Inverse of a matrix

A inverse A^{-1}

$$A^{-1}A = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{array}{cc|cc} & & 5 & 0 \\ A^{-1}\cdot A & & 0 & 2 \\ \hline \frac{1}{5} & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{array}$$

Systems of linear equations

$$\boxed{\begin{array}{ll} (1) & 3x + 4y = 10 \\ (2) & 2x + 4y = 6 \end{array}}$$

$3 \cdot 4 + 4(-\frac{1}{2}) = 10$ ✓
$2 \cdot 4 + 4(-\frac{1}{2}) = 6$ ✓

$$\boxed{\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}} = \begin{bmatrix} 3x+4y \\ 2x+4y \end{bmatrix}$$

$$\begin{array}{cc|c} & & x \\ & & y \\ \hline 3 & 4 & 3x+4y \\ 2 & 4 & 2x+4y \end{array}$$

$$\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{array}{cc|c} & & x \\ & & y \\ \hline 1 & 0 & x \\ 0 & 1 & y \end{array}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -0.5 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \leftarrow \text{unknowns}$$

$$A \cdot x = b$$

$$\boxed{x = A^{-1} \cdot b}$$

if A^{-1} exists

(1) $5x_1 + 3x_2 - x_3 + 2x_4 = 10$
(2) $x_1 - 2x_2 + 14x_3 + x_4 = 22$
(3) $7x_1 + x_2 - 21x_3 + 5x_4 = 41$
(4) $10x_1 - 5x_2 + 4x_3 + 7x_4 = 5$

$$\begin{bmatrix} 5 & 3 & -1 & 2 \\ 1 & -2 & 14 & 1 \\ 7 & 1 & -21 & 5 \\ 10 & -5 & 4 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 22 \\ 41 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 & 3 & -1 & 2 \\ 1 & -2 & 14 & 1 \\ 7 & 1 & -21 & 5 \\ 10 & -5 & 4 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 22 \\ 41 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{27299}{805} \\ \frac{3138}{161} \\ \frac{1907}{805} \\ \frac{49691}{805} \end{bmatrix}$$

(1) $2x + 4y = 6$
(2) $3x + 6y = 9$

$2x = 6 - 4y$
$x = 3 - 2y$

(2) $3(3-2y) + 6y = 9$
$9 - 6y + 6y = 9$
$\underline{\underline{9 = 9}}$

$x = 1$
$y = 1$

$x = 2$
$y = 0.5$

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

Singular matrix = A^{-1} doesn't exist

(1) $2x + 4y + z = 6$
(2) $3x + 6y - 3z = 9$
(3) $5x + 10y - 2z = 15$

(1) + (2) = (3)

$$\begin{bmatrix} 2 & 4 & 1 \\ 3 & 6 & -3 \\ 5 & 10 & -2 \end{bmatrix}^{-1} = ↯$$

Linear independence

(1) $2x + 4y = 6$
(2) $3x + 6y = 9$

$(1) \cdot c = (2)$
↳ if c exists
⇒ linearly dependent

(1) $2x + 4y + z = 6$
(2) $3x + 6y - 3z = 9$
(3) $5x + 10y - 2z = 15$

$(1) \cdot c_1 + (2) \cdot c_2 = (3)$
↳ if it holds for any (c_1, c_2) ⇒ linearly dependent

(3) $7x + 14y - z = 21$ $\quad (1) \cdot 2 + (2) = (3)$

$$(1) \cdot c_1 + (2) \cdot c_2 + (3) \cdot c_3 \neq (4)$$