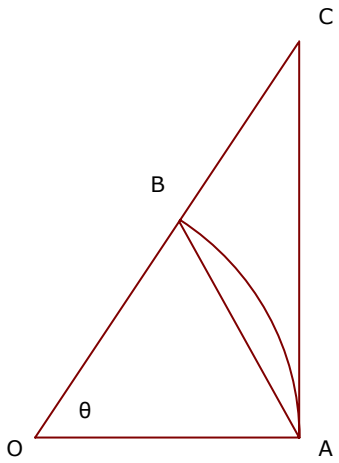


Fundamental Limit (Mathematics Extension 1)

Fundamental Limit



The sector OAB is taken from a circle of unit radius.
Area of ΔAOB < Area of sector AOB < area of ΔOAC

Area of triangle = $\frac{1}{2}ab\sin\theta$

Area of sector = $\frac{1}{2}r\theta$

Note also that $AC = \tan\theta$

Hence,

$$\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{1}{2}\tan\theta$$

$\sin\theta < \theta < \tan\theta$ (Multiplying all the terms by two)

$\frac{1}{\sin\theta} > \frac{1}{\theta} > \frac{1}{\tan\theta}$ (By taking the reciprocals)

$1 > \frac{\sin\theta}{\theta} > \cos\theta$ (Multiplying all the terms by sin θ)

As $\theta \to 0$, $\cos\theta \to 1$, and because sin θ gets sandwiched between 1 and $\cos\theta \to 1$:

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

Example

$$\lim_{x\to 0}\frac{\sin 4x}{2x}$$
$$= 2\times\lim_{x\to 0}\frac{\sin 4x}{4x}$$
$$= 2\times 1$$
$$= 2$$

$$\lim_{x\to 0}\frac{\tan x}{x}$$
$$= \lim_{x\to 0}\frac{\sin x}{x}\div\lim_{x\to 0}\cos x$$
$$= 1\div 1$$
$$= 1$$