Lecture 3: Vector products

Math 195 Section 91

Friday June 26, 2009

Goal: section 13.3 and 13.4

Dot product

definition

properties:

- $v \cdot v = |v|^2$
- $v \cdot (w + u) = v \cdot w + v \cdot u$
- $0 \cdot v = 0.$
- $v \cdot w = w \cdot v$
- $(\alpha v) \cdot w = \alpha \cdot (v \cdot w) = v \cdot (\alpha w).$

use dot product to find length. define unit vector.
what does it measure: the angle $v \cdot w = |v| \cdot |w| \cdot \cos\theta$.
how to find the angle
prop: vectors v and w are orthognoal if $v \cdot w = 0$.
especially fun to find angle with i, j, k.

0.1 Projection

project w onto v.

use projection to write w as a combination of u and v.

0.2 higher dimensions!

we can do dot products in any dimension! can you compute the angle between two four dimensional vectors?

0.3 Cross product

cross product is something special about 3-dimensions—takes two vectors and gives another vector

define cross product using the "circle ijk". mention that the book uses determinants, but that we won't do this.

theorem: $a \times b$ is perpendicular to a and b. proof? dot products!

theorem: $|a \times b| = |a| \cdot |b| \cdot \sin\theta$. This is the same asthe area of the parallelogram.

interactively deduce the varous properties of cross products

commutative? associative?