Problem Set 2 — Math 758

Spring 2012 — Jim Fowler

The exercises below should be handed in on Monday.

Problem 2.1 (Surfaces)

Let Σ_g be the closed surface of genus g; compute the ring structure on $H^\star(\Sigma_g)$.

Problem 2.2 (Surface automorphisms)

A homeomorphism $f : \Sigma_g \to \Sigma_g$ induces a map $f^\star : H^1(\Sigma_g) \to H^1(\Sigma_g)$; can every automorphism of the abelian groups $H^1(\Sigma_g)$ be realized as $f^\star$ for some homeomorphism f?

Problem 2.3 (Not a wedge)

Show that $\mathbb{C}P^2 \not\simeq S^4 \vee S^2$ by using cup products.

Problem 2.4 (Hatcher page 229, problem 4)

Use the Lefschetz fixed point theorem (did you do this last quarter?) to show that every map $f : \mathbb{C}P^n \to \mathbb{C}P^n$ has a fixed point if n is even, using the fact that $f^\star$ is a ring isomorphism; when n is odd, show that there is a fixed point unless $f^\star(\alpha) = -\alpha$ for α a generator of $H^2(\mathbb{C}P^n)$.

Problem 2.5 (Hatcher page 229 problem 6)

Use cup products to compute the map $f^\star : H^\star(\mathbb{C}P^n) \to H^\star(\mathbb{C}P^n)$ induced by the map $f : \mathbb{C}P^n \to \mathbb{C}P^n$ which is the quotient of the map $\mathbb{C}^{n+1} \to \mathbb{C}^{n+1}$ given by raising each coordinate to the dth power.

Problem 2.6 (Suspension kills cup products)

Let ΣX be the suspension of X (recall this is the union of two cones on X glued together along X); for $\alpha \in H^a(\Sigma X)$ and $\beta \in H^b(\Sigma X)$, show that $a \smile b$ vanishes.