4)

$$\lim_{n\to\infty} \frac{1}{n(\sqrt{n^2-1}-n)} = \lim_{n\to\infty} \frac{1}{n(\sqrt{n^2-1}-n)} \cdot \frac{\sqrt{n^2-1}+n}{\sqrt{n^2-1}+n} =$$

$$\lim_{n\to\infty} \frac{\sqrt{n^2-1}+n}{n(n^2-1-n^2)} = \lim_{n\to\infty} \frac{\sqrt{n^2-1}+n}{-n} = \lim_{n\to\infty} \frac{\sqrt{n^2-1}}{-n} + \lim_{n\to\infty} \overbrace{\frac{n}{-n}}^{=-1} =$$

$$\lim_{n\to\infty} \frac{\sqrt{n^2-1}}{-n} - 1 = \lim_{n\to\infty} -\frac{\sqrt{n^2-1}}{\sqrt{n^2}} - 1 = \lim_{n\to\infty} -\sqrt{\frac{n^2-1}{n^2}} - 1 =$$

$$\lim_{n\to\infty} -\sqrt{\frac{n^2}{n^2}-\frac{1}{n^2}} - 1 = \lim_{n\to\infty} -\sqrt{1-\underbrace{\frac{1}{n^2}}_{\downarrow\, 0}} - 1 = -\sqrt{1-0}-1 = -1-1 =$$

$$= \underline{\underline{-2}}$$

5)

$$\lim_{n\to\infty} \sqrt[n]{5} = \lim_{n\to\infty} 5^{\frac{1}{n}\to 0} = 5^0 = \underline{\underline{1}}$$

6.) $\lim_{n\to\infty} \ln\left(\boxed{\frac{1}{n}}\right) \Rightarrow -\infty$ Divergent

$\frac{1}{n} \to 0$

$\ln(1) = 0$

$\ln\left(\frac{1}{2}\right) = \sim -1$

$\ln\left(\frac{1}{3}\right)$

$\ln\left(\frac{1}{4}\right)$:

$\ln\left(\frac{1}{5}\right) = \sim -2$

$\ln(0.000000000001) =$

$e^{-\text{large number}}$

7.1

$$\lim_{n \to \infty} e^{-n} = \lim_{n \to \infty} \frac{1}{e^n} = 0$$

$e^n \to \infty$

$a(1) = e$

$a(2) = e^2$

$a(3) = e^3$

Limits of functions

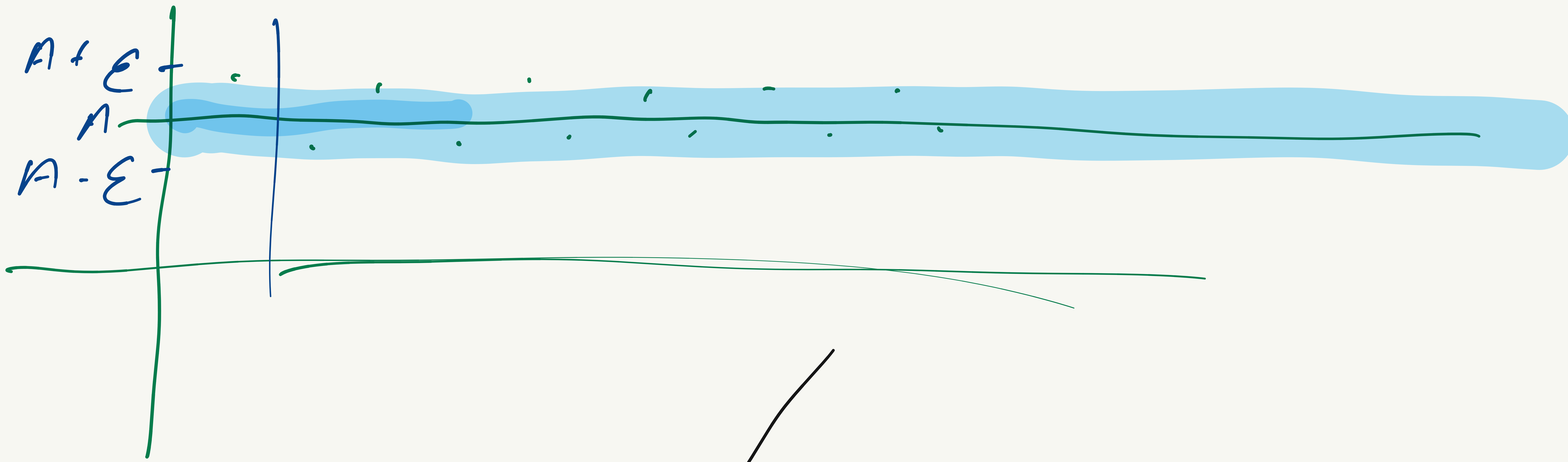
A function $f(x)$ has a limit L at point p if for all $\varepsilon > 0$ $\exists\ \delta > 0$ such that $\forall x$ that satisfies $|x-p| < \delta$ it holds that $|f(x) - L| < \varepsilon$.

$$\lim_{x \to p} f(x) = L$$

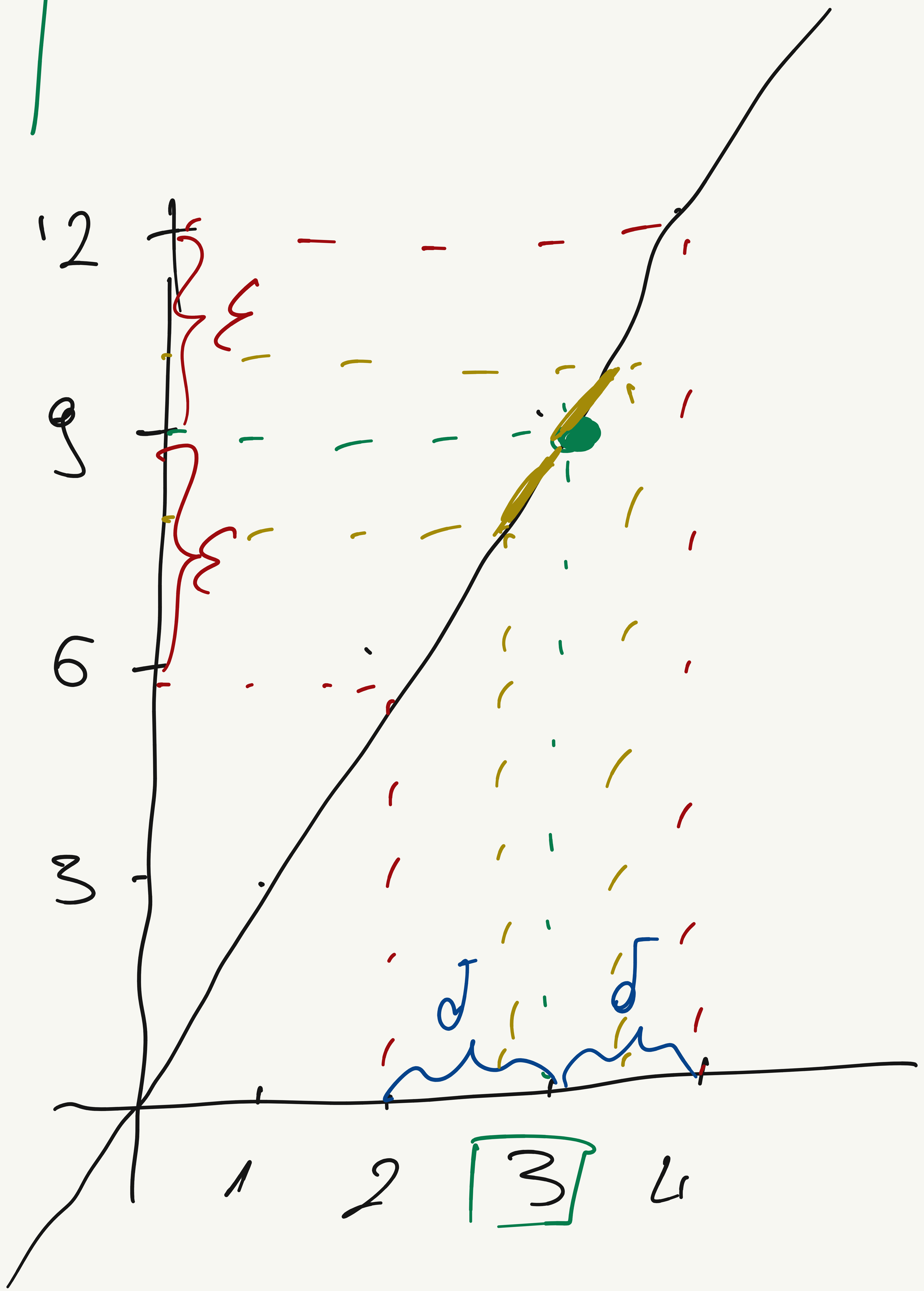
$f(x) = 3x$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} 3x = 9$$

Sequences:



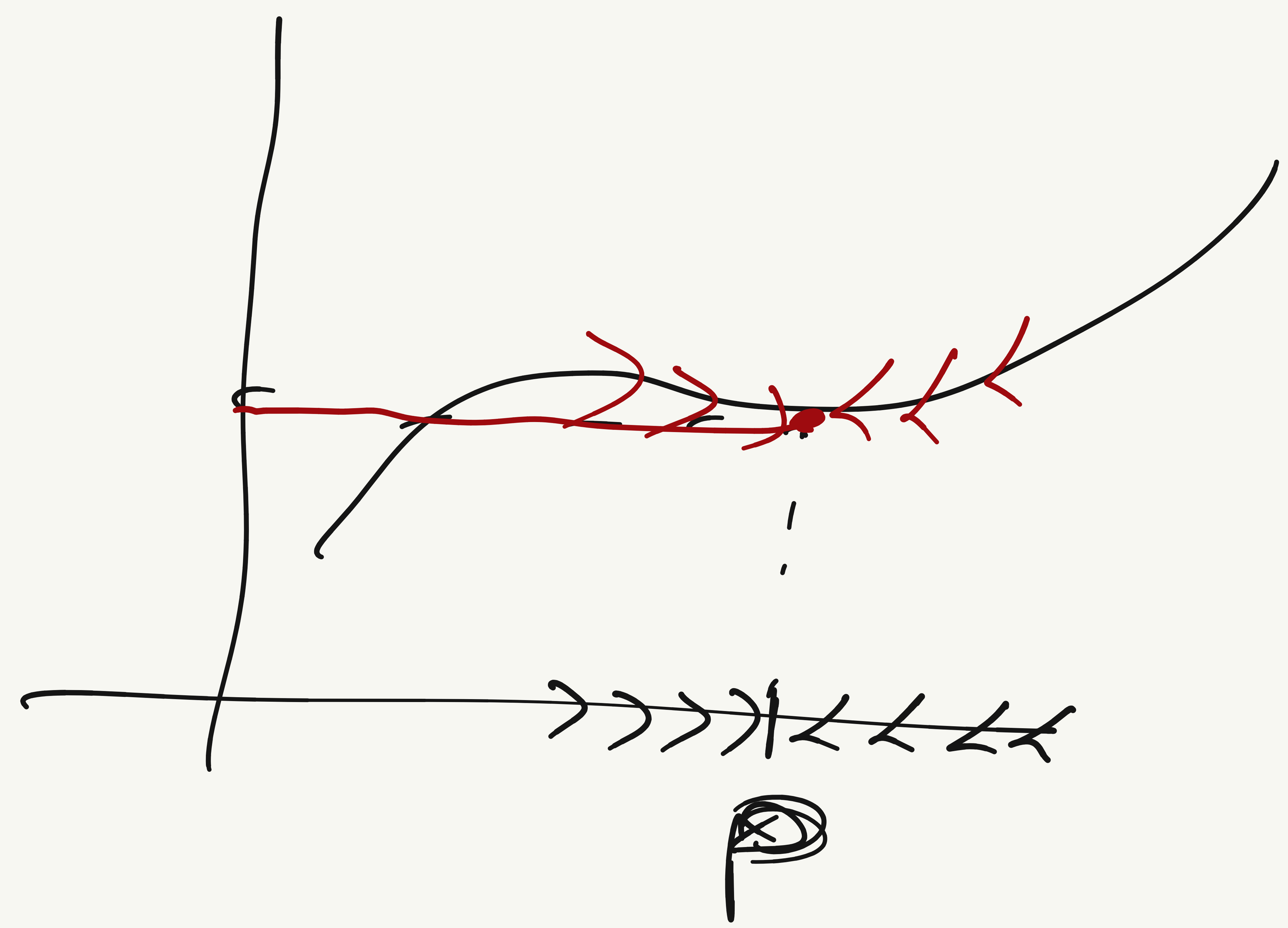
functions:



