Homework 10

Due Friday, November 7, 2008

(a) On page 601, in section 12.5, do problems: 5, 6, 14, 20, 22.

(b) Find a fifth-degree Taylor polynomial (i.e., up to and including an x^5 term) for each of the following functions:

(i) $f(x) = \sin x + \cos x$.

(ii) $g(x) = \cos(x^2)$.

(iii) $h(x) = \tan x$.

(iv) $u(x) = \log \cos x$.

You can find these polynomials using tricks—for example, try substituting x^2 into a Taylor polynomial for $\cos x$ to find a polynomial for $g(x)$.

(c) Consider a function $p : \mathbb{R} \to \mathbb{R}$ defined by a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0.$$

What is the Taylor series for $p(x)$? Justify your answer.

(d) Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 1/(1-x)$. Find the Taylor series $\sum_{n=0}^{\infty} a_n x^n$ for this function, and determine for which values of x the series converges to $f(x)$.