Mathematics problems

1 Elementary algebra

Problem 1.1. Simplify

$$\frac{x^{32}}{x^9 \cdot x^2} \cdot \frac{x^7}{x^2}$$

Problem 1.2. Solve for x:

$$8^2 \cdot 4^x \cdot 2^x = 8^4$$

Problem 1.3. Calculate the missing value. If $\frac{x}{y}$ is 3, then $x^{-4}y^4 =$

Problem 1.4. Calculate

$$\frac{\sqrt{4^{15}}}{\sqrt{16^7}}$$

Problem 1.5. True or False (x and y and z are real numbers):

(a) $x + (y + z) = (y + x) + z$

(b) $y(x + z) = xy + zy$

(c) $x^{y+z} \neq x^z + x^y$

(d) $\frac{x^z}{x^y} \neq x^{y-z}$

Problem 1.6. Find the solution set for the inequality below:

$$\ln(x) \geq e$$

2 Functions of one variable

Problem 2.1 (Based on SYD 2.5.6). The relationship between temperatures measured in Celsius and Fahrenheit is linear. 0°C is equivalent to 32°F and 100°C is the same as 212°F. Which temperature is measured by the same number on both scales?

Problem 2.2. Take the following function $f(x) = 3x - 12$. Find y if $f(y) = 0$.

Problem 2.3. Find all values of x that satisfy:

$$9^{x^2-6x+2} = 81$$

Problem 2.4. Solve the following problem. If the annual GDP growth of a country is 3%, how long does it take the economy to triple its GDP?

Problem 2.5. Calculate the following value

$$\log_\pi\left(\frac{1}{\pi^5}\right)$$

3 Calculus

Problem 3.1. Calculate the following sum

$$\sum_{i=0}^{\infty}\left(\frac{1}{5^i}+0.3^i\right) = \frac{1}{1-\frac{1}{5}}+\frac{1}{1-0.3} = \frac{5}{4}+\frac{10}{7} = \frac{35+40}{28} = \frac{75}{28}$$

Problem 3.2. Find the following limit

$$\lim_{x\to 5}\frac{x^2-25}{x-5} = \lim_{x\to 5}\frac{(x-5)(x+5)}{x-5} = \lim_{x\to 5}(x+5) = 10$$

Problem 3.3. Find the slope of the function $f(x)=x^3-4$ at $(-2,-12)$.

$f'(x)=3x^2 \qquad f'(-2)=12$

Problem 3.4. Find the derivative of the following function:

$$f(x)=\frac{x^5+3}{x^2-1} \qquad f'(x)=\frac{5x^4(x^2-1)-(x^5+3)2x}{x^4-2x^2+1}=\frac{3x^6-11x}{(x^2-1)^2}$$

Problem 3.5. Find the second derivative of the following function:

$$f(x)=x^9+3 \qquad f''(x)=(x^7+3)''=(9x^8)'=72\cdot x^7$$

Problem 3.6. Is the function $f(x)=\frac{1}{x}$ continuous at 0? Why? $0\notin D \Rightarrow$ the question is not applicable

$\lim_{x\to 0^+}\frac{1}{x}=\infty$ & $\lim_{x\to 0^-}\frac{1}{x}=-\infty$

Problem 3.7. Consider the following function. Find all of its local minima, local maxima or inflection points.

$$f(x)=4x^3-12x$$ (on new page)

Problem 3.8. Let $f(x,y)=x^3-y^2$. Calculate $f(2,3)$ $= 2^3-3^2 = 8-9=-1$

Problem 3.9. Consider the following function: $f(x,y)=\ln(x-3y)$. For what combinations of x and y is this function defined?

$x-3y>0 \Leftrightarrow y<\frac{x}{3}$

Problem 3.10. Find the following partial derivative:

$$\frac{\partial}{\partial x}\left(x^5y^7+\frac{x^2}{y^3}\right)=5x^4y^7+\frac{2x}{y^3}$$

Problem 3.11. Find the local maxima or minima of the following function:

$$f(x,y)=\sqrt{xy}-x-y$$ Pls. see separate page

Problem 3.12. Solve the following constrained optimization problem using Lagrange's method: $\max x^2y^2$ s.t. $2x+y=9$ Pls. see top. page

4 Linear algebra

Problem 4.1. Take the following matrices:

