End of the World scenarios

(a) Since the invention of the integrated circuit in 1958, the number of transistors that can be placed on an integrated circuit has doubled approximately every two years.[†] As a result, we have seen computers get faster.

It is January 1, 2010, and a Model 1 self-replicating robot has been built: this robot will spend its first two years building an improved robot that operates twice as quickly. It will take one year for the Model 2 to build a Model 3; it will take six months for the Model 3 to build a Model 4... What horrible thing happens on January 1, 2015?

(b) It is January 1, 2009, and self-replicating nano-bots have escaped from your lab—nano-bots which convert all matter they touch into more of themselves. It takes one day for a nanobot to create two copies of itself.

Knowing that the mass of the earth is 6×10^{24} kilograms, and that each nanobot co

(b)

(a) On page 571, section 11.7, do problems: 1, 6, 9, 10, 17, 31.

(b) Consider the graph of $f(x) = 1/x$, restricted to $x \in [1, \infty)$. Rotate this graph around the x-axis, to produce
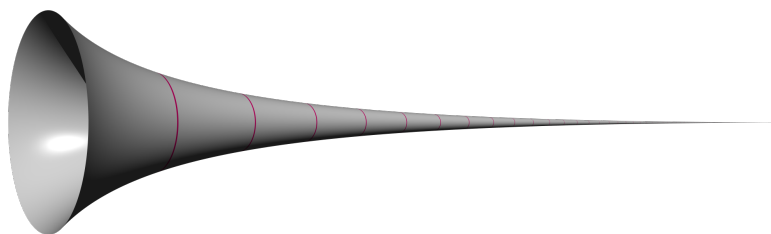
The surface area of this object is infinite, but what is its volume?

(c) The **Gamma function** is defined as follows:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} \, dt.$$

Compute $\Gamma(5)$. *Hint: first compute $\Gamma(1)$, and then integrate by parts to find $\Gamma(z)$ in terms of $\Gamma(z-1)$.*[†]

(d) Evaluate $\int_0^{\pi/2} \frac{x}{\tan x} \, dx$. This is an extremely difficult integral.

[†]See http://en.wikipedia.org/wiki/Moore's_law
[†]If you are very stuck, look at http://en.wikipedia.org/wiki/Gamma_function