Probability

- Random Experiments
- Probability distributions

Random experiments:

Experiments w/ an outcome that cannot be determined in advance.

- Flip a coin
- 2 people having the same birthday
- Dice roll
- Games
- Monthly rainfall
- # times M3 catches on fire

Sample space

the set of all possible outcomes Ω

- coin $\Omega = \{H, T\}$
- birthday $\Omega = \{Yes, No\}$
- M3 fire $\Omega = \{0, 1, 2, 3 \dots\} = \mathbb{Z}_0^+$
- Monthly rainfall $\Omega = \mathbb{R}_0^+$

Events

Pre-defined sets of outcomes. If the outcome is in the set, the event occurs.

- Toss 2 dice
  $\Omega = \{(1,1),(1,2),(1,3) \dots (6,6)\}$ 36 outcomes

  Event: sum is 10.
  $A = \{(4,6),(5,5),(6,4)\}$
- Monthly rainfall
  $\Omega = \mathbb{R}_0^+$

  Event: Rainfall is less than 15 mm.
  $A = [0, 15)$

  [] closed bracket ⇒ number included in the interval

  () : open bracket ⇒ Number is not included

2 events A, B

- $A \cup B$ occurs if A or B or both occur
  - Toss a dice $\Omega = \{1,2,3,4,5,6\}$

    A : outcome is even $A = \{2,4,6\}$

    B : outcome < 3 $B = \{1,2\}$

    $A \cup B = \{1,2,4,6\}$
- $A \cap B$ occurs if both A & B occur

  $A \cap B = \{2\}$

  if $A \cap B = \emptyset$ ⇒ A & B are disjoint events

  A : is one $\{1\}$ B : is three $\{3\}$

  A : odd $\{1,3,5\}$ B : even $\{2,4,6\}$

- A^C occurs if A does not.

  $A = \{1,3,5\}$ $A^C = \{2,4,6\}$

  $\Omega = \{1,2,3,4,5,6\}$

Probability

Number b/w 0 & 1 telling us how likely it is that an event occurs.

$$P(A) \geq 0$$

$$P(\Omega) = 1$$

For disjoint events: $P(\cup_i A_i) = \sum_i P(A_i)$

$\Omega = \{1,2,3,4,5,6\}$ $A_1 = \{1\}$, $A_2 = \{4\}$, $A_3 = \{5,6\}$ disjoint

$P(\cup_i A_i) = \sum_i P(A_i)$

$P(\{1,4,5,6\}) = P(\{1\}) + P(\{4\}) + P(\{5,6\})$

$\Omega = \{1,2,3,4,5,6\}$

$P(\{1\}) = \frac{1}{6}$ $P(\{1,2,3,4\}) = 0.5$ $\{3,4,5,6\}$

$A_1 = \{1\}$ $A_2 = \{2\}$ $A_3 = \{3\}$ $\dots$ $A_6 = \{6\}$

$P(\cup_i A_i) = P(\{1,2,3,4,5,6\}) = P(\Omega) = 1 = P(A_1) + P(A_2) + \dots P(A_6)$

$1 = 6\,P(A_i)$

$\frac{1}{6} = P(A_i)$

$B = \{2,4,6\}$ $P(B) = P(B_1) + P(B_2) + P(B_3)$

$B_1 = \{2\}$ $B_2 = \{4\}$ $B_3 = \{6\}$ $P(B) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3/6 = 0.5$

Often Ω : equally likely disjoint outcomes

C : $\{1,2\}$ $P(C) = \frac{2}{6} = \frac{\#C}{\#\Omega}$

Toss 2 dice $\Omega = \{(1,1),(1,2),(1,3) \dots (6,6)\}$

$\#\Omega = 36$ equally likely disjoint $P(A) = \frac{10}{36}$

$P(\text{sum is} > 8)$

$A = \{(4,5),(5,4),(5,5),(6,5),(5,6),(3,6),(6,3),(6,4),(4,6),(6,6)\}$ $\#A = 10$