Calculus

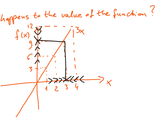
- Limits
- Differentiation
- Optimization

Limits

$f(x) = 3x$ what happens to the value of the function?

$\lim_{x \to 3} f(x) = 9$

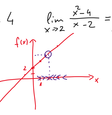
x approaches 3 from positive and negative sides



$\lim_{x \to 2} x^2 = 4$

$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2}$ } $x+2$

$x \neq 2$ $= 4$

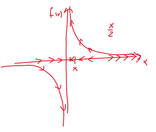


$f(x) = \frac{1}{x}$

$\lim_{x \to 0} f(x)$

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

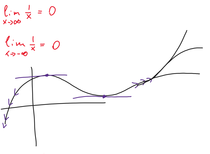


$f(x) = \frac{1}{x}$

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to -\infty} \frac{1}{x} = 0$

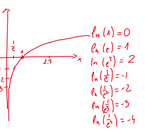
$g(x)$



$\lim_{x \to 0} \ln(x) = -\infty$

$\ln\left(\frac{1}{e^{10000}}\right) = -10000$

$\frac{1}{e^{10001}}$



$\ln(1) = 0$

$\ln(e) = 1$

$\ln(e^2) = 2$

$\ln\left(\frac{1}{e}\right) = -1$

$\ln\left(\frac{1}{e^2}\right) = -2$

$\ln\left(\frac{1}{e^3}\right) = -3$

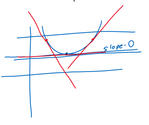
$\ln\left(\frac{1}{e^4}\right) = -4$

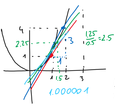
$\lim_{x \to 1} \frac{x^2+7x-8}{x-1} = \lim_{x \to 1} \frac{(x-1)(x+8)}{x-1} = \lim_{x \to 1} x+8$

$x \neq 1$

$= 9$

Differentiation





$f(x) = x^2$

$\lim_{x \to 1} \text{slope} = \frac{x^2-1}{x-1} =$

$\frac{(x-1)(x+1)}{x-1} = x+1 = 2$

derivative function

$f'(x) = \frac{d\,f(x)}{dx}$

$f(x) = x^2$ → $f'(x) = 2x$

$f'(1) = 2 \cdot 1 = 2$

$f'(x) = 2x = 0$

$x = 0$

$f'(4) = 2 \cdot 4 = 8$

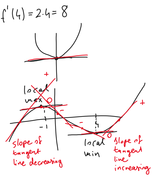
$f(x) = \frac{1}{3}x^3 - 1.5x^2 - 4x + 10$

$f'(x) = x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1$

$x = 4$



$f'(x)$

$f''(x) > 0$ local min

$f''(x) < 0$ local max

$f''(x) = 0$

$f(x) = x^3$

$f'(x) = 3x^2 = 0$

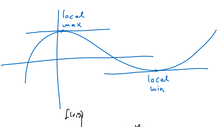
$x = 0$

$f''(x) = 6x$

$f''(0) = 0$

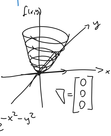
If $f'(x) = 0$

- if $f''(x) > 0$ local min
- if $f''(x) < 0$ local max
- if $f''(x) = 0$ inflection point



$\begin{bmatrix} f'_x(x,y) \\ f'_y(x,y) \end{bmatrix} = \nabla \begin{cases} f'_x(x,y) = 0 \\ f'_y(x,y) = 0 \end{cases}$ GRADIENT

$\begin{bmatrix} f'_x(x,y,z) \\ f'_y(x,y,z) \\ f'_z(x,y,z) \end{bmatrix} = \nabla$

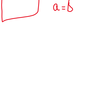


$f(x,y) = -xye^{-x^2-y^2}$

Constrained optimization

$f(a,b) = a \cdot b$ → max

$a + b = 10$ constraint



Lagrangian