I wonder—what will the exam be like?

- There will be **13 questions** on the exam and it will last **50 minutes**. The first question will ask you to give the ϵ-K definition of the limit of a sequence.
- Calculators are **forbidden**.
- There will be **one improper integral** on the exam; it will not involve a challenging integral: I would rather that you make sure you are extremely comfortable with *differentiation* on this exam, so you can handle the questions involving l'Hôpital's rule—future exams will give us plenty of practice with the fancier techniques of integration.
- The last question will be **extra credit** with true/false questions. I think these are **terribly enjoyable!** They will ask you to agree and disagree with statements of theorems, slightly modified statements of theorems, existence of sequences exhibiting certain kinds of phenomena, etc.
- You will lose points if you surround an otherwise convincing argument with false statements (after all, once I have proved $2 = 1$, I can prove anything!). **Erase** or cross-out untrue statements **for full credit.**
- Style matters: if you are taking a limit, write lim. Do not use an "=" between two expressions unless they are, in fact, equal. Do not **under any circumstances** divide by zero during the exam.
- You can refer to our friend the "natural logarithm" by any nickname you prefer.

I wonder—what must I do on this "exam"?

- Give the definition of $\lim_{n\to\infty} a_n = L$.
- Prove that a sequence converges by providing an ϵ-K proof.
- Give the definition for a sequence to be bounded above, bounded below, increasing, decreasing, non-increasing, non-decreasing
- Recognize when a sequence is bounded
- Recognize when a sequence is monotone
- Find limits by using algebraic manipulation
- Use $\lim_{n\to\infty} f(a_n) = f(\lim_{n\to\infty} a_n)$ provided f is continuous.
- Find limits by squeezing sequences between sequences with a common, known limit
- Give the statement of l'Hôpital's rule for indeterminate forms $0/0$ and ∞/∞
- Use l'Hôpital's rule to compute limits (sometimes it takes more than one application!)
- Handle indeterminate forms 0^0, $\infty - \infty$
- Compute an improper integrals by taking limits of definite integrals (and do so carefully, noting where you are taking limits)

How should I write down my answers?

You are not giving answers: you are giving an explanation. For full credit, you must **justify your arguments** and **show the steps you took**: if the central issue in the problem is that the limit of the sum is the sum of the limits, you should be sure to point that out.

You must know the following limits

Example. For $x \in \mathbb{R}$, $x > 0$, $\lim_{n\to\infty} x^{1/n} = 1$.

Example. For $x \in \mathbb{R}$ with $-1 < x < 1$, $\lim_{n\to\infty} x^n = 0$.

Example. For $x \in \mathbb{R}$, $x > 0$, $\lim_{n\to\infty} \frac{1}{n^x} = 0$.

Example. For $x \in \mathbb{R}$, $\lim_{n\to\infty} \frac{x^n}{n!} = 0$.

Example. $\lim_{n\to\infty} \frac{\log n}{n} = 0$.

Example. $\lim_{n\to\infty} n^{1/n} = 1$.

Example. For $x \in \mathbb{R}$, $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$.

You must know the following trigonometric facts

$$\sin(2x) = 2\,\sin x\,\cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

If additional identities are needed, I will provide them (but be warned: when you are one day stranded on desert island, you will have wished you knew as many trigonometric identities as possible—or at least, how to derive them from $e^{i\theta}$).

You should know the derivatives of sin, cos, and tan (which you can remember with the mnemonic "you'll *tan* in two *sec*onds using this product" for $\frac{d}{dx} \tan x = \sec^2 x$).

You must be able to give ϵ-K proofs to show

- $\lim_{n\to\infty} \frac{1}{n} = 0$.
- $\lim_{n\to\infty} \frac{1+n}{n} = 1$.
- $\lim_{n\to\infty} \frac{1}{n^2} = 0$.
- $\lim_{n\to\infty} \frac{3}{n^2} = 0$.
- $\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$.
- $\lim_{n\to\infty} \frac{\cos n}{n} = 0$.
- $\lim_{n\to\infty} \frac{\cos(n^3)}{n} = 0$.
- $\lim_{n\to\infty} \frac{2+3\,n}{n^2} = 0$.