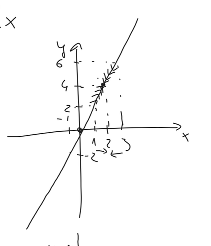
Calculus

- Limits
- Differentiation
- Unconstrained optimization

Limits

- Function $f(x)$, on the x axis we approach p
- Where do the function values go?

$f(x) = 2x$

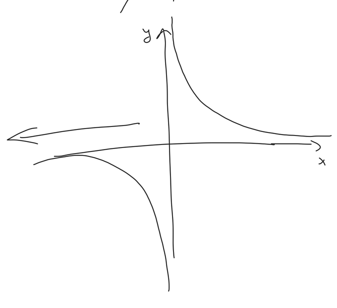


limit at $x=2$

limit $\to 4$

$\lim_{x\to 2} f(x)$

$\lim_{x\to 2} 2x = 4$



$g(x) = \frac{1}{x}$

$\lim_{x\to\infty} \frac{1}{x} = 0$

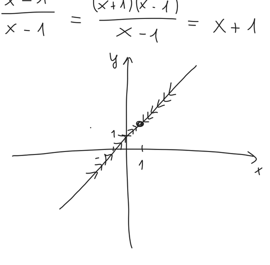
$\lim_{x\to -\infty} \frac{1}{x} = 0$

$\lim_{x\to 0} \frac{1}{x}$ = it depends

$\lim_{x\to 0^+} = \infty$

$\lim_{x\to 0^-} = -\infty$

$h(x) = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{x-1} = x+1 \qquad x \neq 1$



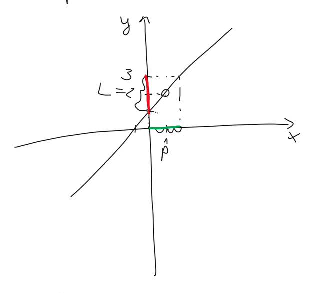
$h(1)$ = undefined

$\lim_{x\to 1} h(x) = 2$

A function $f(x)$ has a limit L when x approaches p if for all $\varepsilon > 0$ there is a $\delta > 0$ such that for all x that satisfies $|x-p| < \delta$ it holds that $|f(x) - L| < \varepsilon$

$\lim_{x\to p} f(x) = L$

$h(x) = \frac{x^2-1}{x-1}$



$\underline{\varepsilon = 1}$

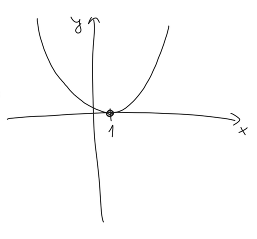
$\underline{\delta = 1}$

$\lim_{x\to 0} 3+2x^2 = 3$

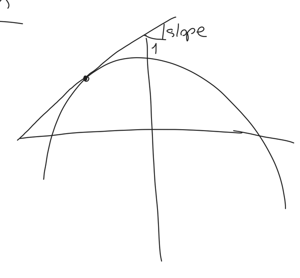
$\lim_{x\to -1} \frac{3+2x}{x-1} = -\frac{1}{2}$

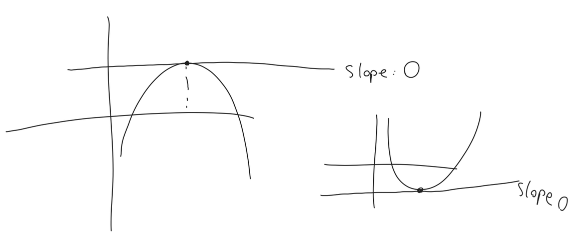
$\lim_{x\to 1} \frac{x^2+7x-8}{x-1} = \lim_{x\to 1} \frac{(x+8)(x-1)}{x-1} = \lim_{x\to 1} x+8 = \underline{\underline{9}}$

$\lim_{x\to 1} \frac{(x-1)^3}{x-1} = \lim_{x\to 1} (x-1)^2 = 0$



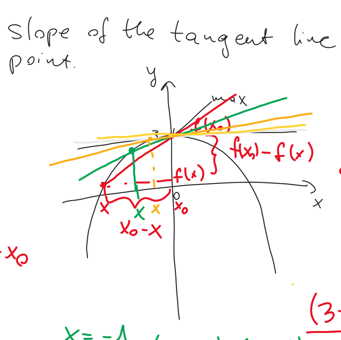
Derivatives





derivative: slope of the tangent line at each point.

