Take-Home Quiz 6

Math 132 Section 22

Due Wednesday, March 8, 2006

Problem 1. (3 points). Find the volume of a ball of radius r with a hole of radius a drilled through it. Specifically, find the volume of

$$\{(x, y, z) \in \mathbb{R}^3 : \sqrt{x^2 + y^2 + z^2} \leq r \text{ and } \sqrt{x^2 + y^2} \geq a\}.$$

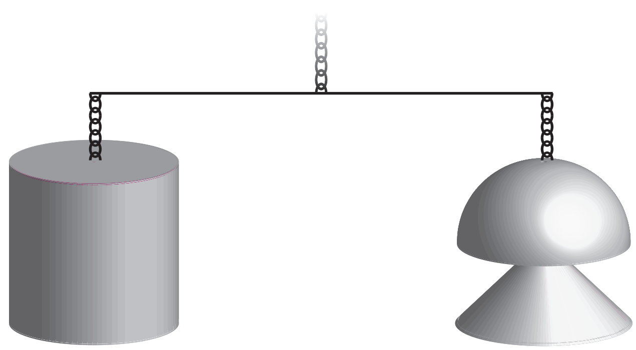
The first condition (i.e., $\sqrt{x^2 + y^2 + z^2}$) picks out those points which are inside the ball of radius r, and the second condition (i.e., $\sqrt{x^2 + y^2} \geq a$) picks out those points which are outside a cylinder of radius a. *Hint:* use the method of shells.

Problem 2. (3 points). Find the length of the curve defined for $0 \leq t \leq 4$ by

$$\begin{aligned} x(t) &= t, \\ y(t) &= \frac{2t^{3/2}}{3}. \end{aligned}$$

Problem 3. (4 points). Where is the center of mass of a hemisphere of radius 1?

Problem 4. (4 points). Archimedes, in a recently discovered book called *The Method*, describes a procedure for computing the volume of a sphere. He begins by building a mobile, with a cylinder of height r and radius r on one side balanced against a hemisphere of radius r and a cone of radius r and height r on the other side:



You may assume that the volume of the cylinder is πr^3, and that the volume of the cone is $\pi r^3/3$, and that the balance point is at the midpoint. Explain why the mobile balances and therefore prove that the volume of a hemisphere is $2\pi r^3/3$.