

Fractions

$$a : b = \frac{a}{b}$$

$a < b$: proper fraction

$a \geq b$: improper fraction

$$\frac{15}{7} = 2 + \frac{1}{7} = 2\frac{1}{7} \rightarrow \text{mixed notation}$$

$$2\frac{1}{7}$$

Reducing fractions

$$\frac{24}{36} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{3} \cdot \cancel{3} \cdot \cancel{2} \cdot 2} = \frac{2}{3}$$

$$\frac{x-1}{x^2-1} = \frac{x-1}{x^2-1^2} = \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \frac{1}{x+1} \neq \frac{1}{x+1}$$

Multiply

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

Add fractions

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b} = \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{d \cdot b} = \frac{a \cdot d + c \cdot b}{b \cdot d}$$

$$bd$$

$$\frac{5}{7} + \frac{3}{7} - \frac{2}{7} = \frac{6}{7}$$

$$\begin{aligned} \frac{\frac{1}{x-1} + \frac{1}{x^2-1}}{x - \frac{2}{x+1}} &= \frac{\frac{1}{x-1} + \frac{1}{(x-1)(x+1)}}{x - \frac{2}{x+1}} = \\ \frac{\frac{1}{x-1} \cdot \frac{x+1}{x+1} + \frac{1}{(x-1)(x+1)}}{\frac{x(x+1)}{x+1} - \frac{2}{x+1}} &= \frac{\frac{x+1+1}{(x-1)(x+1)}}{\frac{x(x+1)-2}{x+1}} = \frac{x+2}{(x-1)(x+1)} \cdot \frac{x+1}{x(x+1)-2} = \\ &= \frac{(x+2)(x+1)}{(x-1)(x+1)[x(x+1)-2]} = \frac{x+2}{(x-1)[x^2+x-2]} = \frac{x+2}{(x-1)(x+2)(x-1)} = \\ &= \frac{1}{(x-1)^2} \end{aligned}$$

Simple equations

- add same number
- subtract same number
- multiply or divide by same non-zero number

$$\begin{array}{rcl} 3x+10 & = & x+4 \quad /-10 \\ 3x & = & x+4-10 \quad /-x \\ 2x & = & -6 \quad / :2 \\ x & = & -3 \end{array}$$

x: hourly wage

38 hours

overtime: twice my hourly wage

48 hours \Rightarrow \$812

$$38x + 10 \cdot 2x = 812$$

$$58x = 812$$

$$x = 14$$

$$\begin{aligned} \frac{x+2}{x-2} - \frac{8}{x(x-2)} &= \frac{2}{x} \quad / \cdot x(x-2) \neq 0 \\ x(x+2) - 8 &= 2(x-2) \\ x^2 + 2x - 8 &= 2x - 4 \\ x^2 - 4 &= 0 \\ (x-2)(x+2) &= 0 \\ x-2 &= 0 \quad x+2 = 0 \\ x &= 2 \quad x = -2 \end{aligned}$$

Quadratic equations

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} 15x - x^2 &= 0 \quad x = \\ (15-x)x &= 0 \\ x=15 \quad x=0 \end{aligned}$$

$$\begin{aligned} x^2 - 4x + 4 &= 0 \\ (x-2)^2 &= 0 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x-3)(x-2) &= 0 \\ x=3 \quad x=2 \end{aligned}$$

Systems of equations

$$(1) \quad 2x + 3y = 18$$

$$(2) \quad 3x - 4y = -7$$

Substitution:

$$\begin{aligned} (1) \quad 2x + 3y &= 18 \\ 2x &= 18 - 3y \\ x &= 9 - 1.5y \end{aligned}$$

$$\begin{aligned} (2) \quad 3(9 - 1.5y) - 4y &= -7 \\ 27 - 4.5y - 4y &= -7 \\ -8.5y &= -34 \\ y &= 4 \\ x &= 9 - 1.5 \cdot 4 = 3 \end{aligned}$$

Elimination:

$$\begin{aligned} (1) \quad 2x + 3y &= 18 \quad / \cdot 1.5 \\ 3x + 4.5y &= 27 \\ (2) \quad 3x - 4y &= -7 \end{aligned}$$

$$(1) - (2) \quad 3x + 4.5y - 3x + 4y = 27 + 7$$

$$8.5y = 34$$

$$y = 4$$

$$(1) \quad 2x + 3 \cdot 4 = 18$$

$$2x = 6$$

$$x = 3$$

$$x = 9 - 1.5 \cdot 4 = 3$$

$$\$10000$$

$$5\% \quad 7.2\%$$

$$\$676 \quad \text{interest}$$

$$\begin{aligned} (1) \quad x + y &= 10000 \\ (2) \quad 0.05x + 0.072y &= 676 \end{aligned}$$

$$(1) \quad y = 10000 - x$$

$$(2) \quad 0.05x + 0.072(10000 - x) = 676$$

$$0.05x + 720 - 0.072x = 676$$

$$-0.022x = -44$$

$$x = 2000$$

$$y = 8000$$