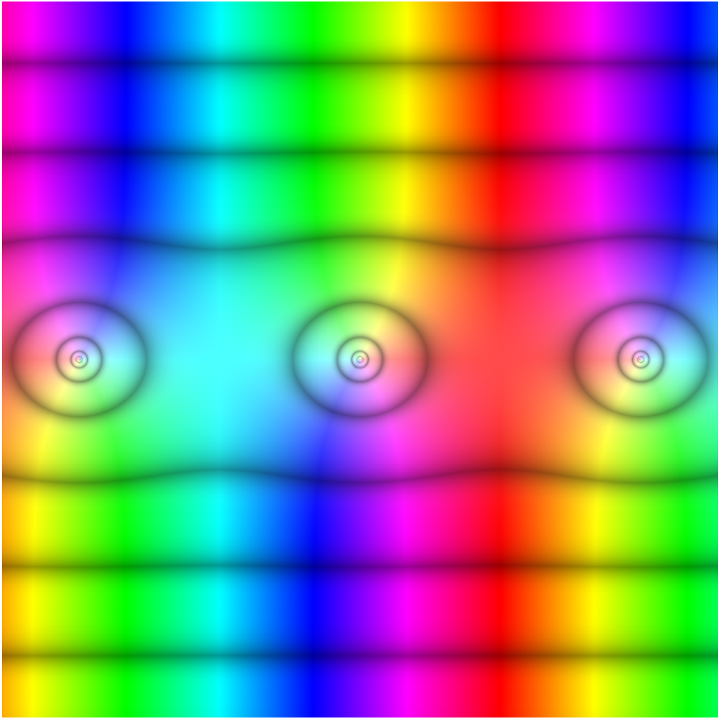
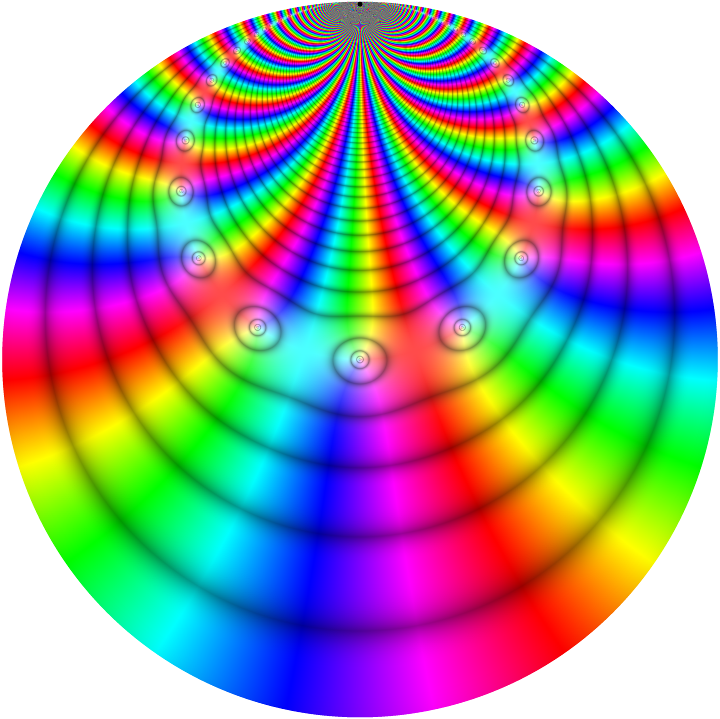
Lecture 5: Power series

Math 660—Jim Fowler

Friday, June 24, 2011





Power series!

section 2.2.4 of the text

Power series

$$\sum_{n=0}^{\infty} a_n z^n$$

or

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n$$

geometric series

$$\sum_{n=0}^{k} z^n = \frac{1 - z^{k+1}}{1 - z}$$

Provided $|z| \leq 1$.

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1 - z}$$

"radius of convergence"

Radius of convergence

For a power series $\sum a_n z^n$,
there exists a number R, so that

- If $|z| < R$, the series converges absolutely.
- If $\rho < R$, then the series converges uniformly for $|z| \leq \rho$.
- If $|z| > R$, the series diverges.
- If $|z| < R$, the series is an analytic function, and the derivative is the sum of the termwise derivatives
  with the same radius of convergence.

Specifically, $1/R = \lim_{n\to\infty} \sup \sqrt[n]{|a_n|}$.