Homework 4

Due Monday, October 13, 2008

(a) On page 552, section 11.4, do problems: 1, 3, 6, 11, 16, 20, 24, 25, 31, 37.

(b) Use the squeezing theorem to prove that

$$\lim_{n\to\infty} \frac{\cos\left(\sqrt{|\sin n|} + 6^n \log n\right)}{n} = 0.$$

(c) This problem is useful if you are stranded and need to compute $\sqrt{2}$. Define a sequence by setting $a_1 = 2$ and $a_{n+1} = a_n/2 + 1/a_n$. Assuming that $\lim_{n\to\infty} a_n = L$ for a positive real number L and that $a_n \neq 0$ for all $n \in \mathbb{N}$, prove that $L = \sqrt{2}$. *Hint:* compute $\lim_{n\to\infty} (a_n/2 + 1/a_n)$ in two different ways.

(d) As a mathematician you must do more than solve problems—you also need to create them! On a separate sheet of paper, define a convergent sequence with an "interesting" limit.