Problem Set 6 — Math 765

Winter 2011 — Jim Fowler

Stokes' theorem. At the end of this week, we'll have Stokes' theorem, a beautiful generalization of the fundamental theorem of Calculus. As usual, email me with questions at `fowler@math.osu.edu`. *The exercises below should be handed in on Monday, February 14, 2011.*

Problem 6.1 (Lee 13–3)

Suppose M and N are oriented smooth manifolds and $F : M \to N$ is a local diffeomorphism. If M is connected, show that F is either orientation-preserving or orientation-reversing.

Problem 6.2 (Lee 14–21)

Suppose M and N are compact, connected, oriented smooth manifolds and $F, G : M \to N$ are diffeomorphisms. If F and G are homotopic, show that they are either both orientation-preserving or both orientation-reversing.

Hint: Use Whitney approximation theorem (page 252) and Stokes' theorem on $M \times I$.

Problem 6.3 (Lee 13–6)

Show that $\mathbb{R}P^n$ is orientable iff n is odd.

Problem 6.4 (Lee 14–5a)

Suppose $\tilde{M}$ and M are smooth n-manifolds, and $\pi : \tilde{M} \to M$ is a smooth k-sheeted covering map. If $\tilde{M}$ and M are oriented and π is orientation-preserving, show that $\int_{\tilde{M}} \pi^\star \omega = k \int_M \omega$ for any compactly supported n-form ω on M.

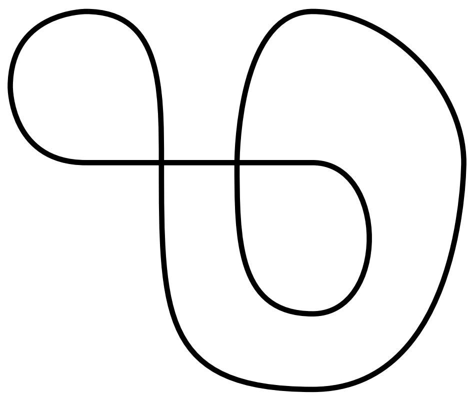
Problem 6.5 (Bicycle chains)

(a) Let $i : S^1 \to \mathbb{R}^2$ be the standard embedding of the circle in the plane, $i(\theta) = (\cos\theta, \sin\theta)$, and let $j : S^1 \to \mathbb{R}^2$ be the figure eight immersion

$$j(\theta) = (\cos\theta, \sin(2\theta)) \,.$$

Can you connect j and i by a smooth family of immersions? In other words, is there a smooth map $F : S^1 \times I \to \mathbb{R}^2$ so that each $f_t : S^1 \to \mathbb{R}^2$ given by $f_t(\theta) = F(\theta, t)$ is an immersion and $f_0 \equiv i$ and $f_1 \equiv j$?

(b) Let $i : S^1 \to \mathbb{R}^2$ be the standard embedding of the circle in the plane, and let $j : S^1 \to \mathbb{R}^2$ be the immersion



Can you connect j and i by a smooth family of immersions?

Problem 6.6 (Lee 14–6)

If M is a compact, smooth, oriented manifold with boundary, show that there does not exist a smooth retraction of M onto its boundary.

Hint: Consider an orientation form on ∂M.