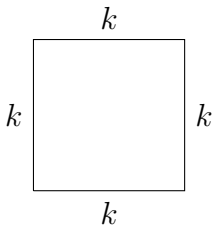
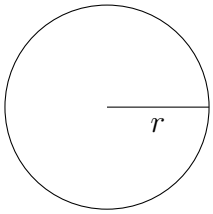
Isoperimetric inequality — Math 1181H

Autumn 2012 — Jim Fowler

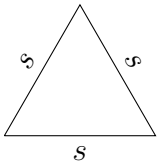
How does perimeter compare to enclosed area? Here are familiar Euclidean examples.



perimeter $= 4k$
area $= k^2$

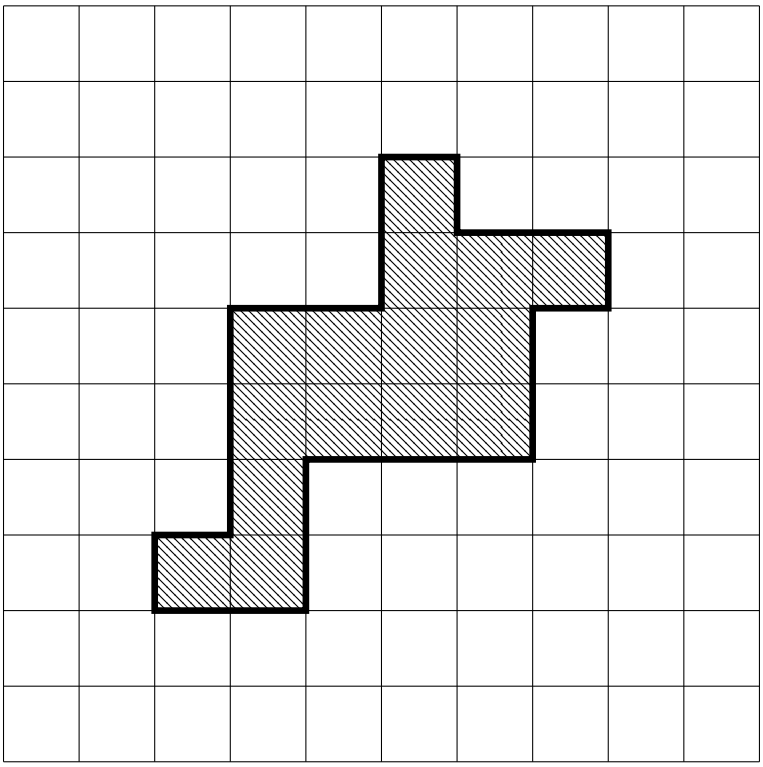


perimeter $= 2\pi r$
area $= \pi r^2$



perimeter $= 3s$
area $= \sqrt{3}s^2/4$

For a given length, how large an area can we enclose? We can ask this question in a combinatorial setting: the "curve" must follow the edges of the graph paper, and each square in the graph paper is a unit square.



perimeter $= 24$
area $= 15$

Can you do better? Yes, you can. There is space on the back for you to try.

length of "curve"	maximum enclosed area

Definition.
The Dehn function $D(n)$ = the maximum area enclosed by a "curve" of length n.