$f(x) = 3x$

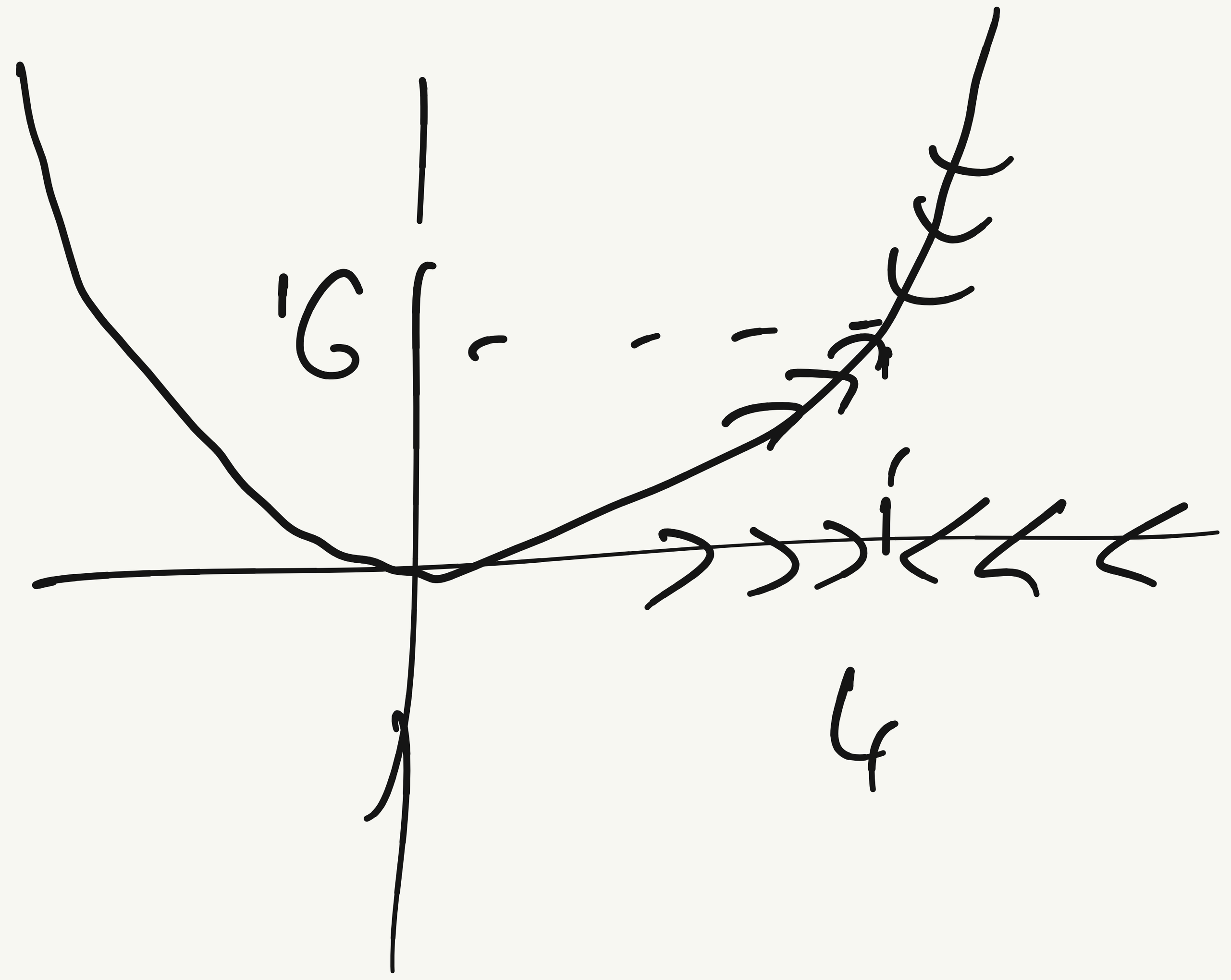
$\varepsilon = 3$

$\delta = 1$



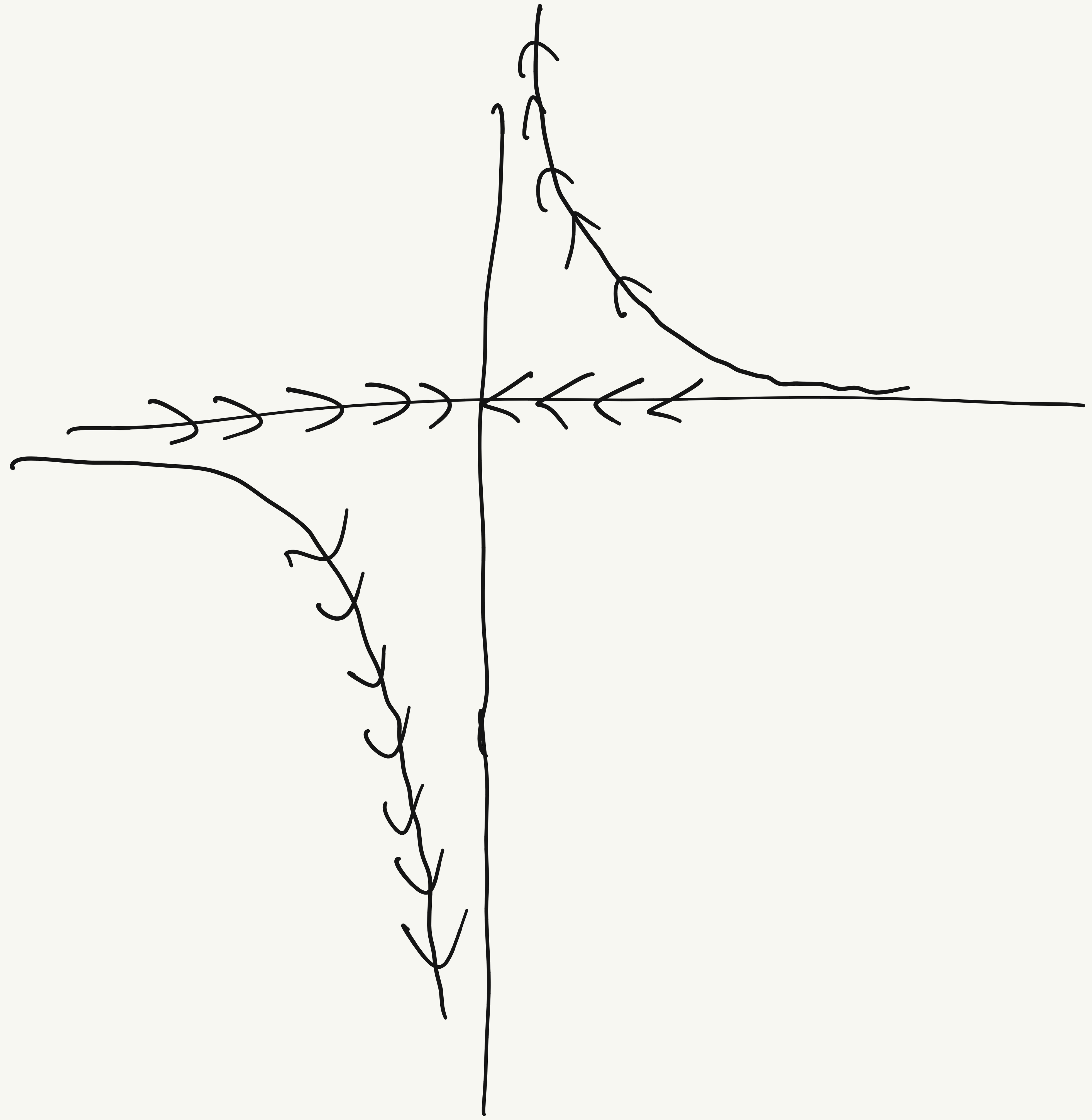
$f(x) = x^2$

$$\lim_{x \to 4} x^2 = 16$$



$f(x) = \frac{1}{x}$

$$\lim_{x \to 0} \frac{1}{x} =$$



Properties

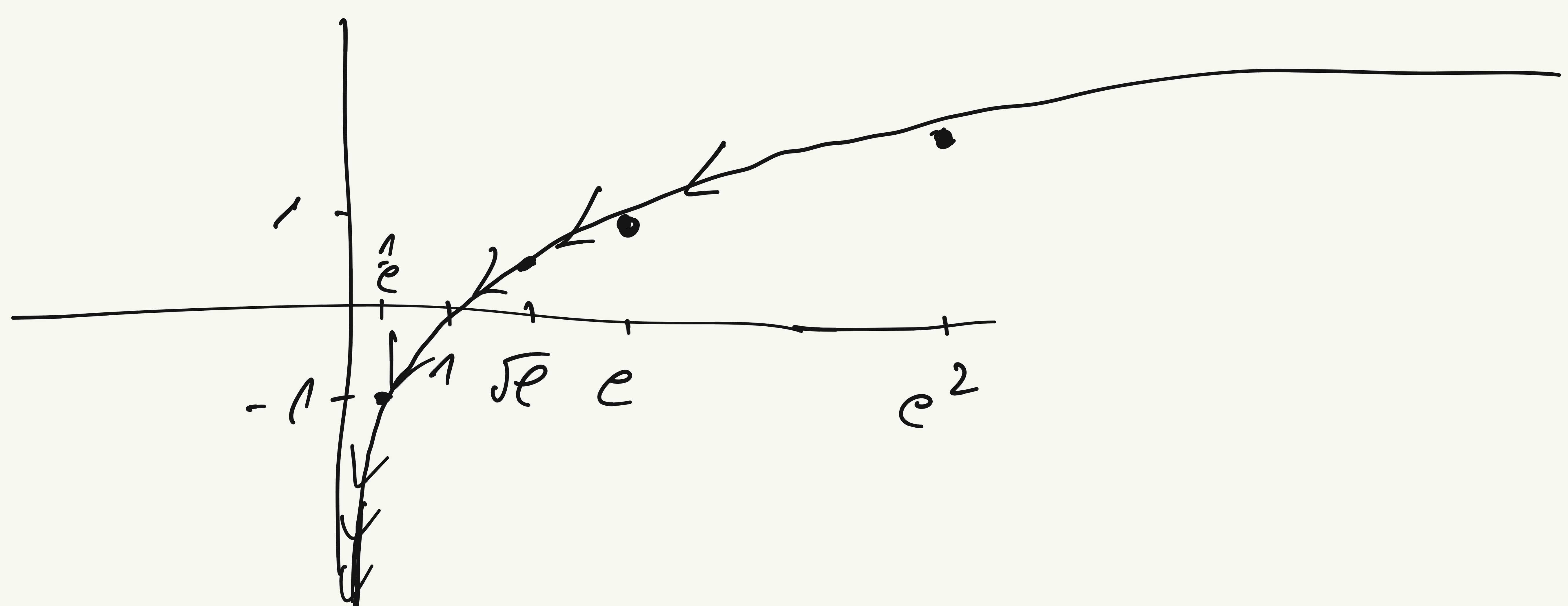
$$\lim_{x \to p} \left(f(x) \pm g(x) \right) = \lim_{x \to p} f(x) \pm \lim_{x \to p} g(x)$$

$$\lim_{x \to p} \left(f(x) \cdot g(x) \right) = \lim_{x \to p} f(x) \cdot \lim_{x \to p} g(x)$$

$$\lim_{x \to p} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to p} f(x)}{\lim_{x \to p} g(x)} \qquad \text{if } \lim_{x \to p} g(x) \neq 0$$

1.1 $\lim_{x \to 5} e^{x-3} = e^{5-3} = e^{2}$

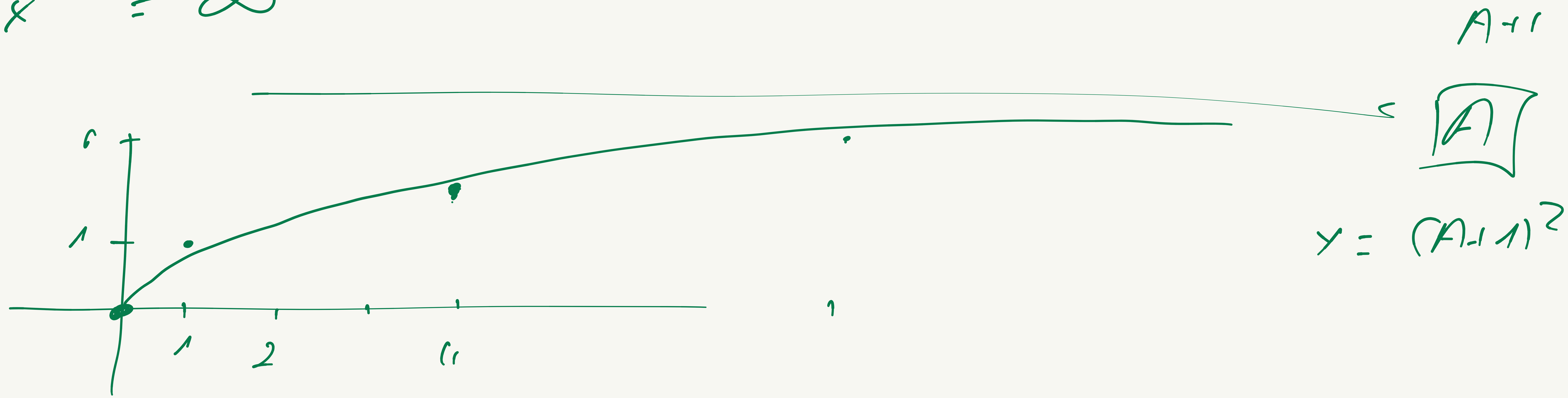
2.1 $\lim_{x \to 0} \ln(x) = -\infty$



3/ $$\lim_{x\to\infty} \frac{x^4-2x^3+x-3}{x^5-2x} = \lim_{x\to\infty} \frac{\frac{1}{x^5}\left(x^4-2x^3+x-3\right)}{\frac{1}{x^5}\left(x^5-2x\right)} =$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}-\frac{2}{x^2}+\frac{1}{x^4}-\frac{3}{x^5}}{1-\frac{2}{x^4}} = \frac{\lim\limits_{x\to\infty} \underbrace{\frac{1}{x}}_{\to 0}-\underbrace{\frac{2}{x^2}}_{\to 0}+\underbrace{\frac{1}{x^4}}_{\to 0}-\underbrace{\frac{3}{x^5}}_{\to 0}}{\lim\limits_{x\to\infty} \underbrace{1}_{1}-\underbrace{\frac{2}{x^4}}_{0}} = \frac{0}{1} = \underline{\underline{0}}$$

4/ $$\lim_{x\to\infty} \sqrt{x} = \infty$$



$$\lim_{x\to\infty} \frac{1}{\sqrt{x}} = \frac{\lim_{x\to\infty} 1}{\lim_{x\to\infty} \sqrt{x}} = \frac{1}{\underbrace{\lim_{x\to\infty} \sqrt{x}}_{\infty}} = 0$$

$$\lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = \infty$$

$$\lim_{x\to\infty} \frac{3x}{x} = \lim_{x\to\infty} 3 = 3$$

$$\lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$$

L'Hôspital's rule

1/ $$\lim_{x\to 0} (3+2x^2) = 3+2\cdot 0^2 = \underline{\underline{3}}$$

2/ $$\lim_{x\to -1} \frac{3+2x}{x-1} = \frac{\lim\limits_{x\to -1} 3+2x}{\lim\limits_{x\to -1} x-1} = \frac{3-2}{-1-1} = \underline{\underline{\frac{1}{-2}}}$$

3/ $$\lim_{x\to 1} \frac{x^2+7x-8}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+8)}{\cancel{(x-1)}} = \lim_{x\to 1} x+8 = \underline{\underline{9}}$$

4/ $$\lim_{x\to \infty} \frac{x^3-3x^2+x-5}{3x^3+5x^2-2} = \lim_{x\to\infty} \frac{\frac{1}{x^3}(x^3-3x^2+x-5)}{\frac{1}{x^3}(3x^3+5x^2-2)} = \frac{\lim\limits_{x\to\infty} \overset{1}{1} - \overset{0}{\frac{3}{x}} + \overset{0}{\frac{1}{x^2}} - \overset{0}{\frac{5}{x^3}}}{\lim\limits_{x\to\infty} \underset{3}{3} + \underset{0}{\frac{5}{x}} - \underset{0}{\frac{2}{x^3}}} =$$

$$= \underline{\underline{\frac{1}{3}}}$$

$$\lim_{x\to 1} \frac{x^2+7x+8}{x-1} = \lim_{x\to 1} \frac{x^2+7x-8+16}{x-1} = \lim_{x\to 1} \frac{(x+8)(x-1)+16}{x-1} =$$

$$= \lim_{x\to 1} \frac{(x+8)(x-1)}{x-1} + \frac{16}{x-1} = \underbrace{\lim_{x\to 1} \frac{(x+8)(x-1)}{x-1}}_{9} + \lim_{x\to 1} \frac{16}{x-1} = 9 + \lim_{x\to 1} \frac{16}{x-1} =$$

$$\Rightarrow \lim_{x\to 1} \frac{16}{\underbrace{x-1}_{0}}$$

$$\lim_{x\to 1^+} \frac{16}{x-1} = \infty$$

$$\lim_{x\to 1^-} \frac{16}{x-1} = -\infty$$

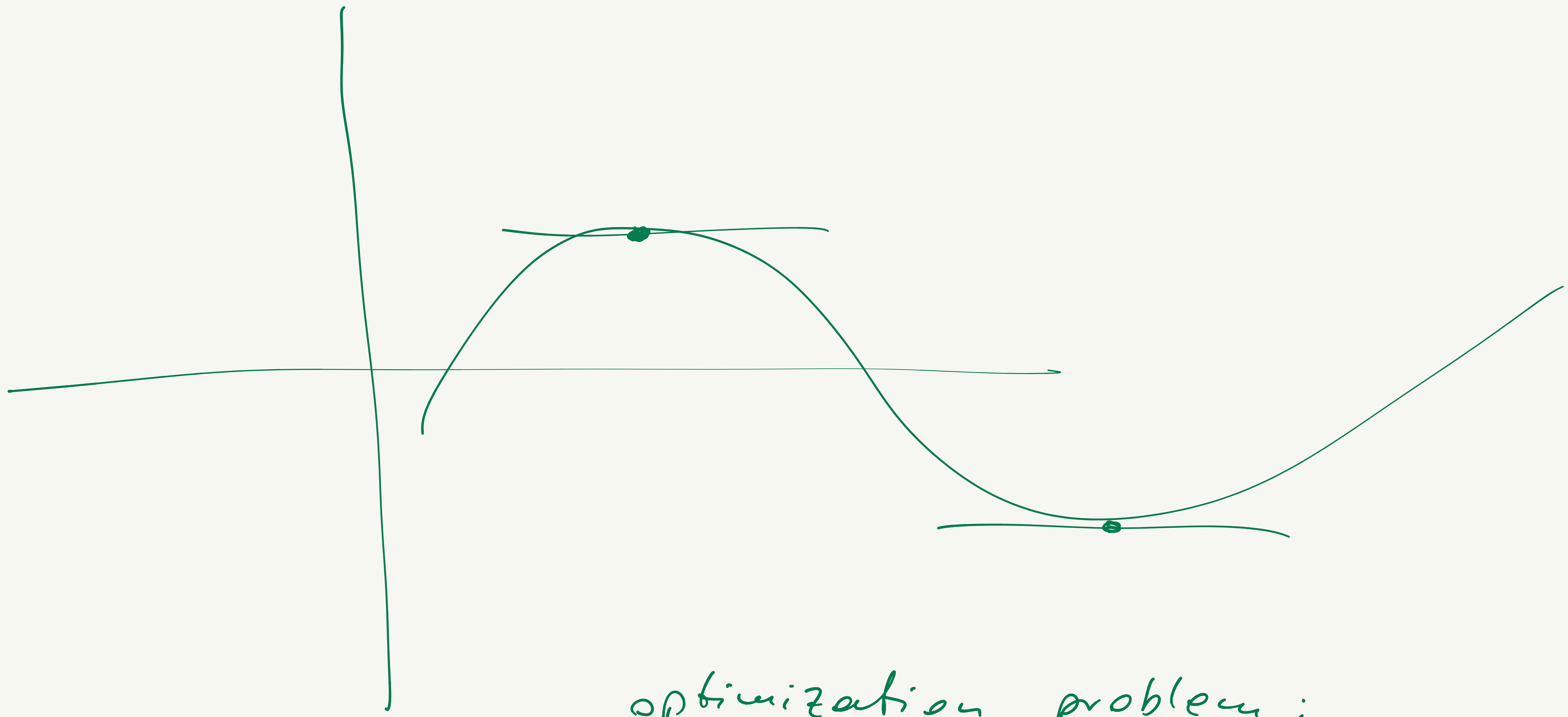
3.1 $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1} x+1 = \underline{\underline{2}}$

6.1 $\lim_{h\to 0} \frac{\sqrt{h+1}-1}{h} = \lim_{h\to 0} \frac{\sqrt{h+1}-1}{h} \cdot \frac{\sqrt{h+1}+1}{\sqrt{h+1}+1} = \lim_{h\to 0} \frac{h+1-1}{h[\sqrt{h+1}+1]} =$

$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}[\sqrt{h+1}+1]} = \lim_{h\to 0} \frac{1}{\underbrace{\sqrt{h+1}}_{1}+1} = \underline{\underline{\frac{1}{2}}}$

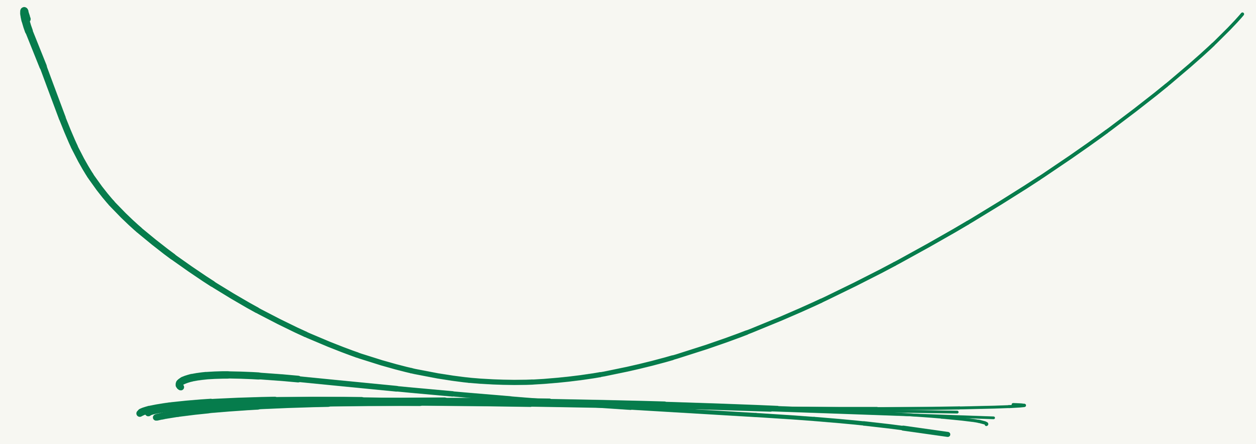
7.1 $\lim_{x\to 5} \frac{3x^2-9x-30}{x-5} = \lim_{x\to 5} \frac{3[x^2-3x-10]}{x-5} = \lim_{x\to 5} \frac{3\cancel{(x-5)}(x+2)}{\cancel{(x-5)}} =$

$\lim_{x\to 5} 3(x+2) = 3(5+2) = \underline{\underline{21}}$



optimization problem:

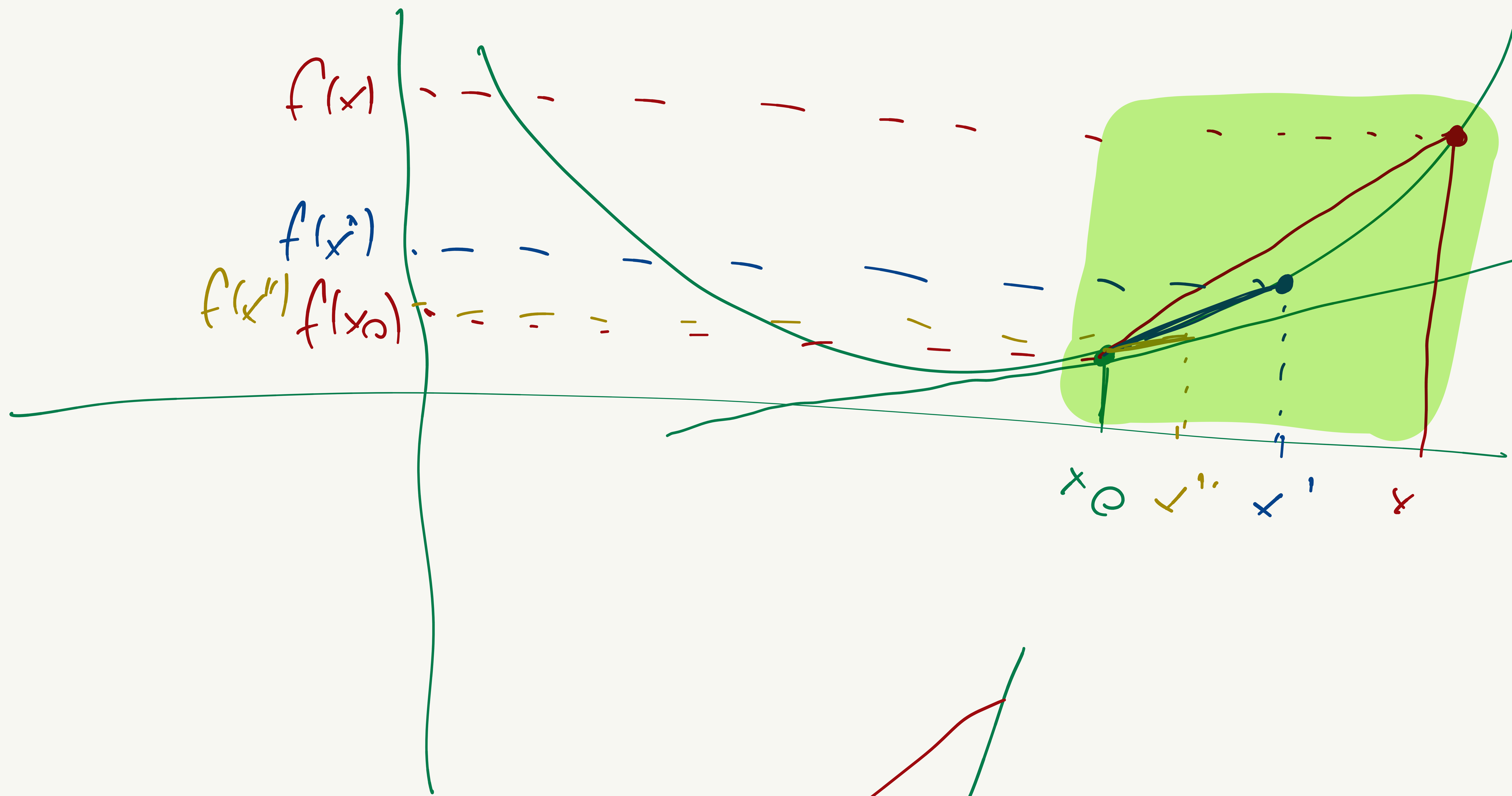
- Loss minimization
- Profit maximization
- Best fitting line



→ Slope of tangent line is 0.

The slope of the tangent line at point P:

= the derivative of the function at point P.

