, s = backgward_euler(f, s0, [2000,2020], h)


plt.figure(figsize = (12, 8))
plt.plot(t, s, 'bo--', label='Approximate')
plt.plot(t, f(s,t), 'g', label='Exact')
plt.title('Approximate by backward euler and Exact Solution \
for Simple ODE')
plt.xlabel('t')
plt.ylabel('f(t)')
plt.grid()
plt.legend(loc='lower right')
plt.show()
```



4th Order Runge Kutta Method

```
def rk4(f,x0,y0,xn,n):
    n = int((xn-x0)/h)
    x = []
    y= []
    print('\n--------SOLUTION--------')
    print('-------------------------')
    print('x0\ty0\tyn')
    print('-------------------------')
    for i in range(n):
        x.append(x0)
        y.append(y0)
        k1 = h * (f(x0, y0))
        k2 = h * (f((x0+h/2), (y0+k1/2)))
        k3 = h * (f((x0+h/2), (y0+k2/2)))
        k4 = h * (f((x0+h), (y0+k3)))
        k = (k1+2*k2+2*k3+k4)/6
        yn = y0 + k
        print('%.4f\t%.4f\t%.4f'% (x0,y0,yn) )
        print('-------------------------')
        y0 = yn
        x0 = x0+h

    return x,y
f = lambda y, t: 0.2*y- 0.01*y*2 + np.sin(2*np.pi*t) # ODE
h = 0.2 # Step size
t = np.arange(2000, 2020, h) # Numerical grid
s0 = 6 # Initial Condition

t, s = rk4(f,2000,6,2020,h)


plt.figure(figsize = (12, 8))
plt.plot(t, s, 'bo--', label='Approximate')
plt.title('Approximate by 4th Order Runge Kutta Method and Exact
Solution \
for Simple ODE')
plt.xlabel('t')
plt.ylabel('f(t)')
plt.grid()
plt.legend(loc='lower right')
plt.show()
```

```
--------SOLUTION--------
-------------------------
x0    y0    yn
-------------------------
2000.0000  6.0000     78.0056
```

2000.2000	78.0056	150.0391
2000.4000	150.0391	222.1586
2000.6000	222.1586	294.3670
2000.8000	294.3670	366.4768
2001.0000	366.4768	438.5154
2001.2000	438.5154	510.5312
2001.4000	510.5312	582.5402
2001.6000	582.5402	654.5471
2001.8000	654.5471	726.5534
2002.0000	726.5534	798.5595
2002.2000	798.5595	870.5657
2002.4000	870.5657	942.5719
2002.6000	942.5719	1014.5782
2002.8000	1014.5782	1086.5847
2003.0000	1086.5847	1158.5913
2003.2000	1158.5913	1230.5980
2003.4000	1230.5980	1302.6050
2003.6000	1302.6050	1374.6121
2003.8000	1374.6121	1446.6195
2004.0000	1446.6195	1518.6272
2004.2000	1518.6272	1590.6352
2004.4000	1590.6352	1662.6436
2004.6000	1662.6436	1734.6525
2004.8000	1734.6525	1806.6620
2005.0000	1806.6620	1878.6722

2005.2000	1878.6722	1950.6833
2005.4000	1950.6833	2022.6957
2005.6000	2022.6957	2094.7098
2005.8000	2094.7098	2166.7263
2006.0000	2166.7263	2238.7468
2006.2000	2238.7468	2310.7743
2006.4000	2310.7743	2382.8162
2006.6000	2382.8162	2454.8985
2006.8000	2454.8985	2527.1370
2007.0000	2527.1370	2599.5172
2007.2000	2599.5172	2671.6655
2007.4000	2671.6655	2743.7448
2007.6000	2743.7448	2815.8288
2007.8000	2815.8288	2888.0001
2008.0000	2888.0001	2960.4123
2008.2000	2960.4123	3032.6541
2008.4000	3032.6541	3104.7694
2008.6000	3104.7694	3176.9155
2008.8000	3176.9155	3249.2923
2009.0000	3249.2923	3321.6292
2009.2000	3321.6292	3393.7781
2009.4000	3393.7781	3465.9780
2009.6000	3465.9780	3538.4340
2009.8000	3538.4340	3610.7035
2010.0000	3610.7035	3682.8817

