Lecture 14: Prime numbers — Math 345

Thursday, October 14, 2010 — Jim Fowler

Textbook

This lecture discusses section 4 of the textbook.

Homework

The homework is due Wednesday, October 20, 2010. From Section 4 of the textbook, do exercise 27.

Suppose that for all x, $y \in \mathbb{Z}$, if $xy \equiv 0 \mod p$, then $x \equiv 0 \pmod p$ or $y \equiv 0 \pmod p$. Show that p is prime.

A message from Professor Falkner

Dear Math 345 Student,

Are you currently majoring in mathematics? If not, are you considering it? If you answered yes to either question, then I would like you to know about an opportunity to get acquainted with a faculty member in the Department of Mathematics whom you might eventually decide you would like to have as your advisor for your major program. A number of mathematics professors are offering to meet this quarter with up to five students each to lead a five-session series of mathematics-related activities. Each of the professors has proposed a different series of activities that they hope will interest students. I strongly encourage you to participate in one of these activities that interests you if your schedule permits it.

an impassioned plea to major in mathematics

definition of prime numbers

A positive integer p is prime means that $p \neq 1$ and that for all $a, b \in \mathbb{N}$, if $p = ab$ then $a = 1$ or $b = 1$.

humorous example

$n^2 + n + 41$ is a prime for each $n \in \mathbb{N}$?

there are infinitely many prime numbers

also, there are infinitely many composite numbers! :-)

divisibility

Let p be an integer, $p \geq 2$.

Suppose that for all $x, y \in \mathbb{Z}$, if p divides xy, then p divides x or p divides y.

Show that p is prime. (the other direction, "if p is prime, then..." requires induction).

inverses modulo p