$f(x) = 3 - x^2$



$x - x_0$

$\lim_{x\to 0} \frac{(3-0^2)-(3-x)^2}{0-x} = \frac{-x^2}{-x} = \lim_{x\to 0} x = \underline{\underline{0}}$

Slope: $\frac{f(x_0) - f(x)}{x_0 - x}$ $\qquad \lim_{x\to x_0} \frac{f(x_0)-f(x)}{x_0 - x}$ = derivative

$x = -2$

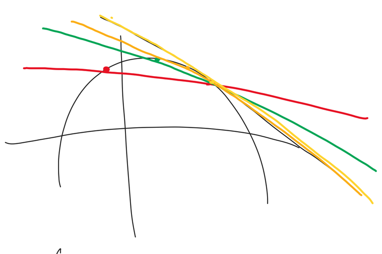
$\frac{(3-0^2)-(3-(-2)^2)}{0-(-2)} = \frac{4}{2} = 2$

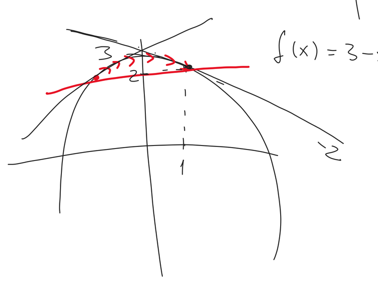
$x = -1$

$\frac{(3-0^2)-(3-(-1)^2)}{0-(-1)} = \frac{1}{1} = 1$

$x = -0.5$

$\frac{(3-0^2)-(3-(-0.5)^2)}{0-(-0.5)} = \frac{1}{2}$





$f(x) = 3-x^2 \qquad f(1) = 3 - 1^2 = 2$

$\lim_{x\to 1} \frac{f(x_0)-f(x)}{x_0 - x} = \frac{2-(3-x^2)}{1-x} = \frac{x^2-1}{1-x} = \frac{(x+1)(x-1)}{-(x-1)} = -(x+1) =$

$\lim_{x\to 1} -(x+1) = \underline{\underline{-2}}$

$f(x) = 3-x^2 \qquad$ derivative at x_0

$\lim_{x\to x_0} \frac{f(x_0)-f(x)}{x_0-x} = \lim_{x\to x_0} \frac{3-x_0^2-(3-x^2)}{x_0-x} = \lim_{x\to x_0} \frac{x^2-x_0^2}{x_0-x} = \lim_{x\to x_0} \frac{(x-x_0)(x+x_0)}{-(x-x_0)} = \lim_{x\to x_0} -(x+x_0) = -2x_0$

$-2x_0 = 0$

$\underline{\underline{x_0 = 0}}$

$3 - x^2$

$\underline{\underline{-2x^1}}$

$f(x) = x^n \qquad$ derivative $\boxed{f'(x)} = \underline{\underline{nx^{n-1}}}$

derivative function

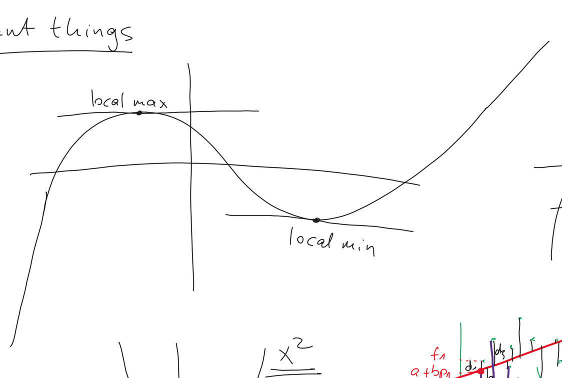
derivative at $x_0 = 0 \quad = -2 \cdot 0 = 0$

