Real-time prediction of bus arrival using joint models of vehicle and road states

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SCIENCE
DEPARTMENT OF STATISTICS

Overview

1. A quick motivation
2. Two real-time models: vehicle (particle filter) & road (Kalman filter)
3. Predicting arrival times

What's wrong with the current[†] system?

What's wrong with the current† system?

† Dealing specifically with Auckland Transport

$\Rightarrow$ applicable to any public transport system using GTFS

What's wrong with the current[†] system?

- Prediction inaccuracy

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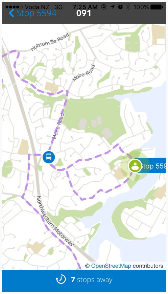

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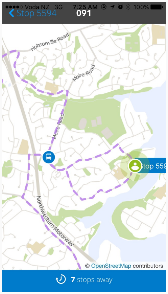

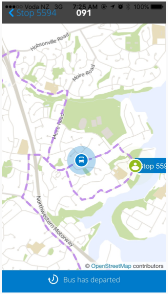

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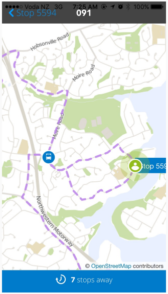
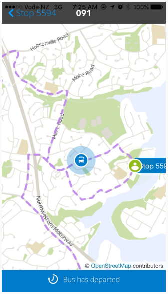
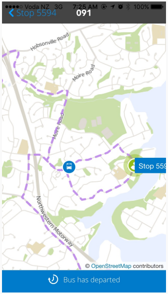
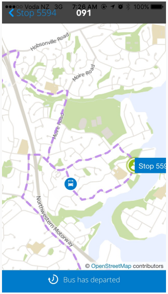


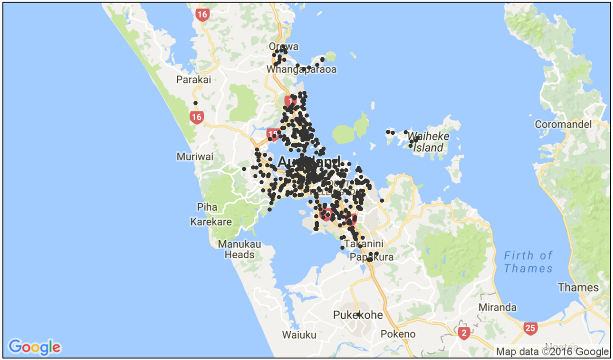






What's wrong with the current† system?

- Prediction inaccuracy
- Prone to errors
- Recent modelling frameworks don't make use of all real-time vehicle data



Vehicle State Model

Vehicle State Model

Goal: use observations of bus location (GPS) ...

$$\mathbf{Y}_k = \begin{bmatrix} \phi_k \\ \lambda_k \\ t_k \end{bmatrix} = \begin{bmatrix} \text{latitude (degrees)} \\ \text{longitude (degrees)} \\ \text{timestamp} \end{bmatrix}$$

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... to infer **unobservable vehicle state** ...

$$\mathbf{X}_k = \begin{bmatrix} d_k \\ v_k \\ s_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \text{distance into trip (meters)} \\ \text{velocity/speed } (ms^{-1}) \\ \text{last visited stop} \\ \vdots \end{bmatrix}$$

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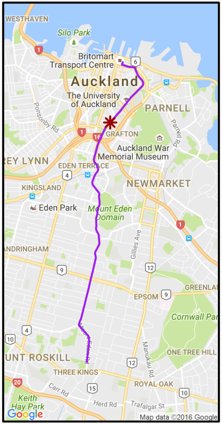
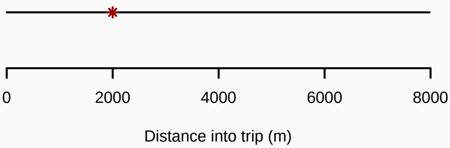
... in real time.

Vehicle State Model

$\mathbf{Y}_k$



$\mathbf{X}_k$ (first component, d_k)



Example: Route 274, Britomart to Three Kings

Vehicle State Model: Particle Filter

- Represent $\mathbf{X}_k$ by a sample of point-estimates (particles) $\mathbf{x}_k^{(i)}$
    1. generate sample of plausible vehicle state predictions
    2. remove predictions no longer plausible, given observation
- Flexible modeling framework, fewer assumptions
- Better coverage of possible states (**multimodality**)
- Intuitive likelihood function

Vehicle State Model: Particle Filter

Step 1: predict

- Start with vehicle state at previous observation, $\mathbf{X}_{k-1} = \{\mathbf{x}_{k-1}^{(i)} : i = 1, \ldots, N\}$

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- **Transition** each particle independently

$$\mathbf{x}_k^{(i)} = f\left(\mathbf{x}_{k-1}^{(i)}, \sigma_v^2, \ldots\right)$$

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1. Add system noise

$$v_k^{(i)} \sim \mathcal{N}_T(v_{k-1}^{(i)}, \sigma_v^2), \qquad 0 \leq v_k^{(i)} \leq \mathbf{v}_{\max}$$

$\mathbf{v}_{\max} \approx$ road speed limit

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$$\mathbf{x}_k^{(i)} = f(\mathbf{x}_{k-1}^{(i)}, \sigma_v^2, \ldots)$$

1. Add system noise
2. Move particles along route (Law of Motion)

$$d_k^{(i)} = d_{k-1}^{(i)} + (t_k - t_{k-1}) v_k^{(i)}$$

Vehicle State Model: Particle Filter

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1. Add system noise
2. Move particles along route (Law of Motion)
3. What about intermediate stop(s)?
   - Does the particle stop? $p_{s_k}^{(i)} \sim \mathrm{Bernoulli}(\pi_{s_k})$
   - If so, for how long? $\bar{t}_{s_k} \sim \mathcal{E}(\tau_{s_k})$

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   - **Dwell time** $= p_{s_k}^{(i)} (\gamma + \bar{t}_{s_k})$

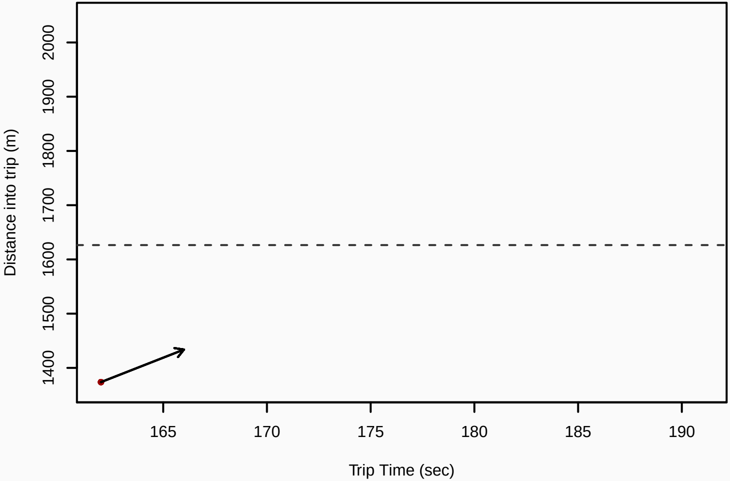
     γ = minimum dwell time (deccelerate/accelerate, open/close doors)

