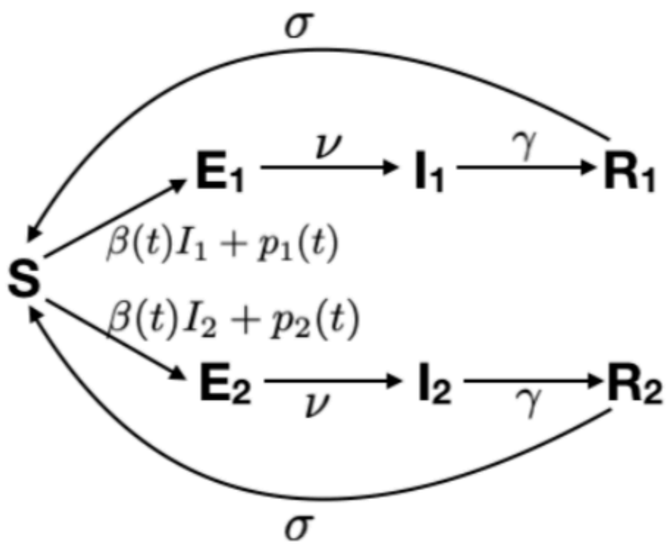
Early models

Two strains with waning immunity



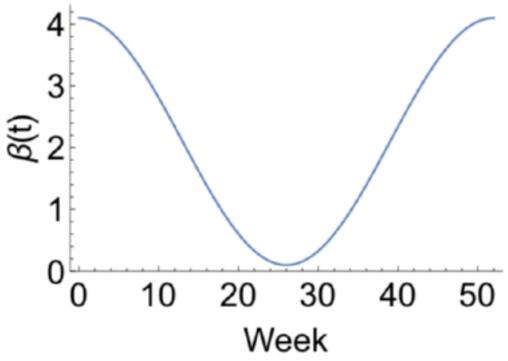
$\beta(t)$: seasonal (cosine with 1-year period) forcing

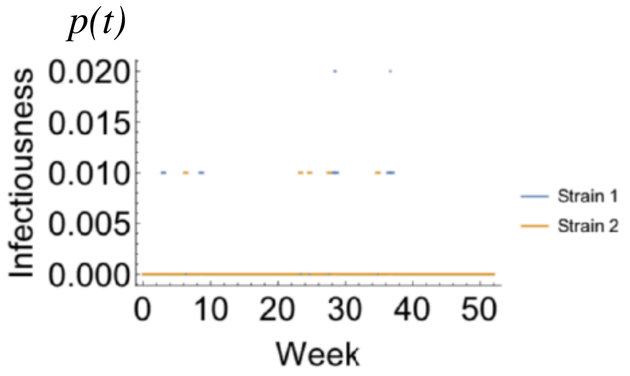
$p_i(t)$: pulses of imported infectiousness from strain *i*

ν: Rate of progression to infectiousness ($\frac{1}{\nu} = 1$ week)

γ: Rate of recovery ($\frac{1}{\gamma} = 1$ week)

σ: Rate of waning immunity ($\frac{1}{\sigma} = 35$ weeks)





$$\frac{dS}{dt} = -(\beta(t)I_1 + p_1(t))S - (\beta(t)I_2 + p_2(t))S + \sigma(R_1 + R_2)$$

$$\frac{dE_*}{dt} = (\beta(t)I_* + p_*(t))S - \nu E_*$$

$$\frac{dI_*}{dt} = \nu E_* - \gamma I_*$$

$$\frac{dR_*}{dt} = \gamma I_* - \sigma R_*$$

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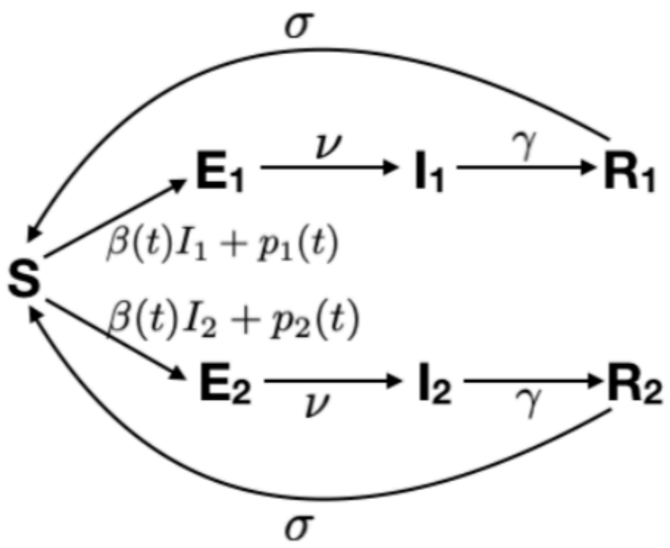

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