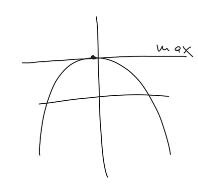
$x_0 = 1 \quad = -2 \cdot 1 = -2$

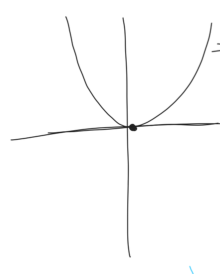
$x_0 = -1 \quad = -2(-1) = 2$

$\frac{d\,f(x)}{dx} = f'(x)$

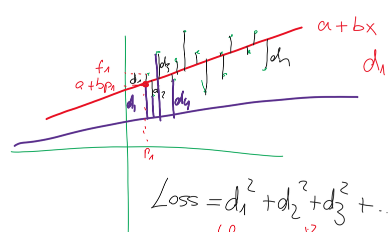
Important things







$\underline{\underline{x^2}}$

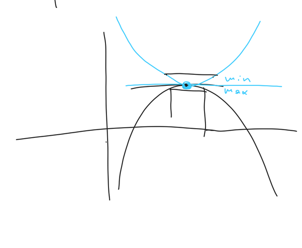


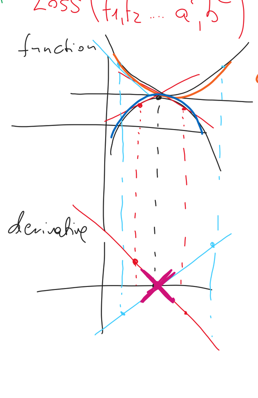
$d_1 = f_1 - (a + b p_1)$

$Loss = d_1^2 + d_2^2 + d_3^2 + \dots + d_n$

$(f_1 - (a+bp_1))^2 + \dots$

$Loss(f_1, f_2 \dots a^2, b^2)$



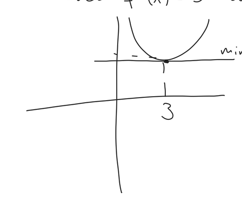


convex $\to$ minimum

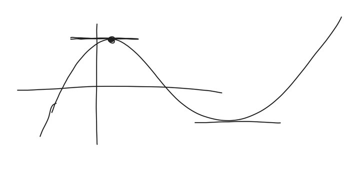
concave $\to$ maximum

Takeaway:

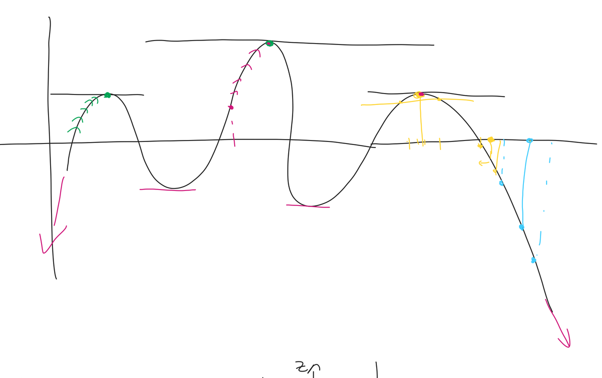
- Most functions $f(x)$ have a derivative $f'(x)$
- Whenever $f'(x) = 0$ we have local minima/maxima



$f'(3) = 0$



profit



$f(x,y) = x^2 + y^2$

$f'_x(x,y)$ = partial derivative w.r.t. x

$f'_y(x,y)$ = partial derivative wrt y

$f'_x(x,y) = \frac{\partial f(x,y)}{\partial x}$

$f'_y(x,y) = \frac{\partial f(x,y)}{\partial y}$