Problem Set 8 — Math 765

Winter 2011 — Jim Fowler

Foliations. Instead of a "smoothly varying vector" (that is, a vector field0, we consider a "smoothly varying subspace." Email me with questions at `fowler@math.osu.edu`. *The exercises below should be handed in on Monday, February 28, 2011.*

Problem 8.1 (Lee 19–3)

Let $D \subset TM$ be a smooth distribution, and let $\mathcal{D}(M) \subset \mathcal{T}(M)$ denote the space of smooth global sections of D. Show that D is involutive if and only if $\mathcal{D}(M)$ is a Lie subalgebra of $\mathcal{T}(M)$

Problem 8.2 (Lee 19–12)

Let D be an involutive distribution on a smooth manifold M, and let N be a connected integral manifold of D. If N is a closed subset of M, show that N is a maximal connected integral manifold and is therefore a leaf of the foliation determined by D.