Signature

Math 758

Spring 2012

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Problem 1. Recall from yesterday's lecture that $\sigma(\partial M) = 0$ for a compact orientable manifold M. What can you say about $\chi(\partial M)$?

Problem 2. Is there a compact 3-manifold M^3 with $\partial M = \mathbb{R}P^2$?

Problem 3. Is there a compact $(2k+1)$-manifold M^{2k+1} with $\partial M = \mathbb{R}P^{2k}$?

Problem 4. Again, recall $\sigma(\partial M) = 0$. Consider $M = \mathbb{C}P^k \times I$, so that ∂M consists of two copies of $\mathbb{C}P^k$. Since $\sigma(\mathbb{C}P^k) = 1$, it appears that $\sigma(\partial M) = 2$. Uh oh!—what is wrong with this argument?

Problem 5. Prove that $\sigma(M \times N) = \sigma(M) \cdot \sigma(N)$.

Problem 6. Find a manifold M^{4k} with $\pi_1 M$ a free group on two generators.

Problem 7. Show that $\sigma(M \# N) = \sigma(M) + \sigma(N)$ by finding a manifold W so that ∂W has three pieces, namely M and N and $M \# N$.

Problem 8. Find a manifold M with $\pi_1 M$ a free group on two generators, and $\sigma(M) = 1$.

Problem 9. Let M be the manifold you produced to solve the previous problem; let N be a k-fold cover of M. Can you compute $\sigma(N)$?

Problem 10. For a manifold M^{4k}, what can you say about the parity of $\chi(M) - \sigma(M)$?

Problem 11. Suppose $f : M \to \mathbb{C}P^4$ is a map which is homotopic to a homeomorphism, transverse to $\mathbb{C}P^2$ so that $f^{-1}(\mathbb{C}P^2)$ is a submanifold of M. Compute $\sigma(f^{-1}(\mathbb{C}P^2))$.