

1 Almost Trigonometric Functions

For convenience, define

$$t(n) := \begin{cases} 1 & \text{if } n \text{ is divisible by 6,} \\ -1 & \text{if } n \text{ has remainder 3 when divided by six,} \\ 0 & \text{otherwise.} \end{cases}$$

Let's define three functions,

$$\begin{aligned} f_0(x) &= \sum_{n=0}^{\infty} \frac{t(n) x^n}{n!} \\ f_1(x) &= \sum_{n=0}^{\infty} \frac{t(n-1) x^n}{n!} \\ f_2(x) &= \sum_{n=0}^{\infty} \frac{t(n-2) x^n}{n!} \end{aligned}$$