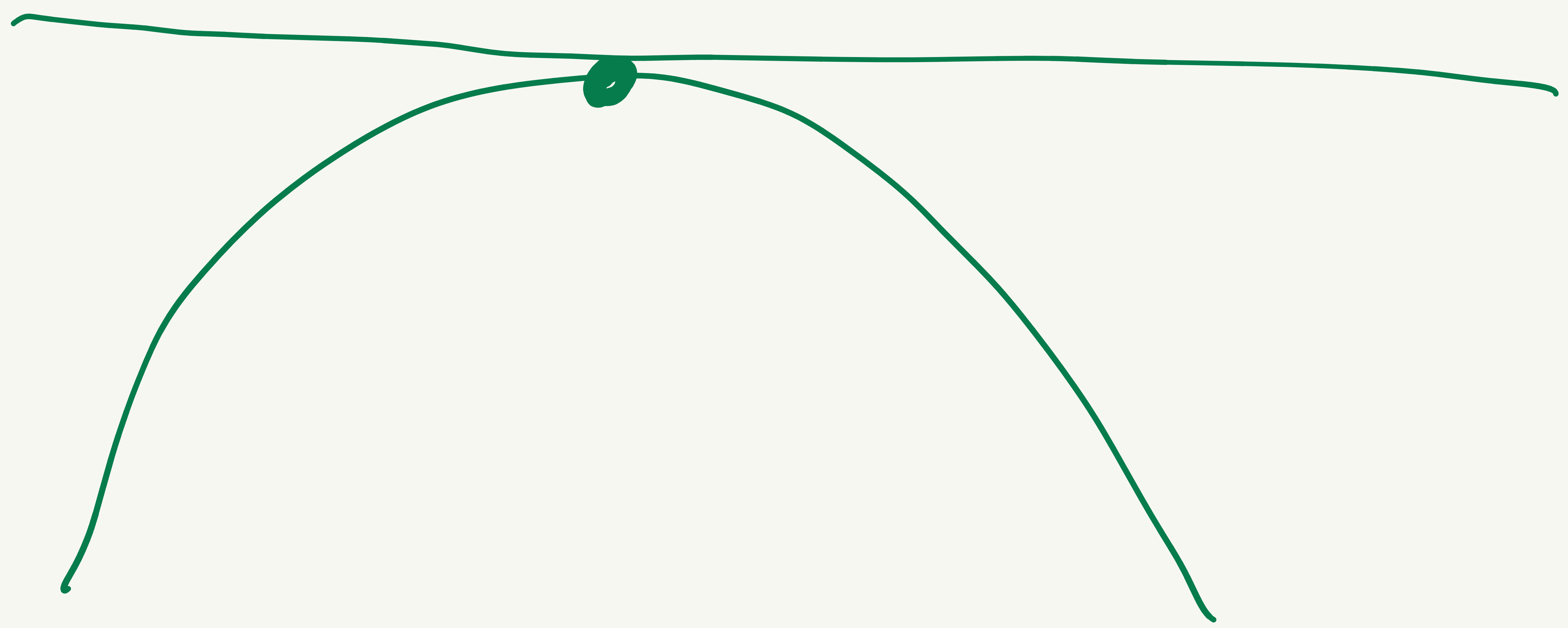


slope: $\dfrac{f(x) - f(x_0)}{x - x_0}$

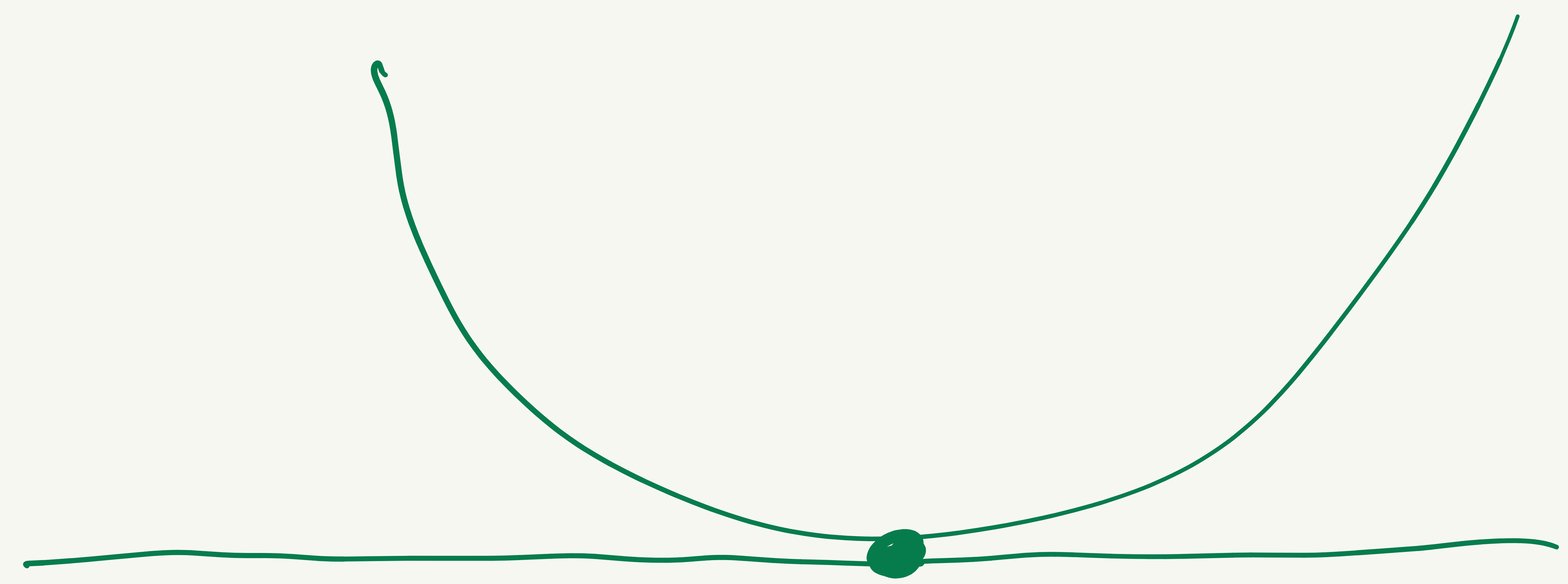
slope: $\dfrac{f(x') - f(x_0)}{x' - x_0}$

slope: $\dfrac{f(x'') - f(x_0)}{x'' - x_0}$

$$\boxed{\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}$$

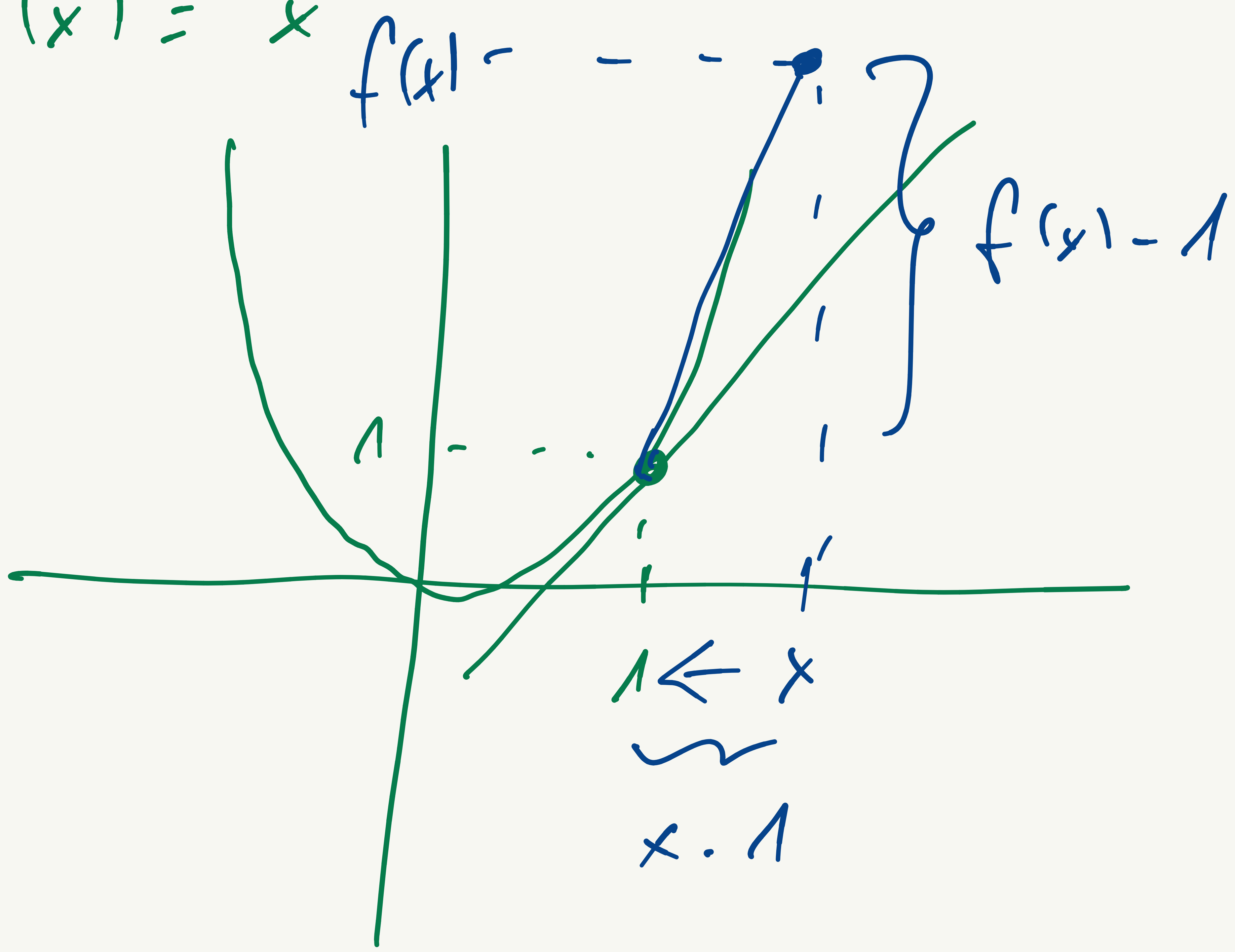


slope : 0



slope : 0

$f(x) = x^2$



Derivative of $f(x) = x^2$ at $x_0 = 1$

$$\boxed{f'(1)}$$

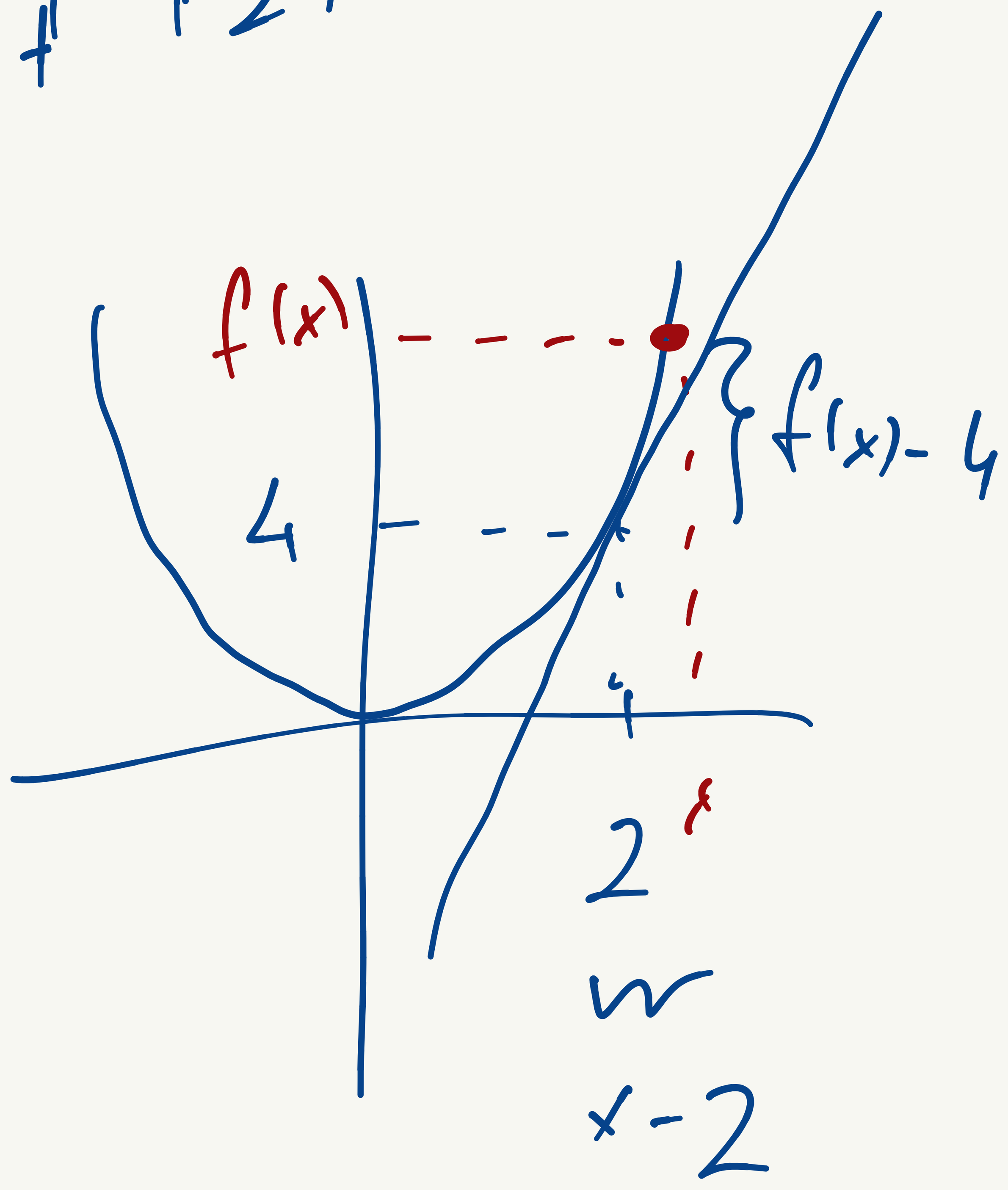
slope: $\dfrac{f(x)-1}{x-1}$

$$\lim_{x\to 1} \frac{f(x)-1}{x-1} = \lim_{x\to 1} \frac{x^2-1}{x-1} =$$

$$= \lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1} x+1 = \underline{\underline{2}}$$

$f(x) = x^2$

$f'(2)$



$$\lim_{x \to 2} \frac{f(x) - 4}{x - 2} = \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} =$$

$$\lim_{x \to 2} x + 2 = \underline{\underline{4}}$$

$$\boxed{f'(2) = 4}$$

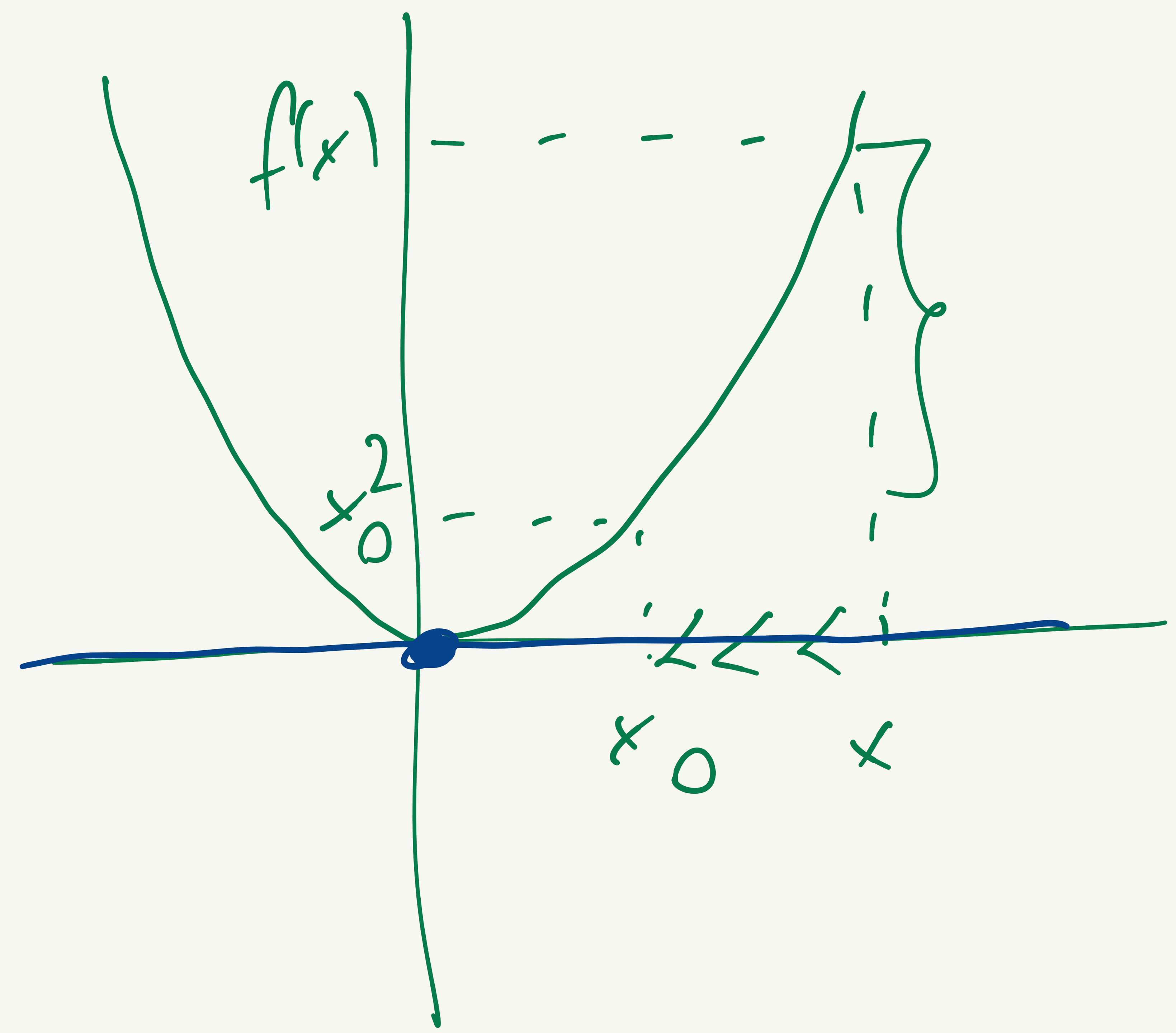
$f(x) = x^2$

$f'(5) =$ $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x - 5} = \lim\limits_{x \to 5} \dfrac{(x-5)(x+5)}{x-5} = \lim\limits_{x \to 5} x + 5 = 10$

$$\underline{\underline{f'(5) = 10}}$$

$f(x) = x^2$

$f'(x_0)$



$$\lim_{x \to x_0} \frac{x^2 - x_0^2}{x - x_0} = \lim_{x \to x_0} \frac{(x - x_0)(x + x_0)}{x - x_0} = \lim_{x \to x_0} x + x_0 =$$

