Lecture 17: More Taylor series

Math 153 Section 57

Wednesday November 5, 2008

Continuing in chapter 12.6.

0.1 Recall

- Start with a function f
- Write down a Taylor series
- Does the series converge?
- Does the series converge to f?

How to find a series? Differentiate the function. Or, maybe use a trick: substitute (e.g., $1/(1+x^2)$) or multiply two well-known series together. As we see soon, differentiation also works.

0.2 Taylor series

promised proofs.

0.3 Approximate π

Here is a series

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

Since $\arctan 1 = \pi/4$, we might try to approximate π this way.

0.4 Things you can see from the series directly

For example, $\sin(-x) = -\sin x$ and $\cos x = \cos(-x)$.

0.5 Relationship between e^x and trig functions

$e^{ix} = \cos x + i \sin x$.

0.6 e is irrational

Assume $e = a/b$.

Clever trick: define

$$x = b!(e - \sum_{n=0}^{b} 1/n!)$$

Step 1: x is an integer. Substitute in $e = a/b$ to see this.

Step 2: x is between 0 and 1. Then,

$$0 < x = \sum_{n=b+1}^{\infty} \frac{b!}{n!} \leq \sum_{n=b+1}^{\infty} \frac{1}{(b+1)^{n-b}} = 1/b$$

But there is no integer between 0 and 1.

Challenge: do something similar for e^2.