Cohomology Operations — Math 758

Spring 2012 — Jim Fowler

Problem 1. Use $0 \to \mathbb{Z}/p \to \mathbb{Z}/p^2 \to \mathbb{Z}/p \to 0$ to produce a map

$$\beta : H^\star(X;\mathbb{Z}/p) \to H^{\star+1}(X;\mathbb{Z}/p)$$

Problem 2. Use $0 \to \mathbb{Z} \to \mathbb{Z} \to \mathbb{Z}/p \to 0$ to produce a map

$$\tilde{\beta} : H^\star(X;\mathbb{Z}/p) \to H^{\star+1}(X;\mathbb{Z})$$

Problem 3. There is a map from $0 \to \mathbb{Z} \to \mathbb{Z} \to \mathbb{Z}/p \to 0$ to $0 \to \mathbb{Z}/p \to \mathbb{Z}/p^2 \to \mathbb{Z}/p \to 0$; what relationship does this provide between β and $\tilde{\beta}$?

Problem 4. For which n is

$$\beta : H^n(\mathbb{R}P^\infty;\mathbb{Z}/2) \to H^{n+1}(\mathbb{R}P^\infty;\mathbb{Z}/2)$$

an isomorphism? Zero?

Problem 5. Determine the sign in the formula

$$\beta(x \smile y) = \beta(x) \smile y \pm x \smile \beta(y).$$

Problem 6. Show that $\beta \circ \beta = 0$.

Problem 7. Compute the homology of the "chain complex" $H^\star(X;\mathbb{Z}/p)$ where the differential is β.

Problem 8. Compute $H^\star(\mathbb{R}P^\infty \times \mathbb{R}P^\infty;\mathbb{Z})$.