Homework 14

Due Monday, November 24, 2008

A common question is: what should I choose for u? What should I choose for dv? You should choose something easier to differentiate for u, and easy to antidifferentiate for dv.

(a) Use integration by parts twice to rewrite

$$\int \cos(2x)\,\sin(3x)\,dx$$

in terms of itself—and therefore, find an antiderivative.

Such problems as these are usually solved by using a trigonometric substitution, but to illustrate the usefulness of integration by parts, refrain from using the angle sum formulas.

(b) Here is our first foray into the world of **Fourier series**. For real numbers $a_1, a_2, \ldots, a_k$, define

$$f(x) = \sum_{n=1}^{k} a_n \sin(nx).$$

For each integer m with $1 \le m \le k$, compute

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\sin(mx)\,dx$$

by using the same trick you used in problem **(a)**.

Hint: you will be helped by first calculating

$$\int_{-\pi}^{\pi} \sin(nx)\,\sin(mx)\,dx.$$

There's a lovely pattern for you to discover.

(c) Use integration by parts to compute, for each natural number $n \in \mathbb{N}$,

$$\int_{-\pi}^{\pi} x\,\sin(nx)\,dx.$$

(d) Use integration by parts to evaluate

$$\int_{0}^{1} \arctan x\,dx.$$

Hint: here you should set $u = \arctan x$, and $dv = dx$.

(e) Replace $\cos^2 x$ with $1 - \sin^2 x$ in order to find

$$\int \cos^5 x\,\sin^4 x\,dx = \int \cos x\,\left(\cos^2 x\right)^2\,\sin^4 x\,dx.$$