Homework 11

Due Monday, November 10, 2008

(a) Without using a calculator, estimate $\sin 2$ to within 0.05.

(b) Without using a calculator, estimate $\cos 2$ to within 0.05.

(c) Find a series representation for $\sin^2 x$. Hint: use a double-angle formula.

(d) The Taylor series for arctan is given by

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

and converges when $|x| \leq 1$. Using the fact that $\arctan 1 = \pi/4$, approximate π to within 0.5.

(e) Use Lagrange's theorem to prove that

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

for all real x.