Homework 5

Due Wednesday, October 15, 2008

Remember: October 20 is the date of the first midterm!

(a) On page 558, section 11.5, do problems: 2, 10, 13, 17, 23, 27.

(b) On page 563, section 11.6, do problems: 3, 9, 14, 15, 21, 31.

(c) Define two functions as follows:

$$\begin{aligned} f(x) &= x + \cos x \, \sin x \\ g(x) &= e^{\sin x} \, (x + \cos x \, \sin x) \, . \end{aligned}$$

We want to calculate $\lim_{x \to \infty} \frac{f(x)}{g(x)}$. Since this is the indeterminate form ∞/∞, we apply l'Hôpital's rule as follows:

$$\begin{aligned} \lim_{x \to \infty} \frac{f'(x)}{g'(x)} &= \lim_{x \to \infty} \frac{1 + \cos^2 x - \sin^2 x}{e^{\sin x} \, \cos x \, (x + \sin x \, \cos x + 2 \, \cos x)} \\ &= \lim_{x \to \infty} \frac{\cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x}{e^{\sin x} \, \cos x \, (x + \sin x \, \cos x + 2 \, \cos x)} \\ &= \lim_{x \to \infty} \frac{2 \cos^2 x}{e^{\sin x} \, \cos x \, (x + \sin x \, \cos x + 2 \, \cos x)} \\ &= \lim_{x \to \infty} \frac{2 \cos x}{e^{\sin x} \, (x + \sin x \, \cos x + 2 \, \cos x)} = 0. \end{aligned}$$

(I left out some steps in computing the derivative of the denominator.) Therefore, by l'Hôpital's rule, the limit vanishes. Or…does it? Note that $g(x) = e^{\sin x} \, f(x)$, hence

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{1}{e^{\sin x}}$$

which does not exist. Where did I make a mistake?