Functions of One Variable Problem Set Solutions

1. For the function $f(x) = 2x + 3$, the domain is $\mathbb{R}$ (all real numbers) and the range is also $\mathbb{R}$ (all real numbers).

2. Using the two points (1,2) and (3,8), we get the slope $m = \frac{8-2}{3-1} = 3$ and the y-intercept $b = 2 - 3 \cdot 1 = -1$. So the equation is $y = 3x - 1$.

3. The vertex of the quadratic function $g(x) = x^2 - 4x + 3$ is given by $(-\frac{b}{2a}, g(-\frac{b}{2a}))$. Here, $a = 1$ and $b = -4$. So the vertex is $(2, -1)$.

4. To sketch the graph of $f(x) = |x - 2|$, plot the vertex at (2, 0) and create a V-shaped graph where y is zero for $x = 2$ and increases as x moves away from 2.

5. For the function $f(x) = \sqrt{x + 4}$, the domain is $x \geq -4$ and solving for x gives $x = y^2 - 4$, where $y \geq 0$.

6. Taking the natural logarithm of both sides, $\ln(V(t)) = \ln(Pe^{rt})$, gives $rt = \ln(2)$, which implies $t = \frac{\ln(2)}{0.05}$.

7. The function $f(x) = x^2$ is not one-to-one because $f(a) = f(-a)$ for any $a \in \mathbb{R}$.

8. The function that models a population that triples every year is $P(t) = P_0 \cdot 3^t$.

9. The x-intercepts of $f(x) = x^2 - 5x + 6$ are found by setting the function equal to zero and factoring, resulting in $x = 2$ and $x = 3$.

10. Evaluating $f(3)$ gives $f(3) = 3^3 - 6(3)^2 + 9(3) - 4 = 27 - 54 + 27 - 4 = -4$.

11. The x-intercept is when $y = 0$, giving $-2x + 5 = 0 \Rightarrow x = 2.5$. The y-intercept is when $x = 0$, giving $y = 5$.

12. For $h(x) = \sqrt{x - 1}$, the domain is $x \geq 1$ and the range is $y \geq 0$ since it is a square root function.

13. The y-coordinate of the vertex (using $\frac{-b}{2a}$) is $y = -3(\frac{-12}{2(-3)})^2 + 12(\frac{-12}{2(-3)}) - 7 = 5$.

14. $5^{2x} = 125 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$.

15. Let $N(t) = 2N_0$. Then $2 = e^{2k}$, and taking the natural logarithm gives $k = \frac{\ln 2}{2}$.

16. $\log_{10} 1000 - \log_{10} 10 = 3 - 1 = 2$.

17. Solving $(x-1)^2 = 9$ for x gives $x - 1 = \pm 3$, therefore $x = 4$ or $x = -2$.

18. $C(25) = 50\sqrt{25} + 400 = 50 \cdot 5 + 400 = 250 + 400 = \$650$.

19. The domain of $g(x) = \frac{2}{x-5}$ is all real numbers x except $x \neq 5$.

20. The domain of $h(x) = \sqrt{5x - 10}$ is all real numbers x such that $5x - 10 \geq 0$.

21. Plugging in x values and solving for $p(x) = 2$ shows that 2 is not in the image of $p(x)$.

22. When $x = 10$, $y = 3(10) + 12 = 30 + 12 = 42$.

23. $$150 \cdot (1.04^{10} - 1.025^{10})$$

24. Solving for x: $e^x = \frac{1}{5}$ gives $x = \ln(\frac{1}{5}) = -\ln(5)$.

25. Solving for x: $\log_3(9x) = 2$ yields $x = \frac{3^2}{9} = 1$.

26. The solution to $4^{2x} = \frac{1}{16}$ is $x = -1$.