Random variable

- Experiment, outcome is a number

⇓

random variable

→ Discrete: Ω is finite or countably infinite

- Dice $\Omega = \{1,2,3,4,5,6\}$
- How many days does humanity has left? $\Omega = \{0,1,2,3,4 \dots\}$

→ Continuous: Ω uncountable

- Rainfall in mm
- Wait times

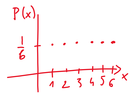
Discrete

Probability mass function.

→ Toss a dice

→ $\Omega = \{1,2,3,4,5,6\}$

$$P(x) = \frac{1}{6} \quad \forall x \in \Omega$$



- 2 dice, sum them up

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

X : sum

$\Omega = \{2,3,4,5, \dots, 12\}$ $P(x) = \frac{6-|7-x|}{36}$

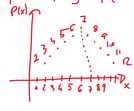
$P(2) = \frac{1}{36}$

$P(3) = \frac{2}{36}$

$P(4) = \frac{3}{36}$

$P(7) = \frac{6}{36}$

$P(12) = \frac{1}{36}$



$P(2) = \frac{6-|7-2|}{36} = \frac{1}{36}$

$P(9) = \frac{6-|7-9|}{36} = \frac{4}{36}$

$P(7) = \frac{6-|7-7|}{36} = \frac{6}{36}$

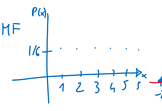
Cumulative density function.

- Probability that x is $\leq X$

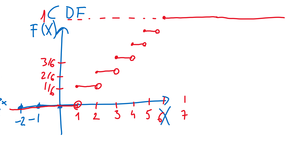
$$F(X) = P(x \leq X)$$

- dice $\Omega = \{1,2,3,4,5,6\}$

PMF



CDF



Expected value

random variable x

$$E(x) = \sum x \, P(x)$$

dice: $E(x) = \sum_{i=1}^{6} i \, P(i) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$

$= 3.5$

Uniform random variable

→ n outcomes DICE

$$P(x) = \frac{1}{n}$$

→ $E(x) = \frac{\sum x}{n} = \frac{1+2+3+4+5+6}{6} = 3.5$

Bernoulli rv

$\Omega = \{0,1\}$ $P(x) = p^x (1-p)^{1-x}$

$P(1) = p$

$P(0) = 1-p$ $E(x) = p \quad = 1 \cdot p + 0 \cdot (1-p) = p$

H 1 $p = 0.5$

0

Binomial rv

- Run experiment n times.
- Outcome is bernoulli, positive outcome w/ prob p
- How many times do we have a positive outcome?

$\Omega = \{0,1,2,3 \dots n\}$

$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$n = 5$ $P(2) = \binom{5}{2} 0.8^2 \cdot 0.2^3$

$x = 2$

$p = 0.8$

$\{0,1,0,0,1\} = 0.2 \cdot 0.8 \cdot 0.2 \cdot 0.2 \cdot 0.8$

$0.8^2 \cdot 0.2^3$

$\{1,1,0,0,0\}$

$E(x) = n \cdot p$

Geometric rv

- Repeat an experiment, Bernoulli
- Success probability p
- Repeat until you succeed
- What is the probability of repeating x times?

$P(x) = (1-p)^{x-1} p$ $E(x) = \frac{1}{p}$

$P(3) = (1-p)^2 p$