     $\bar{t}_{s_k}$ = passengers on/off

Vehicle State Model: Particle Filter

Step 1: predict

Example ($N = 10$ particles)



Vehicle State Model: Particle Filter

Step 1: predict

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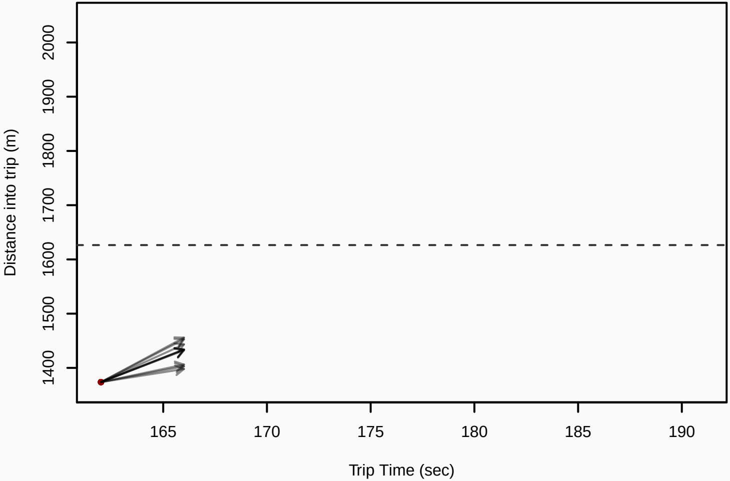

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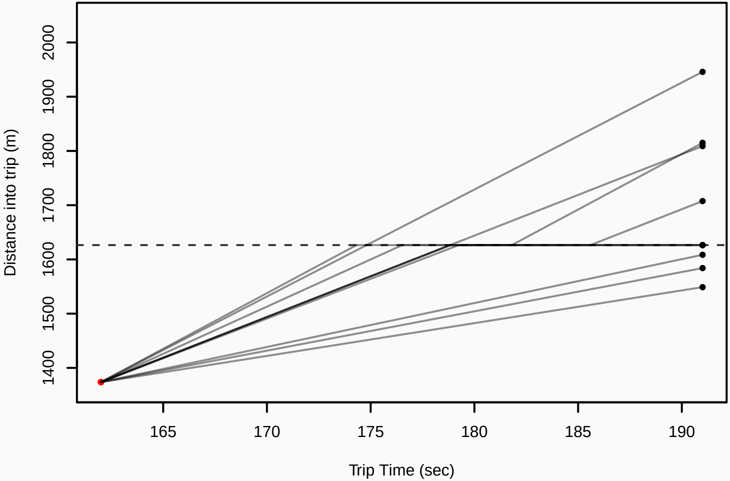


Vehicle State Model: Particle Filter

Step 2: update

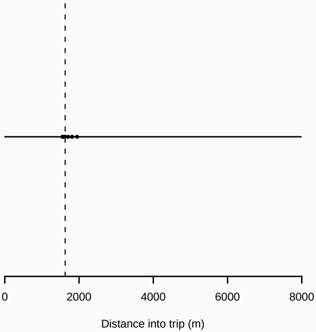
- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2)$

Vehicle State Model: Particle Filter

Step 2: update

- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2)$
    - Transform particles onto flat plane

$$\mathbf{z}_k^{(i)} = g(h(\mathbf{x}_k^{(i)}) | \mathbf{Y}_k)$$



Vehicle State Model: Particle Filter

Step 2: update

- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2, h)$
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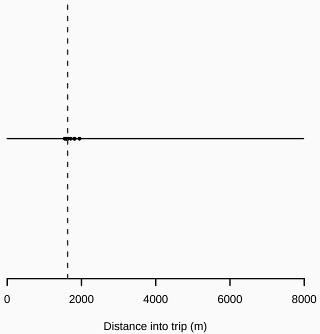
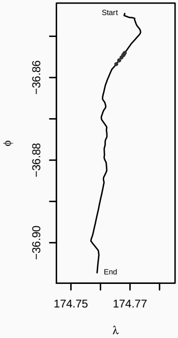
h: measurement function (distance into trip → lat/lon)





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- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
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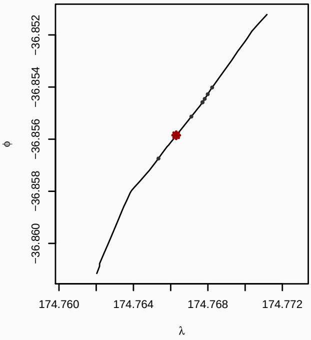
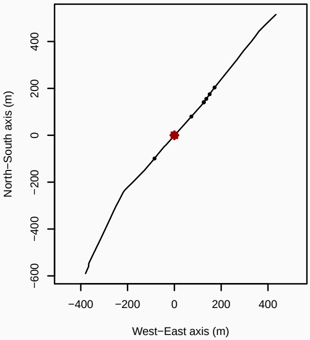
$$\mathbf{z}_k^{(i)} = g(h(\mathbf{x}_k^{(i)}) | \mathbf{Y}_k)$$

$g(\cdot | \mathbf{Y}_k)$: projection centered on $\mathbf{Y}_k$, 1 unit = 1 meter in all directions



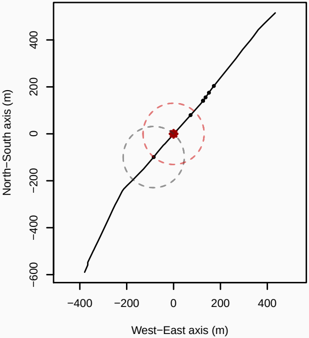


Vehicle State Model: Particle Filter

Step 2: update

- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
  - Transform particles onto flat plane
  - Bivariate normal likelihood, $g(\mathbf{Y}_k|\mathbf{Y}_k) = \mathbf{0}$

$$\mathbf{Y}_k | \mathbf{z}_k^{(i)}, \sigma_y^2 \sim \mathbf{z}_k^{(i)} | \mathbf{Y}_k, \sigma_y^2 \sim \mathcal{N}_2(\mathbf{0}, \sigma_y^2 I_2) \qquad (\sigma_y^2 = \text{GPS error})$$



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  - For each particle

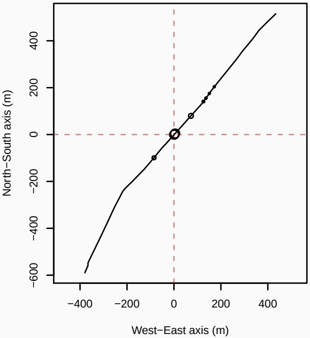
$$\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)}, \sigma_y^2, h, g) \propto e^{-\frac{1}{2\sigma^2}\left((\mathbf{z}_k^{(i)})^T \mathbf{z}_k^{(i)}\right)}$$

Vehicle State Model: Particle Filter

Step 2: update

- Likelihood function $\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
- Compute weights

$$w_i = \frac{\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)})}{\sum_{j=1}^{N} \ell(\mathbf{Y}_k|\mathbf{x}_k^{(j)})}$$



Vehicle State Model: Particle Filter

Step 2: update

- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
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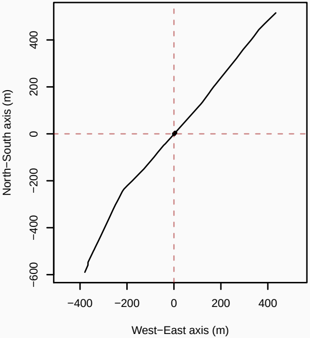
- Weighted resample with replacement



