Lecture 41: Taylor series

Math 660—Jim Fowler

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Taylor series

If $f(z)$ is analytic in the region $\Omega \ni z_0$, then

$$f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n$$

on the largest open disk of center z_0 contained in Ω.

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What is the radius of converegence?

Composition

Find the Taylor series for a composition $f(g(z))$, given the Taylor series for $f(z)$ and $g(z)$.

Inverse function

Find the Taylor series for $f^{-1}(z)$,
given the Taylor series for $f(z)$.

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Convergence occurs in an annulus.