Problem Set 4 — Math 758

Spring 2012 — Jim Fowler

Problem 4.1 (Hatcher page 259, problem 16)

Show that $(\alpha \frown \varphi) \frown \psi = (\alpha \frown (\varphi \smile \psi)$ for all $\alpha \in C_k(X; R)$ and $\varphi \in C^\ell(X; R)$ and $\psi \in C^m(X; R)$.

Problem 4.2 (Natural)

In what sense do cup product and cap product satisfy naturality?

Problem 4.3 (An explicit example)

Let Σ_2 be the orientable genus two surface, and $[\Sigma_2] \in H_2(\Sigma_2)$ its fundamental class. Show that the map $[\Sigma_2] \frown - : H^1(\Sigma_2) \to H_1(\Sigma_2)$ is an isomorphism.

Problem 4.4 (Product of spheres)

For each class $x \in H_n(S^n \times S^n)$, determine $x \frown - : H^n(S^n \times S^n) \to H_n(S^n \times S^n)$.

Problem 4.5 (Hatcher page 258, problem 7)

Show that for any connected closed orientable n-manifold M, there is a degree 1 map $M \to S^n$.

Problem 4.6 (No degree one map)

Show that there is a connected closed orientable n-manifold M for which there is no degree 1 map $S^n \to M$.

☕ Problem 4.7 (Flexible manfiolds)

An oriented closed connected manifold is *flexible* if it admits a self-map that has degree not zero or ± 1. Show that S^k, and $\mathbb{R}P^k$, and $\mathbb{C}P^k$, and any products of such are flexible.