Lecture 40: Weierstrass' theorem

Math 660—Jim Fowler

Monday, August 16, 2010

Pictures of harmonic functions

The Poisson integral produces a harmonic function, given values on the boundary.

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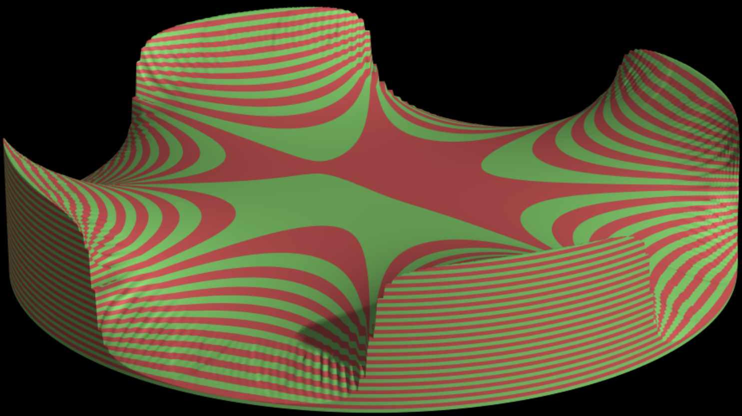
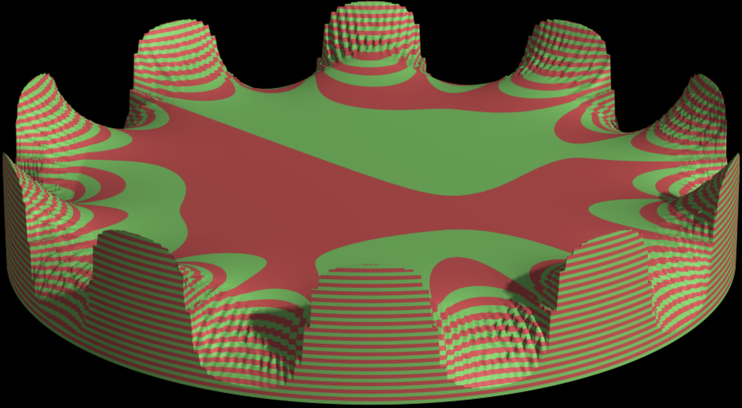
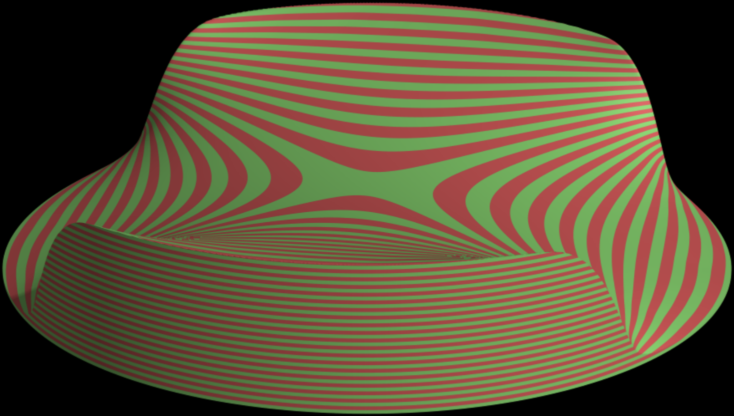
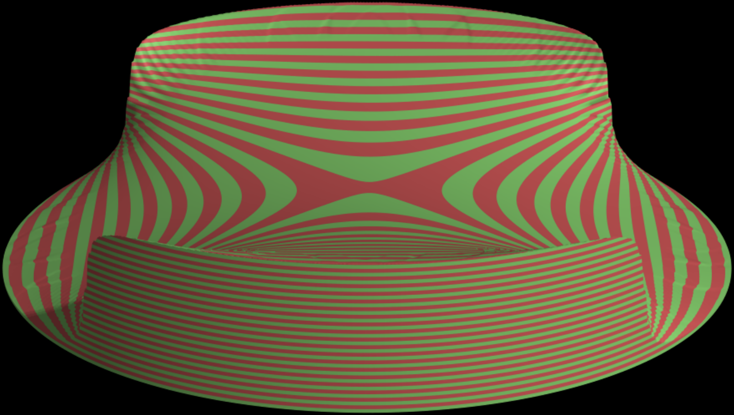
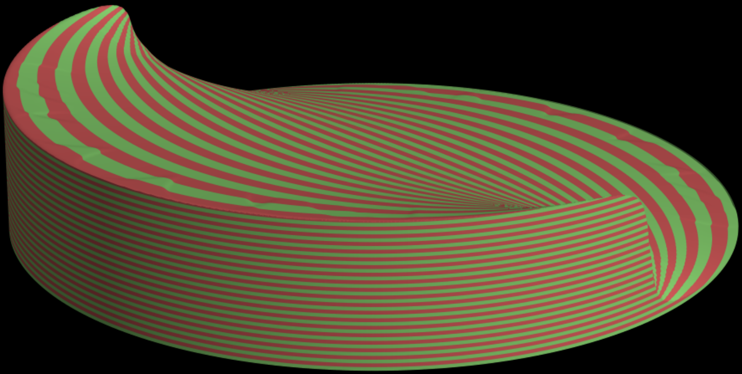
Let's see some examples!

