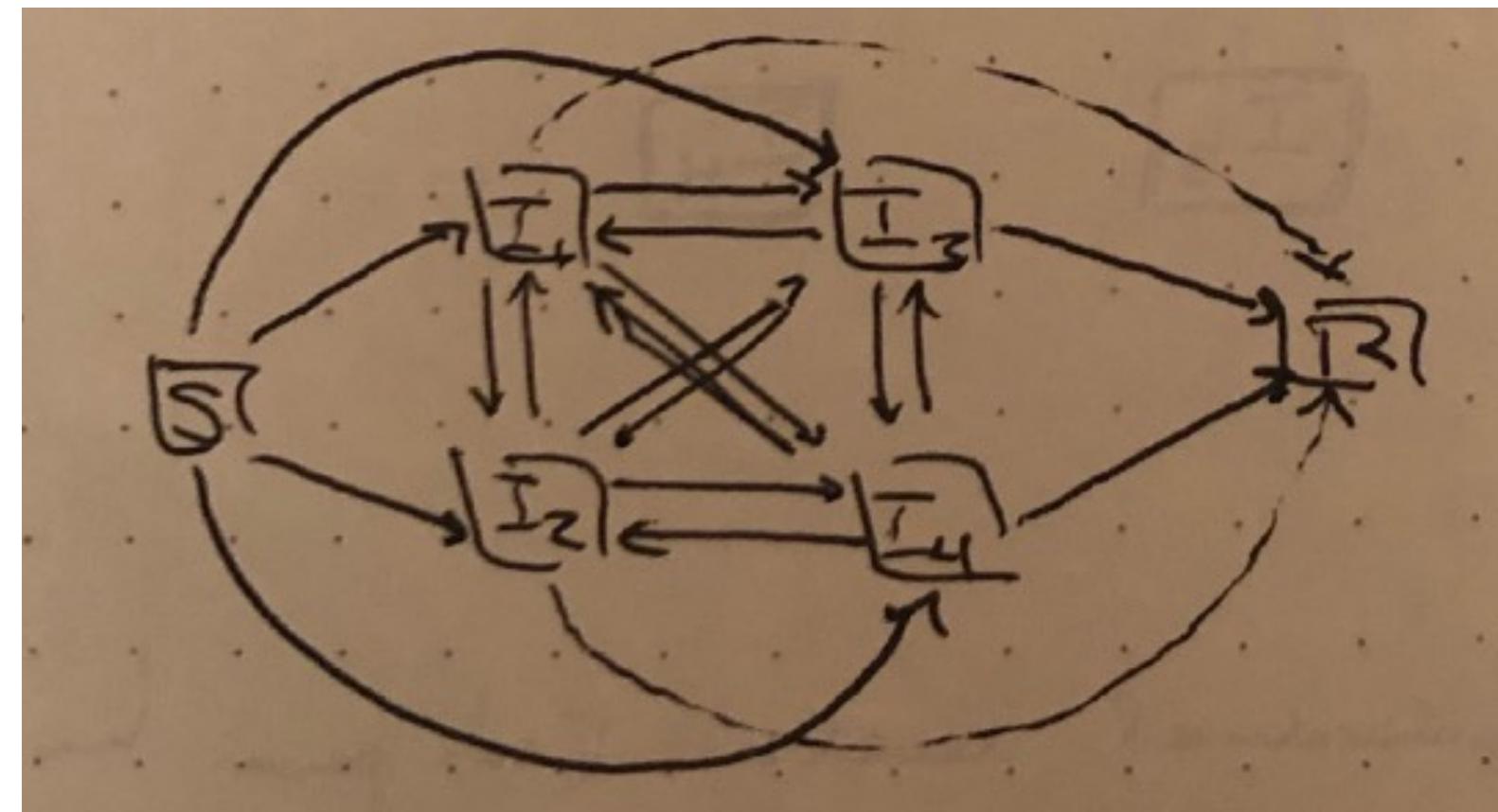


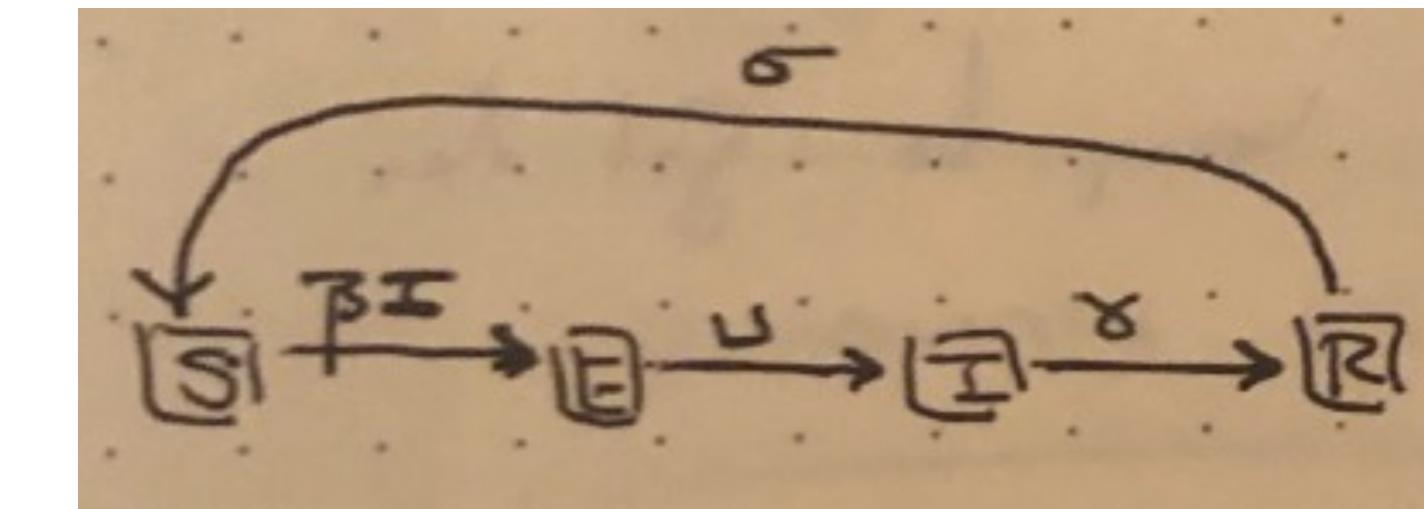
