Lecture 12: Convergence

Math 153 Section 57

Friday October 24, 2008

Following chapters 12.2 and 12.3.

Limit of terms of convergent series

Convergence by boundedness

Theorem: series with nonnegative terms converges iff bounded above

Integral test

If f continuous, positive, decreasing on $[1, \infty)$, then

$$\sum_{k=1}^{\infty} f(k)$$

iff $\int_1^\infty f(x)\,dx$ converges (i.e., as an improper integral).

Draw some pictures

apply integral test to prove harmonic series diverge.

prove harmonic series diverges another way.

apply integral test to p-series.

Comparison theorem

if $a_k \geq 0$ and $b_k \geq 0$, and for k large,

$$\sum a_k \leq \sum b_k$$

Then $\sum b_k$ converges implies $\sum a_k$ converges.

Example: $\sum 1/(2k^2 + 3)$

Example: $\sum 1/\log(k + 5)$ by compariing with $1/2k$.