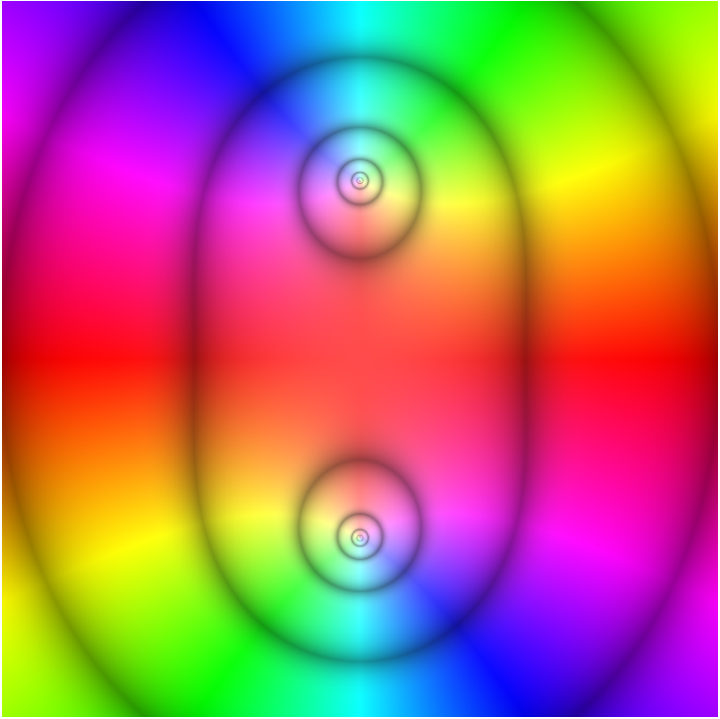
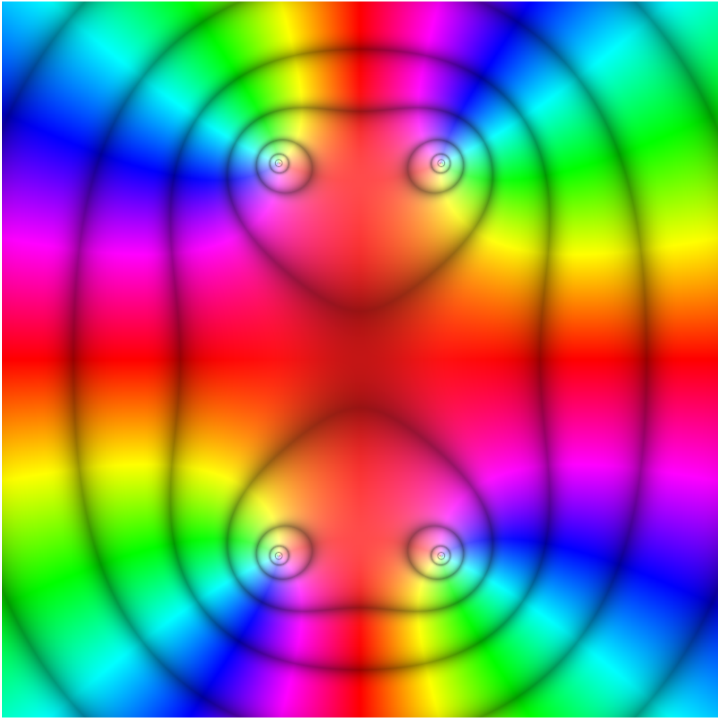
Lecture 10: Continuous functions and topological spaces

