Solution Set 1

Wednesday, October 1, 2008

1. For each of the following sets, find the least upper bound (if it exists) and the greatest lower bound (if it exists).

   (a) $(3, 5)$.
   The least upper bound is 5. The greatest lower bound is 3.

   (b) $[-4, 17)$.
   The least upper bound is 17. The greatest lower bound is -4.

   (c) $[0, \infty)$.
   There is no upper bound. The greatest lower bound is 0.

   (d) $\{x \in \mathbb{R} : x < 6\}$.
   The least upper bound is 6. There is no lower bound.

   (e) $\{x \in \mathbb{R} : x^2 < 2\}$.
   The least upper bound is $\sqrt{2}$. The greatest lower bound is $-\sqrt{2}$.

   (f) $\{x \in \mathbb{R} : |x - 1| < 3\}$.
   The least upper bound is 4. The greatest lower bound is -2.

2. Which of the sets in Problem 1 are bounded?

   Only the sets in parts **(a)**, **(b)**, **(e)**, and **(f)** are bounded.

3. Suppose S is a bounded set of real numbers, and T is a subset of S. Is T also bounded? Why or why not?

   Yes: if S is bounded, and T is a subset of S, then T is bounded. Why? Because an upper (or lower) bound for S is also an upper (or lower) bound for T.

   Suppose b is a upper bound for S; by definition, this means that for all $s \in S$, we have $b \geq s$.

   But T is a subset of S, so for all $t \in T$, we also have $t \in S$; since b is bigger than (or equal to) everything in S, we have $b \geq t$.

   We conclude that for all $t \in T$, we have $b \geq t$. This is what we mean when we say that b is an upper bound for T, so T is bounded above.

   A similar argument shows that a lower bound for S is also a lower bound for T.