Lecture 19: Power series

Math 153 Section 57

Monday November 10, 2008

Following chapter 12.8.

Some loose ends

using Taylor series to instantly solve differential equations
dividing series

Power series

definition of power series
point out that discussing $\sum a_n x^n$ suffices, since $\sum a_n (x-a)^n$ is just a slight modification.

Convergence

If $\sum a_n x^n$ converges at c, then it converges absolutely for all x with $|x| < |c|$.

Proof: use limit comparison test. $|a_n x^n|/|a_n c^n|$ converges to zero, and since $\sum |a_n c^n|$ converges, so does $\sum |a_n x^n|$.

Ask about the set of convergence. The possibilities are

- convergence only at zero (radius 0)
- converges everywhere (radius ∞)
- converges for $(-r, r)$ and diverges for $(-\infty, -r) \cup (r, \infty)$.

interval of convergence

On the endpoints of the interval, we cannot say anything (examples: x^n, $(-1)^n x^n/n$, x^n/n, x^n/n^2)