Problem Set 6 — Math 758

Spring 2012 — Jim Fowler

The exercises below should be handed in on Tuesday.

Problem 6.1 (Hatcher page 260 problem 33)

Show that if M is a compact contractible n-manifold, then ∂M is a homology $(n-1)$-sphere, that is, $H_i(\partial M) = H_i(S^{n-1})$ for all i.

Problem 6.2 (Hatcher page 260 problem 31)

Show that if M is a compact R-orientable n-manifold, then the boundary map $H_n(M, \partial M R) \to H_{n-1}(\partial M; R)$ sends a fundamental class for $(M, \partial M)$ to a fundamental class for ∂M.

Problem 6.3

Does every closed orientable 2-submanifold N^2 of a closed 3-manifold M^3 divide it into two pieces? That is, does $M^3 - N^2$ necessarily have two components?

Problem 6.4 (Embedding real projective space)

For which $n \geq 1$ does $\mathbb{R}P^n$ embed in S^{n+1}?

Problem 6.5 (Embedding complex projective space)

For which $n \geq 1$ does $\mathbb{C}P^n$ embed in S^{2n+1}?

Problem 6.6 (Embedding a pair of circles)

Consider embeddings of $S^1 \sqcup S^1$ in S^3. Use **linking number** to distinguish between two such embeddings up to isotopy.