Vehicle State Model: Particle Filter

Step 2: update

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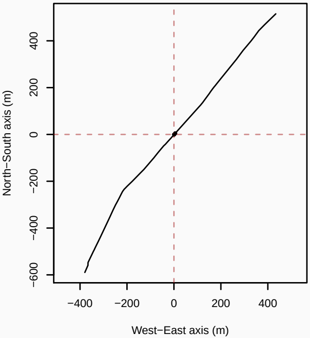
- Compute weights
- Weighted resample with replacement
  $\Rightarrow$ keep particles plausible given data



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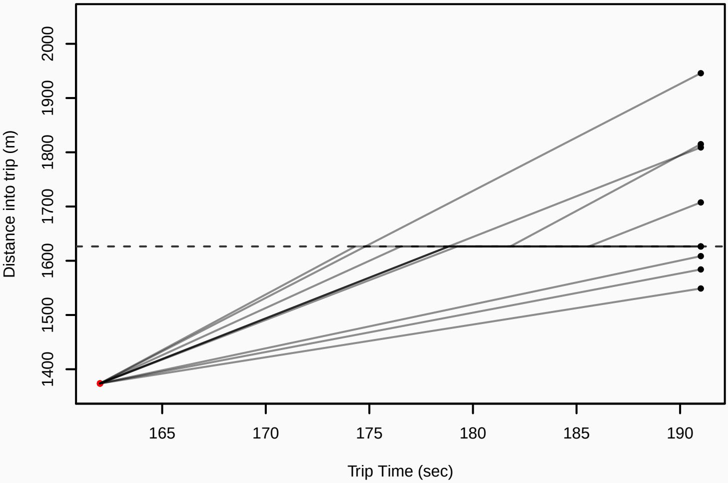

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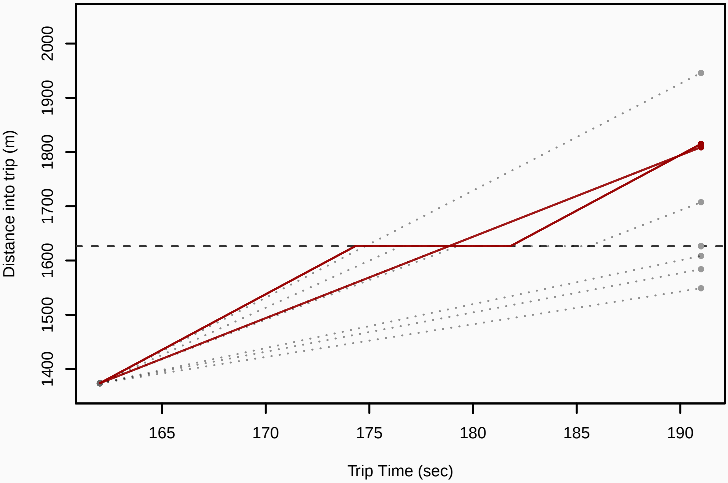

Road State Model

Road State Model

1. Particle filter $\Rightarrow$ speed estimates for a given bus
2. Identify segments of road common to multiple routes
3. Estimate speed along road segments using all busses

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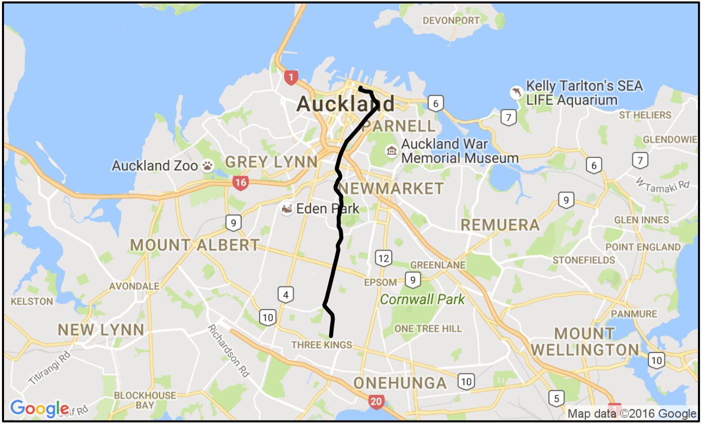
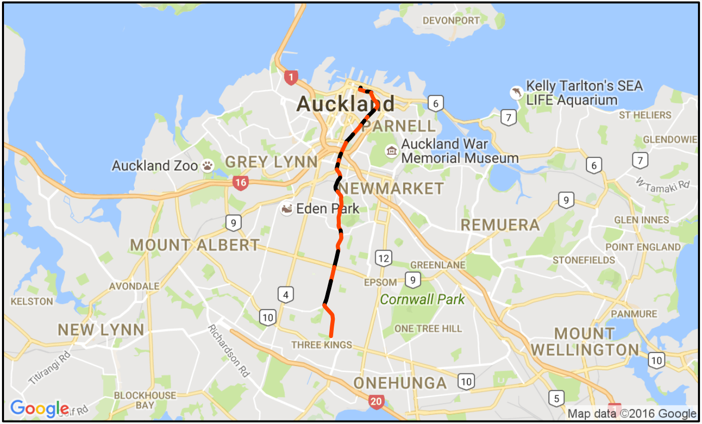
Road State Model

2. Identify segments of road common to multiple routes



Road State Model

2. Identify segments of road common to multiple routes
   $\Rightarrow$ between (major) intersections



Road State Model

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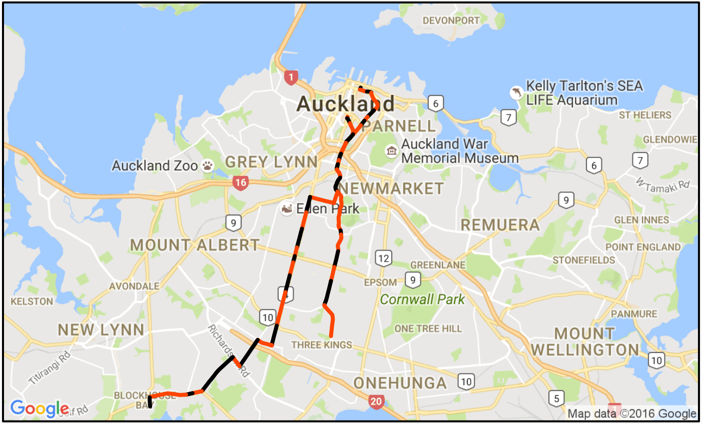

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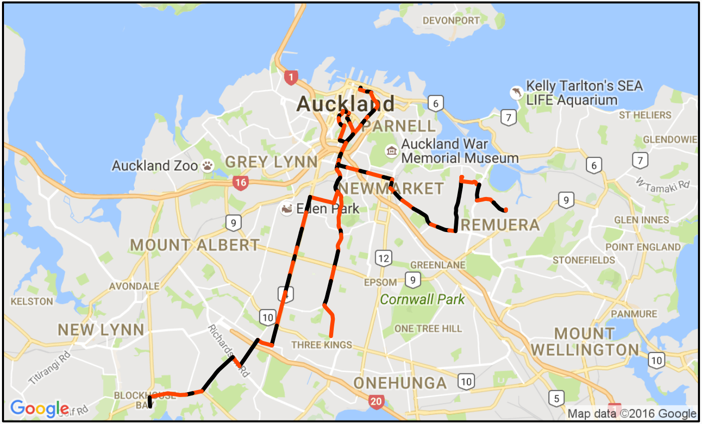