Early models

Multiple strains



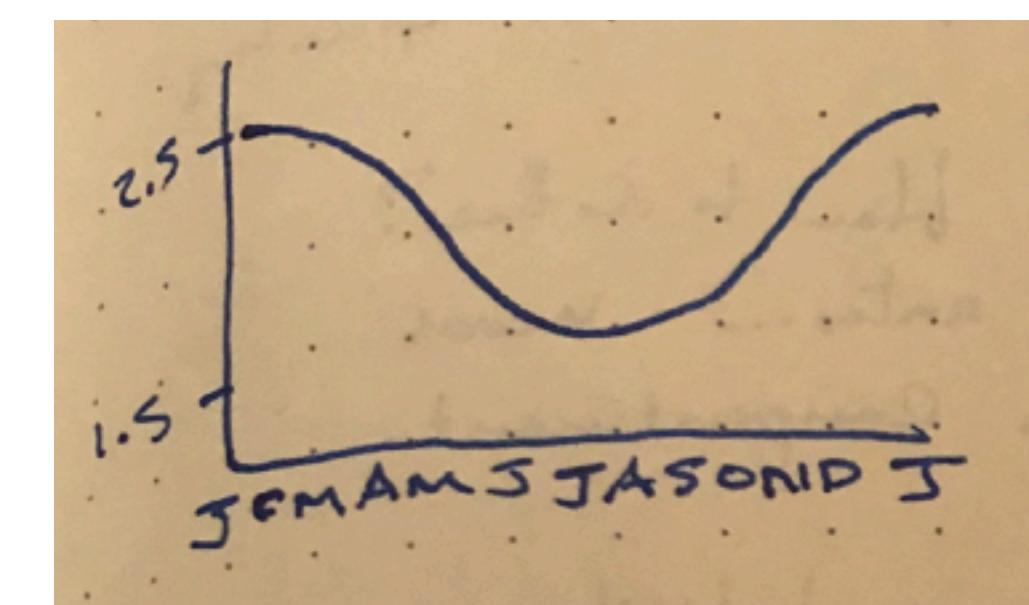
(a bad model)

