Problem Set 1 — Math 765

Winter 2011 — Jim Fowler

The only way to learn the game is to play the game. The following represents a *lower bound* on the number of exercises you should be doing; the textbook is full of great exercises, so I encourage you to do as many as possible. *The exercises below should be handed in on Monday, January 10, 2011.*

Problem 1.1

For each of the following topological spaces, determine whether or not they can be given the structure of a smooth manifold, and, if so, explain how.

(a) The disjoint union of uncountably many copies of $\mathbb{R}$,

(b) Two copies of $\mathbb{R}$ with all non-zero points identified (I mean, x in the first copy is identified witih x in the second copy—in other words, the "line with two origins").

(c) $\mathbb{R}P^n$,

(d) $\mathbb{C}P^n$,

(e) $T^n = \overbrace{S^1 \times \cdots \times S^1}^{n \text{ times}}$.

Problem 1.2 (Lee 2–10)

(a) Show that the quotient map $\pi : \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{C}P^n$ is smooth.

(b) Show that $\mathbb{C}P^1$ is diffeomorphic to S^2.

Problem 1.3

For every pair of positive integers n and m, find infinitely many distinct nonconstant smooth maps from $\mathbb{C}P^n$ to $\mathbb{C}P^m$.

Problem 1.4 (Lee 2-11)

Let G be a connected Lie group, and let $U \subset G$ be any neighborhood of the identity. Show that every $g \in G$ can be written as a finite product of elements of U.

Problem 1.5 (Lee 3–1)

Suppose M and N are smooth manifolds with M connected, and $F : M \to N$ is a smooth map such that $F_\star : T_pM \to T_{F(p)}N$ is the zero map for each $p \in M$. Show that F is a constant map.

Problem 1.6 (Lee 3–3)

If a nonempty smooth n-manifold is diffeomorphic to an m-manifold, prove that $n = m$.