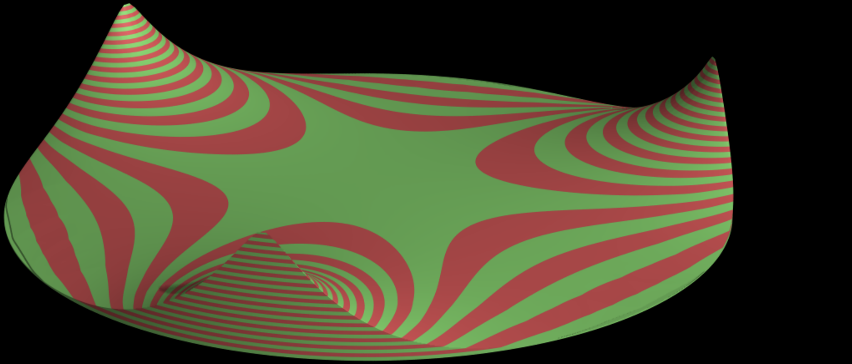


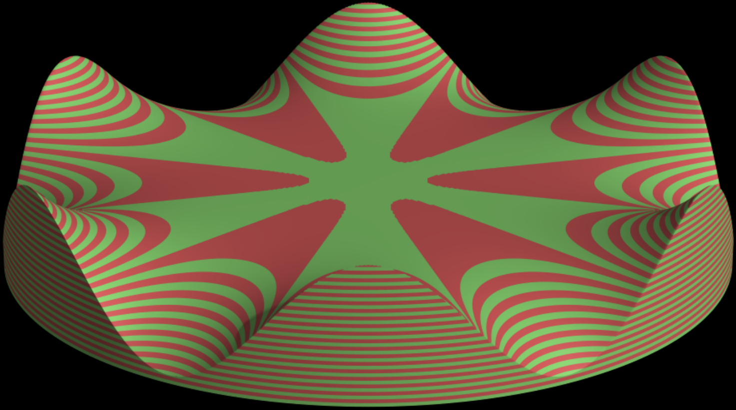


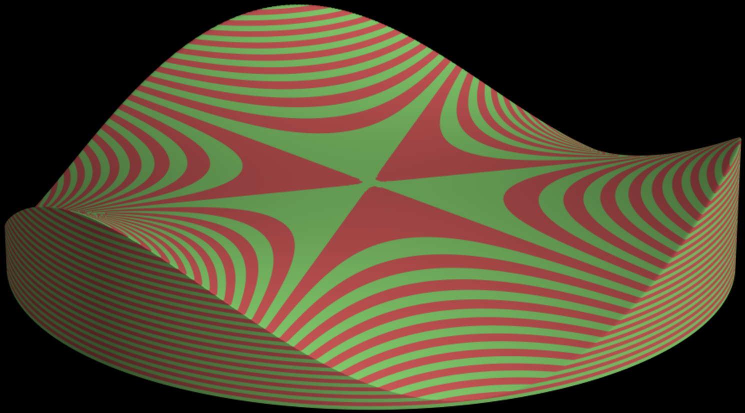


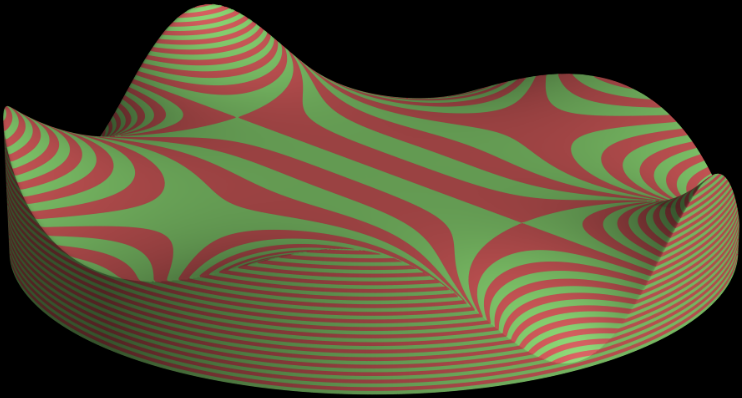


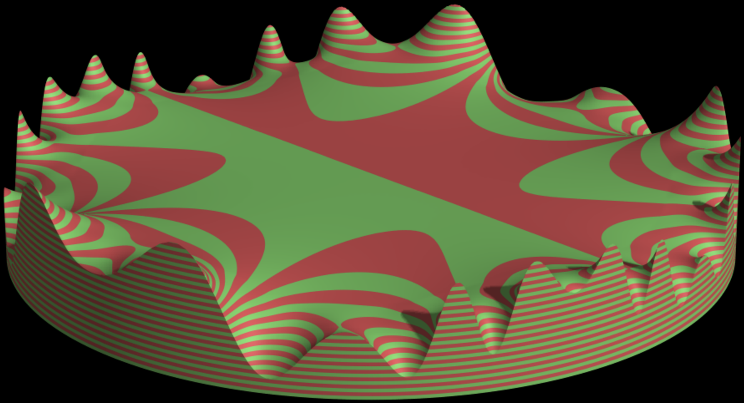












Power series

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Analytic means has a power series.

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Analytic means has a power series.

Now we show that holomorphic implies analytic.

Convergence of analytic functions

Suppose $f_n(z)$ is analytic in Ω_n
and $f_n(z) \to f(z)$ in a region Ω,
uniformly on compact subsets of Ω.
Then $f(z)$ is analytic in Ω, and
$f_n'(z) \to f'(z)$ uniformly on compact subsets.

Convergence of analytic functions

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Proof:

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Proof: Morera's theorem.

Zeroes and convergence

If $f_n(z) \neq 0$ in a region Ω,
and $f_n(z) \to f(z)$ uniformly on compact subsets of Ω
then $f(z)$ is either everywhere or nowhere zero.

Taylor series