Road State Model

3. Estimate speed along road segments using all busses

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$\Rightarrow$ **Kalman filter**

Road state: mean speed for all M road segments at time t_ℓ

$$\boldsymbol{\nu}_\ell = [\nu_{1\ell} \; \nu_{2\ell} \; \cdots \; \nu_{M\ell}]^T$$

with associated covariance matrix

$$\Xi_\ell = \begin{bmatrix} \xi_{1\ell} & 0 & \cdots & 0 \\ 0 & \xi_{2\ell} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi_{M\ell} \end{bmatrix}$$

Road State Model

3. Estimate speed along road segments using all busses

⇒ **Kalman filter**

- no complex model necessary (Normal distribution adequate)

Road State Model

3. Estimate speed along road segments using all busses

$\Rightarrow$ **Kalman filter**

- no complex model necessary (Normal distribution adequate)
- updated using particle filter estimates

$$\mathbf{v}_\ell = \boldsymbol{\nu}_\ell + \mathbf{r}_\ell$$

  - $\mathbf{v}_\ell$: mean speed of particles
  - $\mathbf{r}_\ell \sim \mathcal{N}(0, \hat{\mathbf{R}}_\ell)$, $\hat{\mathbf{R}}_\ell$: variance of particle speeds

Predicting Arrival Time

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1. Schedule
2. Schedule deviation (AT?)
3. Vehicle state
4. Road state

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2. Schedule deviation (AT?)
3. Vehicle state
4. Road state

Some notation:

- S_j^t = scheduled arrival time at stop j
- $\hat{A}_j$ = (predicted) arrival time at stop j
- $\tilde{T}_{s_k}^a, \tilde{T}_{s_k}^d$ = observed arrival/departure delay at last stop (from Auckland Transport's API)

Prediction Arrival Time

1. **Schedule**
   - $\hat{A}_j = S_j^t$
   - Baseline for other predictors

Prediction Arrival Time

2. **Schedule deviation**

- $\hat{A}_j = \begin{cases} S_j^t + \tilde{T}_{s_k}^d & \text{if departed stop } s_k \\ S_j^t + \tilde{T}_{s_k}^a & \text{if not departed stop } s_k \end{cases}$
- **OR** use particle estimates of arrival/departure delay, $\tilde{A}_{s_k}^{(i)}$ and $\tilde{D}_{s_k}^{(i)}$

Prediction Arrival Time

3. **Vehicle state**

- S_j^d = distance along route of stop j

Prediction Arrival Time

3. Vehicle state

- S_j^d = distance along route of stop j
- $\hat{A}_j = t_k + \frac{S_j^d - d_k}{v_k}$

Prediction Arrival Time

3. Vehicle state

- S_j^d = distance along route of stop j
- $\hat{A}_j{}^{(i)} = t_k + \frac{S_j^d - d_k{}^{(i)}}{v_k{}^{(i)}}$
- Prediction for each particle

Prediction Arrival Time

3. Vehicle state

- S_j^d = distance along route of stop j
- $\hat{A}_j{}^{(i)} = t_k + \frac{S_j^d - d_k{}^{(i)}}{v_k{}^{(i)}} + \sum_{\ell=s_k+1}^{j-1} p_\ell^{(i)}(\gamma + \bar{t}_\ell^{(i)})$
- Prediction for each particle
- Allow for dwell time uncertainty

Prediction Arrival Time

4. Road state

- $r_k =$ route segment index
  $R_b =$ distance along route of start of segment b

Prediction Arrival Time

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- $r_k =$ route segment index
  $R_b =$ distance along route of start of segment b
- $\hat{A}_j = t_k +$

Prediction Arrival Time

4. Road state

- r_k = route segment index
  R_b = distance along route of start of segment b
- $\hat{A}_j = t_k + \frac{R_{s_k+1} - d_k}{\nu_{s_k}} +$
- travel time until end of current segment

Prediction Arrival Time

4. Road state

- $r_k =$ route segment index
  $R_b =$ distance along route of start of segment b
- $\hat{A}_j = t_k + \frac{R_{s_k+1} - d_k}{\nu_{s_k}} + \sum_{b \in B^\star} \frac{R_{b+1} - R_b}{\nu_b} +$
- travel time through intermediate segments ($B^\star$)

Prediction Arrival Time

4. Road state

- r_k = route segment index
  R_b = distance along route of start of segment b
- $\hat{A}_j = t_k + \frac{R_{s_k+1} - d_k}{\nu_{s_k}} + \sum_{b \in B^\star} \frac{R_{b+1} - R_b}{\nu_b} + \frac{S_j^d - R_{b'}}{\nu_{b'}}$
- travel time along segment b' to stop j

Prediction Arrival Time

4. Road state

- r_k = route segment index
  R_b = distance along route of start of segment b
- $\hat{A}_j^{(i)} = t_k + \frac{R_{{s_k}^{(i)}+1} - {d_k}^{(i)}}{v_{s_k}^{(i)}} + \sum_{b \in B^\star} \frac{R_{b+1} - R_b}{v_b^{(i)}} + \frac{S_j^d - R_{b'}}{v_{b'}^{(i)}}$
- For each particle, sample $v_b^{(i)} \sim \mathcal{N}(\nu_b, \xi_b)$

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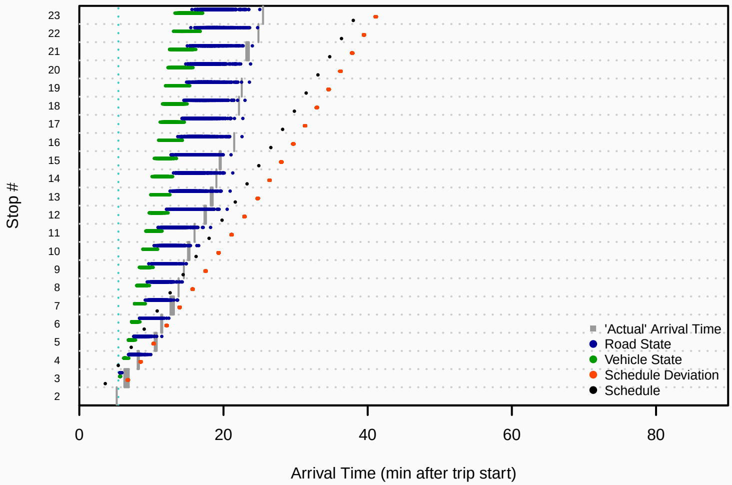
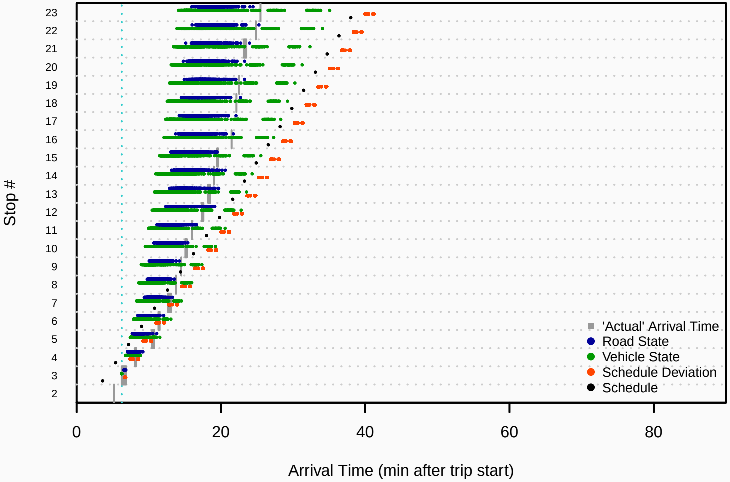
  $+ \sum_{\ell = s_k + 1}^{j-1} p_\ell^{(i)} (\gamma + \bar{t}_\ell^{(i)})$
- For each particle, sample $v_b^{(i)} \sim \mathcal{N}(\nu_b, \xi_b)$
- Allow for dwell time uncertainty