One strain with seasonality and waning immunity



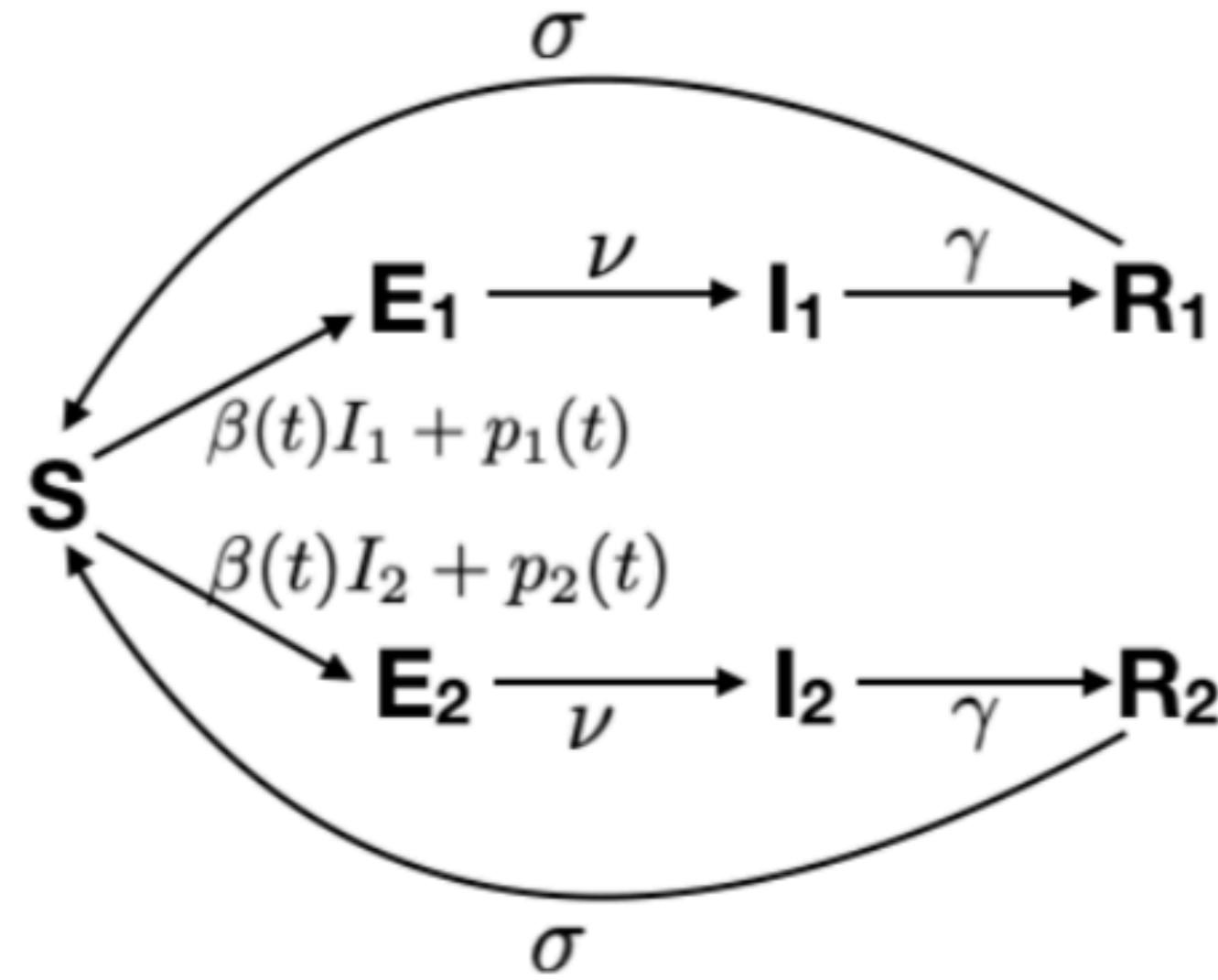
(a better model, but not enough detail)

