# Take-Home Quiz 3

Math 132 Section 22

Due Monday, January 30, 2006

For the following problems, if you claim that something doesn't exist, you must *prove* it doesn't exist.

In some sense, problems three and four are actually about soap bubbles: problem three, for instance, is about minimizing the surface area enclosing a particular volume, and this is exactly what bubbles do.

**Problem 1. (3 points).** Find (if they exist) the local and global minima and maxima of the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = x^4 - x^2.$$

**Problem 2. (3 points).** Find (if they exist) the local and global minima and maxima of the function $f : (-\infty, 0) \cup (0, 4) \cup (4, \infty) \to \mathbb{R}$ given by

$$f(x) = \frac{1}{x \cdot (4 - x)}.$$

**Problem 3. (4 points).** You are again working for a soup company. This time, build a soup can (i.e., a cylinder) holding $16\pi$ cubic units of soup, with the surface area as small as possible.

**Problem 4. (4 points).** You are a highway planner. You must connect three cities,

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| City A | located at | $(-1, 1)$, |
| City B | located at | $(-1, -1)$, |
| City C | located at | $(0, 0)$, |

by three straight roads all meeting at a 3-way intersection located at $(x, 0)$. Where should the junction be placed to minimize the total length of road needed?