Predicting Arrival Time



Predicting Arrival Time



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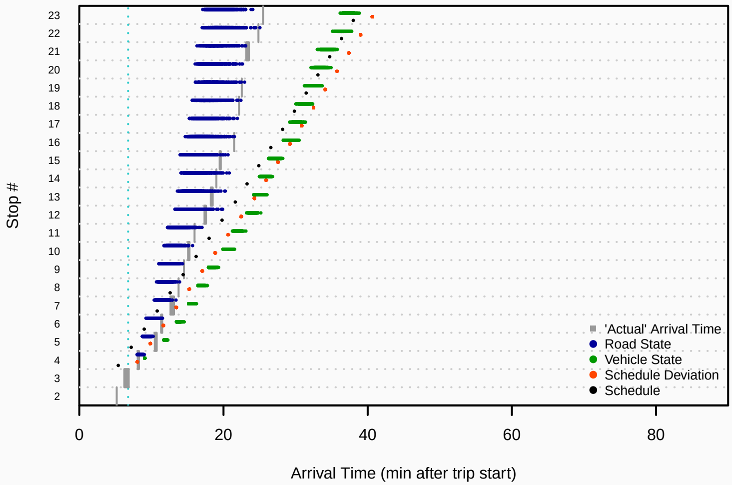

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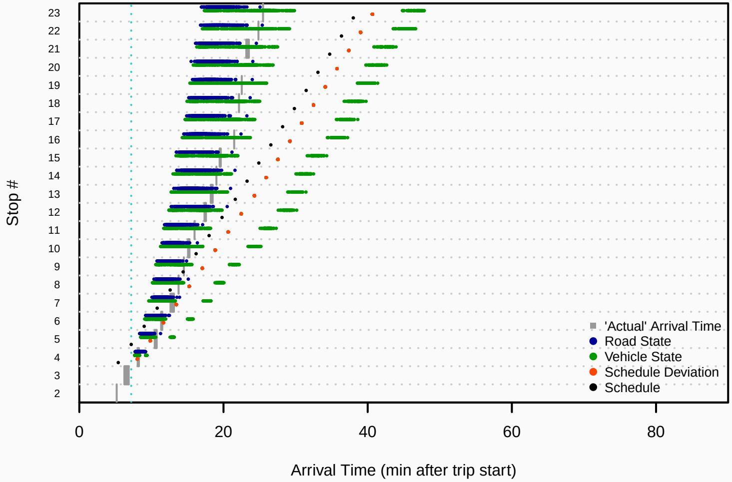

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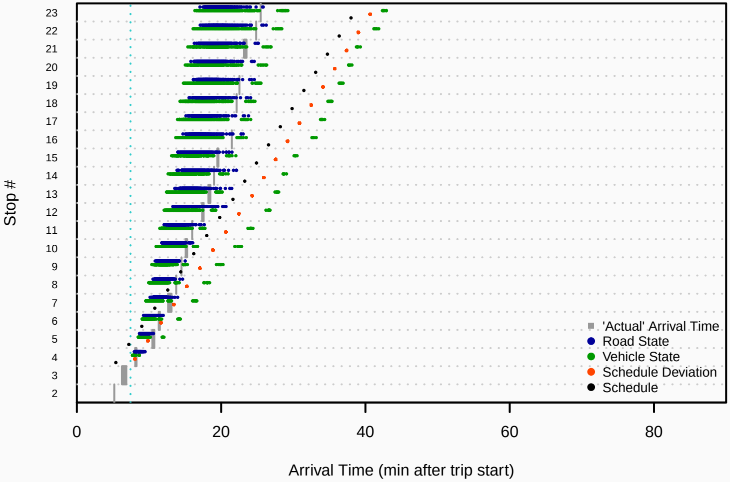

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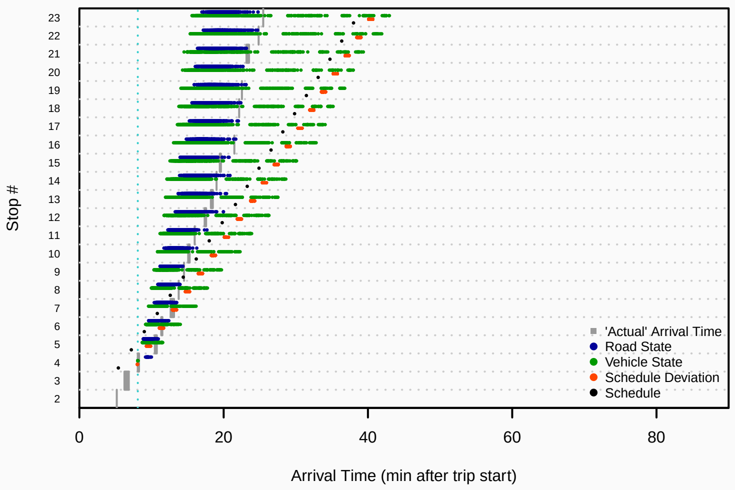

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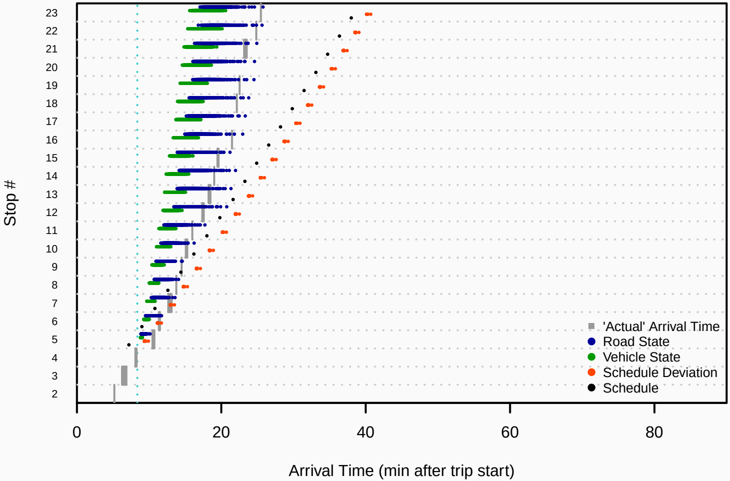

