Problem Set 9 — Math 758

Spring 2012 — Jim Fowler

This is the last problem set in the course. The final examination will be a take-home exam, distributed at the last lecture and due the following Thursday.

Problem 9.1 (Lens spaces)

The cyclic group $\mathbb{Z}/m$ acts on the circle S^1 by rotation through an angle of $2\pi/m$, so $\mathbb{Z}/m$ acts on the k-fold join $S^1 * S^1 * \cdots * S^1$, which is a $(2k-1)$-sphere. Let L^{2k-1} be the quotient of S^{2k-1} by this action of $\mathbb{Z}/m$, and compute (either using Poincaré duality or Bocksteins) the cup product structure in $H^\star(L; \mathbb{Z}/m)$.

Problem 9.2 (Product of real projective space)

Calculate $H^\star(\mathbb{R}P^\infty \times \mathbb{R}P^\infty; \mathbb{Z})$ as a ring.

Problem 9.3 (Hatcher exercise 4 page 310)

Using the cup product structure in $H^\star(\mathrm{SO}(5); \mathbb{Z})$, show that $\mathrm{SO}(5)$ is not homotopy equivalent to the product of any two CW complexes with nontrivial cohomology.