Lecture 9: More improper integrals

Math 153 Section 57

Friday October 17, 2008

Continuing in chapter 11.7.
Also, review for the test.

1 Insufficiently impressed with logarithms?

How about quartersquares?

2 Review for the test

3 Existence without computation

Sometimes we can show that an improper integral converges without actually doing the calculation. This is the comparison test.

If f and g are cts on $[a, \infty)$ and $0 \leq f(x) \leq g(x)$, then $\int_a^\infty g(x)\, dx$ converges implies $\int_a^\infty f(x)\, dx$ converges. And the contrapositive.

Example: $\int_1^\infty dx/\sqrt{1 + x^{100}}$ converges because

$$\frac{1}{\sqrt{1 + x^{100}}} < 1/x^{50}$$

as long as $x \geq 1$.

Example: $\int_1^\infty dx/\sqrt{1 + x^{100} + \sin^2 x}$ converges because

$$\frac{1}{\sqrt{1 + x^{100} + \sin^2 x}} < \frac{1}{\sqrt{1 + x^{100}}}$$

as long as $x \geq 1$.

Example: $\int_2^\infty dx/\sqrt{x^2 - \sin^2 x}$ diverges because

$$\frac{1}{x} < \frac{1}{\sqrt{x^2 - \sin^2 x}}$$

4 If only I could l'Hôpital sequences...

Now you can!

The following might be called a "l'Hôpital's rule for sequences." Here, the differences of successive terms take the place of the derivative:

Theorem (Stolz-Cesàro)**.** *Let a_n and b_n be sequences of real numbers, both unbounded. Assume b_n is positive and increasing. If*

$$\lim_{n\to\infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = L$$

then $\lim\limits_{n\to\infty} a_n/b_n = L$.

5 Review ϵ-K proofs