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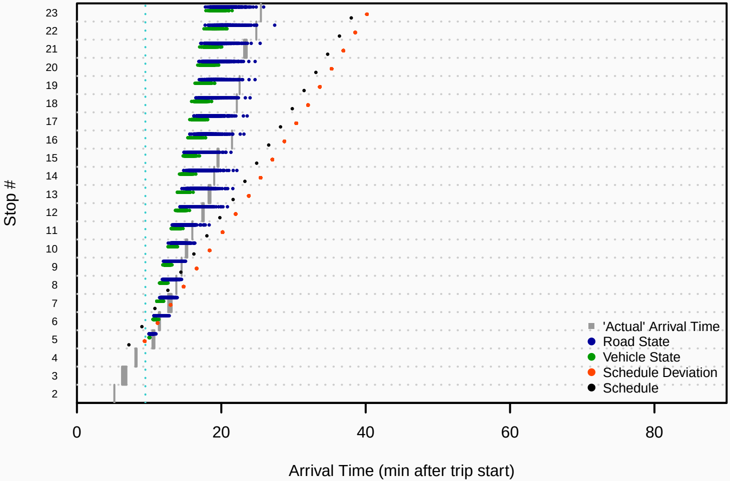

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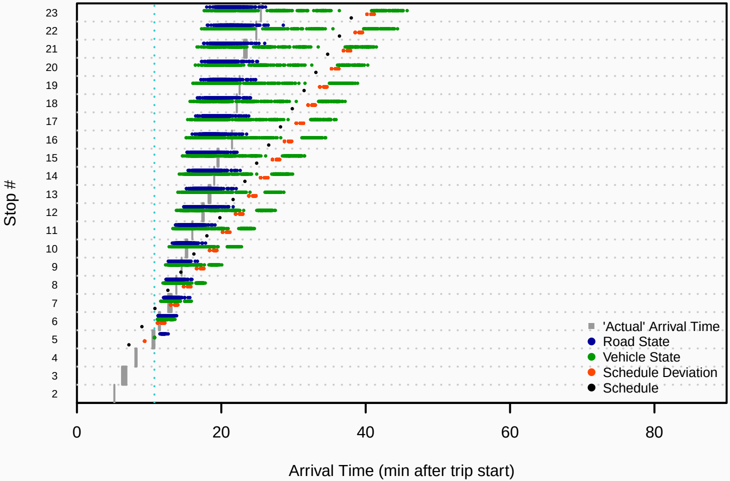

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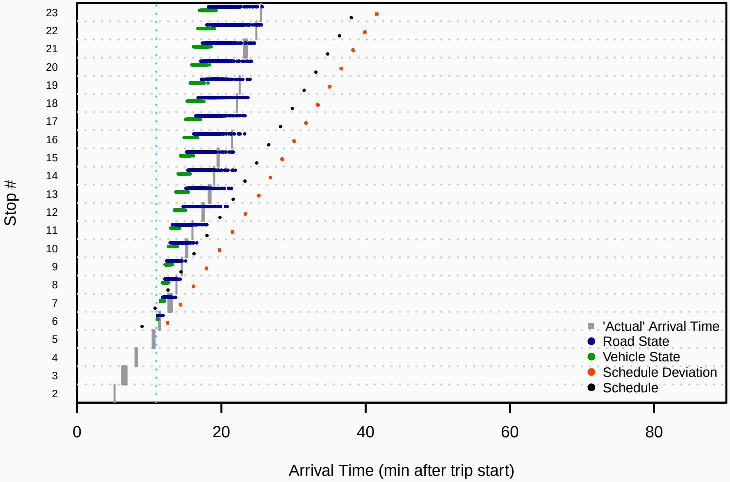

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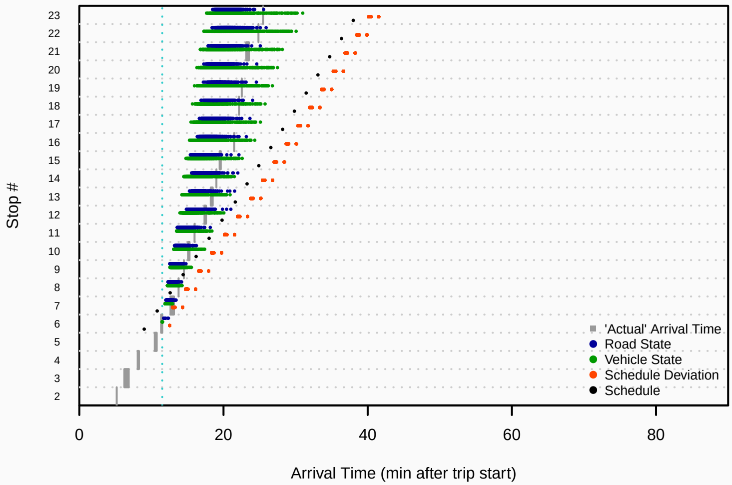

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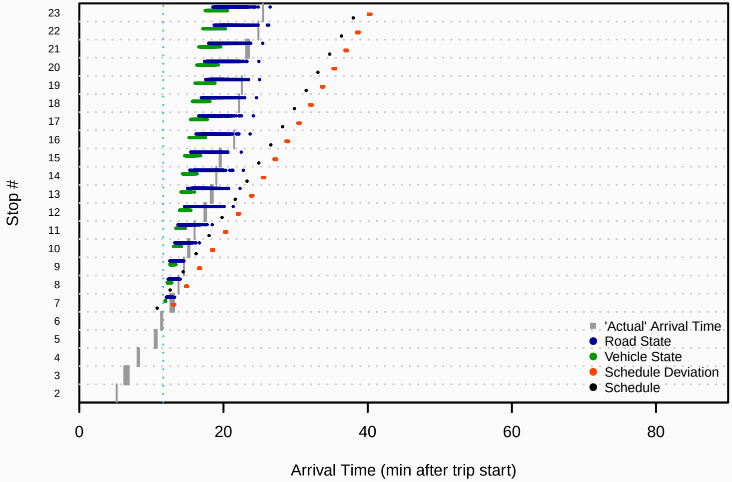

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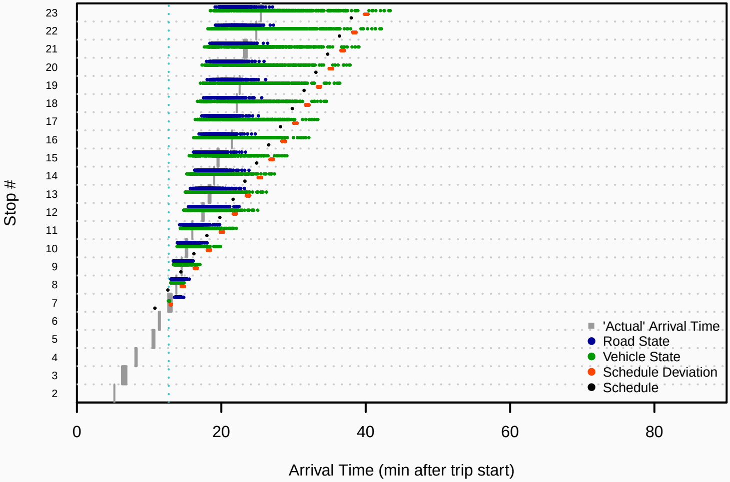

