Problem Set 7 — Math 758

Spring 2012 — Jim Fowler

The exercises below should be handed in on Monday. The first four questions are "easy" once we've discussed the "signature" of a manifold; the last two are concrete examples.

Problem 7.1 (Euler characteristic)

Let M be a closed, simply-connected, oriented 4-manifold; show that $|\sigma(M)| \leq |\chi(M)|$.

Problem 7.2 (Does it bound?)

Is there a compact oriented manifold M^5 so that $\partial M = \mathbb{C}P^2$?

Problem 7.3 (Odd-dimensional signature)

Is $(\mathbb{C}P^2)^{17}$ homeomorphic to a product of odd-dimensional manifolds?

Problem 7.4 (Splitting)

Can you find a smoothly embedded submanifold N^3 of $\mathbb{C}P^2$ so that $\mathbb{C}P^2 - N^3$ has two components? Can you arrange it so that each of those two components have zero signature?

Problem 7.5 (Connected sum)

Suppose M^4 is a connected, orientable, closed 4-manifold with $H^2(M) = \mathbb{Z}^2$, generated by α and β so that

$$\begin{aligned} \alpha \smile \alpha &= [M], \\ \alpha \smile \beta &= [M], \\ \beta \smile \beta &= 0. \end{aligned}$$

Is it possible that there are manifolds N_1 and N_2 so that $N_1 \# N_2 = M$? If not, explain why not; if so, exhibit such manifolds.

Problem 7.6 (Linking number redux)

Recall that $S^3 = S^1 \star S^1$, where $\star$ denotes the "join" operation. What is the linking number (up to sign) of the left and right hand S^1 and S^1 in the join?