

Trigonometric Equations (Mathematics Extension 1)

Transformations ($a \sin x + b \cos x = c$)

For these, use the transformation:

$$a \sin x + b \cos x = r \sin(x + \alpha) = r \sin x \cos \alpha + r \cos x \sin \alpha$$

$$\therefore a = r \cos \alpha$$

$$b = r \sin \alpha$$

$$\Rightarrow \frac{r \sin \alpha}{r \cos \alpha} =$$

$$\tan \alpha = \frac{b}{a}$$

$$a^2 + b^2 = r^2(\cos^2 \alpha + \sin^2 \alpha) = r^2 \Rightarrow$$

$$r = \sqrt{a^2 + b^2}$$

Always have $a, b > 0$ i.e. α is in the first quadrant. Use the following forms:

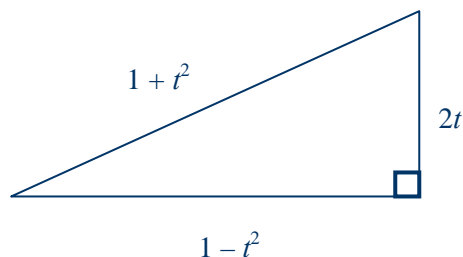
$$a \sin x \pm b \cos x = r \sin(x \pm \alpha)$$

$$a \cos x \pm b \sin x = r \cos(x \mp \alpha)$$

Alternate method using t-results

If $\tan \frac{\theta}{2} = t$,

$$\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \frac{2t}{1 - t^2}$$



Using Pythagoras' Theorem,

$$\sin \theta = \frac{2t}{1 + t^2}$$

$$\cos \theta = \frac{1 - t^2}{1 + t^2}$$

IMPORTANT:

You must always test whether π is a solution if you are using the t -results.

General solutions

- If $\sin \theta = x$, then $\theta = n\pi + (-1)^n \sin^{-1} x$
- If $\cos \theta = x$, then $\theta = 2n\pi \pm \cos^{-1} x$
- If $\tan \theta = x$, then $\theta = n\pi + \tan^{-1} x$

In general, $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ is the result on your calculator display.