```
--------------------------
2010.2000  3682.8817  3755.2845
--------------------------
2010.4000  3755.2845  3827.6615
--------------------------
2010.6000  3827.6615  3899.8581
--------------------------
2010.8000  3899.8581  3972.2618
--------------------------
2011.0000  3972.2618  4044.6672
--------------------------
2011.2000  4044.6672  4116.8903
--------------------------
2011.4000  4116.8903  4189.3538
--------------------------
2011.6000  4189.3538  4261.7230
--------------------------
2011.8000  4261.7230  4334.0062
--------------------------
2012.0000  4334.0062  4406.5458
--------------------------
2012.2000  4406.5458  4478.8242
--------------------------
2012.4000  4478.8242  4551.2681
--------------------------
2012.6000  4551.2681  4623.7136
--------------------------
2012.8000  4623.7136  4696.0460
--------------------------
2013.0000  4696.0460  4768.6084
--------------------------
2013.2000  4768.6084  4840.9126
--------------------------
2013.4000  4840.9126  4913.4675
--------------------------
2013.6000  4913.4675  4985.8278
--------------------------
2013.8000  4985.8278  5058.3395
--------------------------
2014.0000  5058.3395  5130.7862
--------------------------
2014.2000  5130.7862  5203.2803
--------------------------
2014.4000  5203.2803  5275.7713
--------------------------
2014.6000  5275.7713  5348.2713
--------------------------
2014.8000  5348.2713  5420.7787
--------------------------
2015.0000  5420.7787  5493.3046
```

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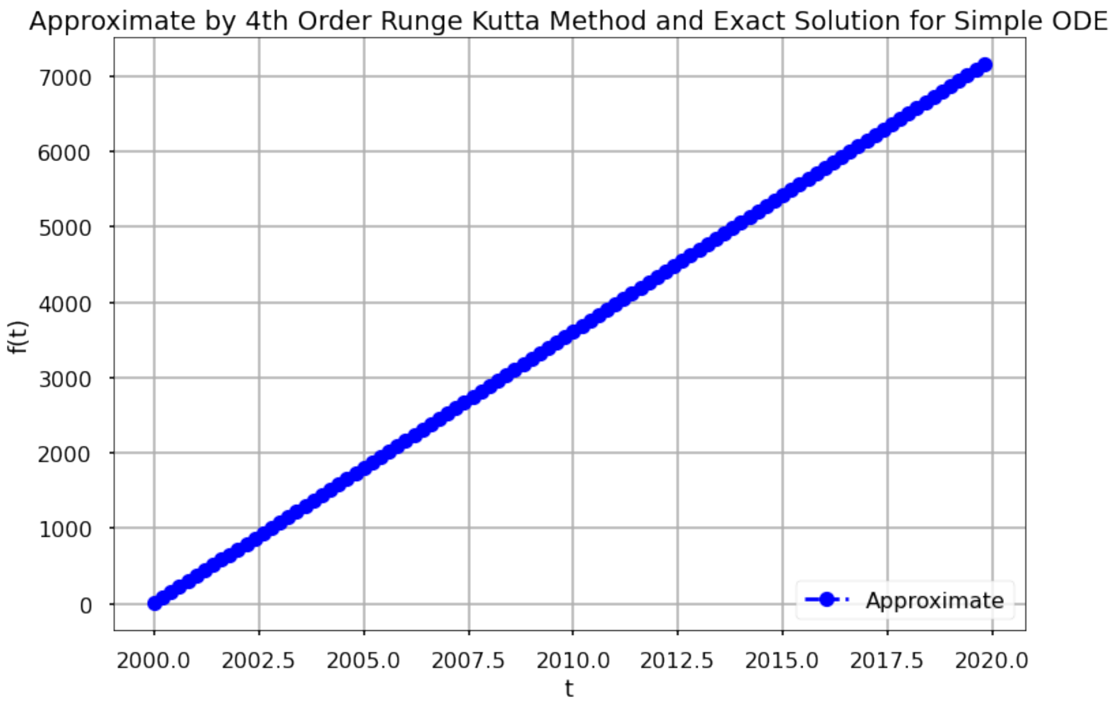
-------------------------
2015.2000  5493.3046  5565.8065
-------------------------
2015.4000  5565.8065  5638.3692
-------------------------
2015.6000  5638.3692  5710.8467
-------------------------
2015.8000  5710.8467  5783.4511
-------------------------
2016.0000  5783.4511  5855.8993
-------------------------
2016.2000  5855.8993  5928.5512
-------------------------
2016.4000  5928.5512  6000.9956
-------------------------
2016.6000  6000.9956  6073.6690
-------------------------
2016.8000  6073.6690  6146.1959
-------------------------
2017.0000  6146.1959  6218.8036
-------------------------
2017.2000  6218.8036  6291.4365
-------------------------
2017.4000  6291.4365  6363.9404
-------------------------
2017.6000  6363.9404  6436.6498
-------------------------
2017.8000  6436.6498  6509.2187
-------------------------
2018.0000  6509.2187  6581.8475
-------------------------
2018.2000  6581.8475  6654.5501
-------------------------
2018.4000  6654.5501  6727.0944
-------------------------
2018.6000  6727.0944  6799.7888
-------------------------
2018.8000  6799.7888  6872.4787
-------------------------
2019.0000  6872.4787  6945.0399
-------------------------
2019.2000  6945.0399  7017.7716
-------------------------
2019.4000  7017.7716  7090.4765
-------------------------
2019.6000  7090.4765  7163.0656
-------------------------
2019.8000  7163.0656  7235.8054
-------------------------
```



Trapezoidal

```
def trapz(f,a,b,t,N=50):

    x = np.linspace(a,b,N) # N+1 points make N subintervals
    y = f(x,t)
    y_right = y[1:] # right endpoints
    y_left = y[:-1] # left endpoints
    dx = (b - a)/N
    T = (dx/2) * np.sum(y_right + y_left)
    return x,y
f = lambda y, t: 0.2*y- 0.01*y*2 + np.sin(2*np.pi*t) # ODE
h = 0.2 # Step size
t = np.arange(2000, 2020, h) # Numerical grid
s0 = 6 # Initial Condition

t, s = trapz(f, 2000, 2020, t, int((2020-2000)/h))


plt.figure(figsize = (12, 8))
plt.plot(t, s, 'bo--', label='Approximate')
plt.plot(t, f(s,t), 'g', label='Exact')
plt.title('Approximate by trapezoid and Exact Solution \
for Simple ODE')
plt.xlabel('t')
plt.ylabel('f(t)')
plt.grid()
```

```
plt.legend(loc='lower right')
plt.show()
```