Lecture 1: Welcome, parametric equations

Math 195 Section 91

Monday June 22, 2009

Goal: Sections 11.1, and 11.2.

Welcome to the course

Syllabus

Goal

Differentiation as a "wiggle"

Machine with n inputs and k outputs—wiggle one input.

Goal: Expand both our intuition and our technical skill from one variable to many variables.

Stop using your calculator. You are Neo, and when you are good enough...

The book does a lot of great examples which are more complicated than what I intend to do in class. You should look at these examples. You should read the book. In contrast, I will tell you the secrets that this book—that every book—leaves out.

Objects

Review: functions—what is a function?

We often broad the class of objects under consideration (history of "number")

Introduce section 11.1: Parametric curve (i.e., curve defind using a parameter)

example: $x(t) = 3t$ and $y(t) = 4t$—remark the goal of calculus to reduce evertying to a straight line.

example: $x(t) = t^2$ and $y(t) = t^3$—what happens at 0?

example: $x(t) = \sin t$ and $y(t) = \cos t$—why is that a circle?

concerned more with patterns, with qualitative data, than quantative results. A common problem is the following: you have a parametric equation, and you must find a cartesian equation for part of the curve.

Better terminology than the books: reparameterization—sweep out the same curve, but in a different amount of time

Thinking about parameters

Think about the parameter as looking down at a 3-dimensional object from above...

sometimes we might have more than one parameter: example: $x(u,t) = u \cdot \sin t$ and $y(u,t) = u \cdot \cos t$—a family of circles!

there's a great deal of flexibility about how you explore these objects

Calculus on curves

take derivatives at a point

elliptic curve $x(t) = t^2$ and $y(t) = t^3 - 3t$. graph through $(0,0)$ and $(3,0)$ and $(1,2)$ and $(1,-2)$.

$dy/dx = (dy/dt)/(dx/dt)$. Why?

calculate derivatives (and second derivatives) elsewhere to finish the sketch

better than using a calculator—we're not just sampling points! We're using calculus to understand the qualitative features of a graph

what is the deriative at 0 when $x(t) = t^2$ and $y(t) = t^3$?

arc length

under suitable conditions

$$L = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

confess: i could never remember this formula

challenge: try to find situations where the formula is incorrect. this is the "understand biology by killing things" or "learn how cars work by removing parts until the car stops working" method

why should this be true?

example... can we calculate the circumference of a circle?