Lecture 30: Chains and cycles

Math 660—Jim Fowler

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Homology

Replace integrals over arcs by integrals over chains.

What is a chain?

Cycles

If a chain is a sum of closed curves, we call it a cycle.

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Theorem

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Define winding number of cycles.

Simply connected

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Examples?

Theorem

A region Ω is simply connected if and only if $n(\gamma, a)$ for all cycles in Ω and all points $a \notin \Omega$.

Homology

A cycle in a region Ω is homologous to zero inside Ω if $n(\gamma, a)$ for all $a \notin \Omega$.

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We write $[\gamma_1] = [\gamma_2] \in H_1(\Omega)$ if $[\gamma_1 - \gamma_2] = [0]$.

Cauchy's theorem

Theorem

If $f(z)$ is analytic in Ω, then

$$\int_\gamma f(z)\,dz = 0$$

for every cycle γ which is homologus to zero in Ω.

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In other words, if the property holds for $1/(z-a)$ with $a \notin \Omega$, then it holds for all analytic f.

By Cauchy's theorem, there exists a single-valued analytic function $F(z)$ so that $F'(z) = f(z)$.

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Corollary

If $f(z)$ is analyic and nonzero in a simply connected region Ω, then it is possible to define single valued analytic branches of $\log f(z)$ and $\sqrt[n]{f(z)}$.