Functions

$f(x) = y$ $\quad x \mapsto y$

Examples

$y = mx$ $\quad$ distance = elapsed time × speed

$y = x^2$ $\quad$ $P(x) = \text{Number}(x)$ and $P \in \mathbb{R}$

Function

an object with elements of a set with an element of another set

size of a person (input) $\quad$ domain $\quad X$

size of which is can take $\quad$ range $\quad Y$

function $\quad$ a set of ordered pairs (x, y) where $x \in X$, $y \in Y$

$f: X \to Y$ $\quad$ $y = f(x)$

$f(\{x_1, x_2\}) = \{x_1, x_2\}$

$f(\{x_1, x_2\}) = \{y_1, y_2\}$ $\quad$ injective

$f(\{y_1, y_2\}) = \{x_1, x_2\}$ $\quad$ surjective

$f(\{x_1, x_2\}) = \{y_1, y_2\}$ $\quad$ bijective

f $\quad$ second in new speed

$f(\text{Cristiano}) = 103\ \text{km/h}$

$f(\text{Lee Jeonmin}) = 0.004\ \text{km/h}$

Inverse functions

$f(x) = x^2$

$f(x) = x^3$

$f: \mathbb{R} \to \mathbb{R}_0^+$

$f(x) = \frac{1}{x}$ $\quad$ Domain: $X = \mathbb{R} \setminus \{0\}$

range for $-$

$\{\}$ set of numbers

range: $Y = \mathbb{R} \setminus \{0\}$

$0 = \frac{1}{x}$

$0(x) = 1$

$0 = 1$

$g(x) = f(x+1)$

$X: \quad x \in \mathbb{R} \quad x \geq -2$

$Y: \mathbb{R}_0^+$

$f(x) = \frac{3x+6}{x-2}$ $\quad 5 \in X$, $3 \in Y$

$5 = \frac{3x+6}{x-2}$

$5 \cdot (x-2) = 3x+6$

$2x = 16$

$x = 8$

$3 = \frac{3x+6}{x-2}$

$3x - 6 = 3x + 6$

$-6 = 6$

Linear functions

$f(x) = ax + b$

a: slope

b: shift

$C \to F$

$0°C = 32°F$ $\quad \subseteq F$

$100°C = 212°F$ $\quad$ temp $\to$ temp

$C = ax + b$

(1) $0 = a \cdot 0 + b \Rightarrow 32$

(2) $100 = a \cdot 100 + 32$

(2)-(1) $\quad 100 = 100a + 32 + 0$

$100 = 100a$

$b = \frac{100}{100} = 1$

(1) $0 = a \cdot 0 + 32$

$-32 = a \cdot 0$

-0.32

$C = 0.32°F$

$C + 0.32 = °F$ $\quad / \frac{1}{2}$

$\frac{18}{5}C + 32 = F$

$\sim 3.6 + 2C$

Quadratic functions

$y = ax^2 + bx + c$

$\lambda(a), (x, y) \mid (y \geq \lambda(a) \cdot x + b \cdot a)$

$\forall a, b \in X \ \& \ \lambda \in (0, 1)$

$f(x) = 2^x$ $\quad$ convex

Polynomials

a polynomial of degree $n \geq 0$

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

each $a_i \in \mathbb{R}$, $a_n \neq 0$

degree 5: $f(x) = 2x^5 - x + 2$

Polynomial of degree $n \geq 1$

max n roots or pairs where $a = 0$

max $n-1$ local extrema

$n-2$ inflection points

Power functions

$\to \frac{1}{x^2}$ $\quad f(x) = a \cdot x^b$

$a \cdot x^b$

Exponential functions

$f(x) = b \cdot \frac{(1+r)^x}{\text{interest}}$

\$100 5% $\quad f(x) = 100 \cdot 1.05^x$

Euler's number $\quad e = 2.71828$

$f(x) = e^{0.05x} = 2.71828^{0.05x}$

\$1 $\quad$ 100% annual interest

$f(1+1) = 2$

then $\quad f(1+\frac{1}{2})^2 = 2.25$

then $\quad f(1+\frac{1}{4})^4 = 2.44$

when $\quad f(1+\frac{1}{n})^n$

Continuously $\quad \lim_{n \to \infty} \quad \$e$

Logarithm functions

$a^x = n$

$x = \log_a n$

$\log_a n = \log_a n$

$b^{\log_b n}$

$x = n$

$b^{\log_b x} = x$

$f(1) = 0$ $\quad b^{\log_b 1} = 1 = b^0$

$f(2) = 1$ $\quad b^{\log_b 2} = 2 = b^1$

$f(10^3) = 3$ $\quad b^{\log_b 10^3} = 10^3$

$f(\frac{1}{2}) = -1$ $\quad b^{\log_b \frac{1}{2}} = \frac{1}{2} = b^{-1}$

$f(\frac{1}{10}) = -3$ $\quad b^{\log_b \frac{1}{10}} = \frac{1}{10} = b^{-3}$

$f(-1) = \text{undefined}$ $\quad b^{\log_b -1} = -1 = b^{-1}$

$X: \mathbb{R}^+$

$Y: \mathbb{R}$

$f(xy) = f(x) + f(y)$ $\quad b^{\log_b(xy)} = b^{\log_b x} \cdot b^{\log_b y} = xy$

$b^{\log_b(xy)} = xy$

$f(\frac{x}{y}) = f(x) - f(y)$ $\quad b^{\log_b(\frac{x}{y})} = \frac{b^{\log_b x}}{b^{\log_b y}} = \frac{x}{y}$

$f(x^n) = n \cdot f(x)$ $\quad b^{\log_b(x^n)} = (b^{\log_b x})^n = x^n$

$5^x \cdot 2^{x+1} = 14$ $\quad f(10^{x^2}) = 20$

$5^x \cdot 2^x \cdot 2 = 14$ $\quad f(10^{x^2}) = f(20)$

$f(5^x) = f(7)$ $\quad x^2 f(10) = f(20)$

$-x = f(7)$ $\quad \boxed{x = \sqrt{\frac{f(20)}{f(10)}}}$

$x = -\frac{f(7)}{2}$

$5^x \cdot 2^{x+1} = 8$

$f(5^x \cdot 2^{x+1}) = f(8)$

$f(5^x) + f(2^{x+1}) = f(8)$

$x f(5) + (x+1) f(2) = f(8)$

$x f(5) + x f(2) + 2 f(2) = f(8)$

$x f(2 \cdot 5) = f(8) - 2 f(2)$

$x = \frac{f(8) - f(2 \cdot 5)}{f(2 \cdot 5)}$

$\log_3 x = 4$ $\quad \log_3 x = b$

$\log_3(xy) = \log_3 x + \log_3 y$

$\log_3(\frac{x}{y}) = \log_3 x - \log_3 y$

$\log_3(x^n) = y \log_3 x$ $\quad \log_{10} \log_3 x$

$\log_3 x = \frac{\log_{10} x}{\log_{10} 3}$ $\quad \log_3 x = \log_{10} x$

$\log_3 x = 3$ $\quad \log_3 x^{0.5} = 0.5 = 5^{-1}$

$\log_3 x^{-1} = \frac{1}{3}$ $\quad 3^{\log_3 x^{0.5}} = 5^{-1} = 3^{\log_3 x}$

$\log_3(xy) = -1$

$\log_3(xy) = -1$

$xy = 3^{-1}$

$xy = \frac{1}{3}$

$x + \frac{1}{5} = -\frac{5}{3}$