$$A=\begin{bmatrix}2&5\\2&1\\7&6\end{bmatrix}$$

$$B=\begin{bmatrix}1&0&1\\9&1&5\end{bmatrix}$$

What is $B\cdot A$? $=\begin{bmatrix}10&11\\55&70\end{bmatrix}$

3.7 $f(x) = 4x^3 - 12x$ dok

$f'(x) = 12x^2 - 12 = 0$

$x^2 = 1$

$x_{1,2} = \pm 1$ MAX MIN

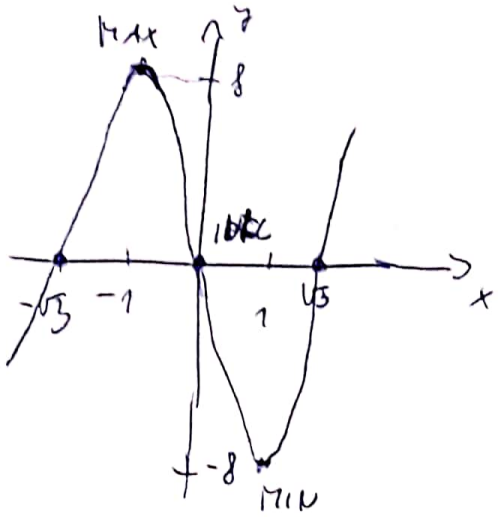
$f''(x) = 24x = 0$

$x = 0$ infleksna

$f(x) = 0$

$4x^3 - 12x = 0$

$x(4x^2 - 12) = 0$ $x_{1,2,3}$ / $\pm\sqrt{3}$ \ 0



3.12

$\max x^2y^2$ p. $2x+y=9$ (! $x \geq 0, y \geq 0$, ograničenje na MAX!)

$f(x,y) = x^2y^2$ $g(x,y) = 2x+y-9$

$\mathcal{L}(x,y,\lambda) = f(x,y) - \lambda g(x,y) = x^2y^2 - 2\lambda x - \lambda y + 9\lambda$

1) $\frac{\partial}{\partial x}\mathcal{L} = 2y^2x - 2\lambda = 0$

2) $\frac{\partial}{\partial y}\mathcal{L} = 2x^2y - \lambda = 0$

3) $\frac{\partial}{\partial \lambda}\mathcal{L} = -2x - y + 9 = 0 \rightarrow y = -2x + 9$

1) $\Rightarrow x(4x^2 - 36x + 81)x = 2\lambda$

2) $\Rightarrow 2x^2(-2x+9) = \lambda$

$4x^3 - 36x^2 + 81x = -4x^3 + 18x^2$

$8x^3 - 54x^2 + 81x = 0$

$x(8x^2 - 54x + 81) = 0 \Rightarrow x_1 = 0, y_1 = 9, \lambda_1 = 0$

$8x^2 - 54x + 81 = 0$

$x_{2,3} = \begin{cases} 4{,}5 & y_2 = 0 \quad \lambda_2 = 0 \\ 2{,}25 & y_3 = 4 \quad \lambda_3 = 40{,}5 \end{cases}$

$\max x^2y^2 = 81$

3.11 $xy \geq 0$!!!

$$\frac{\partial}{\partial x}\left(\sqrt{xy} - x - y\right) = \frac{\partial}{\partial x}\left(\sqrt{y} \cdot x^{\frac{1}{2}} - x - y\right) = \sqrt{y} \cdot \frac{1}{2} x^{-\frac{1}{2}} - 1 =$$

$$= \frac{1}{2}\sqrt{y} \cdot \frac{1}{\sqrt{x}} - 1$$

$$\frac{\partial}{\partial y}\left(\sqrt{xy} - x - y\right) = \sqrt{x} \cdot \frac{1}{2} y^{-\frac{1}{2}} - 1$$

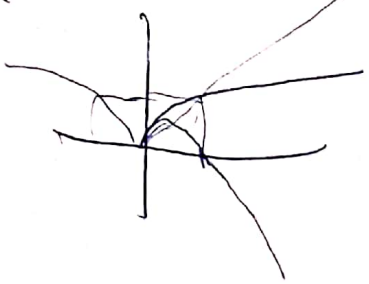
$$\left.\begin{aligned} \frac{1}{2}\sqrt{y} \cdot \frac{1}{\sqrt{x}} &= 1 \\ \frac{1}{2}\sqrt{x} \cdot \frac{1}{\sqrt{y}} &= 1 \end{aligned}\right\} \quad \sqrt{x} = 2\sqrt{y} \quad \left\} \; \frac{1}{2}\sqrt{y} \cdot \frac{1}{2\sqrt{y}} = 1 \right.$$

No solution

⇓

no MAX/MIN

(Or if $x \geq 0, y \geq 0$) the max at $(0,0)$



Problem 4.2. Take the following matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

What is $A \cdot B$?

Problem 4.3. What is the transpose of the following matrix?

$$\begin{bmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{bmatrix}^T$$

Problem 4.4. Calculate the determinant of

$$\left| \begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix} \right|$$

5 Probability theory

Problem 5.1. You run an experiment where you toss a dice two times. Each time you get either 1. 2, 3, 4, 5 or 6. What is the sample space of your experiment?

Problem 5.2. Assume that in a certain country 0.1% of the population uses a certain drug. You have a way to test drug use, which will give you a positive result in 98% of the cases where the individual is indeed a drug user and a negative result in 99.7% of the cases where the individual doesn't use the drug. What is the probability that someone with a positive drug test is indeed a drug user?

Problem 5.3. You run an experiment in which you toss a dice 20 times and record how many times you ended up with a 1, 2, 3. 4, 5 or 6. Your random variable is the number of times you ended up with a 5. What is expected value of this random variable?