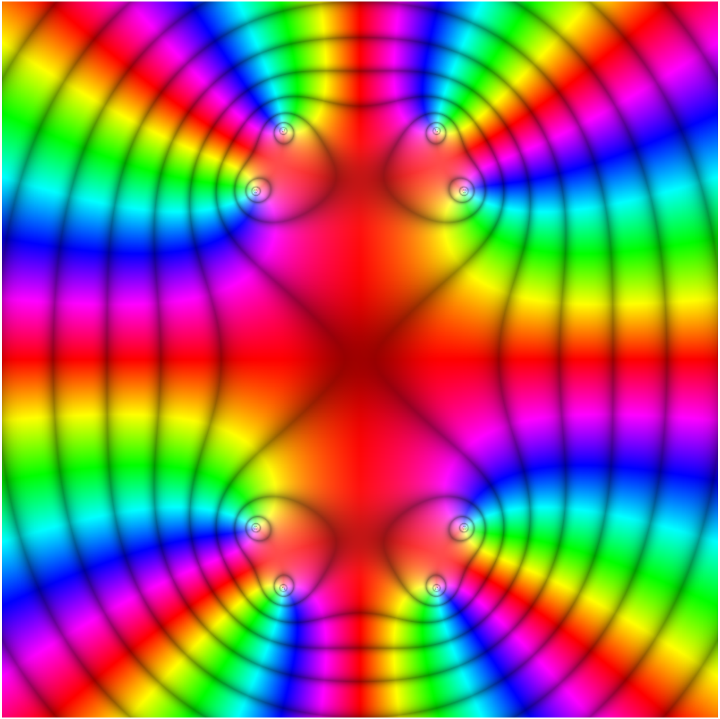
Math 660—Jim Fowler

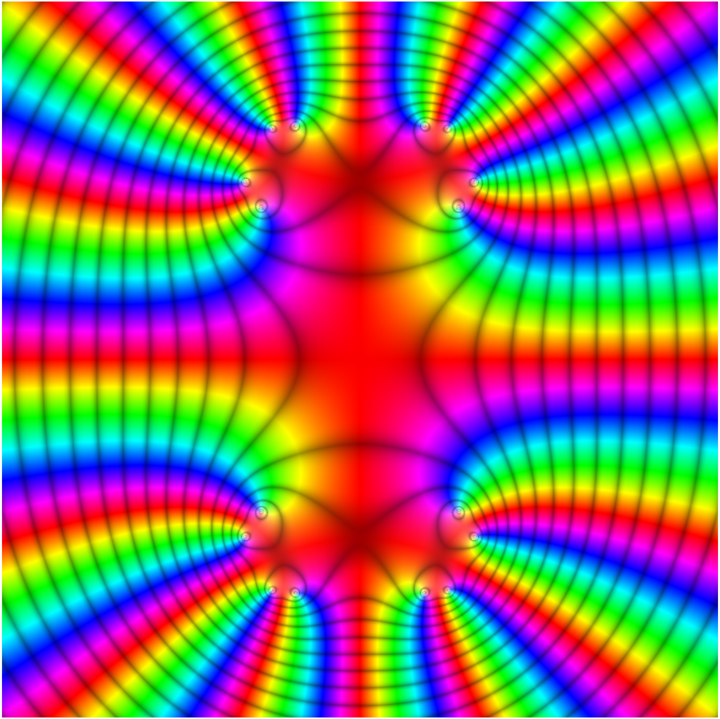
Friday, July 1, 2011

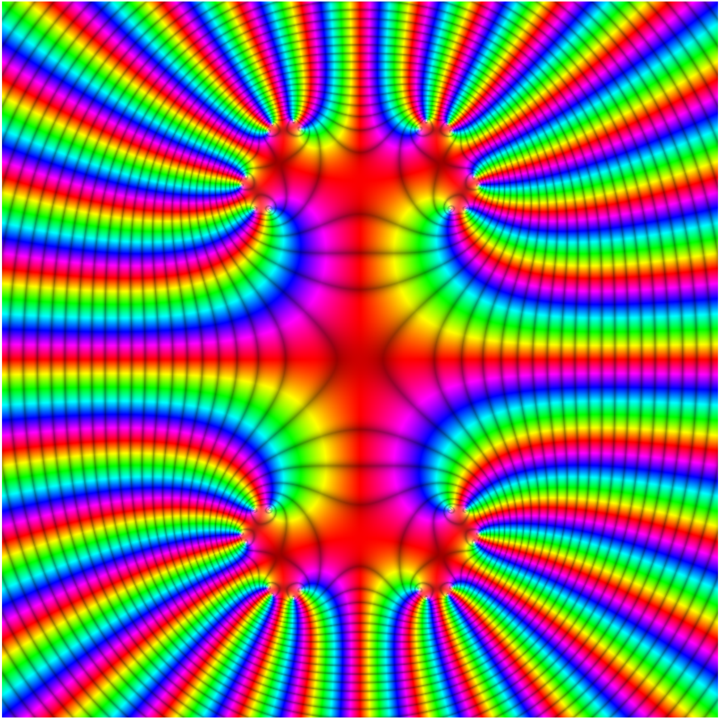
$$f(z) = z^2 + 1$$

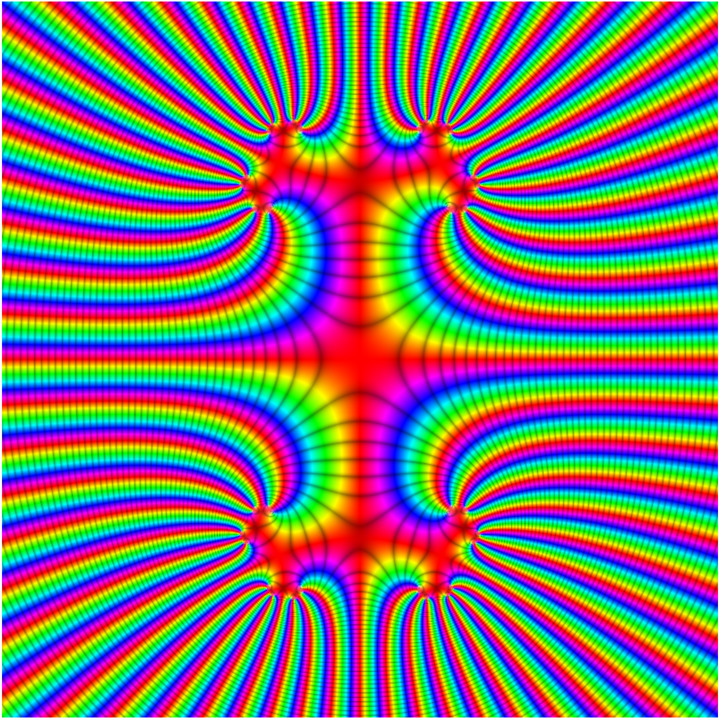


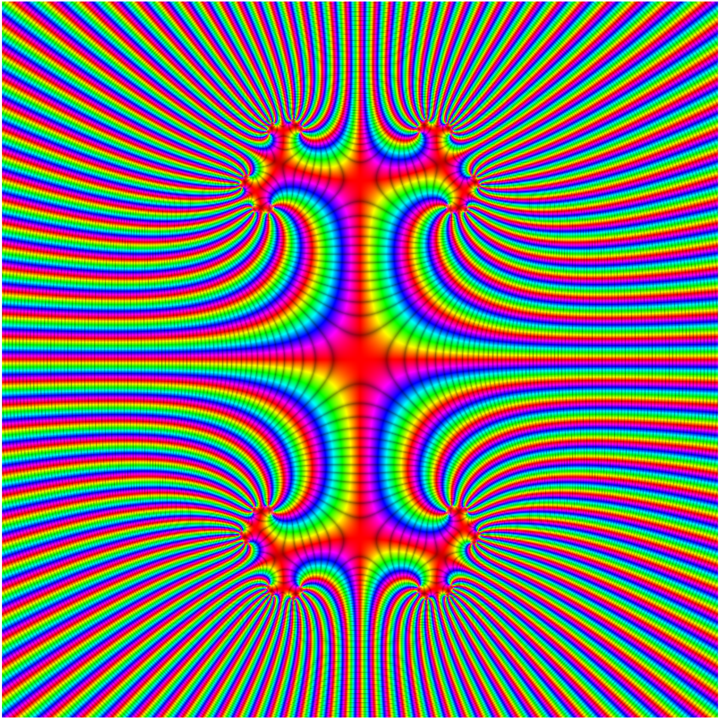


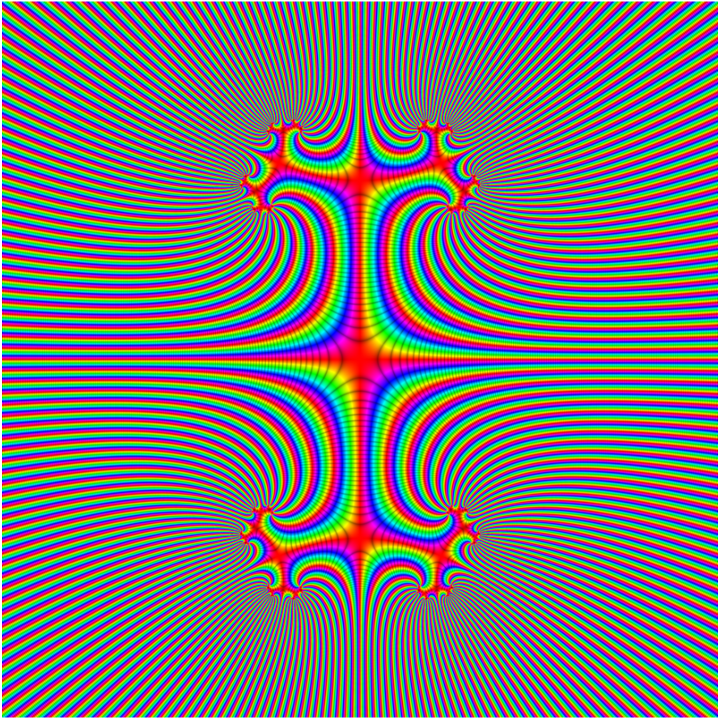


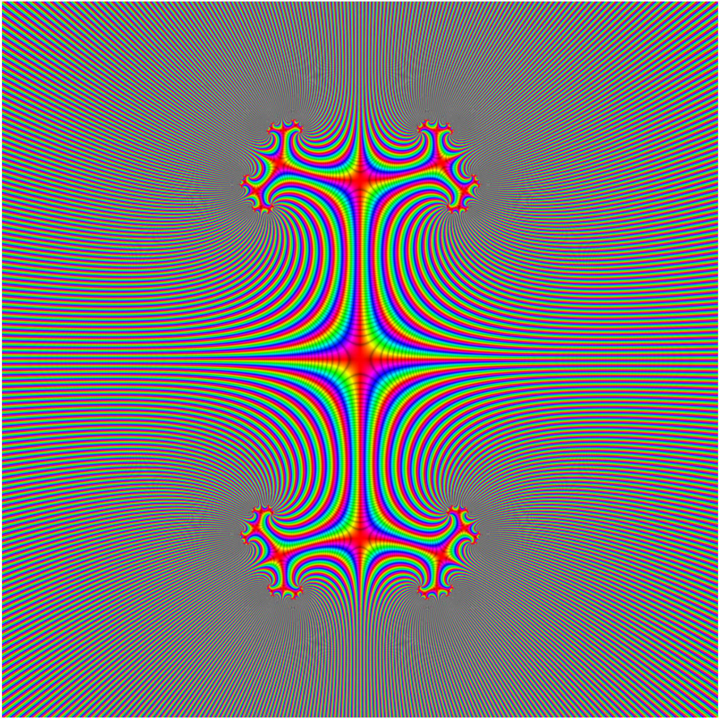




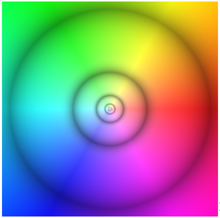


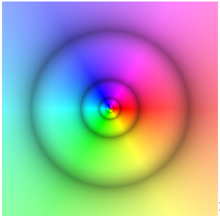


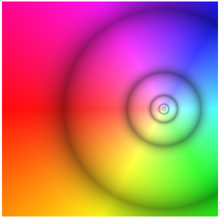


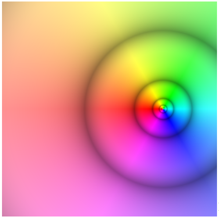


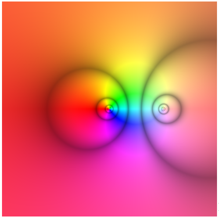
Consider $f(z) = 1/z$ and $g(z) = 1 - z$.

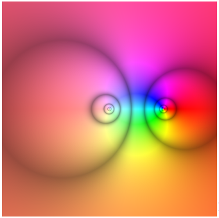

$= \mathrm{id}$


$= f$


$= g$


$= f \circ g$


$= g \circ f$


$= g \circ f \circ f$

- Simple arc?
- Closed curve?
- Simple closed curve?
- Opposite arc?

Arcs and closed curves

Slogan: probe an object by considering curves on the object.

