Quartersquares: an old-fashioned way to multiply

Recall the fact that

$$ab = \frac{(a+b)^2}{4} - \frac{(a-b)^2}{4}.$$

To multiply a and b,

- Look up $a+b$ in the table.
- Look up $a-b$ in the table.
- Subtract the quartersquare of $a-b$ from the quartersquare of $a+b$.

From a complexity point of view, this is an improvement: adding (subtracting) an n digit number to (from) an n digit number requires n operations, but multiplying an n digit number by an n digit number using the usual algorithm requires n^2 operations. **Multiplication is expensive.** By using quartersquares, we can replace expensive multiplications with cheaper additions, subtractions, and table lookups.

n	$n^2/4$	n	$n^2/4$	n	$n^2/4$	n	$n^2/4$	n	$n^2/4$	n	$n^2/4$
1	$\frac{1}{4}$	26	169	51	$650\frac{1}{4}$	76	1444	101	$2550\frac{1}{4}$	126	3969
2	1	27	$182\frac{1}{4}$	52	676	77	$1482\frac{1}{4}$	102	2601	127	$4032\frac{1}{4}$
3	$2\frac{1}{4}$	28	196	53	$702\frac{1}{4}$	78	1521	103	$2652\frac{1}{4}$	128	4096
4	4	29	$210\frac{1}{4}$	54	729	79	$1560\frac{1}{4}$	104	2704	129	$4160\frac{1}{4}$
5	$6\frac{1}{4}$	30	225	55	$756\frac{1}{4}$	80	1600	105	$2756\frac{1}{4}$	130	4225
6	9	31	$240\frac{1}{4}$	56	784	81	$1640\frac{1}{4}$	106	2809	131	$4290\frac{1}{4}$
7	$12\frac{1}{4}$	32	256	57	$812\frac{1}{4}$	82	1681	107	$2862\frac{1}{4}$	132	4356
8	16	33	$272\frac{1}{4}$	58	841	83	$1722\frac{1}{4}$	108	2916	133	$4422\frac{1}{4}$
9	$20\frac{1}{4}$	34	289	59	$870\frac{1}{4}$	84	1764	109	$2970\frac{1}{4}$	134	4489
10	25	35	$306\frac{1}{4}$	60	900	85	$1806\frac{1}{4}$	110	3025	135	$4556\frac{1}{4}$
11	$30\frac{1}{4}$	36	324	61	$930\frac{1}{4}$	86	1849	111	$3080\frac{1}{4}$	136	4624
12	36	37	$342\frac{1}{4}$	62	961	87	$1892\frac{1}{4}$	112	3136	137	$4692\frac{1}{4}$
13	$42\frac{1}{4}$	38	361	63	$992\frac{1}{4}$	88	1936	113	$3192\frac{1}{4}$	138	4761
14	49	39	$380\frac{1}{4}$	64	1024	89	$1980\frac{1}{4}$	114	3249	139	$4830\frac{1}{4}$
15	$56\frac{1}{4}$	40	400	65	$1056\frac{1}{4}$	90	2025	115	$3306\frac{1}{4}$	140	4900
16	64	41	$420\frac{1}{4}$	66	1089	91	$2070\frac{1}{4}$	116	3364	141	$4970\frac{1}{4}$
17	$72\frac{1}{4}$	42	441	67	$1122\frac{1}{4}$	92	2116	117	$3422\frac{1}{4}$	142	5041
18	81	43	$462\frac{1}{4}$	68	1156	93	$2162\frac{1}{4}$	118	3481	143	$5112\frac{1}{4}$
19	$90\frac{1}{4}$	44	484	69	$1190\frac{1}{4}$	94	2209	119	$3540\frac{1}{4}$	144	5184
20	100	45	$506\frac{1}{4}$	70	1225	95	$2256\frac{1}{4}$	120	3600	145	$5256\frac{1}{4}$
21	$110\frac{1}{4}$	46	529	71	$1260\frac{1}{4}$	96	2304	121	$3660\frac{1}{4}$	146	5329
22	121	47	$552\frac{1}{4}$	72	1296	97	$2352\frac{1}{4}$	122	3721	147	$5402\frac{1}{4}$
23	$132\frac{1}{4}$	48	576	73	$1332\frac{1}{4}$	98	2401	123	$3782\frac{1}{4}$	148	5476
24	144	49	$600\frac{1}{4}$	74	1369	99	$2450\frac{1}{4}$	124	3844	149	$5550\frac{1}{4}$
25	$156\frac{1}{4}$	50	625	75	$1406\frac{1}{4}$	100	2500	125	$3906\frac{1}{4}$	150	5625