$$= \underline{\underline{2x_0}}$$

$$\boxed{f'(x_0) = 2x_0}$$ Derivative function

$f'(1) = 2 \cdot 1 = 2$

$f'(2) = 2 \cdot 2 = 4$

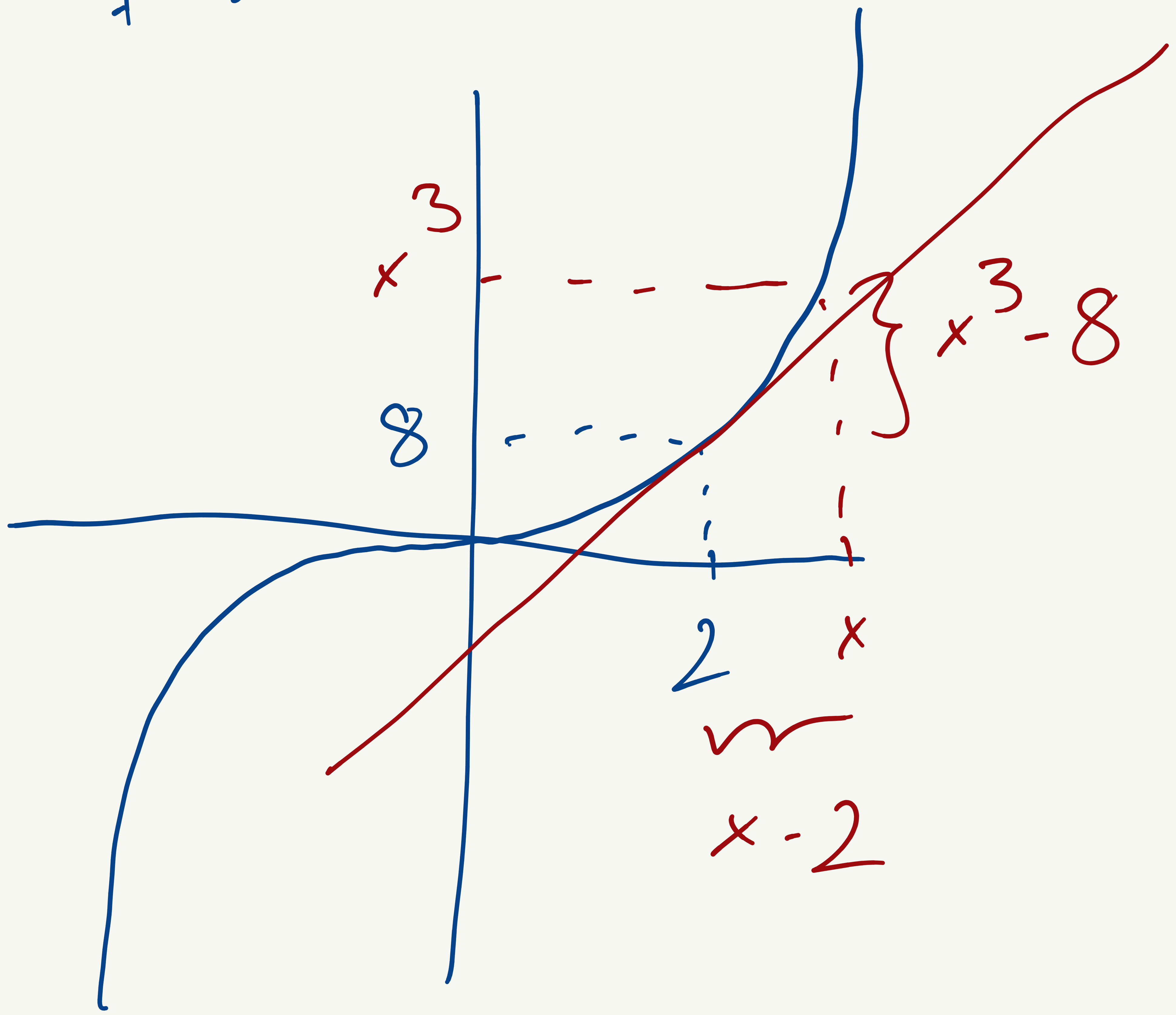
$f'(5) = 2 \cdot 5 = 10$

$f'(x_0) = 2x_0 = 0$

$\underline{\underline{x_0 = 0}}$

$f(x) = x^3$

$f'(2) = ?$



$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4)}{x - 2} =$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^n - b^n$$

$$\lim_{x \to 2} x^2 + 2x + 4 = 4 + 4 + 4 = \underline{\underline{12}}$$

$$\underline{\underline{f'(2) = 12}}$$

$f(x) = x^3$

$f'(5) = ?$

$$\lim_{x \to 5} \frac{x^3 - 125}{x - 5} = \lim_{x \to 5} \frac{(x-5)(x^2+5x+25)}{x-5} =$$

$$\lim_{x \to 5} x^2 + 5x + 25 = 25 + 25 + 25 = \underline{\underline{75}}$$

$f(x) = x^3$

$f'(x_0)$

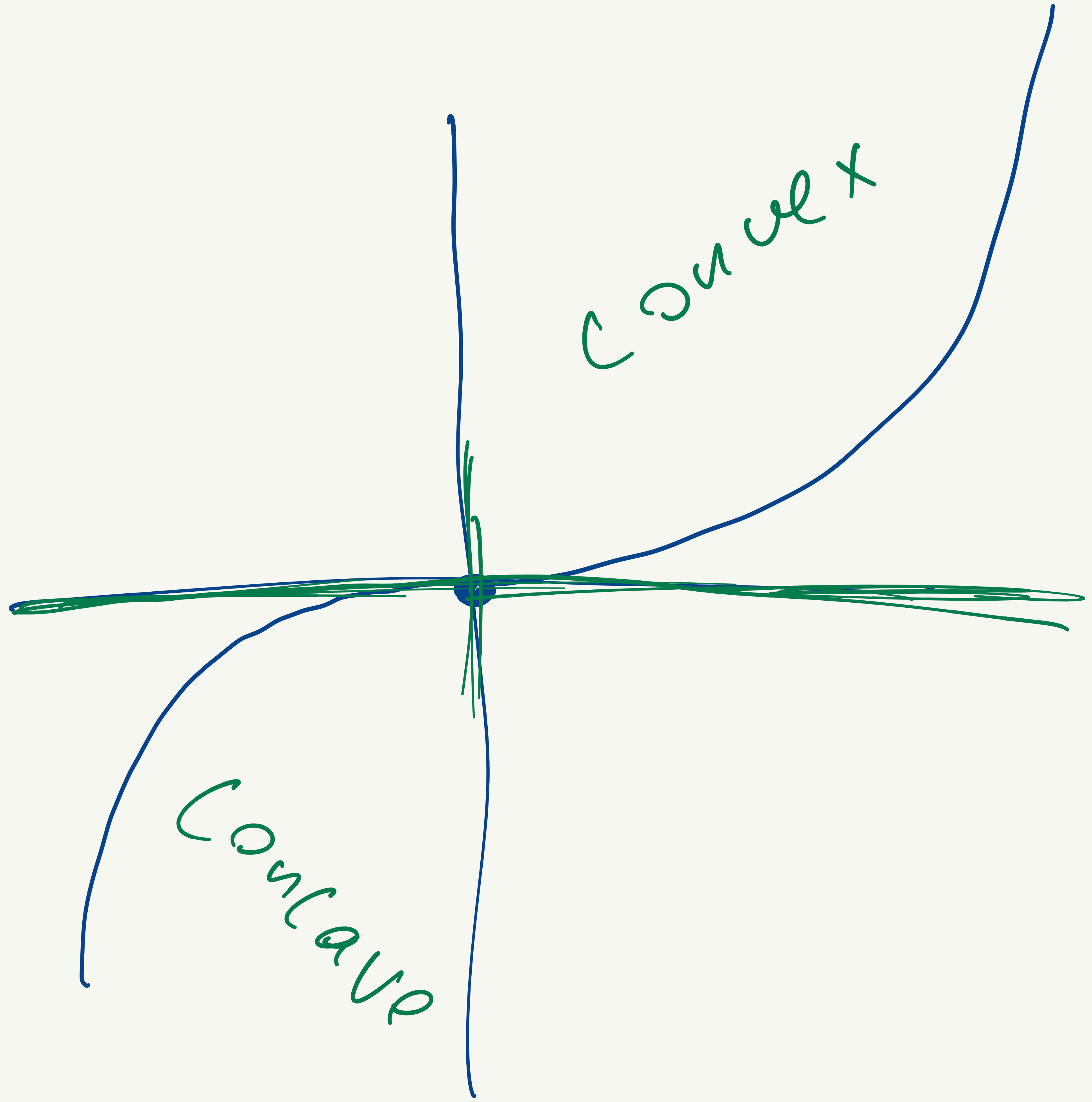
$$\lim_{x \to x_0} \frac{x^3 - x_0^3}{x - x_0} = \lim_{x \to x_0} \frac{(x-x_0)(x^2 + x x_0 + x_0^2)}{x - x_0} =$$

$$\lim_{x \to x_0} x^2 + x x_0 + x_0^2 = x_0^2 + x_0^2 + x_0^2 = \underline{\underline{3x_0^2}}$$

$f'(x_0) = 3x_0^2$ $\qquad$ $f'(7) = 3 \cdot 49 = 147$

$f(x) = x^3$

$f'(x_0) = 3x_0^2$



$f'(x_0) = 0$

$3x_0^2 = 0$

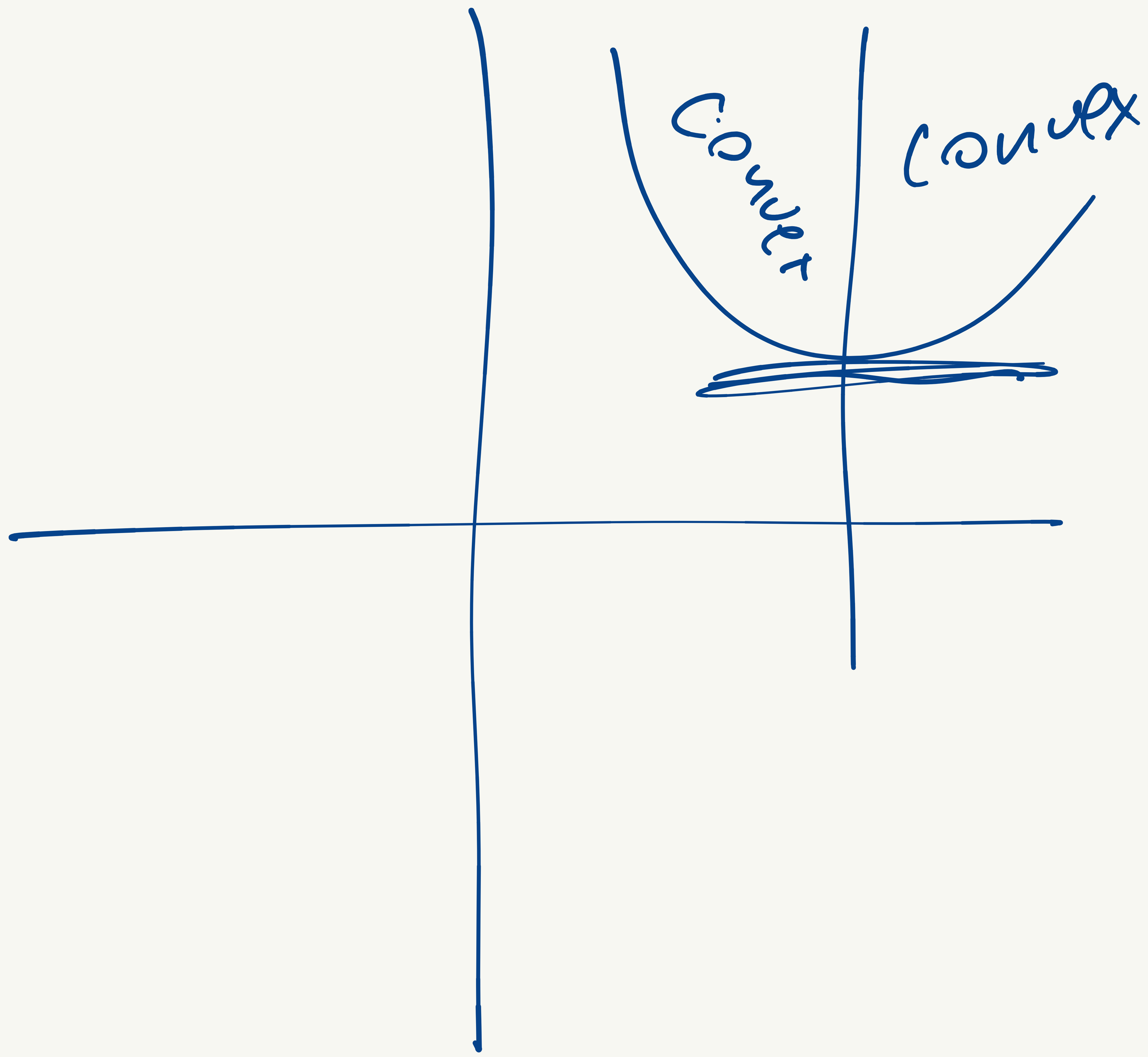
$\underline{\underline{x_0 = 0}}$

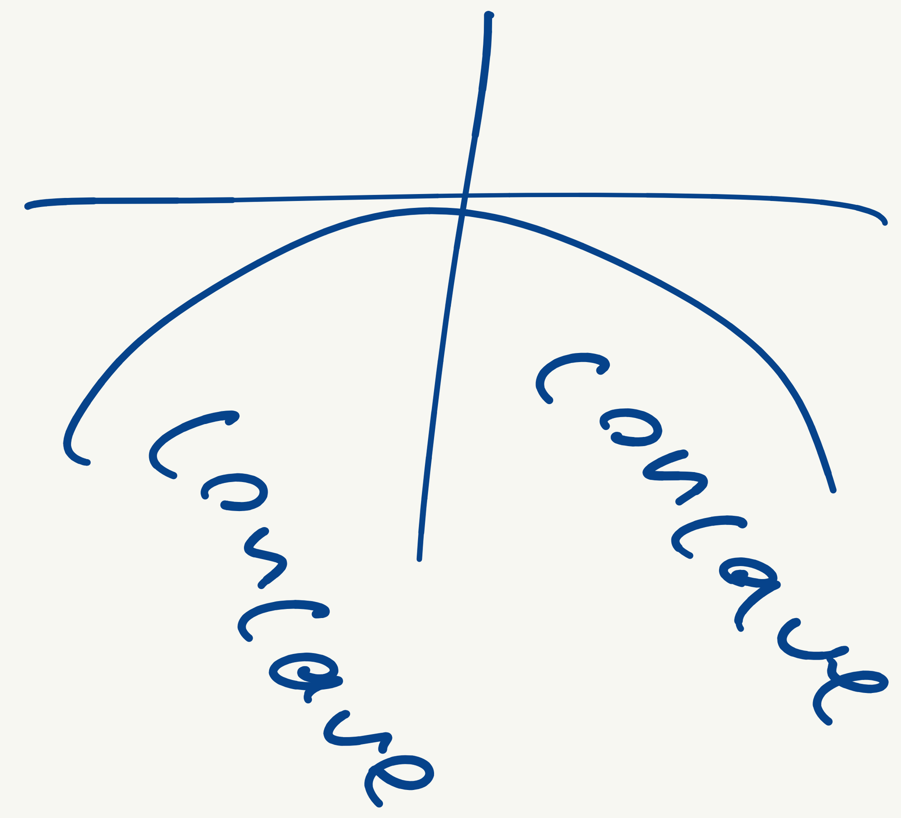
Inflection point:

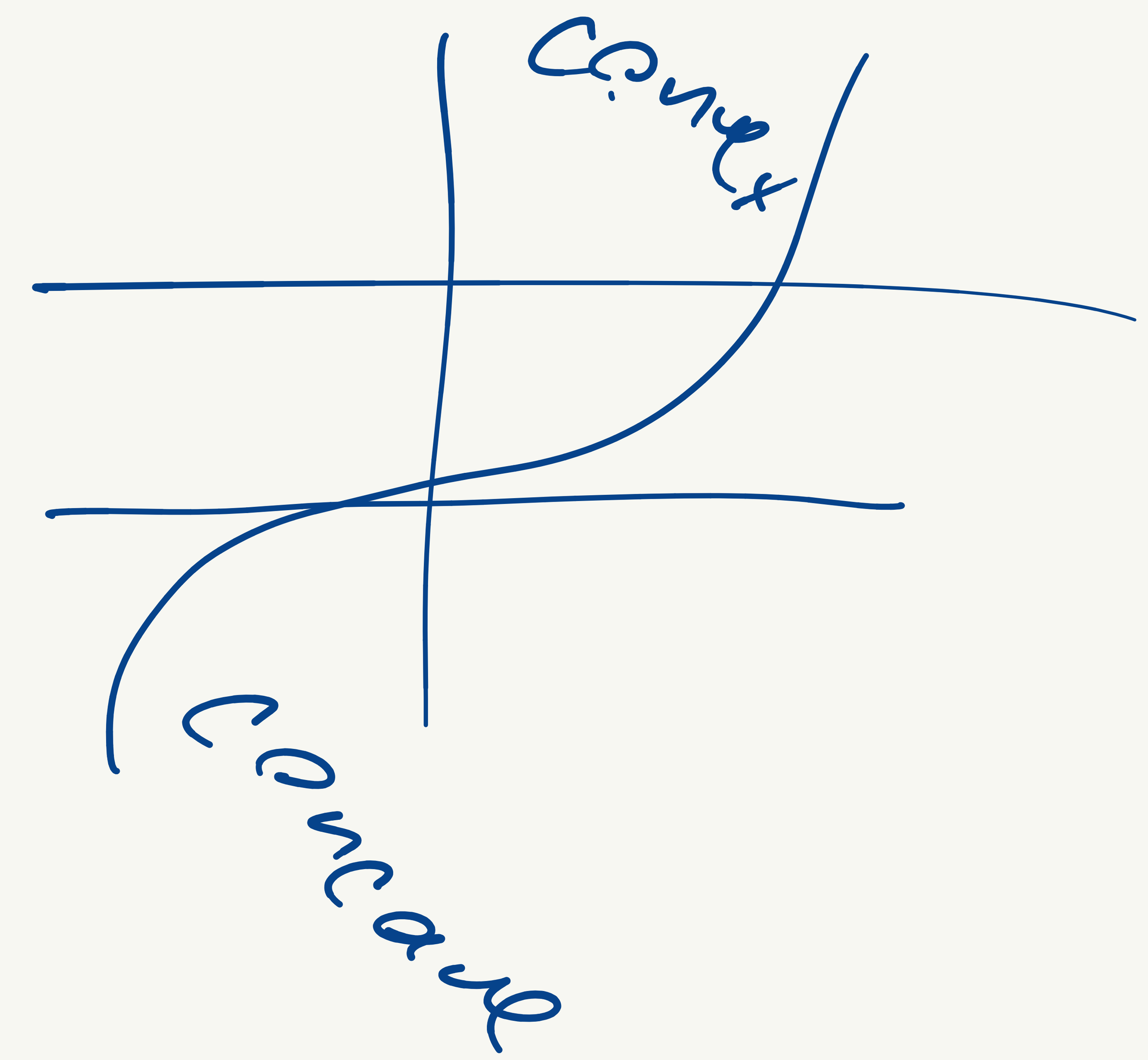
function changes convexity

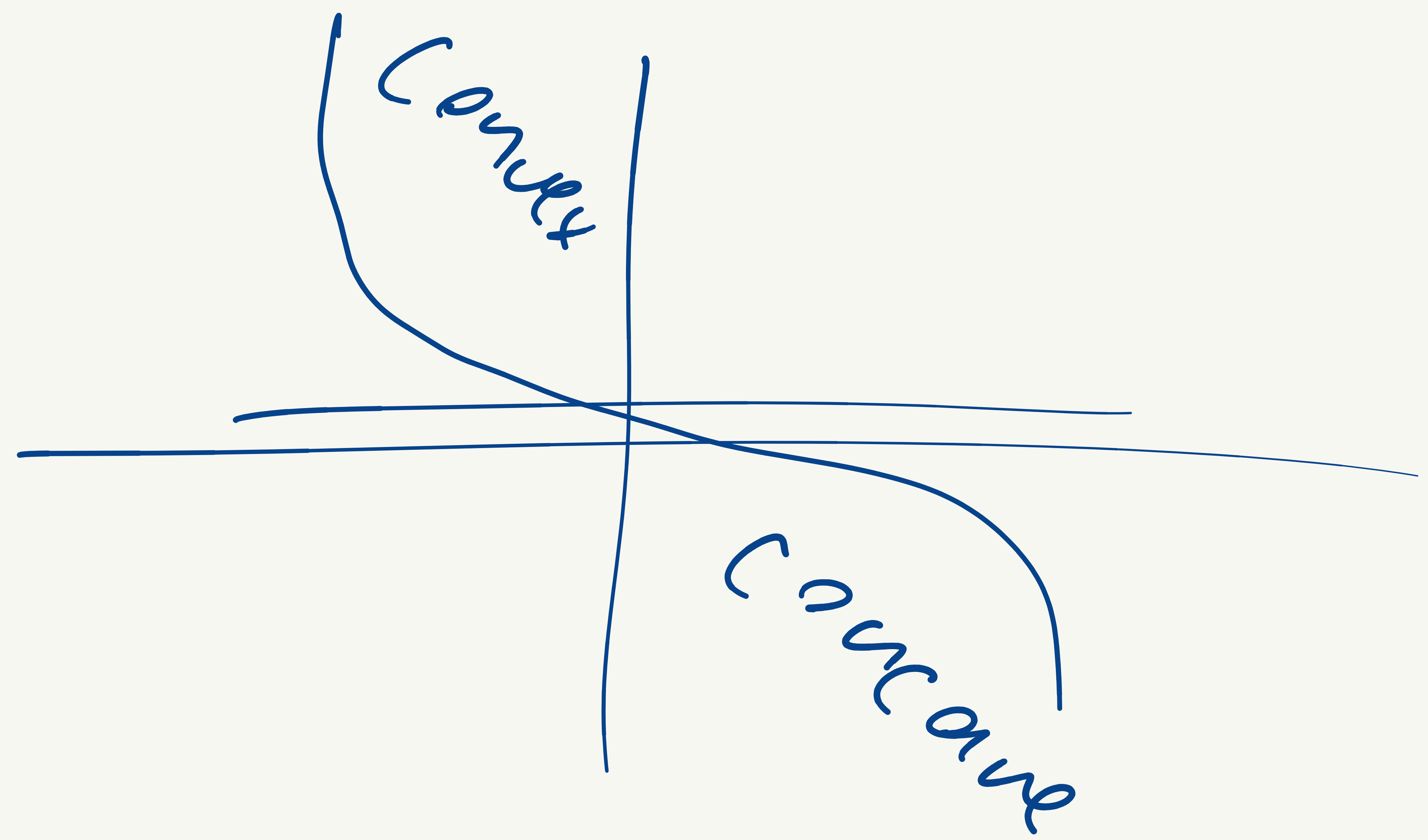
If $f'(x_0) = 0$ there MIGHT be a min/max

