Lecture 23: Integration by parts

Math 153 Section 57

Wednesday November 19, 2008

Following chapter 8.2.

Jeopardy—integration as an inverse operation. 295927 is 541 times 547.

Overview about integration—emphasize again the fundamental theorem and its amazing power

antidifferentiation and the challenge of inverse operations

u substitution is the chain rule—backwards!

parts is the product rule—backwards! not a rule, but a technique—something like a skilled warrior who transforms the situation into something more advantengeous rather than outright solving it

what is u? what is v? Try to make choices to have easy antiderivatives.

$$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \int (1/x)(-1/x)\,dx.$$

$$\int \frac{\ln(\sin x)}{(\cos x)^2}\,dx = \ln(\sin x)\tan x - \int \frac{1}{\tan x}\tan x\,dx.$$

$$\int x\,dx = x^2 - \int x\,dx$$

so we can find an antiderivative for x^2. Same trick works for $\log x$.

$$\int x^3 e^{x^2}\,dx = \frac{1}{2}e^{x^2}(x^2-1)+C.$$

Repeated integration

$$\int uv = uv_1 - u'v_2 + u''v_3 - \cdots + (-1)^n\ u^{(n)}\ v_{n+1}$$

Integrals of products of sines and cosines

Do

$$\int \sin(ax)\ \cos(bx)\,dx$$

by using an angle sum formula? No! Use parts twice.

Powers of trig functions? Replace $\sin^2 x$ with $1 - \cos^2 x$ as follows

$$\int \cos^3 x\, \sin^5 x\, dx = \int \cos x(1 - \sin^2 x)\, \sin^5 x\, dx = \int (1 - u^2)\, u^5\, du =$$