Lecture 9: Number theory — Math 345

Wednesday, October 6, 2010 — Jim Fowler

Textbook

This lecture discusses section 4 of the textbook.

Homework

The homework is due Monday, October 11, 2010.

From Section 4 of the textbook, do exercises 1 and 2.

number theory

number theory: study the properties of whole numbers

e.g., prime numbers

easy-to-state questions remain open

perfect numbers

$6 = 1 + 2 + 3$

$28 = 1 + 2 + 4 + 7 + 14$

$496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248$

8128 is perfect, too. 47 such numbers are known today.

infinitely many perfect numbers? unknown.

any odd perfect numbers? unknown. it is known that such a number must have more than 300 digits.

Descartes writes in 1638 [1]

> I think I am able to prove that there are no even numbers which are perfect apart from those of Euclid; and that there are no odd perfect numbers, unless they are composed of a single prime number, multiplied by a square whose root is composed of several other prime number. But I can see nothing which would prevent one from finding numbers of this sort.

[1] http://www-history.mcs.st-andrews.ac.uk/HistTopics/Perfect_numbers.html

collatz conjecture

unknown

definitions for today

x is even if there exists $k \in \mathbb{Z}$ so that $x = 2k$.

x is odd if there exists $k \in \mathbb{Z}$ so that $x = 2k + 1$.

example: odd plus odd is even

what do we need to use? distributivity.

properties you are allowed to use

two operations ($+$ and $\cdot$) both associative, commutative, with identities. distributivity.

can you prove that an integer is either even or odd?

no? why not?

a deeper reason: there are mathematical systems which satisfy all the properties we can use, but for which "even" and "odd" don't make sense.