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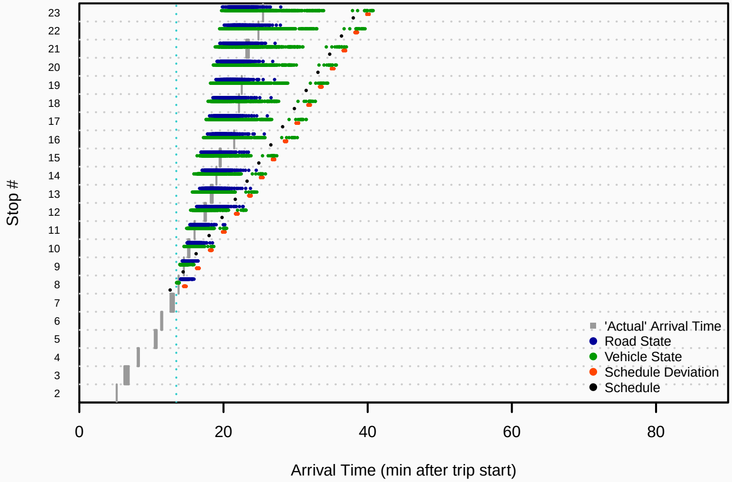

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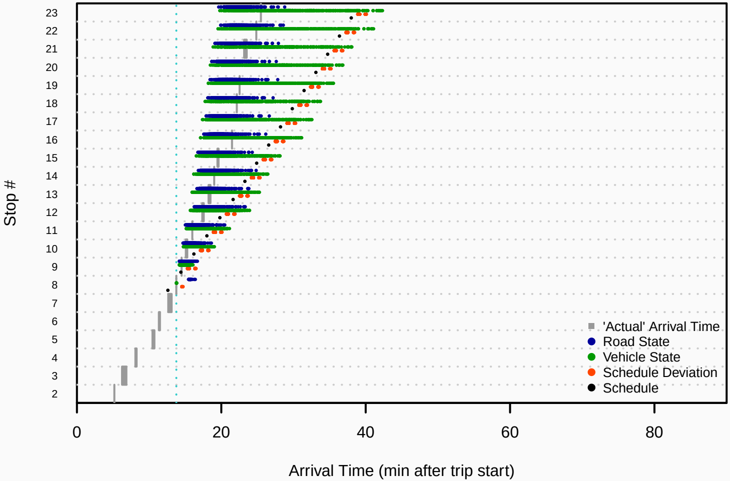

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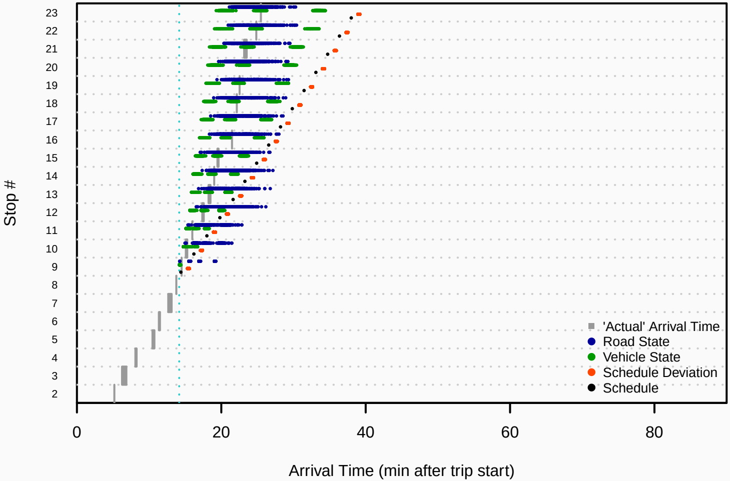

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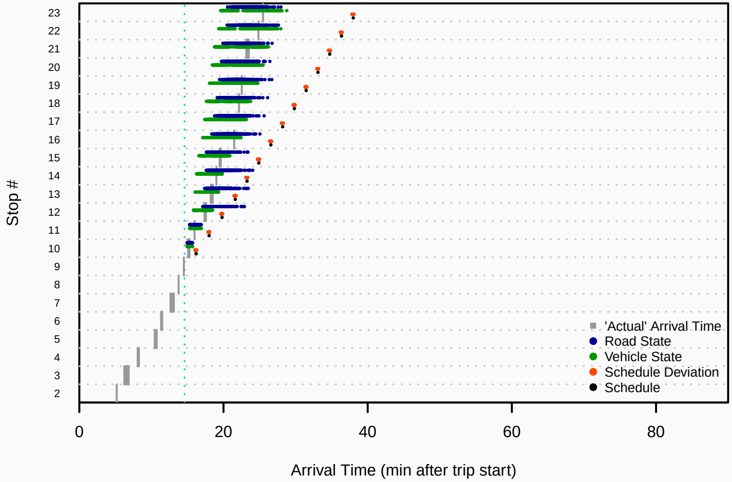

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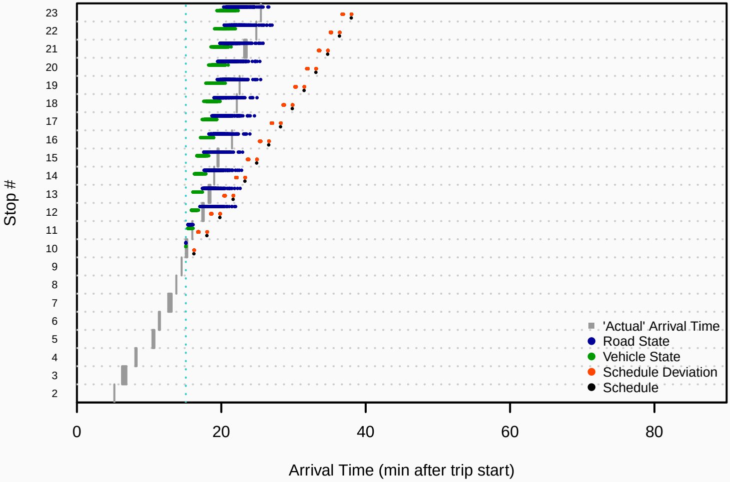

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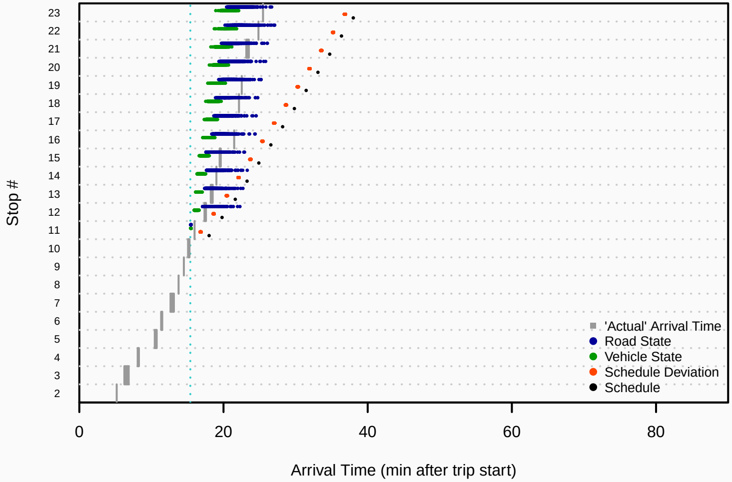

