Solution Set 2

Due Monday, October 6, 2008

1. For each sequence, state whether it is bounded (and if so, above or below) and whether it is monotone (and if so, (non)increasing or (non)decreasing).

   (a) $a_n = 2^n$. *Bounded below but not above. Increasing.*

   (b) $b_n = \sin n$. *Bounded both below and above. Not monotone.*

   (c) $c_n = \dfrac{4n}{n+1}$. *Bounded both below and above. Increasing.*

   (d) $d_n = \dfrac{\sqrt{n+1}}{\sqrt{n}}$. *Bounded both below and above. Decreasing.*

   (e) $e_n = (-1)^n \cdot n!$. *Not bounded. Not monotone.*

   (f) $f_n = \cos\left(\dfrac{1}{n}\right)$. *Bounded both below and above. Increasing.*

   (g) $g_n = 17$. *Bounded both below and above. Nondecreasing and nonincreasing.*

   (h) $h_n = |5-n| - n$. *Bounded both below and above. Nonincreasing.*

2. Suppose a_n is a bounded sequence. Is the sequence $b_n = 17 \cdot a_n$ also bounded? Why or why not?

   Yes. If b_n is bounded above by U and below by L, then for all $n \in \mathbb{N}$, we have $L \leq a_n \leq U$. Multiplying this inequality by 17 gives
   $$17 \cdot L \leq 17 \cdot a_n \leq 17 \cdot U$$
   for all $n \in \mathbb{N}$. This means b_n is bounded above by $17 \cdot U$ and below by $17 \cdot L$.

3. Suppose a_n is a bounded sequence, and $a_n \neq 0$. Is the sequence $b_n = 1/a_n$ also bounded? Why or why not?

   No. Let $a_n = 1/n$. Then a_n is bounded, but $b_n = 1/a_n = n$, which is not a bounded sequence.

4. Suppose a_n is a decreasing sequence. Is the sequence $b_n = 2a_n + 3$ also decreasing? Why or why not?

   Yes. We assume for all $n \in \mathbb{N}$ that $a_n > a_{n+1}$. Multiplying both sides by 2 and adding 3 gives the inequality
   $$2a_n + 3 > 2a_{n+1} + 3 \text{ for all } n \in \mathbb{N}.$$
   This means $b_n > b_{n+1}$ for all $n \in \mathbb{N}$, so b_n is decreasing.

5. Suppose a_n is an increasing sequence. Is the sequence $b_n = {a_n}^2$ also increasing? Why or why not?

   No. Suppose $a_n = -1/n$. This is an increasing sequence, but ${a_n}^2 = 1/n^2$ is not increasing.