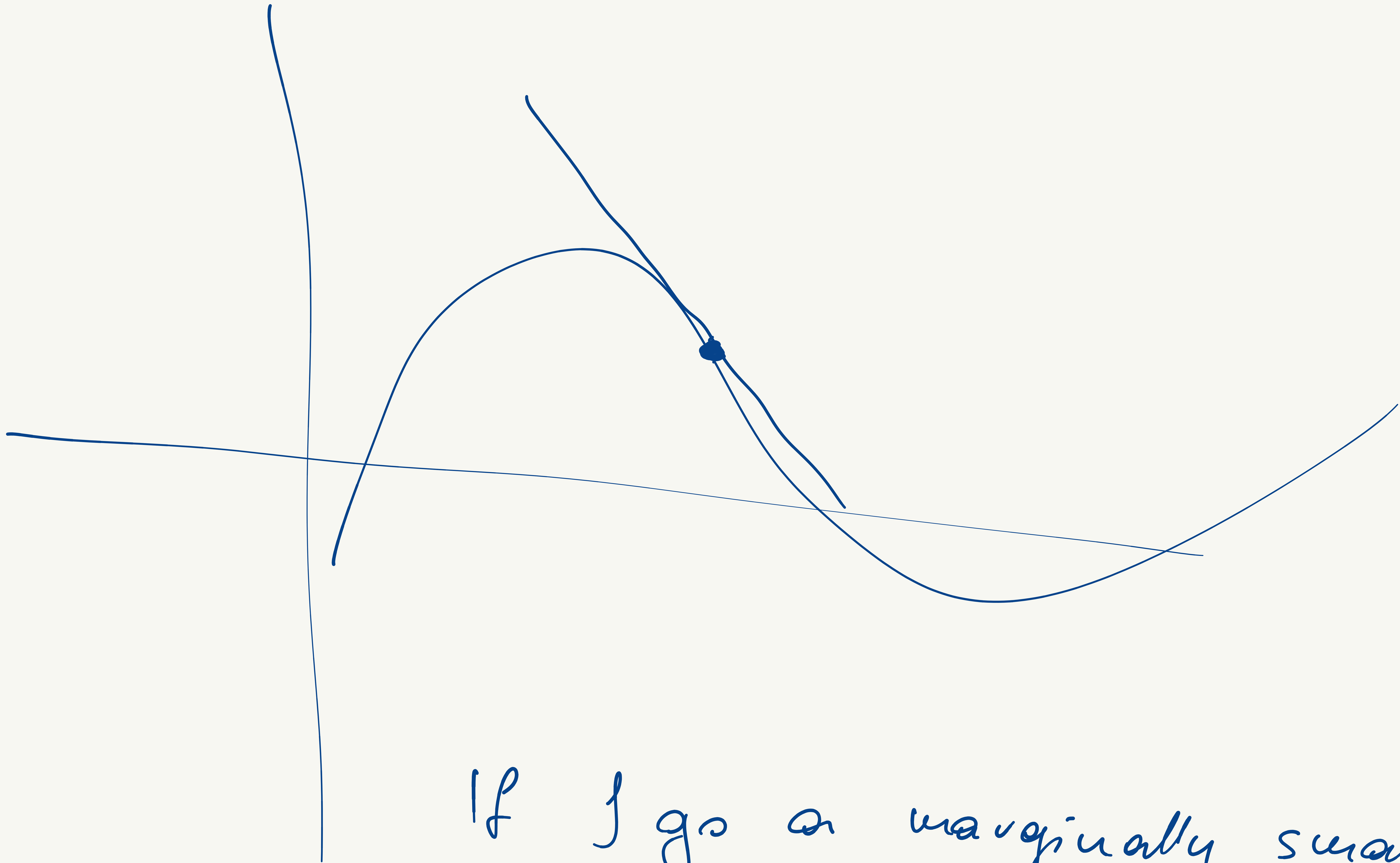
If $f(x)$ has a min/max at x_0 $f'(x)$ will be 0.

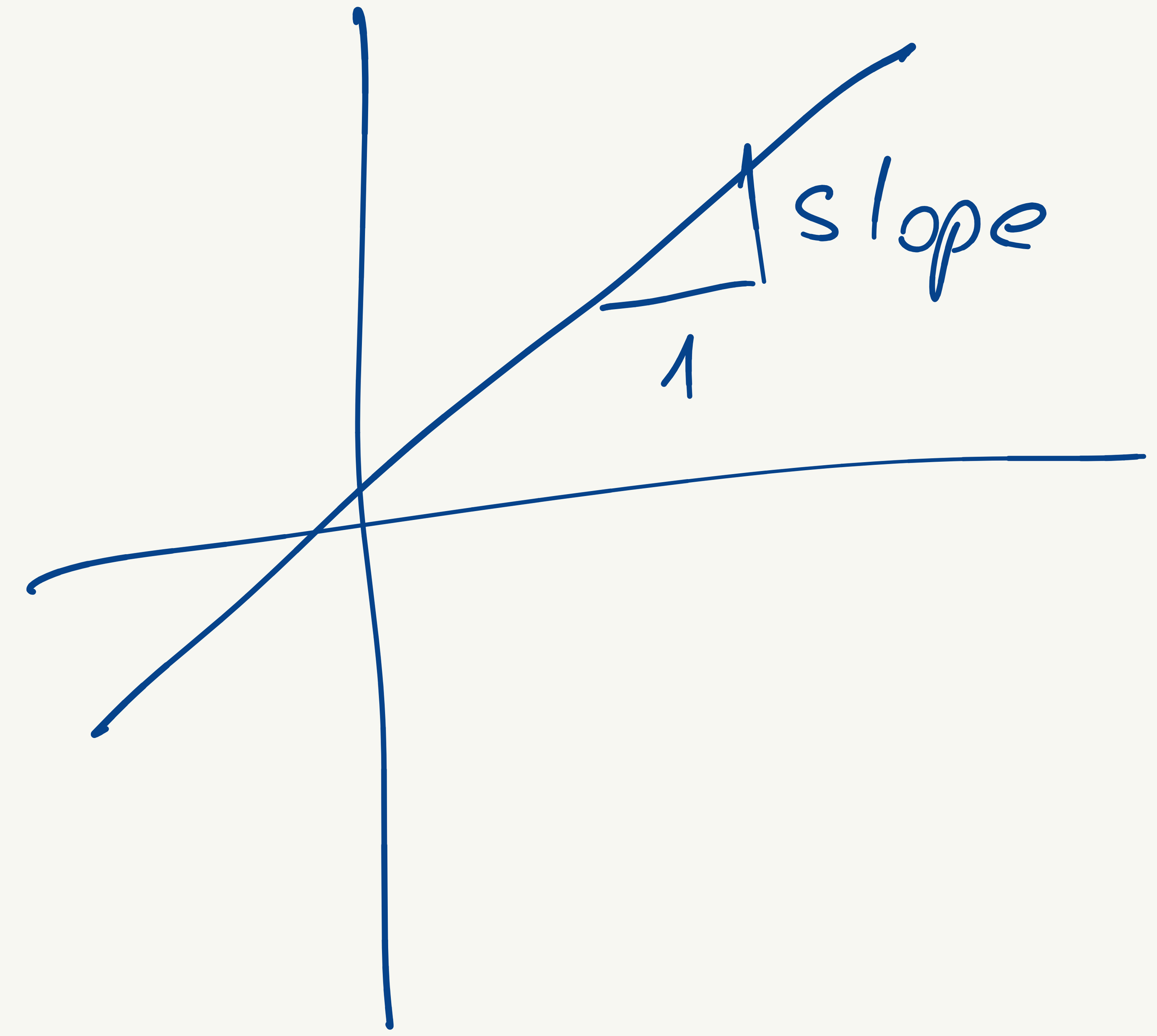

Convex
Convex


Concave
Concave


Convex
Concave


Convex
Concave





If I go a marginally small unit →

then I go $f' \cdot$ small unit ↑

$f(x) = x^n$

$f'(x_0) = ?$

$$\lim_{x \to x_0} \frac{x^n - x_0^n}{x - x_0}$$

$n = 2$ $\quad a^2 - b^2$

$$a^2 - b^2 = (a-b) \cdot \sum_{i=0}^{2-1} \left(a^{2-1-i} b^i\right) =$$

$$a^n - b^n = (a-b) \sum_{i=0}^{n-1} a^{n-1-i} b^i$$

$$= (a-b) \cdot \left[a^{2-1-0} \cdot b^0 + a^{2-1-1} \cdot b^1\right]$$

$$\sum_{\boxed{i=0}}^{5} i = 0 + 1 + 2 + 3 + 4 + 5$$

$$= (a-b) \cdot \left[a \cdot 1 + 1 \cdot b\right] =$$

$$= (a-b)(a+b)$$

$$\sum_{i=3}^{6} \frac{1}{i^2} = \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2}$$

$$a^3 - b^3 = (a-b) \sum_{i=0}^{3-1} \left(a^{3-1-i} b^i\right) =$$

$$(a-b) \sum_{i=0}^{2} \left(a^{2-i} b^i\right) = (a-b)\left[a^{2-0} \cdot b^0 + a^{2-1} \cdot b^1 + a^{2-2} \cdot b^2\right] = (a-b)\left[a^2 + ab + b^2\right]$$

$f(x) = x^n$

$$\lim_{x \to x_0} \frac{x^n - x_0^n}{x - x_0} = \lim_{x \to x_0} \frac{(x - x_0) \sum_{i=0}^{n-1} \left(x^{n-1-i} x_0^i \right)}{x - x_0} =$$

$$\lim_{x \to x_0} \sum_{i=0}^{n-1} x^{n-1-i} x_0^i = \sum_{i=0}^{n-1} x_0^{n-1-i} x_0^i =$$

$$\sum_{i=0}^{n-1} x_0^{n-1-i+i} = \sum_{i=0}^{n-1} x_0^{n-1} = \underbrace{x_0^{n-1} + x_0^{n-1} + \dots + x_0^{n-1}}_{} =$$

$$\left.\begin{array}{l} i = 0 \\ i = 1 \\ i = 2 \\ \vdots \\ i = n-1 \end{array}\right\} n$$

$$= n x_0^{n-1}$$

$f(x) = x^2$

$f'(x_0) = 2x_0^{2-1}$

$n = 2$

$$\boxed{f'(x_0) = 2x_0}$$

$f(x) = x^3$

$n = 3$

$f'(x_0) = 3x_0^{3-1}$

$f'(x_0) = 3x_0^2$

$$f(x) = x^{153}$$

$$n = 153$$

$$\boxed{f'(x_0) = 153\, x_0^{152}}$$

Derivative function

$$\left.\begin{array}{c} \boxed{f'(x_0)} \\ \boxed{f'(x)} \\ \boxed{\dfrac{d f(x)}{d x}} \end{array}\right\} \text{SAME}$$

Properties for derivative functions

$$c' = 0 \qquad \forall c \in \mathbb{R}$$

$$(a f(x))' = a \cdot f'(x)$$

$$f(x) = 5x^2 \qquad (5x^2)' = 5\,(x^2)' = 5 \cdot 2x = 10x$$

$$(af(x) + bg(x))' = af'(x) + bg'(x)$$

$$(2x^2 + 5x^3)' = 2\,(x^2)' + 5\,(x^3)' = 2 \cdot 2x + 5 \cdot 3x^2 = 4x + 15x^2$$

$$(f(x)g(x))' = f'(x)g(x) + f(x) \cdot g'(x)$$

$$(\underbrace{x^2 \cdot x^3}_{x^5})' = (x^2)'x^3 + x^2 \cdot (x^3)' = 2x\,x^3 + x^2 \cdot 3x^2 = 2x^4 + 3x^4 = 5x^4$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\left(\frac{x^7}{x^2}\right)' = \frac{(x^7)'x^2 - x^7 \cdot (x^2)'}{(x^2)^2} = \frac{7x^6x^2 - x^7 \cdot 2x}{x^4} = \frac{7x^8 - 2x^8}{x^4} = \frac{5x^8}{x^4} = \underline{5x^4}$$

$$(x^3+2x^2-x)' = (x^3)' + (2x^2)' - (x)' = 3x^2 + 2(x^2)' - 1 = 3x^2 + 2\cdot 2x - 1 =$$

$$x = x^1 = 1\cdot x^0 = 1$$

$$= \underline{\underline{3x^2 + 4x - 1}}$$

$$[(x^2+2)(x-4)]' = (x^2+2)'(x-4) + (x^2+2)(x-4)' = [(x^2)' + (2)'](x-4) + (x^2+2)[(x)' - (4)']$$

$$= [2x + 0](x-4) + (x^2+2)[1-0] = 2x(x-4) + x^2 + 2 =$$

$$= 2x^2 - 8x + x^2 + 2 = \underline{\underline{3x^2 - 8x + 2}}$$

$$\left(\frac{x^{12} - 15x^2}{x-5}\right)' = \frac{(x^{12} - 15x^2)'(x-5) - (x^{12} - 15x^2)(x-5)'}{(x-5)^2} = \frac{[12x^{11} - 15\cdot 2\cdot x](x-5) - \dots}{}$$

$$\frac{\dots (x^{12} - 15x^2)(1-0)}{(x-5)^2} = \frac{(12x^{11} - 30x)(x-5) - (x^{12} - 15x^2)}{(x-5)^2}$$

Useful derivatives

$$(e^x)' = e^x$$

$$(a^x)' = a^x \cdot \ln(a)$$

$$(\ln(x))' = \frac{1}{x}$$

$$(\log_a x)' = \frac{1}{x\ln(a)}$$

$$f(x) = \frac{x^2}{\ln(x)}$$

$$g(x) = e^x(x^3 - x^2)$$

$$h(x) = \frac{5^x}{x^2 - 2}$$

$$\left(\frac{x^2}{\ln(x)}\right)' = \frac{(x^2)'\ln(x) - x^2\cdot(\ln(x))'}{(\ln(x))^2} = \frac{2x\ln(x) - x^2\cdot\frac{1}{x}}{(\ln(x))^2} = \frac{2x\ln(x) - x}{(\ln(x))^2}$$

$$\left(e^x(x^3-x^2)\right)' = (e^x)'(x^3-x^2) + e^x(x^3-x^2)' = e^x(x^3-x^2) + e^x(3x^2-2x) =$$

$$= e^x(x^3+2x^2-2x)$$

$$\left(\frac{5^x}{x^2-2}\right)' = \frac{(5^x)'(x^2-2) - 5^x(x^2-2)'}{(x^2-2)^2} = \frac{5^x\cdot\ln(5)(x^2-2) - 5^x(2x-0)}{(x^2-2)^2} =$$

$$= \frac{5^x(\ln(5)(x^2-2) - 2x)}{(x^2-2)^2}$$

$$(f(g(x)))' = f'(g(x))\,g'(x) \quad : \text{Chain rule}$$

1) $$\left(e^{-x^2}\right)' = e^{-x^2} \cdot (-2x)$$

$$f(x) = e^x \qquad \boxed{f'(x) = e^x}$$

$$g(x) = -x^2 \qquad g'(x) = -2x$$

2) $$\left[\ln(x^2+2x)\right]' = \frac{1}{x^2+2x} \cdot (2x+2)$$

$$f(x) = \ln(x) \qquad f'(x) = \frac{1}{x}$$

$$g(x) = x^2+2x \qquad g'(x) = 2x+2$$

3/

$$\left(5^{x^2-x}\right)' = 5^{x^2-x} \cdot \ln(5) \cdot (2x-1)$$

$$f(x) = 5^x \qquad f'(x) = 5^x \ln(5)$$

$$g(x) = x^2 - x \qquad g'(x) = 2x - 1$$

4.)

$$\left(e^{\frac{x^2-1}{x^3+2x^2}}\right)' = e^{\frac{x^2-1}{x^3+2x^2}} \cdot \left[\frac{2x(x^3+2x^2) - (x^2-1)(3x^2+4x)}{(x^3+2x^2)^2}\right]$$

$$f(x) = e^x \qquad f'(x) = e^x$$

$$g(x) = \frac{x^2-1}{x^3+2x^2} \qquad g'(x) = \frac{(x^2-1)'(x^3+2x^2) - (x^2-1)(x^3+2x^2)'}{(x^3+2x^2)^2} = \frac{2x(x^3+2x^2) - (x^2-1)(3x^2+4x)}{(x^3+2x^2)^2}$$

Unconstrained optimization:

Find minima / maxima

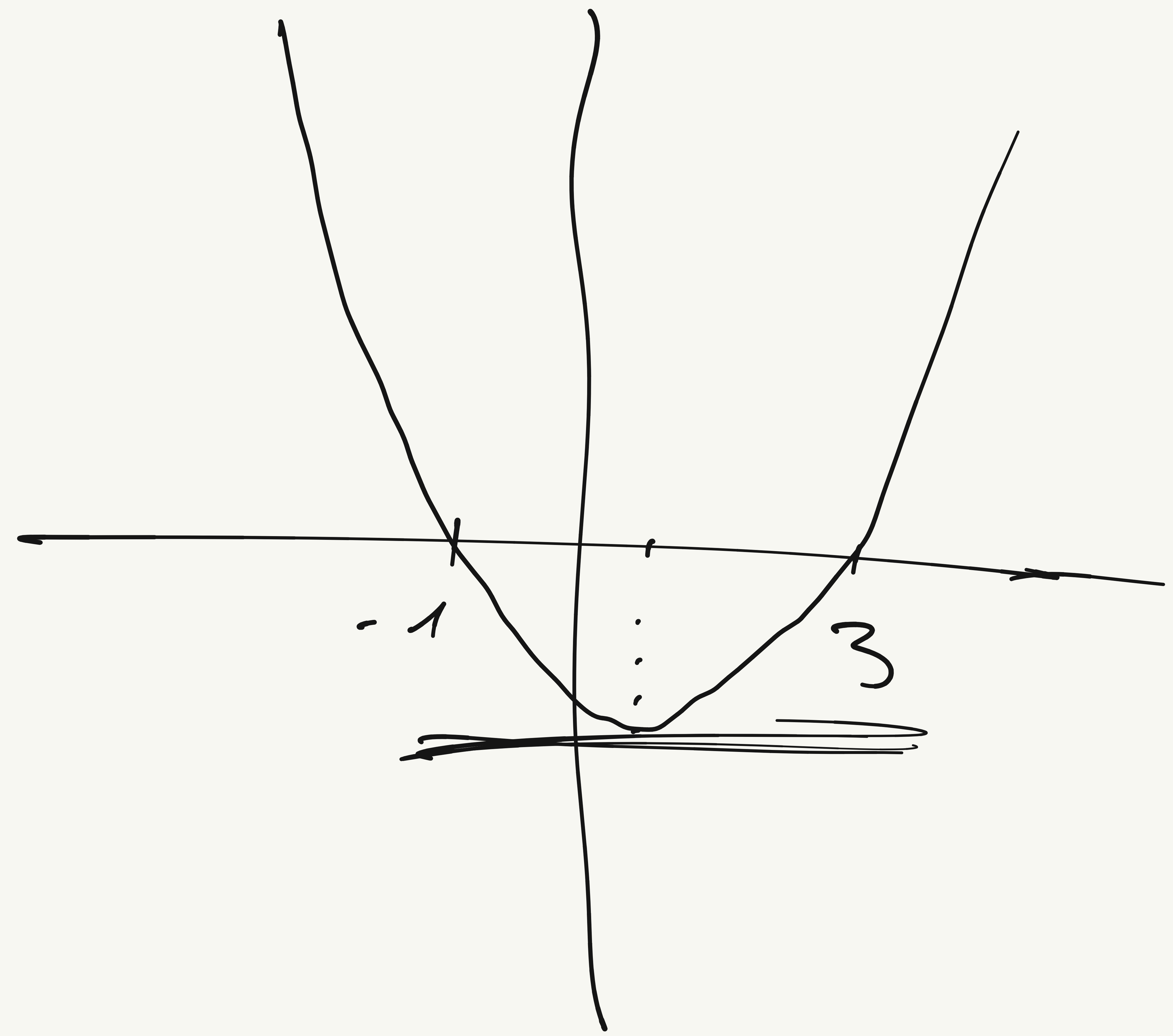
1. Derivative function
2. Set it equal to zero
3. Potential set of minima / maxima
4. Figure out if they are in fact minima / maxima

