I have finally finished grading the exams. And I must admit: the exam was **very long**—so long that my compatriots said it could not be done! But we proved them wrong!

You did very well. To quantify:

25% of you scored above 220.
50% of you scored above 210.
75% of you scored above 200.

The median was 210. The mean was 206. The standard deviation was 21. This means that many of you scored over 90%, which is very good.

Here is the breakdown per problem:

Problem	**Average Score**
Problem 10	79%
Problem 9	80%
Problem 6	81%
Problem 11	83%
Problem 4	86%
Problem 1	90%
Problem 8	90%
Problem 2	92%
Problem 5	92%
Problem 3	93%
Problem 12	96%
Problem 7	99%

Problem 1

Quanitifiers (those "for all" and "there exists") are very important, and their order matters. "For every cat C, there is a bowl B, so that C eats from B" is not the same as "There is a bowl of food B, such that for every cat C, C eats from B" In the first world, every cat has a bowl of food. In the other, there is one bowl of food that all the cats must eat from.

In the same way, "for every $\epsilon > 0$, there is a $K\ldots$" is not at all the same thing as "there is a K, for every $\epsilon > 0$."

Problem 2

Your proofs should begin "Let $\epsilon > 0$." This problem proved tricky because the limit was not equal to one.

Problem 3

The definition should include for all $n \in \mathbb{N}$.

Problem 4

I found it difficult to grade this problem, as very few people gave convincing arguments for c_n being unbounded above: I was fairly generous.

In any case, be careful not to use circular logic (i.e., do not claim that c_n diverges because c_n is unbounded, and then claim that c_n is unbounded because it diverges).

Problem 5

You should have shown $d_n > d_{n+1}$ algebraically, or at least claimed that n^2 is increasing, so $-n^2$ is decreasing, so $20 - n^2$ is decreasing.

Problem 6

Many people failed to notice $\sin(\pi n) = 0$ when $n \in \mathbb{N}$ (admittedly, it might feel like me tricking you, but this question hits at the difference between limits of sequences and of functions).

Many people tried to use l'Hôpital's rule on this sequence—but l'Hôpital is only for functions (unless you do something clever to explain why the limit of a sequence is equal to the limit of some similarly defined function).

Problem 7

People did very well on this problem. Be careful to mention that cosine is continuous.

Problem 8

Some people found fancier things to squeeze between, but $\frac{n^4 \pm 1}{n^4}$ works—you don't need to make things harder than they already are.

Problem 9

Many people forgot that additional assumption: that $g'(x) \neq 0$ for x near a. The theorem is not true without this assumption.

Problem 10

This problem turned out to be harder than I expected. I think many people ran out of time; other people seemed to have trouble applying the chain rule (though, again, this problem was designed to test that: applying the chain rule to $\sin^2(2x)$ can be tricky).

Problem 11

One person did a very nice thing by pointing out

$$\lim_{x \to 0^+} x^{2x} = \left(\lim_{x \to 0^+} x^x \right)^2 = 1^2 = 1.$$

Most people just used l'Hôpital to evaluate this limit, but many of you forgot to undo the logarithm at the end:

$$\log \lim_{x \to 0^+} x^{2x} = 0 \text{ implies } \lim_{x \to 0^+} x^{2x} = e^0 = 1.$$

Of course, the problem is designed to lead you into trap (as one of you pointed out explicitly!), since the other limit equals 0.

Problem 12

People mostly did well on this problem, though I think some of you ran out of time.

Overall impressions.

Your arguments in problems 4 and 5 should have been better.

Many of you can do a better job presenting your answers. I'm very sloppy human being, but mathematics (which comes from the Greek word for "discipline" or "disciple") does demand a certain amount of care, lest you make mistakes along the way. The chain rule, for instance, is a powerful tool, and we must be careful not to hurt ourselves when wielding such tools.

Additionally, your mathematics should more closely resemble billiards than, say, tennis: your goal is to announce your next move clearly, and then perform it properly—not to surprise me with fast-moving shots precisely aimed to the place I am not looking.