Inequations (Mathematics Extension 1)

Properties of Inequalities

1. When taking reciprocals, the inequality sign is reversed if both sides have the same sign, but not if the signs are different:

$$4 < 7 \quad \text{but} \quad \frac{1}{4} > \frac{1}{7}$$

$$5 > -3 \quad \text{and} \quad \frac{1}{5} > -\frac{1}{7}$$

2. After squaring, there will be positive numbers on both sides of the inequality. The direction of the inequality sign will depend on which original side had the larger magnitude.

3. The process of taking the square root is only defined if both sides are positive. The inequality sign remains the same.

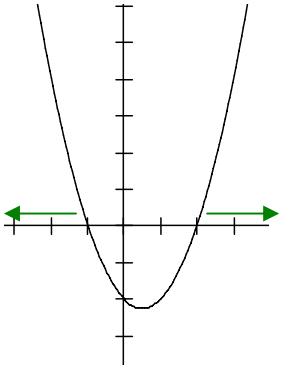
$$a > b \Rightarrow \sqrt{a} > \sqrt{b} \text{ if } a \text{ is positive and } b \text{ is positive or zero}$$

Examples

1. For what values of x is $\frac{x+1}{x-2} \geq 0$?

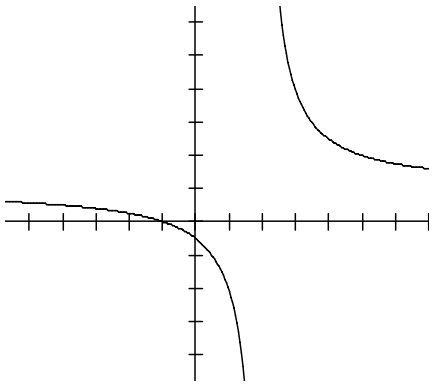
Multiply both sides by $(x-2)^2$ since this is positive. Hence, $(x+1)(x-2) \geq 0$ provided $x \neq 2$

Sketch the graph of $y = (x+1)(x-2)$:



Thus the required solution is $x \leq -1$ or $x > 2$.

Alternatively, sketch the graph of $y = \frac{x+1}{x-2}$:



The x intercepts can be easily found, and the solution corresponds to the parts of the graph above the x-axis. There is an asymptote where the denominator equals zero.

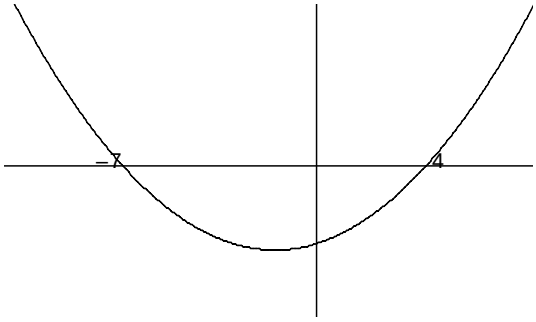
2. For what values of x is $\frac{2x+3}{x-4} > 1$?

$(2x+3)(x-4) > (x-4)^2$

$2x^2 - 5x - 12 > x^2 - 8x + 16$

$x^2 + 3x - 28 > 0$

$\therefore (x+7)(x-4) > 0$



Hence $x < -7, x > 4$

REMEMBER:

- When dealing with fractional inequalities, multiply both sides by the square of the denominator.
- The denominator of the fraction cannot equal to zero, producing an asymptote.