$$\frac{dS}{dt} = -\beta(t) SI + \sigma R$$
$$\frac{dE}{dt} = \beta(t) SI - \nu E$$
$$\frac{dI}{dt} = \nu E - \gamma I$$
$$\frac{dR}{dt} = \gamma I - \sigma R$$



Early models

Two strains with waning immunity



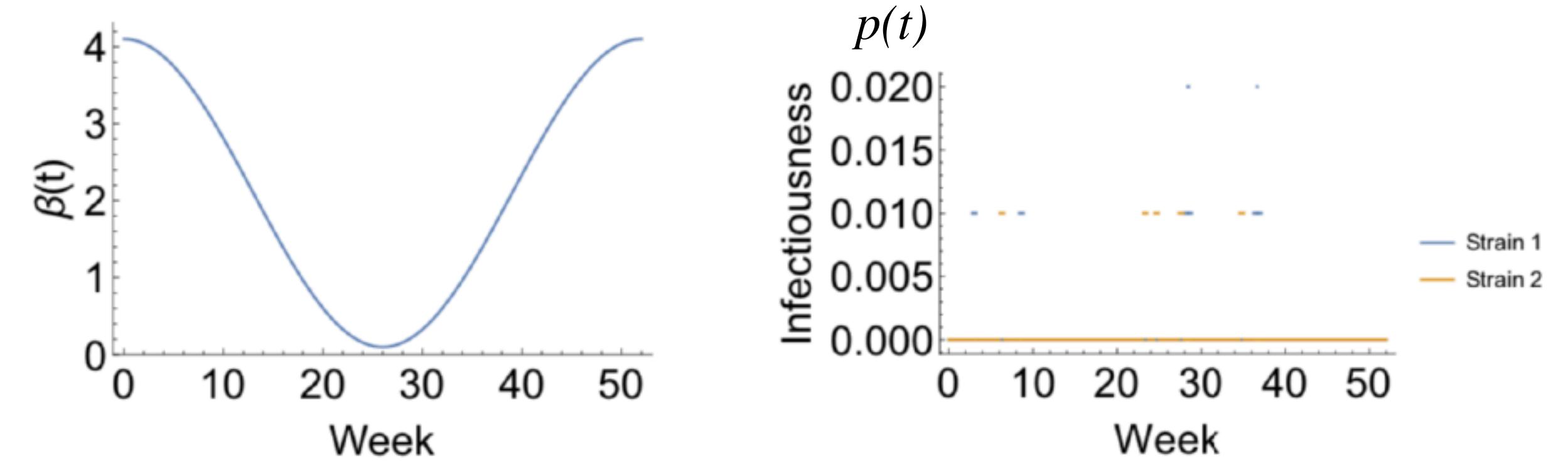
$\beta(t)$: seasonal (cosine with 1-year period) forcing

$p_i(t)$: pulses of imported infectiousness from strain i

ν : Rate of progression to infectiousness ($\frac{1}{\nu} = 1$ week)

γ : Rate of recovery ($\frac{1}{\gamma} = 1$ week)

σ : Rate of waning immunity ($\frac{1}{\sigma} = 35$ weeks)



$$\begin{aligned} \frac{dS}{dt} &= -(\beta(t)I_1 + p_1(t))S - (\beta(t)I_2 + p_2(t))S \\ &\quad + \sigma(R_1 + R_2) \\ \frac{dE_*}{dt} &= (\beta(t)I_* + p_*(t))S - \nu E_* \\ \frac{dI_*}{dt} &= \nu E_* - \gamma I_* \\ \frac{dR_*}{dt} &= \gamma I_* - \sigma R_* \end{aligned}$$