Lecture 21: Differentiating and integrating series

Math 153 Section 57

Friday November 14, 2008

Stirling's formula

Try to estimate $k!$ by

$$\log(k!) = \sum_{n=1}^{k} \log n \approx \int_1^k \log x \, dx$$

But this integral we can do

$$\log(k!) \approx k \log k - k + 1$$

so this means that

$$k! = e^{\log(k!)} \approx e^{k \log k - k + 1} \approx= (k/e)^k$$

A better estimate:

$$k! \approx \sqrt{2\pi k}(k/e)^k$$

For example,

$$270! \approx \sqrt{540\pi}(270/e)^{270} \approx \sqrt{1600}(100)^{270} \approx 40 \cdot 10^{540} \approx 4 \cdot 10^{541}$$

The true value is about $6 \cdot 10^{540}$.

Limits do not commute

$$\frac{1}{2} = \lim_{x\to 0} \lim_{y\to 0} \frac{x+2y}{2x+y} \neq \lim_{y\to 0} \lim_{x\to 0} \frac{x+2y}{2x+y} = 2$$