$f(x) = x^2 - 2x - 3 = (x+1)(x-3)$

$f'(x) = 2x - 2 - 0 = 2x - 2 = 0$

$2x = 2$

$\underline{\underline{x = 1}}$

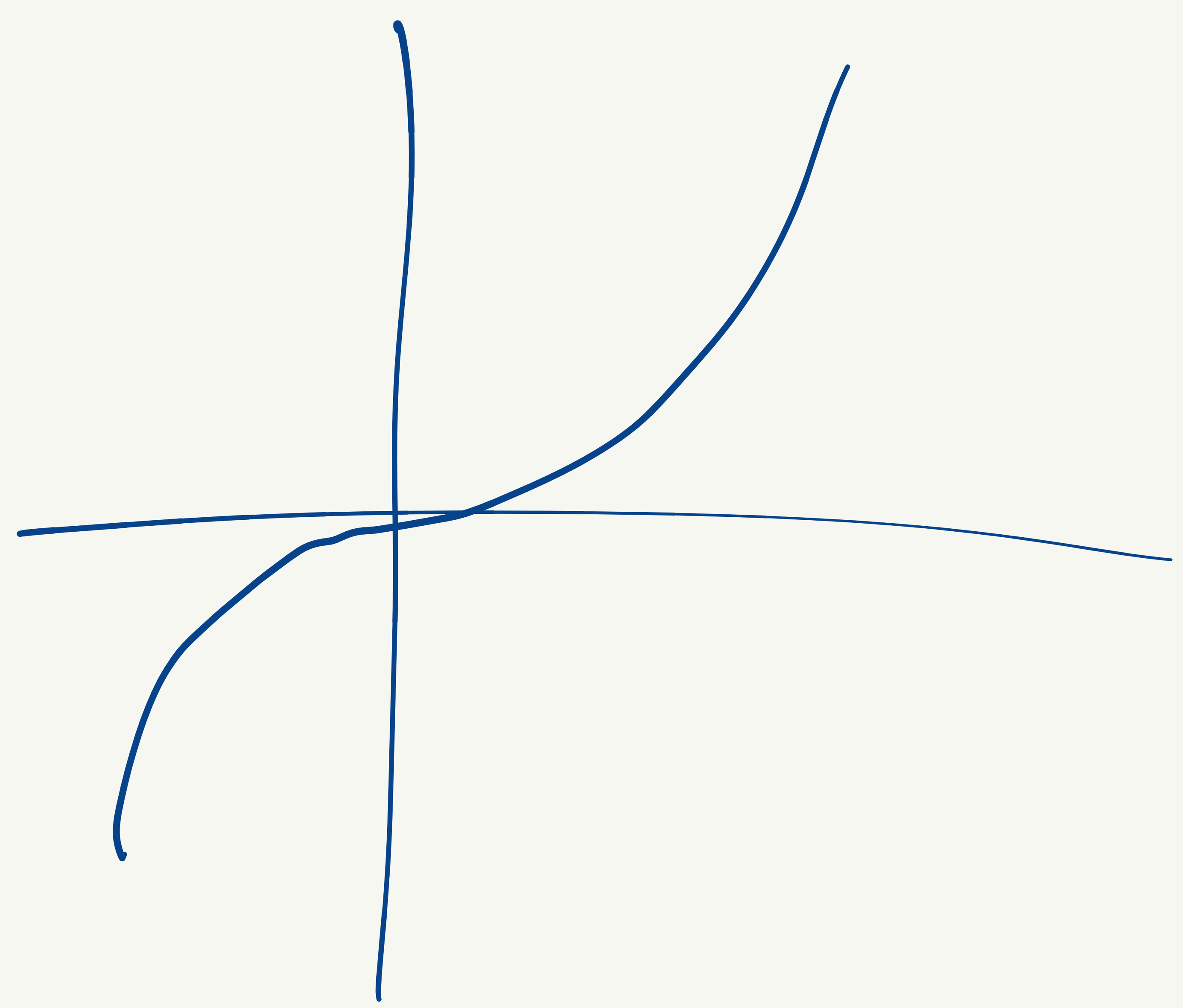


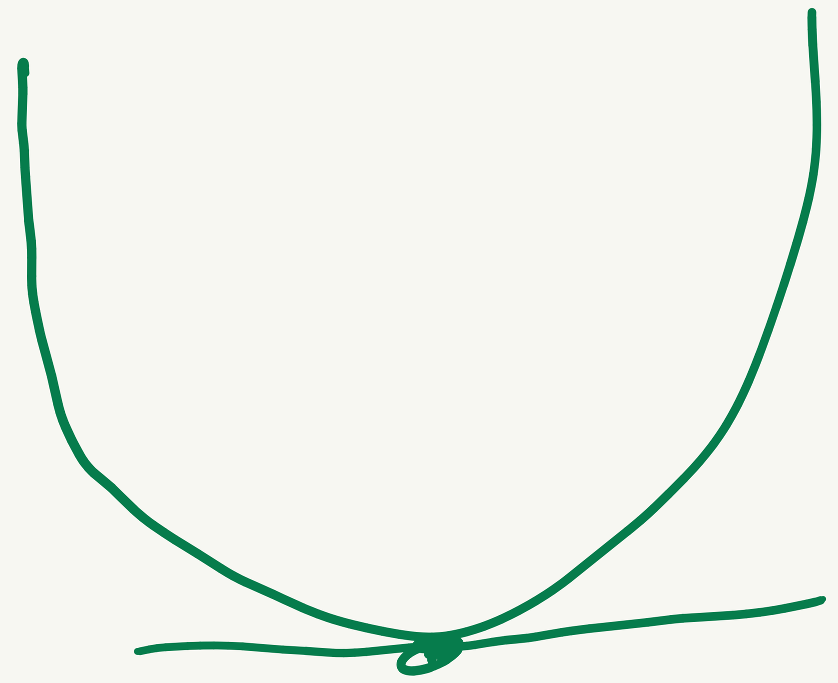
minimum

$g(x) = x^3$

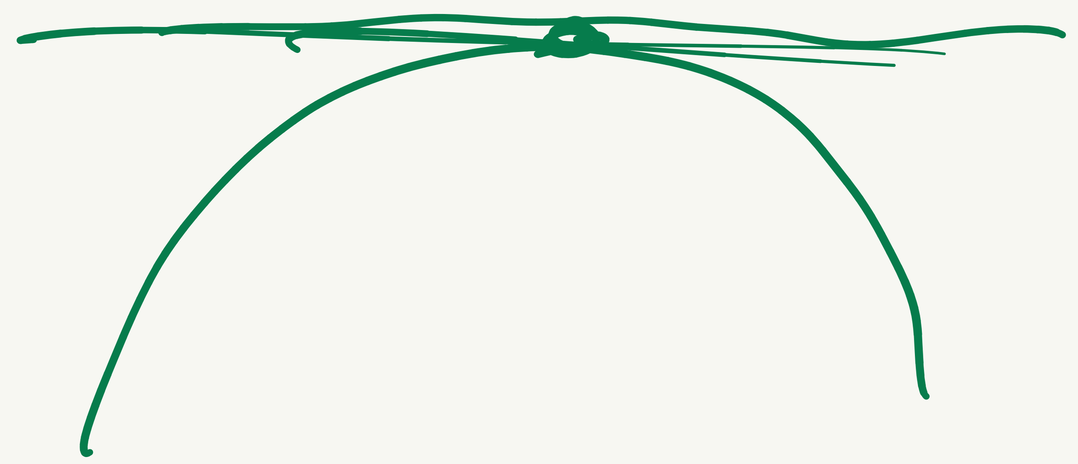
$g'(x) = 3x^2 = 0$

$\underline{\underline{x = 0}}$

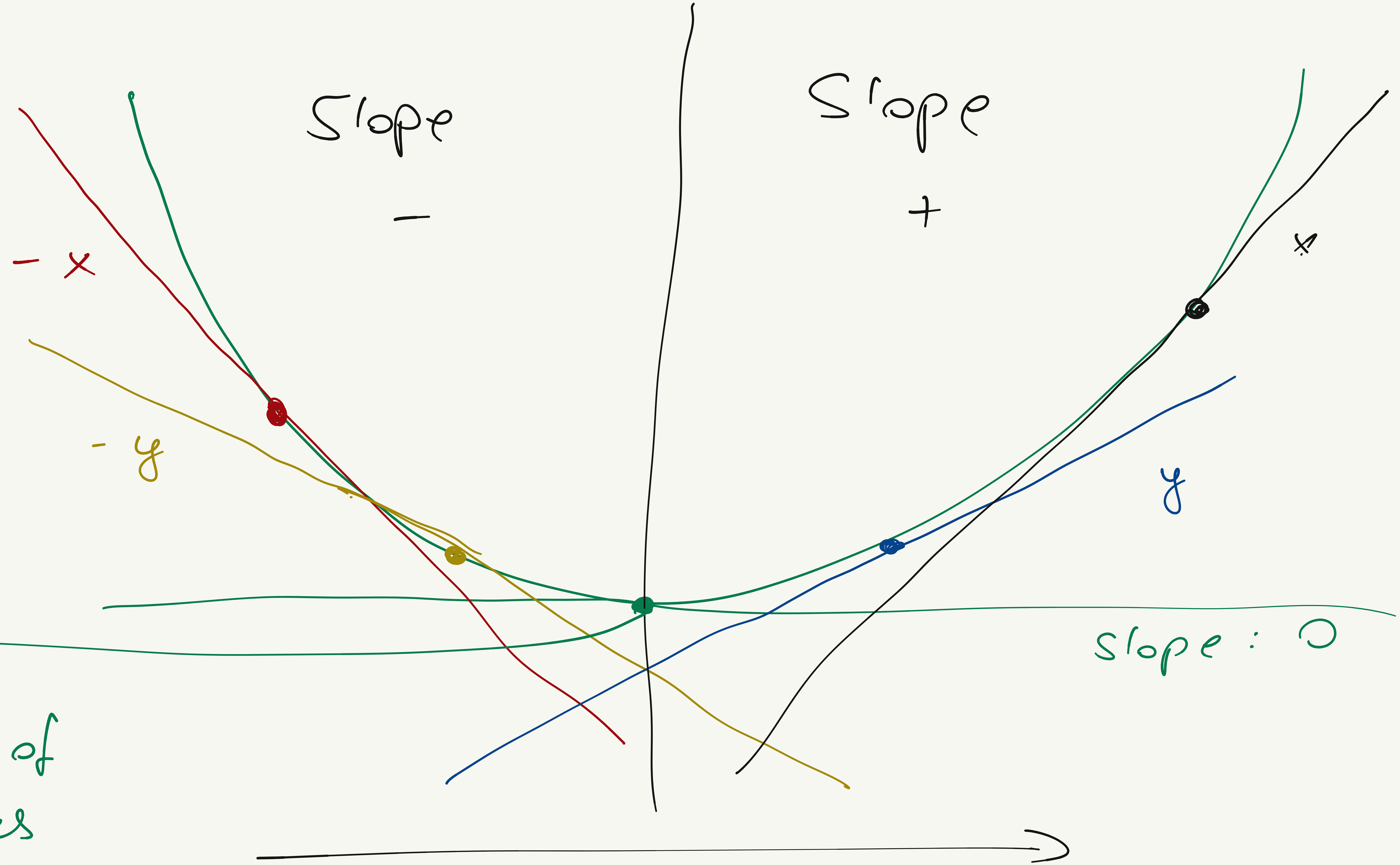




convex => minimum



concave => maximum

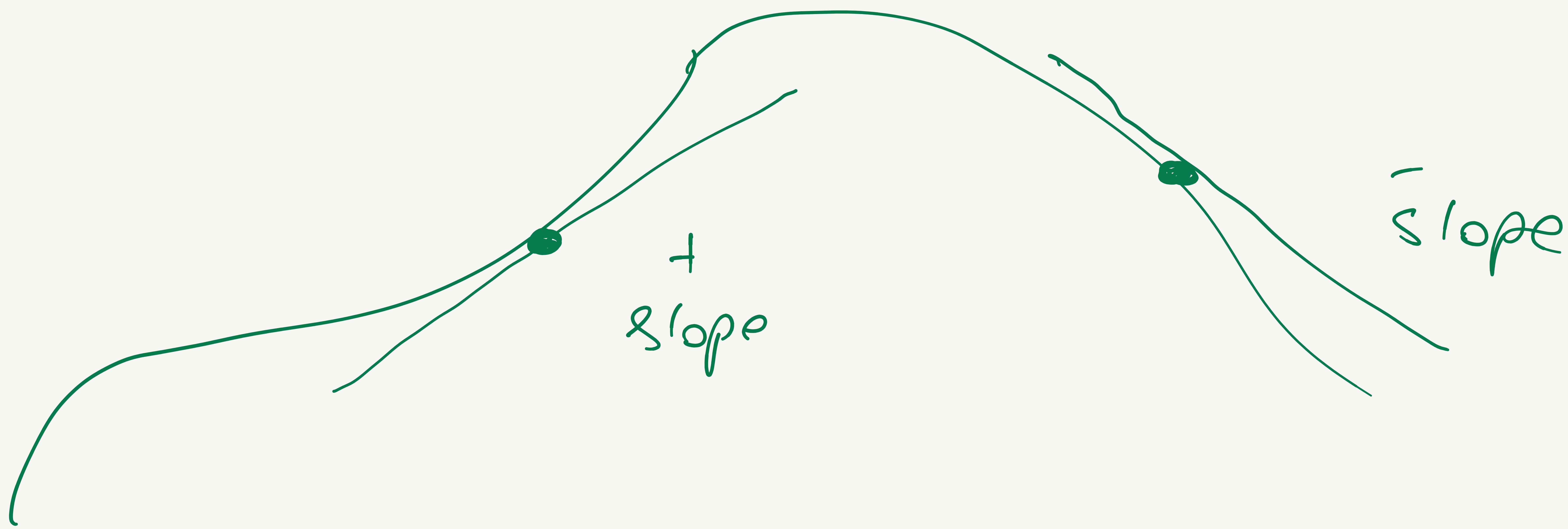


are the slopes of tangent lines increasing?

slopes of tangent lines are increasing.

slopes of tangent lines are decreasing.

convex: slopes of tangent lines => increasing
concave: slopes of tangent lines => decreasing



derivative of the derivative function: How does the slope change?

