Matrices

Rectangular array of numbers.

A, q rows, n columns. ($q \times n$ matrix)

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & & & \vdots \\ a_{q,1} & a_{q,2} & \cdots & a_{q,n} \end{bmatrix}$$

$a_{i,j}$: (i,j)th element of A

3×4 $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ e & \pi & -4 & 2.3 \\ 21 & -2 & 1 & 0 \end{bmatrix}$

$X = Y$ if X is $q \times n$, Y is $q \times n$, $x_{i,j} = y_{i,j}$ $\forall i,j$

Transpose

A, $q \times n$ $\rightarrow$ A^T, $n \times q$ $\quad A^T_{i,j} = A_{j,i}$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \quad (A^T)^T = A$$

Add/subtract

A, B same order $q \times n$

$$A \pm B = \begin{bmatrix} a_{1,1} \pm b_{1,1} & a_{1,2} \pm b_{1,2} & \cdots & a_{1,n} \pm b_{1,n} \\ a_{2,1} \pm b_{2,1} & a_{2,2} \pm b_{2,2} & \cdots & a_{2,n} \pm b_{2,n} \\ \vdots & & & \vdots \\ a_{q,1} \pm b_{q,1} & a_{q,2} \pm b_{q,2} & \cdots & a_{q,n} \pm b_{q,n} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} -1 & -4 \\ -7 & 8 \end{bmatrix} \quad A + B = \begin{bmatrix} 1 & 0 \\ -2 & 15 \end{bmatrix}$$

Multiply by scalar

$$c \cdot A = \begin{bmatrix} c \cdot a_{1,1} & c \cdot a_{1,2} & \cdots & c \cdot a_{1,n} \\ \vdots & & & \\ c \cdot a_{q,1} & \cdots & \cdots & c \cdot a_{q,n} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} \quad 1.5 \cdot A = \begin{bmatrix} 3 & 4.5 & 6 \\ 7.5 & 9 & 10.5 \end{bmatrix}$$

Multiplication

A: $q \times n$ #cols in A = # rows in B

B: $n \times m$

$A \cdot B \rightarrow q \times m$ matrix

$$AB_{i,j} = \sum_{h=1}^{n} A_{i,h} \cdot B_{h,j}$$

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 6 \\ 7 & 1 \end{bmatrix} \quad 3 \times 2 \qquad 2 \times 4 \quad B = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 5 & 6 \end{bmatrix}$$

$1 \cdot 4 + 2 \cdot 6 = 16$
$2 \cdot 4 + 4 \cdot 6 = 32$
$3 \cdot 4 + 5 \cdot 6 = 42$
$1 \cdot 4 + 6 \cdot 6 = 40$

$1 \cdot 2 + 2 \cdot 1 = 4$
$2 \cdot 2 + 4 \cdot 1 = 8$
$3 \cdot 2 + 5 \cdot 1 = 11$
$1 \cdot 2 + 6 \cdot 1 = 8$

3×4

$1 \cdot 7 + 2 \cdot 1 = 9$
$2 \cdot 7 + 4 \cdot 1 = 18$
$3 \cdot 7 + 5 \cdot 1 = 26$
$1 \cdot 7 + 6 \cdot 1 = 13$

Square matrix : $q = n$

Diagonal matrix : A, only $a_{i,i}$ are non-zero.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Upper triangular : $a_{i,j} = 0$ if $i > j$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Lower triangular : $a_{i,j} = 0$ if $i < j$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

Symmetric matrix : $A = A^T$