Quiz 5

Name:

Winter 2011 Math 765

The two-torus $T^2 = S^1 \times S^1$ admits a pair of projection maps

$$p_1 : T^2 \to S^1 \text{ and } p_2 : T^2 \to S^1$$

onto the first and second factor, respectively; the circle embeds in $\mathbb{R}^2$ via the inclusion map $i : S^1 \to \mathbb{R}^2$ sending θ to $(\cos\theta, \sin\theta)$.

Consider the 1-form $\omega = y\,dx$ on $\mathbb{R}^2$, and define $\eta = {p_1}^\star i^\star\omega + {p_2}^\star i^\star\omega$. Is there a smooth function $f : T^2 \to \mathbb{R}$ so that $df = \eta$? Why or why not?

Solution