Lecture 11: Series

Math 153 Section 57

Wednesday October 22, 2008

Following chapters 12.1 and 12.2.

Sigma notation

Write

$$\sum_{n=0}^{N} f_n$$

or

$$\sum_{n=a}^{b} f_n$$

Draw an analogy with integration.

Some properties:

$$\sum_{n=a}^{b} (\alpha f_n + \beta g_n) = \alpha \sum_{n=a}^{b} f_n + \beta \sum_{n=a}^{b} g_n$$

and

$$\sum_{n=a}^{b} f_n + \sum_{n=b+1}^{c} f_n = \sum_{n=a}^{c} f_n$$

Classic example: $\sum_{n=1}^{b} n = (b)(b+1)/2$

Infinite series

Consider a series $\sum_{k=0}^{\infty} a_n$.

A partial sum is

$$s_n = \sum_{k=0}^{n} a_n.$$

If $\lim_{n\to\infty} s_n = L$, then we write

$$\sum_{k=0}^{\infty} a_k = L$$

We call L the sum of the series.

This is not the same as "adding up all the terms in the sequence."

Example

The series $\sum_{n=1}^{\infty} 1$ diverges.

The series $\sum_{n=1}^{\infty} 1/2^n$ converges to 1 (show the picture). Mention Zeno's paradox, and the Singularity.

The series $\sum_{n=1}^{\infty}(-1)^n$ diverges. Thompson's Lamp (every interval, the lamp switches on or off).

The most important example

Geometric series.

If $-1 < x < 1$, then $\sum_{n=1}^{\infty} x^n = 1/(1-x)$. For other values of x, the series diverges.

Proof: compute $s_k = \sum_{n=1}^{k} x^n$. Multiply by $(1-x)$ to get $(1-x)s_k = 1 - x^{k+1}$, so

$$s_k = \frac{1-x^{k+1}}{1-x}$$

And take the limt as $k \to \infty$.

Application

$\sum_{n=1}^{\infty} \frac{9}{10^n} = 1$.

Wrong applications

$\sum_{n=0}^{\infty} 2^n = -1$? Uhm, no.

Theorems

Sum of convergent series are convergent.

Products of convergent series by a constant are convergent.

Simplest criterion

If $\sum_{k=0}^{\infty} a_k$ converges, then $\lim a_k = 0$.