Angle between arcs?

A **region** is an open, connected set.
Usually denoted by Ω.

Analytic in a region

A function $f : \Omega \to \mathbb{C}$ is analytic at $x \in \Omega$ if it is the restriction to Ω of a function analytic in an open set $U \supset \Omega$.

An analytic function in a region Ω whose derivative vanishes identically is a constant.

An analytic function in a region Ω whose real part is constant, is constant.

An analytic function in a region Ω whose real part is constant, is constant.

If $f = u + iv$, and u is constant, then

$$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} = 0$$

so f is constant.

An analytic function in a region Ω whose imaginary part is constant, is constant.

An analytic function in a region Ω whose imaginary part is constant, is constant.

If $f = u + iv$, and v is constant, then

$$f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x} = 0$$

so f is constant.

An analytic function in a region Ω whose modulus is constant, is constant.

An analytic function in a region Ω whose modulus is constant, is constant.

If $f = u + iv$, and $u^2 + v^2$ is constant, then

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = 0 \text{ and } u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} = 0$$

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If $f = u + iv$, and $u^2 + v^2$ is constant, then

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Now use Cauchy-Riemann equations—but how?

An analytic function in a region Ω whose argument is constant, is constant.

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Choose k so that $u = kv$.

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Choose k so that $u = kv$.

Then the real part of $(1 + ik)f$ is constant, so f is constant.