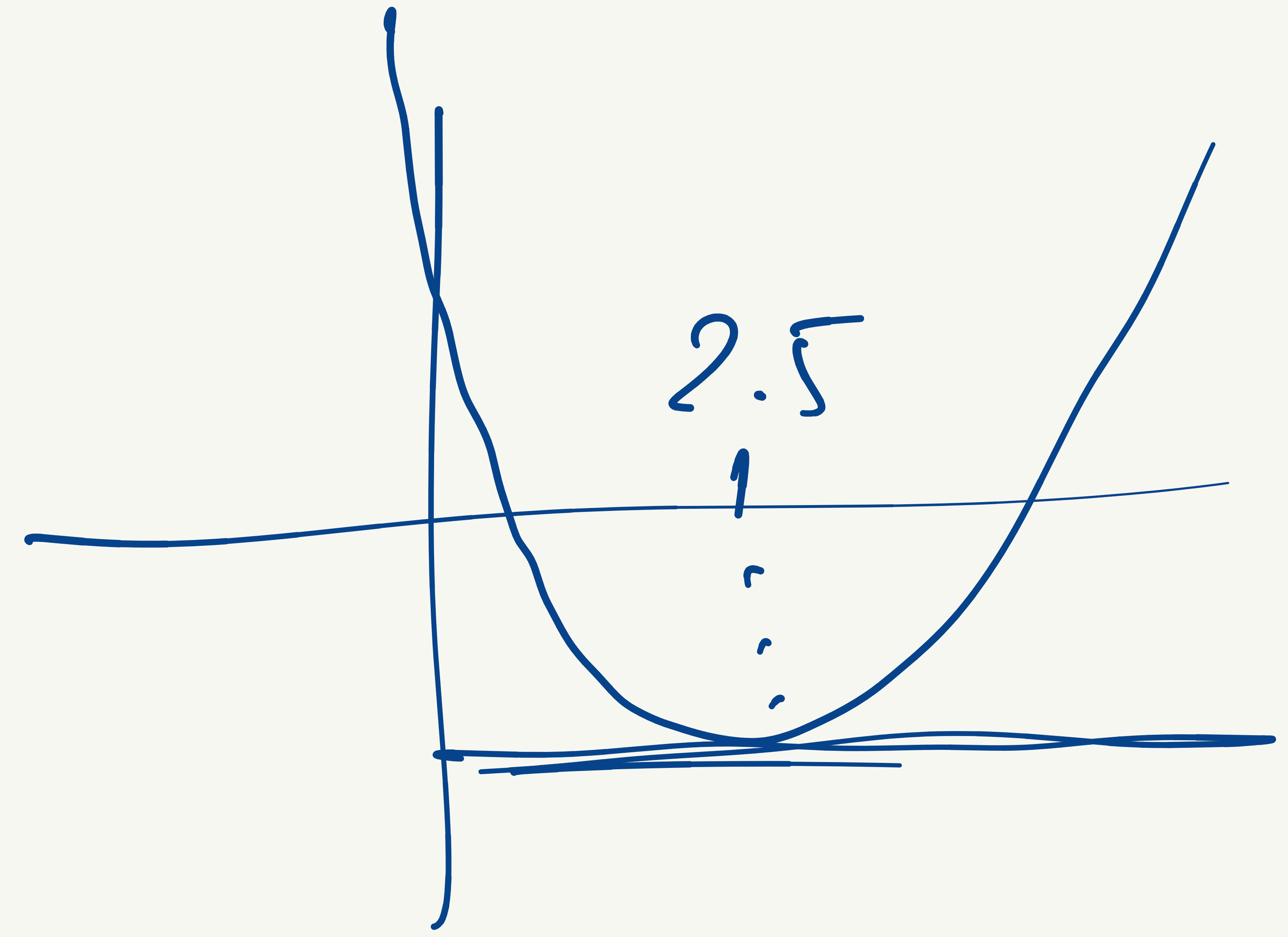
positive => convex
negative => concave

$$f(x) = x^2 - 5x + 7$$

$$f'(x) = 2x - 5 = 0$$

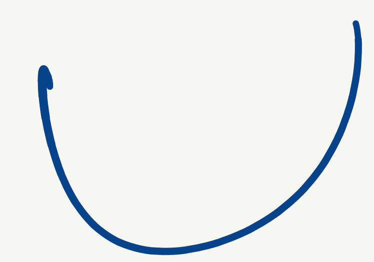
$$2x = 5$$

$$\underline{\underline{x = 2.5}}$$



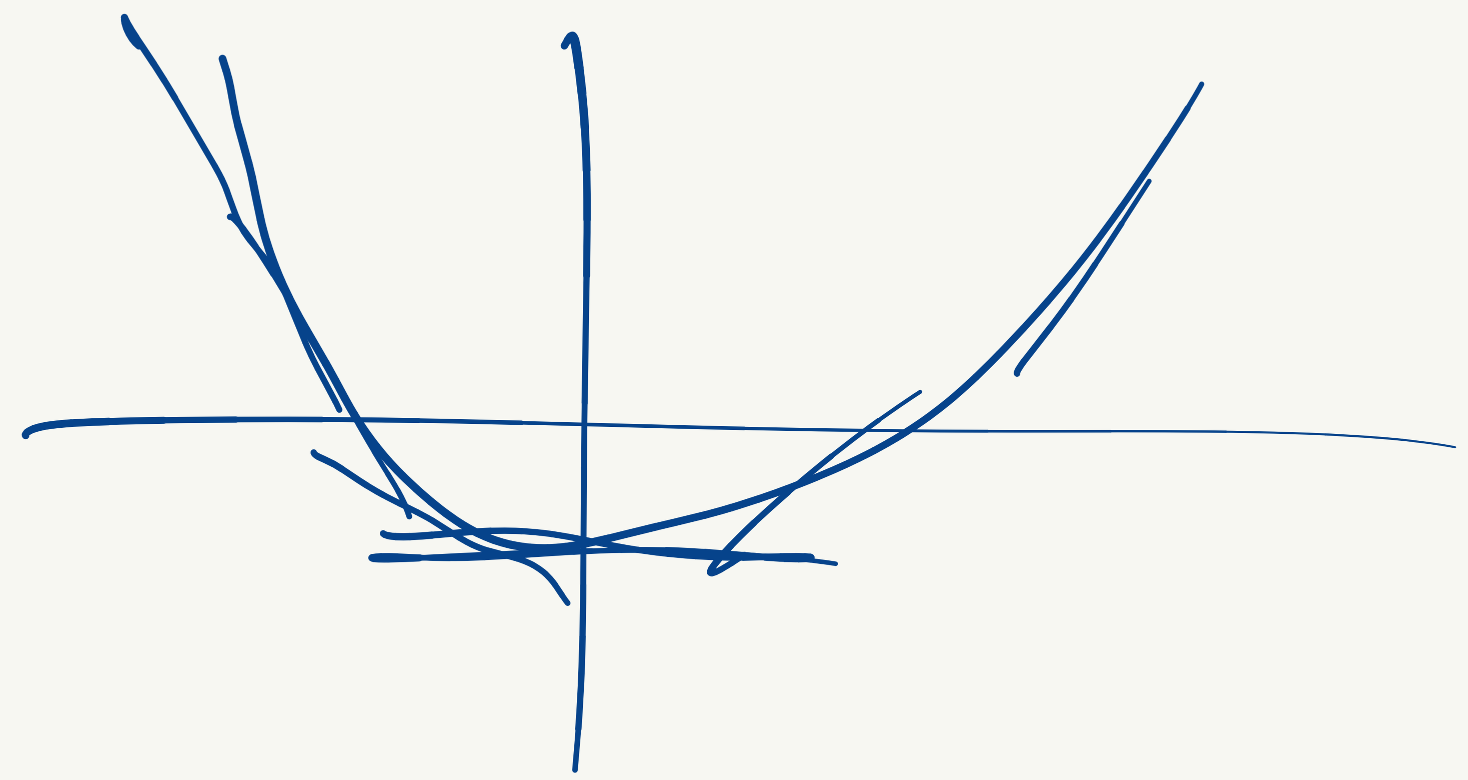
$$(f'(x))' = f''(x) = 2 - 0 = \underline{\underline{2}}$$

second derivative

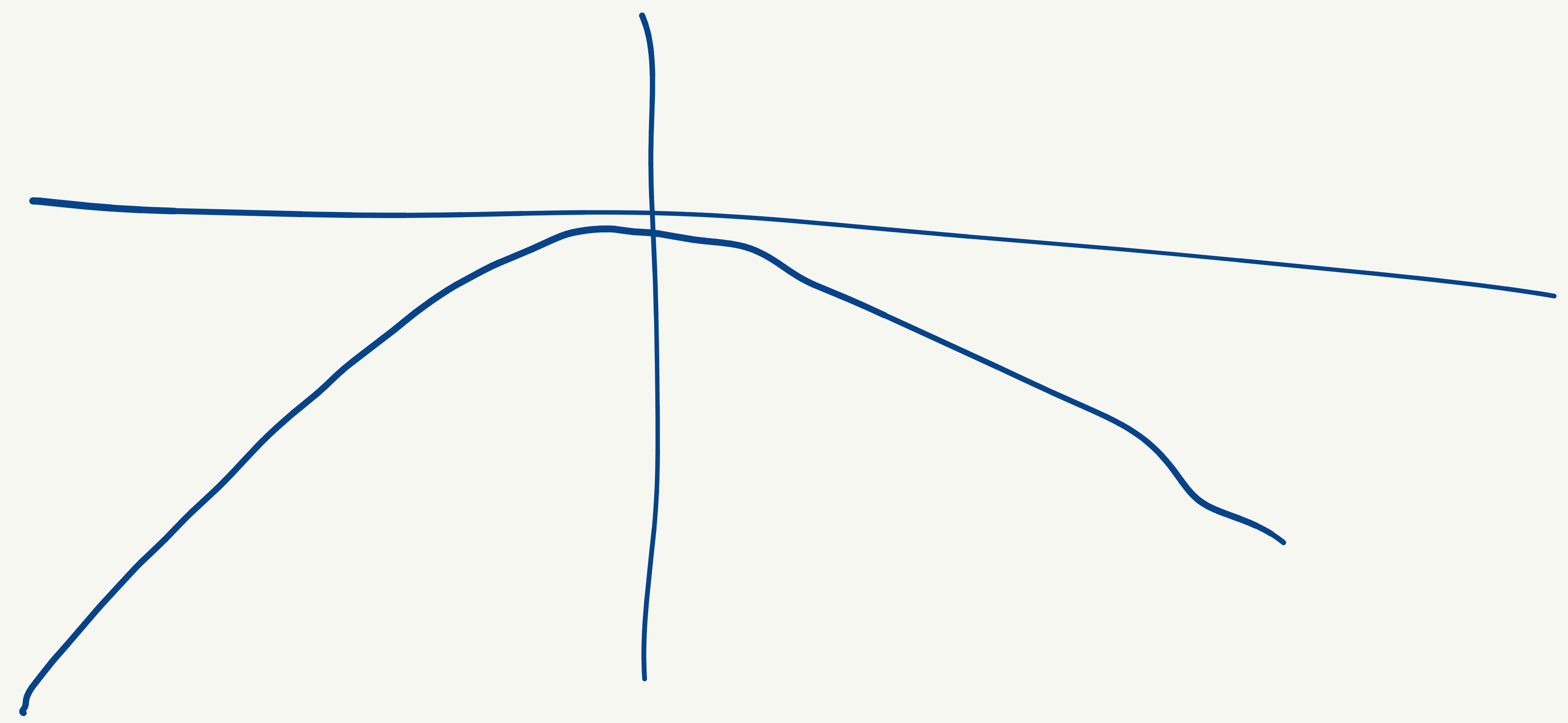
positive => convex => 

=> minimum

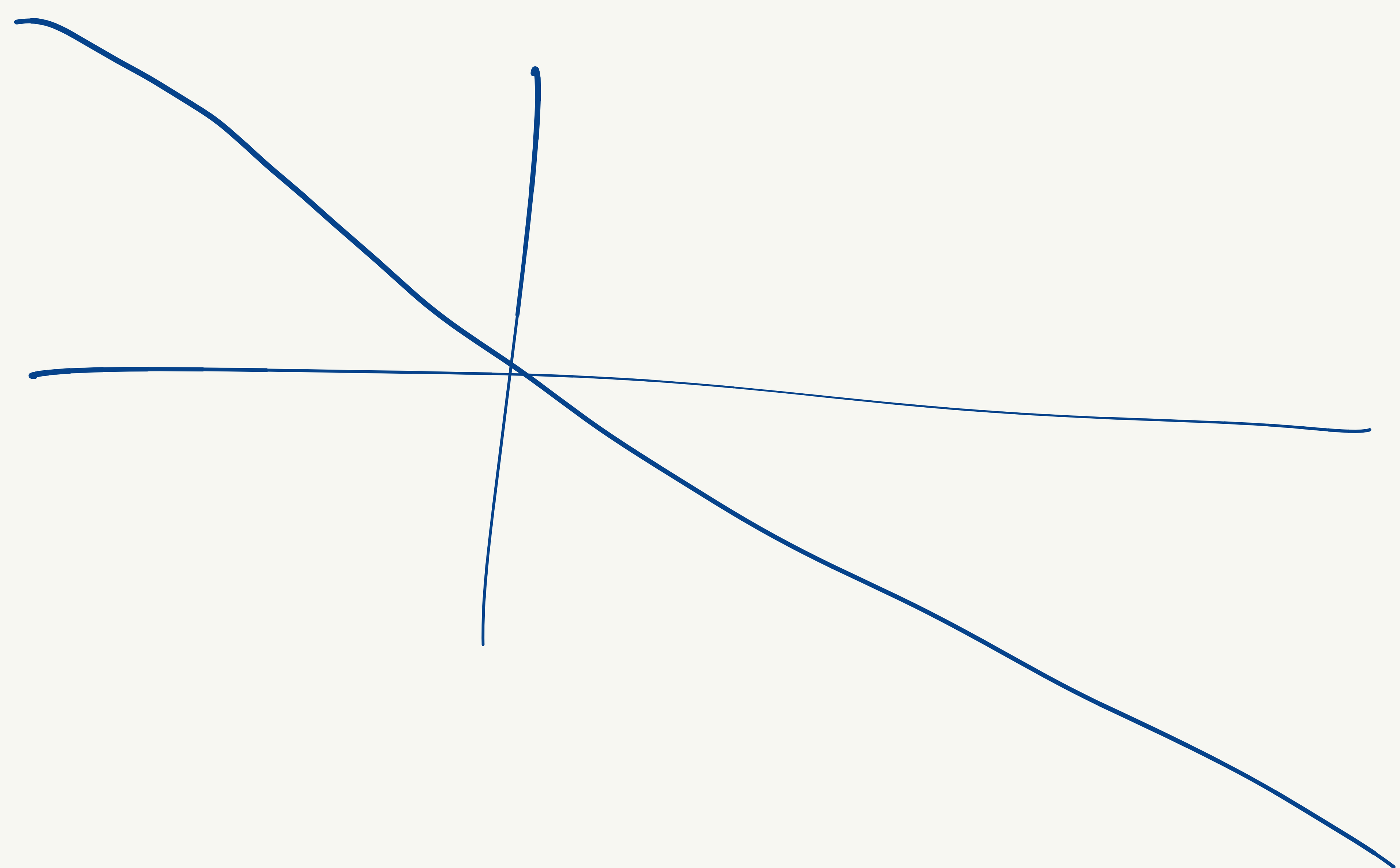
$f(x)$



$f'(x)$



$f''(x)$



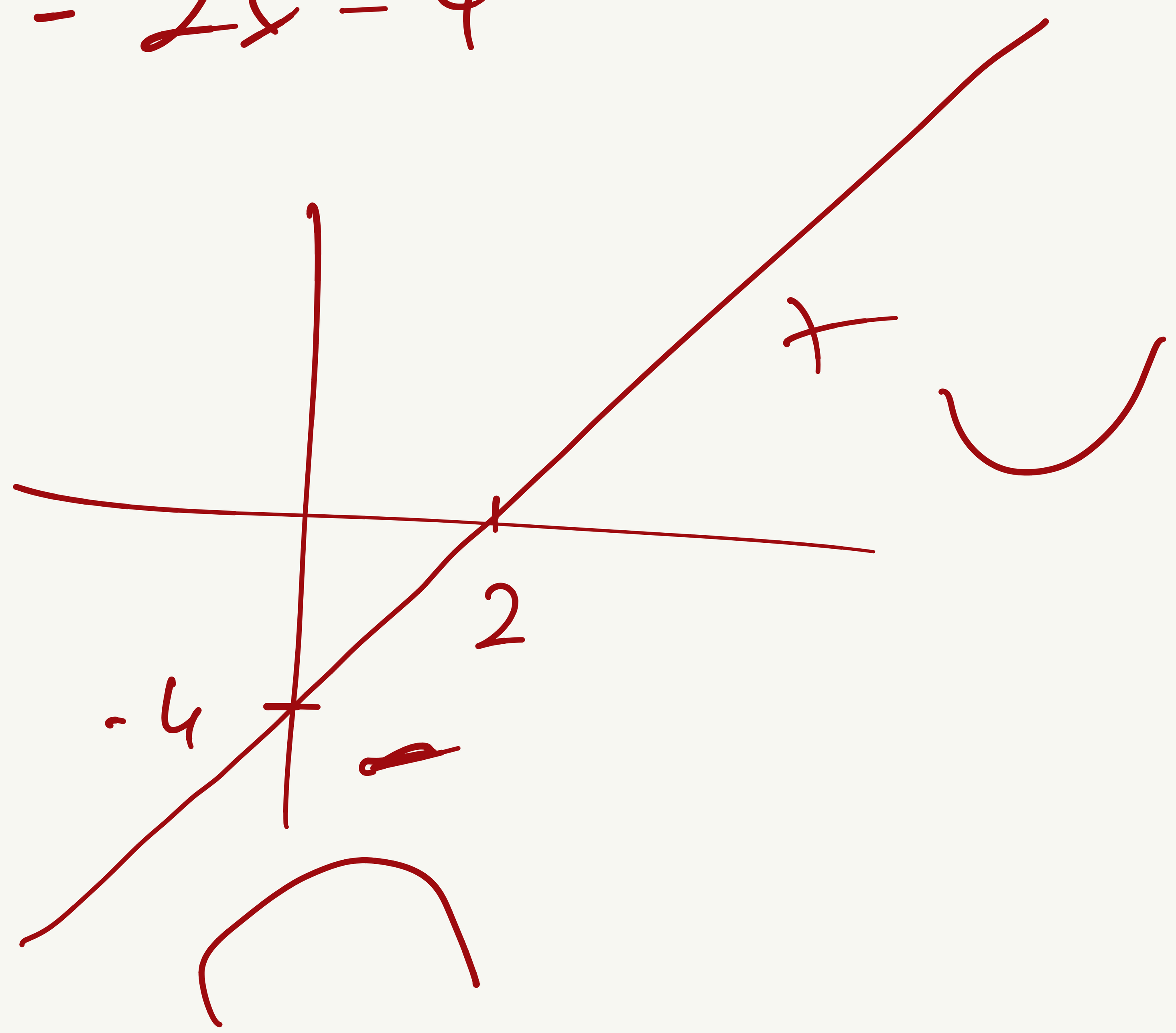
$$f(x) = \frac{x^3}{3} - 2x^2 + 4x - 7$$

$$f'(x) = \frac{3x^2}{3} - 4x + 4 = x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$\underline{\underline{x = 2}}$ Might have min/max

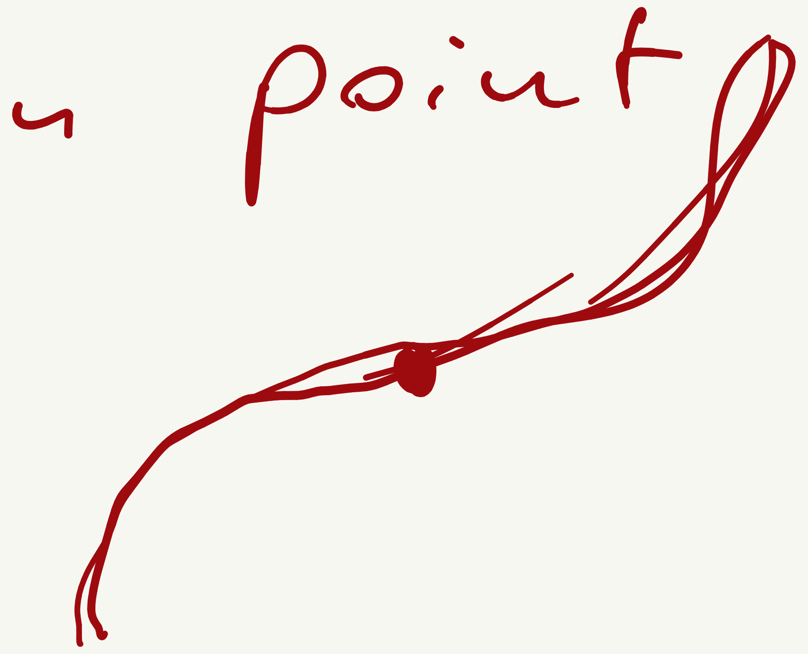
$$f''(x) = 2x - 4$$



$$f''(2) = 2 \cdot 2 - 4 = \underline{\underline{0}}$$

if second derivative $= 0$

$\Rightarrow$ inflection point



$$f(x) = \frac{1}{3}x^3 - 1.5x^2 - 4x + 10$$

$$f'(x) = x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0 \rightarrow x_1 = 4 \qquad x_2 = -1$$

