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Elementary algebra

- Powers
- Roots
- Fractions
- Equations
- Systems of equations

Powers

$a^n \Rightarrow$ a : base
n : exponent

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$$

$a^0 = ?$ $\quad a^0 = 1$

$$\boxed{a^n = a^{n-1} \cdot a}$$
$$a^1 = a^0 \cdot a$$
$$a = a^0 \cdot a \quad / : a$$
$$\frac{a}{a} = a^0$$
$$1 = a^0$$

$$a \cdot a^{-1} = a^0$$
$$a^{-1} = \frac{1}{a}$$

$0^x = 0$ if $x \neq 0$

$0^0 = \lightning$ undefined

$$a^n \cdot a^m = a^{n+m}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$(a^n)^m = a^{n \cdot m}$$
$$(a \cdot b)^n = a^n \cdot b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

1. $(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$
2. $(a-b)(a-b)(a-b) = (a-b)^3$
3. $\frac{z^2 : z^5}{z^3 \cdot z^{-4}} = \frac{z^{-3}}{z^{-1}} = z^{-3-(-1)} = z^{-2}$
4. $\frac{4^2 \cdot 6^2}{3^3 \cdot 2^3} = \frac{(2\cdot 2)^2 \cdot (2 \cdot 3)^2}{3^3 \cdot 2^3} = \frac{2^6 \cdot 3^2}{2^3\, 3^3} = 2^{6-3} \cdot 3^{2-3} = 2^3 \cdot 3^{-1} = \frac{8}{3}$
5. $\frac{10^x}{10^5} = 10^{-2}$

   $10^{x-5} = 10^{-2}$

   $x-5 = -2$

   $x = 3$
6. $\left(\frac{xy}{z}\right)^{-2} = 3 \qquad \left(\frac{z}{xy}\right)^6 = ?$

   $\left(\frac{z}{xy}\right)^2 = 3 \qquad \left(\left(\frac{z}{xy}\right)^2\right)^3 = 3^3 = \underline{\underline{27}}$

Roots:

$\sqrt[n]{x}$: nth root of x
x : base
n : degree

$$\boxed{\begin{array}{l} \sqrt[n]{x} \equiv z \\ z^n = x \end{array}}$$

$$z^n = x$$
$$(z^n)^{\frac{1}{n}} = x^{\frac{1}{n}}$$
$$\sqrt[n]{x} = z = x^{\frac{1}{n}}$$
$$\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$$

$\sqrt{\ }$: radical
$\Rightarrow$ positive

$$\sqrt[2]{4} = \sqrt{4} = 2$$
$$4^{\frac{1}{2}} = \begin{cases} 2 \\ -2 \end{cases}$$

$$\sqrt[b]{x^a} = x^{\frac{a}{b}}$$

$$\sqrt[b]{x^a} = z$$
$$z^b = x^a$$
$$(z^b)^{\frac{1}{b}} = (x^a)^{\frac{1}{b}}$$
$$z^{b \cdot \frac{1}{b}} = x^{a \cdot \frac{1}{b}}$$
$$\sqrt[b]{x^a} = z = x^{\frac{a}{b}}$$
$$\sqrt[b]{x^a} = x^{\frac{a}{b}}$$

$$\sqrt[b]{x^a} = x^{\frac{a}{b}}$$
$$\sqrt[n]{a \cdot b} = \sqrt[n]{a}\,\sqrt[n]{b} \qquad a, b > 0$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \qquad a, b > 0$$

1.) $\sqrt{1600} = \sqrt{16 \cdot 100} = \sqrt{16}\,\sqrt{100} = 4 \cdot 10 = 40$

2) $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

3) $x^{0.25} = 2$

   $x^{\frac{1}{4}} = 2$

   $x = 2^4 = 16$

4) $(a+b)^{-0.5} = \frac{1}{\sqrt{a+b}}$ T

$$\frac{1}{(a+b)^{0.5}} = \frac{1}{\sqrt{a+b}}$$

$\sqrt{-1} = i \rightarrow$ imaginary unit

$$\sqrt{-4} = \sqrt{4 \cdot (-1)} = \sqrt{4} \cdot \sqrt{-1} = 2 \cdot i$$

$\boxed{2 + 4i}$ Complex number

$a + b = b + a$ commutativity
$a \cdot b = b \cdot a$

$a(b+c) = ab + ac$ multiplication is distributive over addition

$(a+b) + c = a + (b+c)$
$(ab)c = a(bc)$

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$