Lecture 7: Bound variables — Math 345

Monday, October 4, 2010 — Jim Fowler

Textbook

This lecture discusses section 3 of the textbook.

Homework

The homework is due Wednesday, October 6, 2010.

From Section 3 of the textbook, do exercises 2, 4, and 7.

Have some people come up and present solutions to homework from before

Bound and free variables

In the statement $P(x)$, the variable x is free.

In the statement $\forall x\, P(x)$, the variable x is bound. In other words, "$\forall x\, P(x)$" is not a statement about x.

Possible (but poor style) to use the same variable name for a bound variable and a free variable.

The scope of a quantifier is the part of the proposition it applies to.

Example

For all integers x, x is even or odd.

(For all integers x, x is even) or (For all integers x, x is odd).

Nested quantifiers

Does the order of quantifiers matter?

$P(x, y)$ means "x wants to date y."

Then, $\exists y \forall x P(y, x)$ means there is someone who wants to date everybody.
Then, $\forall x \exists y P(y, x)$ means everybody has someone interested in them.
These are different.
How does $\forall x \exists y$ differ from $\exists y \forall x$? The latter implies the former.

Algebraic example

think about sum and products