Power functions

$f(x) = a x^b \qquad a, b \in \mathbb{R}$

$S = 4.84 V^{2/3}$

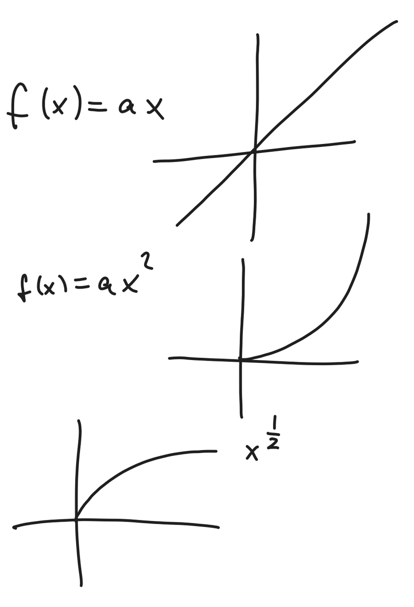
Convexity depends on b

$b = 1 \qquad f(x) = ax$

$b > 1 \qquad b=2 \; f(x) = ax^2$
Convex

$b < 1$
Concave

$x^{\frac{1}{2}}$



Exponential function

$f(x) = a \cdot \boxed{b}^x \qquad a, b \in \mathbb{R}$

→ base

$\$100 \qquad 5\% \qquad x$ years

$f(x) = 100 \cdot 1.05^x$

$e = 2.71828$ Euler's number

$\boxed{f(x) = e^x}$ Natural exponential function

$\$1 \qquad 100\%$

$1 \cdot (1+1) = 2$

$1(1+0.5)^2 = 2.25$

$1\left(1+\frac{1}{3}\right)^3 = 2.44$

$1\left(1+\frac{1}{n}\right)^n = \$e$

$n = \infty$

$\$100 < \begin{matrix} 2\% \\ 3\% \end{matrix} \Big\} 20$ years

$100 \cdot 1.02^{20} \qquad 100 \cdot 1.03^{20} \qquad \$32$

$100(1.03^{20} - 1.02^{20}) = 32.02$

Logarithmic functions

Problem:

$b^x = a \qquad a, b \in \mathbb{R}$

$b = e$

$e^x = a$

$x = \ln a$

↳ natural logarithm

$\ln a$ is a number s.t.

$e^{\ln a} = a \qquad \ln 1 = 0$

$e^{\ln 1} = 1 \qquad \ln e = 1$

$e^{\ln e} = e \qquad \ln e^5 = 5$

$e^{\ln e^5} = e^5 \qquad \ln \frac{1}{e} = -1$

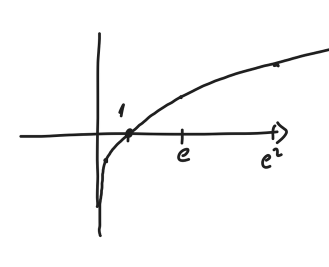
$e^{\ln \frac{1}{e}} = \frac{1}{e} = e^{-1} \qquad \ln(-6) = \cancel{\varnothing}$

$e^{\ln(-6)} = -6$

$f(x) = \ln(x) \qquad$ domain $\mathbb{R}^+$, image $\mathbb{R}$

0.0000000001

e^-



1. $\ln(x \cdot y) = \ln(x) + \ln(y)$

$e^{\boxed{\ln(xy)}} = xy = e^{\ln x} \cdot e^{\ln y} = e^{\boxed{(\ln x + \ln y)}}$

$x = e^{\ln x}$

$y = e^{\ln y}$

2. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$e^{\boxed{\ln \frac{x}{y}}} = \frac{x}{y} = \frac{e^{\ln x}}{e^{\ln y}} = e^{\boxed{\ln x - \ln y}}$

3. $\ln x^y = y \ln x$

$e^{\boxed{\ln x^y}} = x^y = \left(e^{\ln x}\right)^y = e^{\boxed{y \cdot \ln x}}$

$x = e^{\ln x}$

$5e^{-3x} = 16$

$e^{-3x} = 3.2$

$-3x = \ln 3.2$

$\boxed{x = -\frac{\ln 3.2}{3}}$

$(1.08)^x = 10$

$\ln 1.08^x = \ln 10$

$x \cdot \ln 1.08 = \ln 10$

$x = \frac{\ln 10}{\ln 1.08}$

$3^x 4^{x+2} = 8$

$\ln[3^x 4^{x+2}] = \ln 8$

$\ln 3^x + \ln 4^{x+2} = \ln 8$

$x \ln 3 + (x+2)\ln 4 = \ln 8$

$x \ln 3 + x \ln 4 + 2 \ln 4 = \ln 8$

$x(\ln 3 + \ln 4) = \ln 8 - 2\ln 4$

$\boxed{x = \frac{\ln 8 - 2\ln 4}{\ln 3 + \ln 4}}$

$8^2 \qquad x^2$

$\ln 8 \qquad \ln x$

$e^x = a$

$b^x = a$

$x = \log_b a$

$10^x = 100$

$x = \log_{10} 100 = 2$

$7^x = 24$

$x = \log_7 24$

$\pi^x = 8$

$x = \log_\pi 8$

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^y = y \log_a x$

$\log_a b = \frac{\log_c b}{\log_c a}$

$\log_{e^2} e^5 = \frac{\ln e^5}{\ln e^2} = \frac{5}{2}$

$\log_5 125 = 3 \qquad 5^{\log_5 125} = 125$

$\log_{125} 5 = \frac{1}{3} \qquad 125^{\log_{125} 5} = 5$

$\log_{11}(x+5) = -1$

$11^{\log_{11}(x+5)} = 11^{-1}$

$x + 5 = \frac{1}{11}$

$x = \frac{1}{11} - 5 = -\frac{54}{11}$

$\log_7(-8r) = 1$

$7^{\log_7(-8r)} = 7^1$

$-8r = 7$

$\boxed{r = -\frac{7}{8}}$