$$f''(x) = 2x - 3$$

$$f''(4) = 2 \cdot 4 - 3 = 5$$

positive

convex

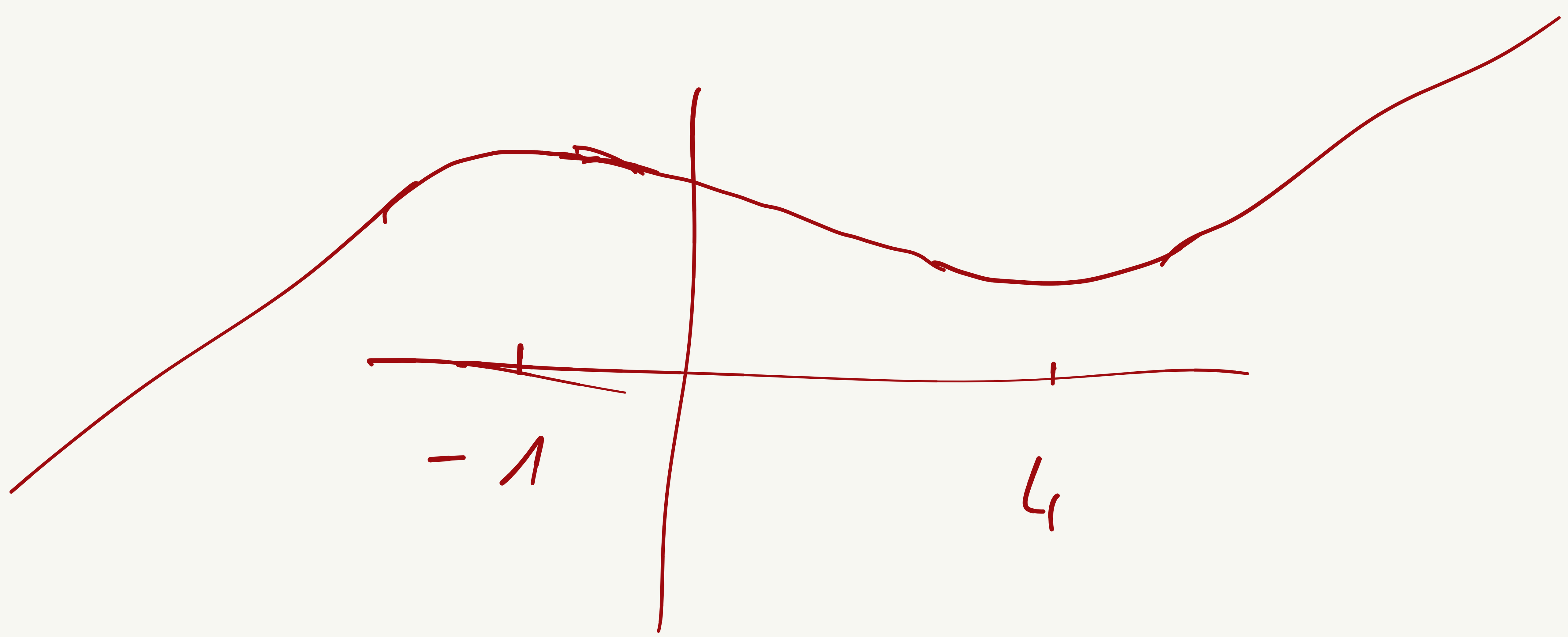
minimum

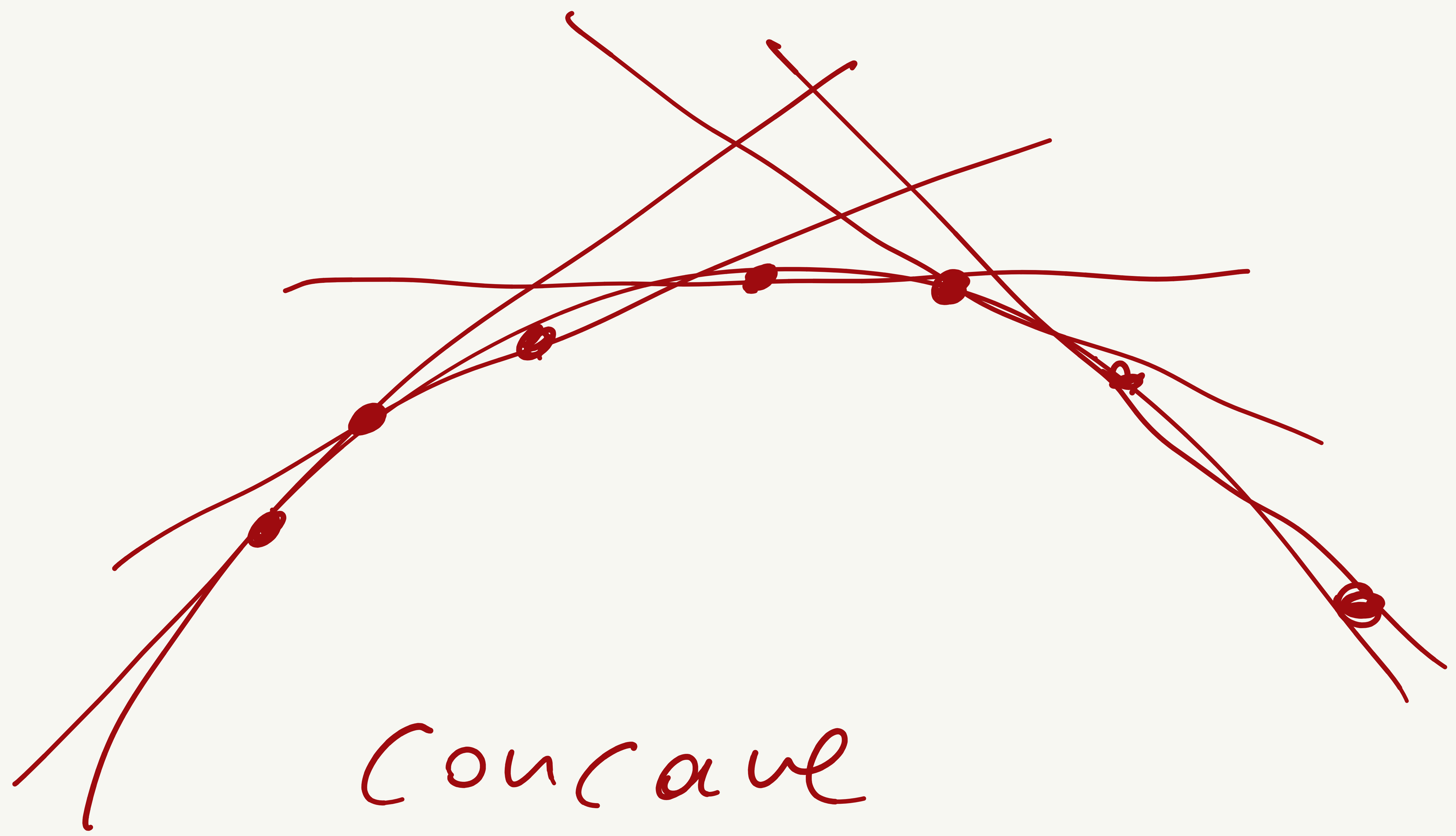
$$f''(-1) = 2 \cdot (-1) - 3 = -5$$

negative

concave

maximum





concave

$f'' < 0$

$f'' > 0$

7 5 3 0 -3 -5 -7

$\longrightarrow$

decreasing

second derivative negative

$f'' < 0$ -> slopes decreasing -> concave

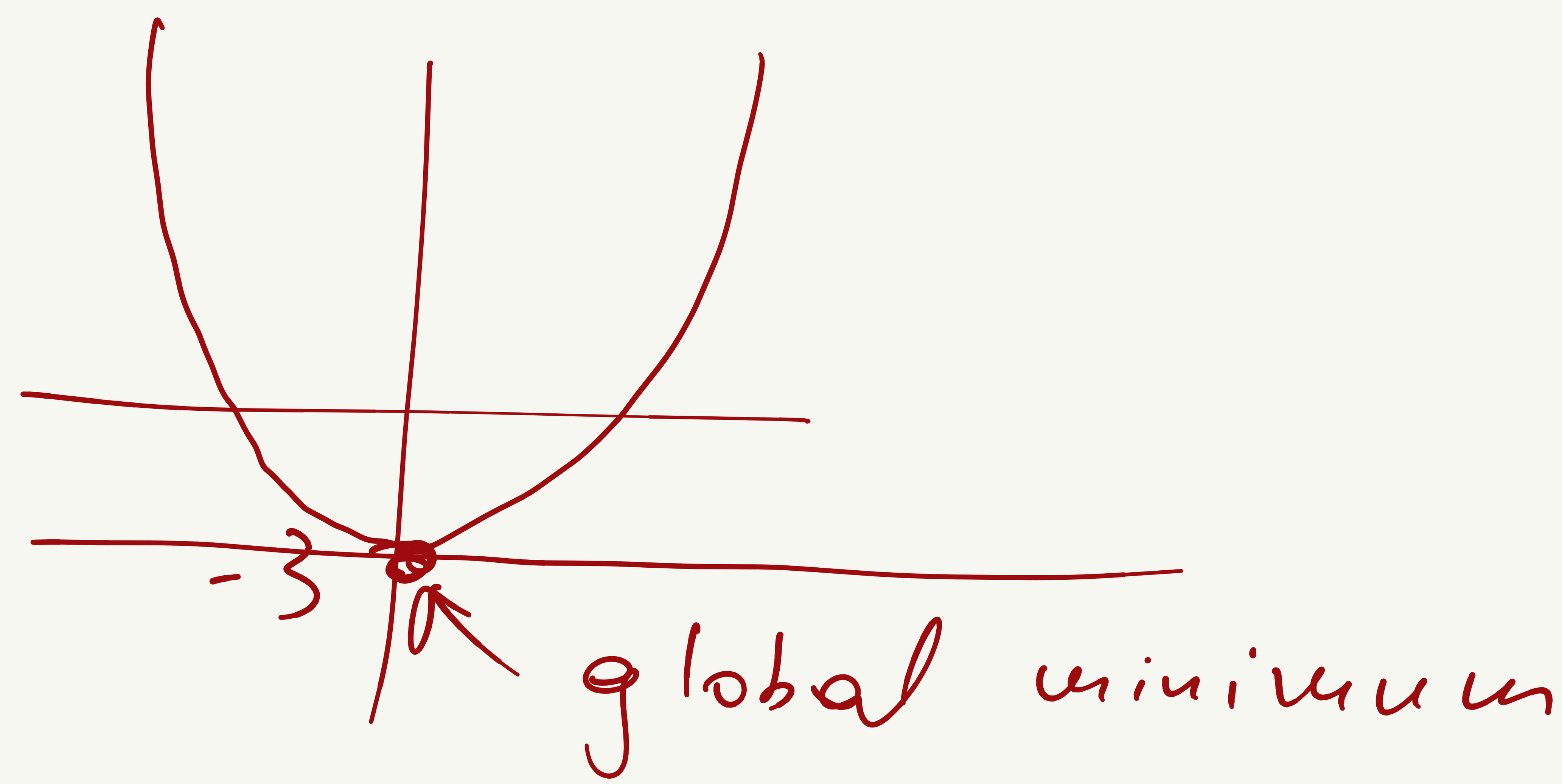
Local vs global

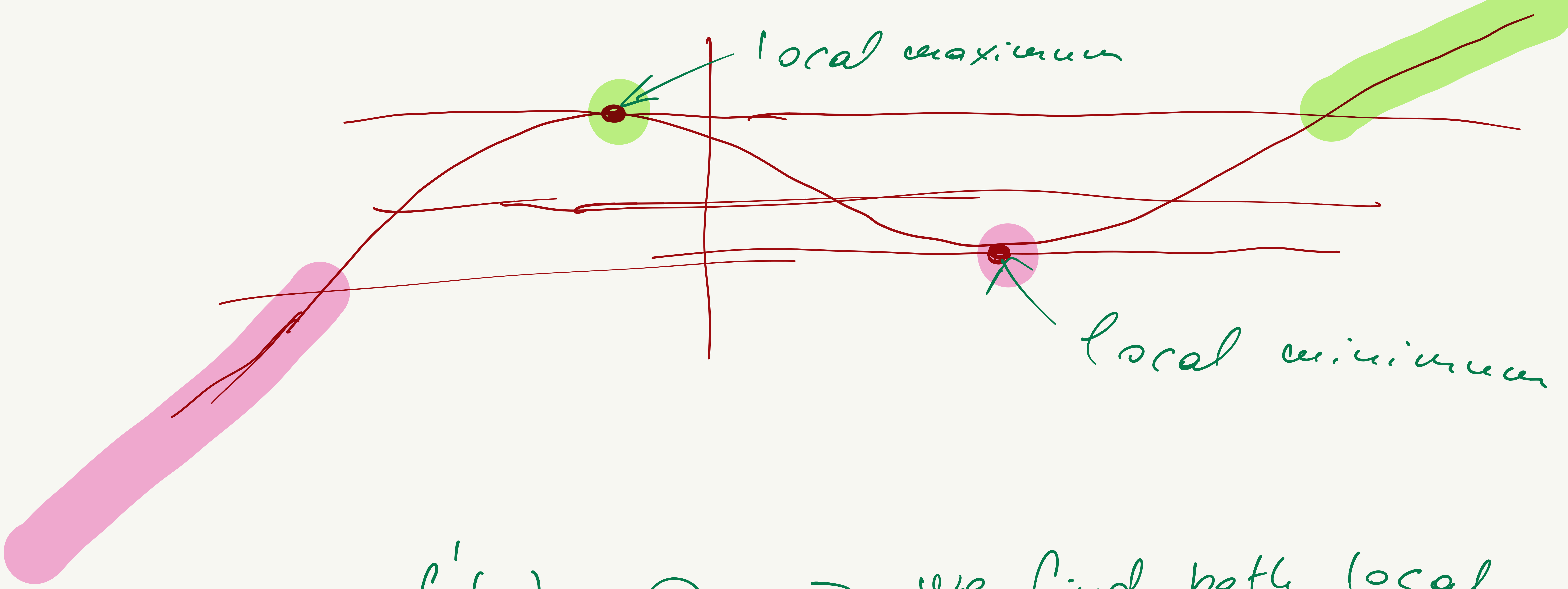
global maximum : no function value is larger than this

$$f(x) = -x^2 + 3$$

global minimum : no function value is lower than this

$$g(x) = x^2 - 3$$

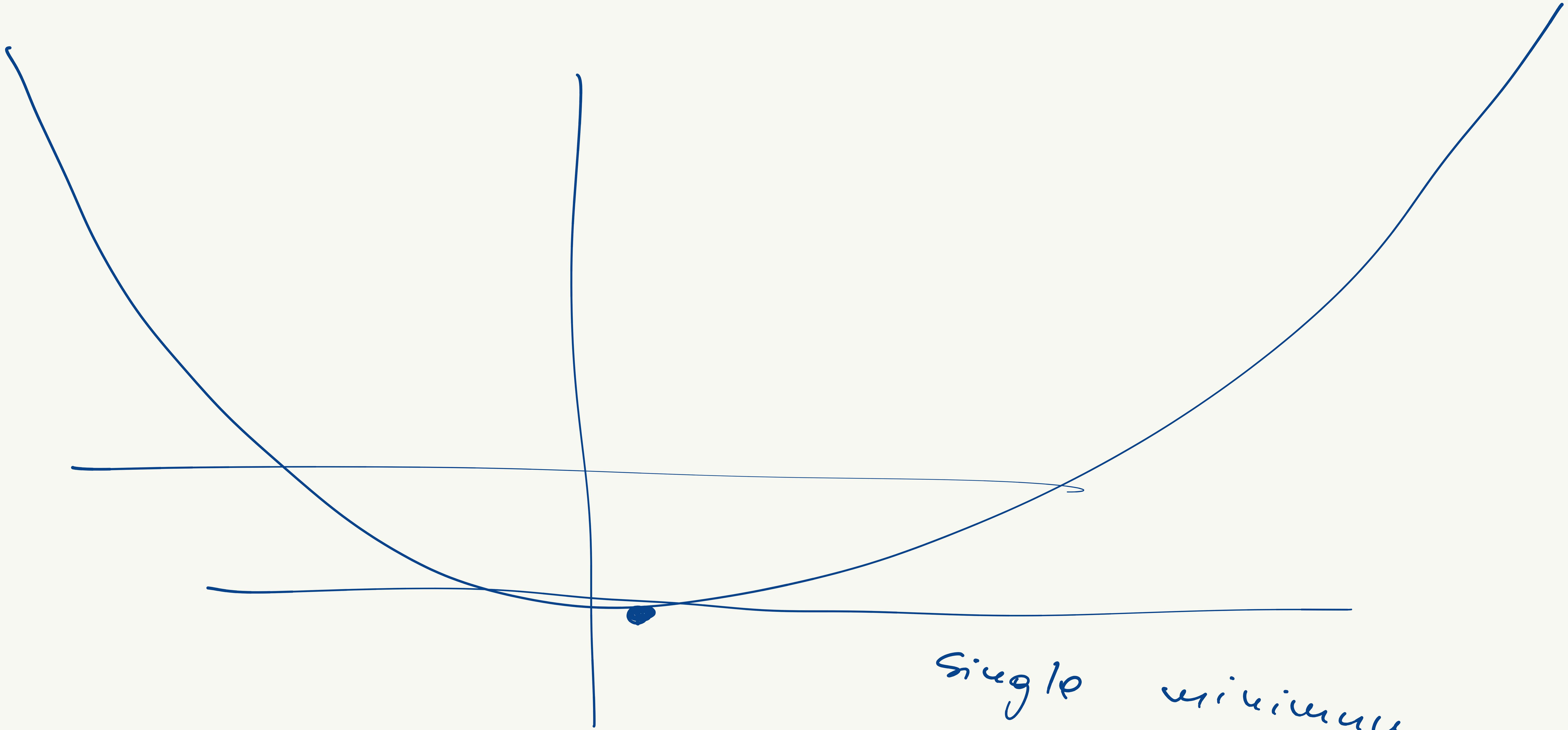




$f'(x) = 0 \Rightarrow$ we find both local and global minima/maxima goal : maximize function value

-> derivative = 0

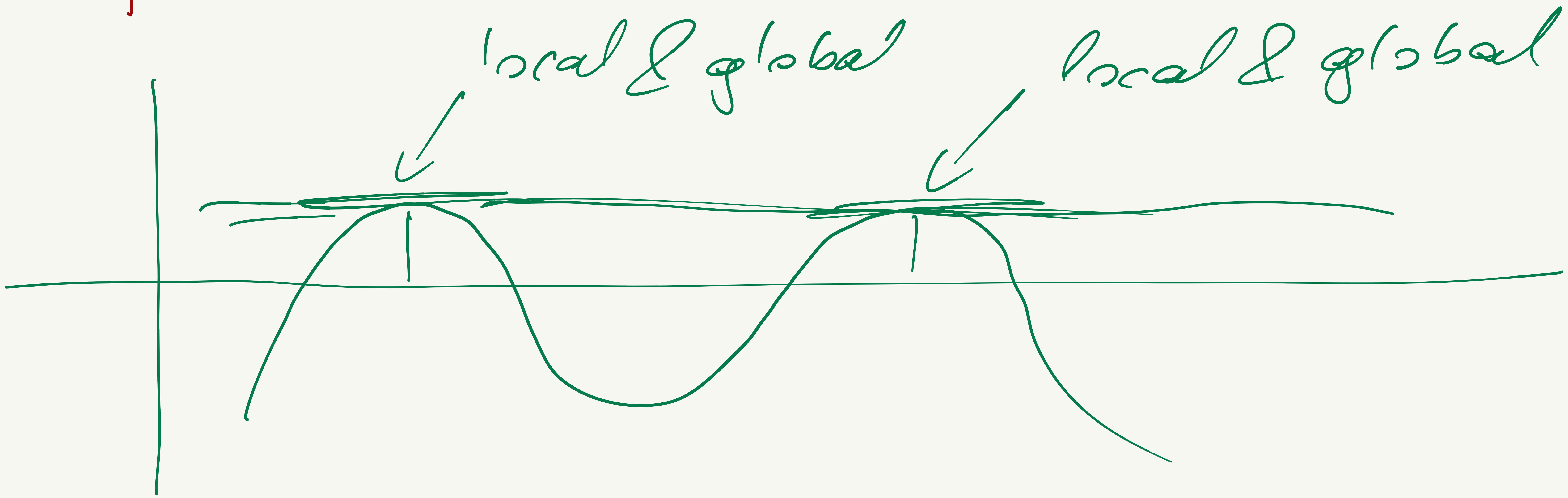
Convex problem: function optimized is convex



single minimum

=> global minimum


only local
local & global


local & global
local & global

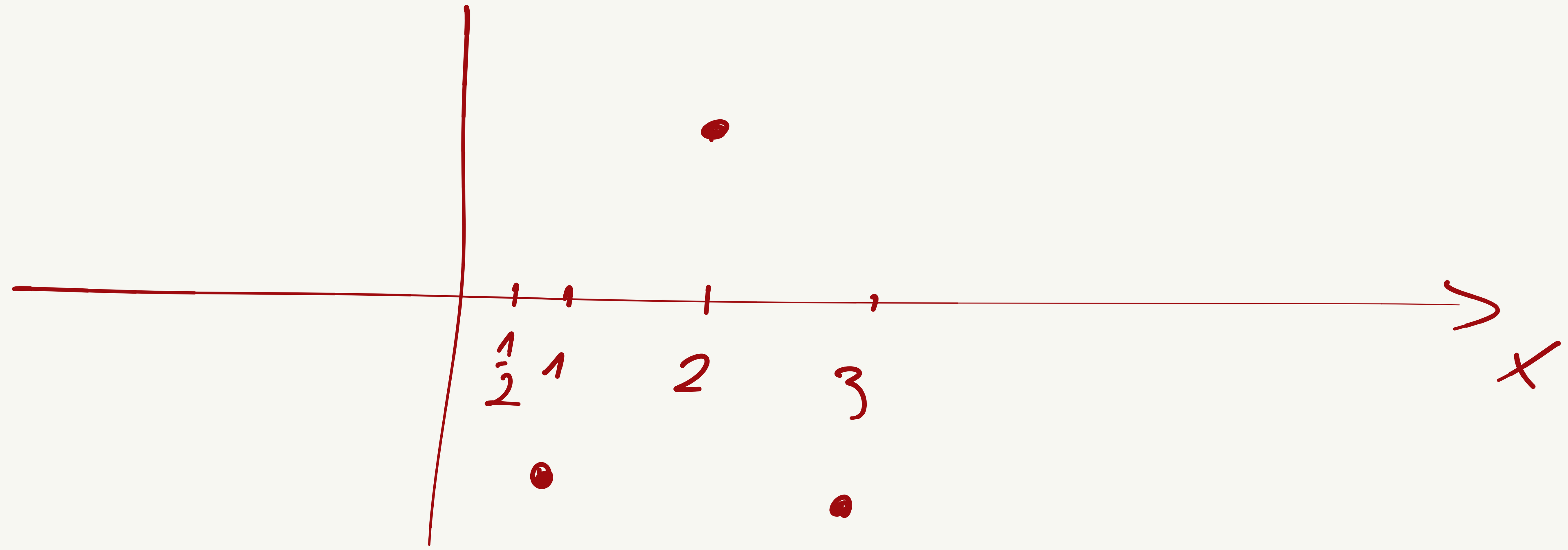
$f(x) = \sin(x)$

global max $\Rightarrow$ local max

local max might be global max

global max doesn't have to be unique.

$f(x) = (-1)^x$



$f(x) = a^x$

$f'(x) = a^x \ln|a|$

if $a > 0$

$f(x) = (-1)^{\frac{1}{2}} = \sqrt{-1} = i$

$(-1)^x \Rightarrow$ complex valued function

Domain : $\mathbb{R}$

Image : $\mathbb{C}$ $\to$ complex numbers