Oliver Kiss
kiss_oliver@phd.ceu.edu
oliverkiss@pm.me
Budapest 1991 SeeYou@CEU
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Elementary algebra

- Powers
- Roots
- Fractions
- Equations
- Systems of equations

Powers

a^n → n th power of a

n → exponent

a → base

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$$

$$\boxed{a^0 = 1}$$

$a^{n+1} = a^n \cdot a$
$a^1 = a^0 \cdot a$
$a = a^0 \cdot a$
$\frac{a}{a} = 1 = a^0$

0^0 → undefined

$\forall$: for all

$$a^{-n} = \frac{1}{a^n} \quad \forall\, a \neq 0$$

0^{-n} → undefined

$\cancel{a^{-1} = a^0 \cdot a}$

$a^0 = a^{-1} \cdot a$
$1 = a^{-1} \cdot a$
$\frac{1}{a} = a^{-1}$

Rules:

$a^n \cdot a^m = a^{n+m}$
$\frac{a^n}{a^m} = a^{n-m}$

$(a^n)^m = a^{n \cdot m}$
$(a \cdot b)^n = a^n \cdot b^n$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$(-1)^5 = -1$

$$\frac{z^2 : z^5}{z^3 \cdot z^{-4}} = z^{2-5-3-(-4)} = z^{-2}$$

$$\frac{4^2 \cdot 6^2}{3^3 \cdot 2^3} = \frac{2^4 \cdot 3^2 \cdot 2^2}{3^3 \cdot 2^3} = \frac{2^6 \cdot 3^2}{2^3 \cdot 3^3} = \frac{2^3}{3} = \frac{8}{3}$$

$10^x : 10^5 = 10^{-2} \quad / \cdot 10^5$
$10^x = 10^{-2} \cdot 10^5$
$10^x = 10^3$
$x = 3$

$\left(\frac{xy}{z}\right)^{-2} = 3 \qquad \left(\frac{z}{xy}\right)^6 = ?$

$\left(\frac{z}{xy}\right)^2 = 3$
$\left(\left(\frac{z}{xy}\right)^2\right)^3 = \left(\frac{z}{xy}\right)^{2 \cdot 3} = \left(\frac{z}{xy}\right)^6 = 3^3 = 27$

Roots

$\sqrt[n]{x}$ → n th root of x

$\sqrt[n]{x}$ → z

$z^n = x$

$\sqrt[72]{196}$

$\sqrt[2]{4} = 2$ → z (n = 2, x = 4)

$2^2 = 4$

$z^n = x \quad / \wedge \frac{1}{n}$
$(z^n)^{\frac{1}{n}} = x^{\frac{1}{n}}$
$z^{n \cdot \frac{1}{n}} = x^{\frac{1}{n}}$
$\sqrt[n]{x} = z = x^{\frac{1}{n}}$
$\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$

radical notation

$\sqrt{}$: radical
n : degree
x : base

$\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$

positive root

$\sqrt{4} = 2$

$4^{\frac{1}{2}} = \begin{cases} 2 \\ -2 \end{cases}$

Properties

$\sqrt[b]{x^a} = x^{\frac{a}{b}}$

$\sqrt[n]{a \cdot b} = \sqrt[n]{a}\,\sqrt[n]{b} \quad \forall\, a, b \in \mathbb{R}_0^+$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \forall\, a \in \mathbb{R}_0^+ \ \& \ b \in \mathbb{R}^+$

$\mathbb{R}$: real number
7 -4 2.14

$\sqrt{1600} = 40 = \sqrt{16 \cdot 100} = \sqrt{16} \cdot \sqrt{100} = 4 \cdot 10$

$125^{\frac{1}{3}} = 5$
$\sqrt[3]{125} \qquad z^3 = 125$

$x^{0.25} = 2 \quad / \wedge 4$
$(x^{0.25})^4 = 2^4$
$x^1 = \boxed{x = 16}$

$(a+b)^{-0.5} = \frac{1}{\sqrt{a+b}} \qquad a+b > 0$

$(a+b)^{-0.5} = \frac{1}{(a+b)^{0.5}} = \frac{1}{(a+b)^{1/2}} = \frac{1}{\sqrt[2]{a+b}}$

$\sqrt{} = \sqrt[2]{}$

$\sqrt{-1} = \boxed{i} \in \mathbb{C}$

$4^{\frac{1}{2}} = \begin{cases} \boxed{2} \\ \boxed{-2} \end{cases}$

Rules of algebra

$a + b = b + a$: commutativity
$a \cdot b = b \cdot a$
$a(b+c) = ab + ac$: multiplication is distributive over addition

$(a+b)+c = a+(b+c)$: associativity
$(ab) \cdot c = a \cdot (b \cdot c)$

Useful identities

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

$$\frac{1000^2}{252^2 - 248^2} = \frac{1000^2}{(252+248)(252-248)} = \frac{1000^2}{500 \cdot 4} = \frac{1000^2}{2 \cdot 1000} =$$

$$\frac{1000}{2} = \boxed{500}$$

$\left(\frac{1}{2}x + \frac{1}{3}y\right)\left(\frac{1}{2}x - \frac{1}{3}y\right) = \frac{1}{4}x^2 - \frac{1}{9}y^2$
$(a+b)(a-b)$

Fractions:

$a : b = \frac{a}{b}$ (a → numerator, b → denominator)

$a < b$: proper fraction
$a \geq b$: improper fraction

$\frac{15}{7} = 2 + \frac{1}{7} = 2\frac{1}{7}$

mixed notation

AVOID!

Reducing fractions:

$$\frac{24}{36} = \frac{\cancel{3} \cdot 2 \cdot \cancel{2} \cdot \cancel{2}}{\cancel{3} \cdot 3 \cdot \cancel{2} \cdot \cancel{2}} = \frac{2}{3}$$

$$\frac{xy + x^2y}{xy^2 + x^3y^2} = \frac{\cancel{xy}(1+x)}{\cancel{xy}(y+x^2y)} = \frac{1+x}{y+x^2y}$$

$$\frac{x-1}{x^2-1} = \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \frac{1}{x+1} \neq \frac{0}{\cancel{x}+1}$$