Functions of one variable

one variable systematically depends on another

- $y = x+1$
- $y = x^2$
- total price = quantity × unit price
- distance = speed × time

Formally

Associating elements of a set w/ elements of another set

- Arguments (inputs): domain X
- Values: image Y
- A function is a set of pairs (x, y) where $x \in X$, $y \in Y$
- $y = f(x)$ $\quad f: X \to Y$

$f(\text{fruit}) = \text{color}$

$f(\text{lemon}) = \text{yellow}$ $\quad f(\text{apple}) = \text{red}$ green

$f(\text{plum}) = \text{purple}$

$f(x) = x^2$

$f(x) = x^{\frac{1}{2}}$

Numeric function

- square
- absolute value
- Types of numbers:
  - Natural number $(\mathbb{N})$: 0, 1, 2, 3, 4 ...
  - Integer number $(\mathbb{Z})$: .... -3, -2, -1, 0, 1, 2, 3 ...
  - Real number $(\mathbb{R})$: every number with a decimal point representation
  - Complex number $(\mathbb{C})$: $c + d\,i$ (i: imaginary unit)

RUN!

- $f(x) = x^2$ $\quad$ domain: $\mathbb{R}$, image: $\mathbb{R}_0^+$

$f(x) = \frac{1}{x+3}$ $\quad$ domain: $\mathbb{R} \setminus \{-3\}$

$\{\}$: set of numbers

$\setminus$: subtract

$f(x) = \sqrt{2x+4}$ $\quad$ domain: $x \geq -2$

$f(x) = \frac{3x+6}{x-2}$ $\quad$ 5 is in the image.

$5 = \frac{3x+6}{x-2}$ $\quad x \neq 2$ $\quad f(8) = \frac{3\cdot 8+6}{8-2} = \frac{30}{6} = 5$

$5x - 10 = 3x + 6$

$2x = 16$

$x = 8$

3 is not!

$3 = \frac{3x+6}{x-2}$

$3x - 6 = 3x + 6$

$-6 = 6$

$\nexists$

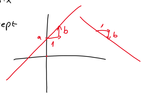
Linear functions

$y = a + b \cdot x$ $\quad a, b \in \mathbb{R}$

$f(x) = a + b \cdot x$

a: intercept

b: slope



C: temp in °C

F: — in °F

$C = a + bF$

(1) $0 = a + b \cdot 32$

(2) $100 = a + b \cdot 212$

(2)-(1) $100 = 180\,b$

$\frac{5}{9} = \frac{10}{18} = b$

(1) $0 = a + \frac{5}{9} 32$

$a = -\frac{160}{9} = -17.777$

$C = -17.777 + \frac{5}{9} F$

$C + 17.777 = \frac{5}{9} F$

$\frac{9}{5} C + 32 = F$

$C = F$

$\frac{9}{5} F + 32 = F$

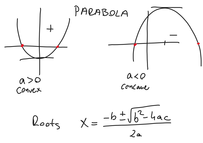
$32 = -0.8\,F$

$-40 = F$

Quadratic functions

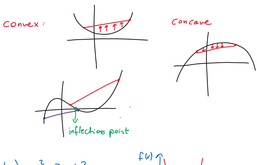
$f(x) = ax^2 + bx + c = 0$ $\quad a, b, c \in \mathbb{R}$

PARABOLA



Roots $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Minimum / Maximum : $\quad x = \frac{-b}{2a}$



$f(x) = x^2 - 3x + 2$

$f(x) = (x-2)(x-1)$

$x = 2$

$x = 1$

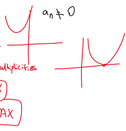
Polynomials:

degree n

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$a_0, a_1, \dots a_n \in \mathbb{R}$

$a_n \neq 0$



n roots

↳ some are complex

↳ there might be multiplicities

n-1 local extrema MAX

n-2 inflection points MAX