

Conditional probability

- A, B, Ω
- $P(A), P(B)$
- We know that B occurred. What is the probability that A also occurred?

$$\underbrace{P(A|B)}_{B \text{ occurred}} = \frac{P(A \cap B)}{P(B)}$$

$$\begin{aligned}\Omega &= \{1, 2, 3, 4, 5, 6\} & A &= \{1, 2\} & P(A) &= \frac{1}{3} \\ & & B &= \{2, 3\} & P(B) &= \frac{1}{3} \\ & & A \cap B &= \{2\} \\ & & P(A \cap B) &= \frac{1}{6}\end{aligned}$$

} Know that B occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/3} = 0.5$$

- $P(\text{throw 2 3s} \mid \text{first is 3})$

$$A: 2 \text{ 3s} = \{(3, 3)\} \quad P(A) = \frac{1}{36}$$

$$B: 1\text{st is 3} = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\} = \frac{6}{36} = \frac{1}{6}$$

$$A \cap B = \{(3, 3)\} \quad P(A \cap B) = \frac{1}{36}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{1/6} = \underline{\underline{\frac{1}{6}}}$$

- Monty Hall Problem