# Homework 15

Due Monday, December 1, 2008

**(a)** Recall our techniques: integration by parts, two half-angle formulas for exchanging $\sin^2 x$ or $\cos^2 x$ for something involving $\cos(2x)$, and the Pythagorean theorem for trading $\sin^2 x$ for $\cos^2 x$ (and vice versa). Use these techniques to find:

$$\int \cos^5 x \, \sin^5 x \, dx \quad \text{and} \quad \int \cos^4 x \, \sin^4 x \, dx$$

**(b)** Use the technique of partial fractions to evaluate the following integrals:

**(i)** $\displaystyle\int \frac{x+27}{x^2-9}\, dx.$

**(ii)** $\displaystyle\int \frac{8x-36}{(x-5)^2}\, dx.$

**(iii)** $\displaystyle\int \frac{6x^2+20x+9}{x(x^2+2x+1)}\, dx.$

**(iv)** $\displaystyle\int \frac{2x^3-7x^2+6x-21}{(x+1)(x-2)(x-3)}\, dx.$

**(v)** $\displaystyle\int \frac{x^2-3x+46}{(x+3)\left((x-1)^2+16\right)}\, dx.$

**(c)** Find a function $f : \mathbb{R} \to \mathbb{R}$ so that $f'(x) = f(x) \cdot (1 - f(x))$. *Hint:* Use partial fractions! Divide both sides of $f'(x) = f(x) \cdot (1 - f(x))$ by $f(x) \cdot (1 - f(x))$ and integrate to discover

$$\int \frac{f'(x)}{f(x)\,(1-f(x))}\, dx = \int 1\, dx = x + C.$$

Make the substitution $u = f(x)$, evaluate the integral on the left hand side, and then solve for $f(x)$.