Problem Set 1 — Piecewise-Linear Topology

Summer 2010 — Jim Fowler

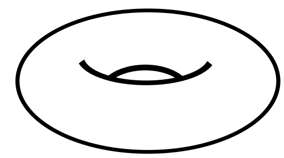
Problem Set 1 is *intentionally* vague: I want you to think a bit about how you would make these notions precise. Problems marked with a • should be written up.

Problem 1. Build a circle S^1 by gluing some line segments together along their vertices. How many different ways are there of doing this?

Problem 2. Build a sphere S^2 by gluing triangles together along their boundaries. How are the various ways of doing this related to each other?

Definition. An object built by gluing together triangles is called a *simplicial complex*.

Problem 3. Build a torus T^2 by gluing triangles together.



a torus

Definition. A function $f : K \to L$ sending

- vertices to vertices,
- edges to either edges or vertices, and
- triangles to triangles, or
  to edges, or
  to vertices

is called a *simplicial map*.

• **Problem 4.** Find a simplicial map $f : T^2 \to S^1$ so that, for every vertex $x \in S^1$, the preimage $f^{-1}(x)$ is a circle.

• **Problem 5.** Let $k \in \mathbb{N}$. Find a simplicial map $f : T^2 \to T^2$ so that, for every point $x \in T^2$, the preimage $f^{-1}(x)$ consists of k points? And consider: what does this even mean when x is not a vertex?

Remark. You might be worried that, because there are different ways of building T^2 out of triangles, the notion of "simplicial map" is ill-defined. You're right: we'll have to fix that. Nevertheless…

• **Problem 6.** Find a simplicial map $f : T^2 \to S^2$ which doesn't crush any edges (i.e., edges are sent to edges, not to vertices).

• **Problem 7.** Find a simplicial map $f : S^2 \to T^2$ which doesn't crush any edges.

Problem 8. Suppose $f : S^1 \to S^2$ is an injective simplicial map (i.e., distinct simplexes are sent to distinct simplexes). Does the image of f necessarily separate S^2 into two pieces?

Problem 9. Suppose $f : S^1 \to T^2$ is an injective simplicial map. Into how many pieces can the image of f separate T^2?

Problem 10. Do there exist simplicial maps $f : T^2 \to S^2$ and $g : S^2 \to T^2$ which are inverses of each other? If not, why not?