Homework 3

Due Wednesday, October 8, 2008

1. State whether the following sequences converge (and, if so, state the limit).

   (a) $a_n = \sqrt{n}$.

   (b) $b_n = 2^{-n} + \frac{1}{n}$.

   (c) $c_n = \dfrac{n+1}{n^2}$.

   (d) $d_n = \dfrac{n + (-1)^n}{n}$.

   (e) $e_n = (-1)^n \cdot n^3$.

   (f) $f_n = \cos \dfrac{1}{n}$.

   (g) $g_n = |10 - n| - n$.

   (h) $h_n = \dfrac{3^n}{4^n + 1}$.

   (i) $i_n = \log{(n+1)} - \log n$.

   (j) $j_n = \sqrt{n^2 + n} - n$.[‡]

2. Give an ϵ-K proof that the sequence $a_n = \dfrac{3}{n}$ converges,

3. Prove that if $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$, then $\lim_{n\to\infty} a_n + b_n = L + M$.

4. Let a_n be a sequence of real numbers, and set $b_n = |a_n|$. If b_n converges, does a_n converge? If so, prove it; if not, provide a counterexample.

5. Let a_n and b_n be sequences of real numbers; suppose $\lim_{n\to\infty} a_n = 0$. Is it the case that

$$\lim_{n\to\infty} a_n \cdot b_n = 0?$$

   If so, prove it; if not, provide a counterexample.

[‡]This is rather tricky; if you don't answer it, you will not be penalized.