Decentralized Optimization

▼ 2022 — Gong — Push-Pull Based Distributed Primal-Dual Algorithm for Coupled Constrained Convex Optimization in Multi-Agent Networks

2022_Gong_Push_Pull_Based_Distributed_Primal_Dual_Algorithm_for.pdf

💡 Primal-Dual

- **Постановка задачи:**

$$\min_{x \in X} \sum_{i=1}^{m} f_i(x)$$
$$\text{s.t.} \sum_{i=1}^{m} h_i(x) \in \mathbb{K}$$

Граф: directed, time-varying

Множество минимизации: convex, compact

- **Вычислительный эксперимент:**

$$f_i(x) = a_i^\top x + b_i + c_i \log(1 + e^{d_i^\top x}),$$
$$h_i^1(x) = \alpha_i \|x\|^2 + \beta_i,$$
$$h_i^2(x) = \gamma_i^\top x + \delta_i,$$
$$X = [-3, 3] \times [-3, 3]$$

▼ 2020 — Carli — Distributed Alternating Direction Method of Multipliers for Linearly Constrained Optimization Over a Network

10.1109@LCSYS.2019.2923078.pdf

💡 ADMM

- **Постановка задачи:**

$$\min_{x_n \in X_n} \sum_{n=1}^{N} f_n(x_n)$$
$$\text{s.t.} \sum_{n=1}^{N} A_n x_n = b$$

Граф: undirected, static

Множество минимизации: convex, compact

- **Вычислительный эксперимент:**

$$\min_{x \in X} x^\top C x + q^\top x$$
$$\text{s.t.} Ax = b$$

▼ 2018 — Alghunaim — Dual Coupled Diffusion for Distributed Optimization with Affine Constraints

2018_Alghunaim_Dual_Coupled_Diffusion_for_Distributed_Optimization.pdf

💡 Dual with sub-networks

- **Постановка задачи:**

$$\min_{w_1,\dots,w_K} \sum_{k=1}^{K} J_k(w_k)$$
$$\text{s.t.} \sum_{k \in C_e} B_{e,k} w_k = b_e, \quad \forall e = 1, \dots, E$$

Граф: undirected, each sub-network C_e is connected

Множество минимизации: —

- **Вычислительный эксперимент:**

$$\min_{w_1,\dots,w_K} \sum_{k=1}^{K} (w_k^\top R_k w_k + r_k^\top w_k)$$
$$\text{s.t.} \sum_{k \in N_e} B_{e,k} w_k = b_e, \quad \forall e = 1, \dots, E$$

▼ 2019 — Alghunaim — A Proximal Diffusion Strategy for Multi-Agent Optimization with Sparse Affine Constraints

10.1109@TAC.2019.2960265.pdf

💡 Proximal primal-dual with sub-networks

- **Постановка задачи:**

$$\min_{w_1,\dots,w_K} \sum_{k=1}^{K} J_k(w_k) + R_k(w_k)$$
$$\text{s.t.} \sum_{k \in C_e} B_{e,k} w_k = b_e, \quad \forall e = 1, \dots, E$$

$J_k(\cdot) : \mathbb{R}^{Q_k} \to \mathbb{R}$ is a smooth function, while $R_k(\cdot) : \mathbb{R}^{Q_k} \to \mathbb{R} \cup \{+\infty\}$ is a convex function possibly non-smooth.

Граф: undirected, each sub-network C_e is connected

Множество минимизации: —

- **Вычислительный эксперимент:**

$$\min_{w_1,\dots,w_K} \sum_{k=1}^{K} (w_k^\top R_k w_k + r_k^\top w_k) + \mathcal{I}_k(w_k)$$
$$\text{s.t.} \sum_{k \in N_e} B_{e,k} w_k = b_e, \quad \forall e = 1, \dots, E$$

$\mathcal{I}_k(\cdot) : \mathbb{R}^{Q_k} \to \mathbb{R} \cup \{+\infty\}$ is an indicator of a cube $[-1, 1] \times \ldots \times [-1, 1]$.

▼ 2022 — Huang — Distributed Event-Triggered Algorithm for Convex Optimization with Coupled Constraints

2210.14415.pdf

💡 Primal-dual with event-triggered communication

❓ **Event-triggered:** each agent only communicates with the neighbors at the event-triggering times determined by some triggering rules

✅ In contrast to the distributed algorithm given in **2019 — Liang — Distributed Smooth Convex Optimization with Coupled Constraints** that includes two gradient pairs and requires twice communications in each iteration, the proposed algorithm only involves one gradient pair and only need once communication in each iteration. In addition, we also introduce the event-triggered mechanism and therefore our algorithm can significantly reduce communication overhead than that of **Liang**.

- **Постановка задачи:**

$$\begin{aligned}\min_{x_i\in\Omega_i}&\sum_{i=1}^{N}f_i(x_i)\\ \text{s.t.}&\sum_{i=1}^{N}g_i(x_i)\leq 0\\ &\sum_{i=1}^{N}h_i(x_i)=0\end{aligned}$$

  $f_i(x_i)$ are just convex

  $h_i(x_i)$ are affine and $g_i(x_i)$ are nonlinear

  Граф: undirected, connected

  Множество минимизации: closed, convex

- **Вычислительный эксперимент:**

$$\begin{aligned}f_i(x_i)&=a_ix_i^2+b_ix_i+c_i\log(1+e^{d_ix_i})\\ g_i(x_i)&=\pi_ix_i^2+\zeta_i\\ h_i(x_i)&=\gamma_ix_i+\delta_i\end{aligned}$$