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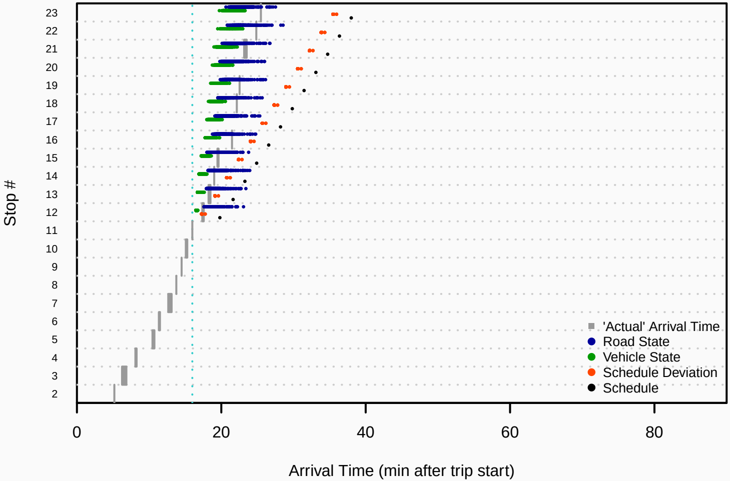

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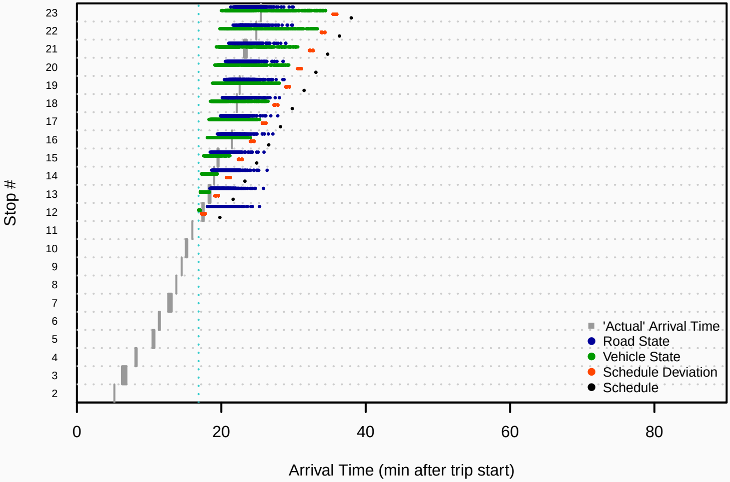

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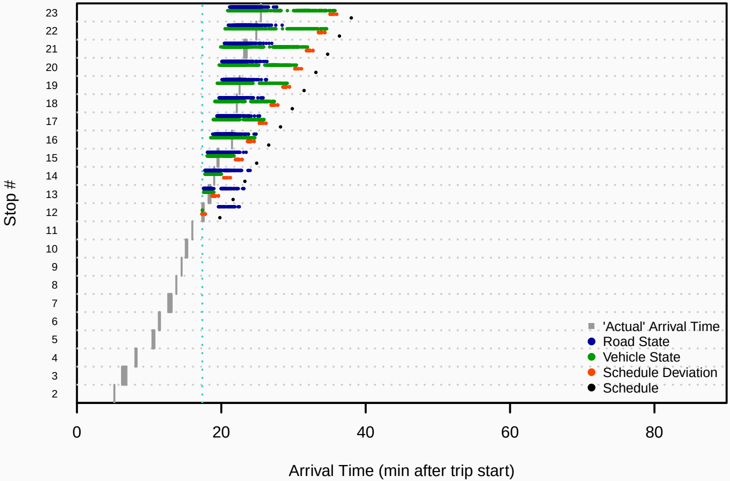

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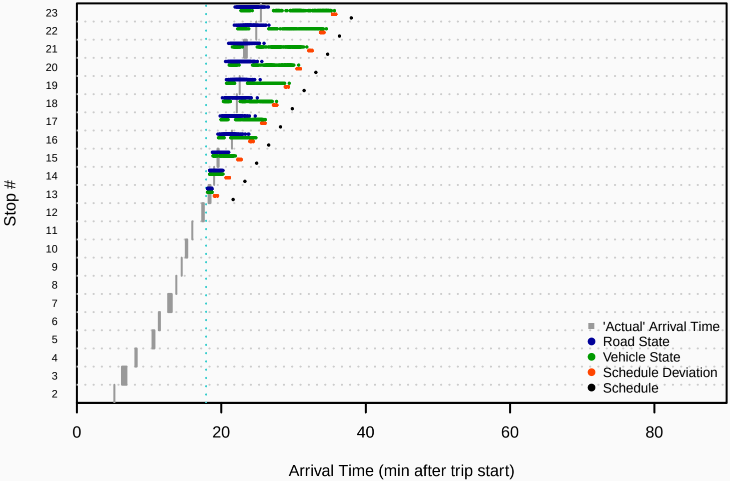

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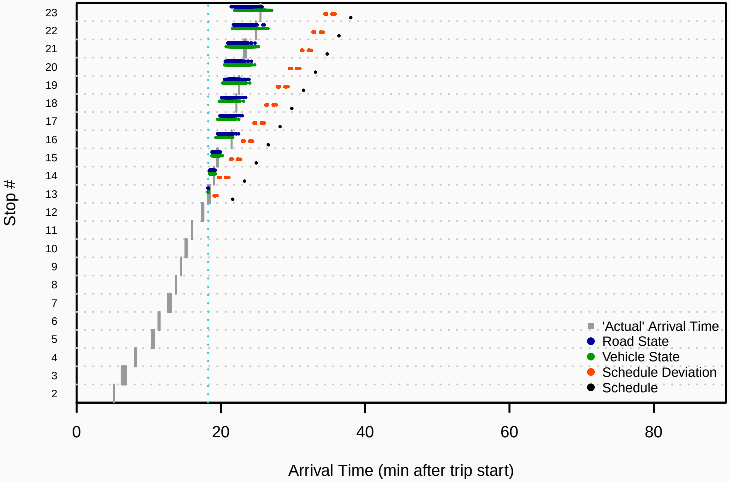

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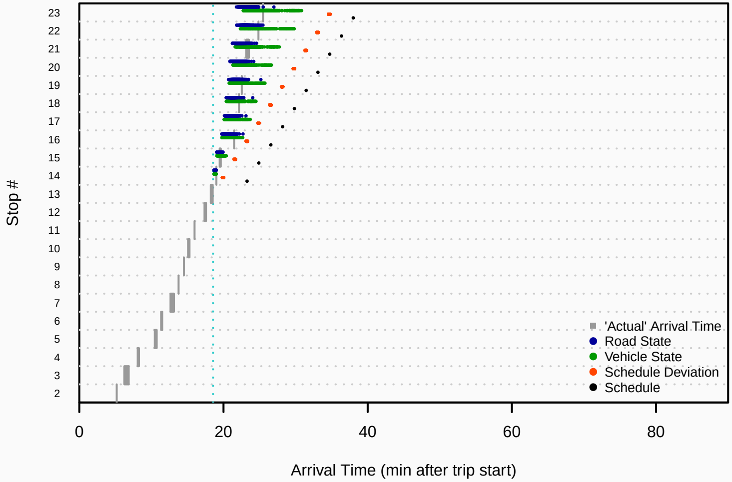

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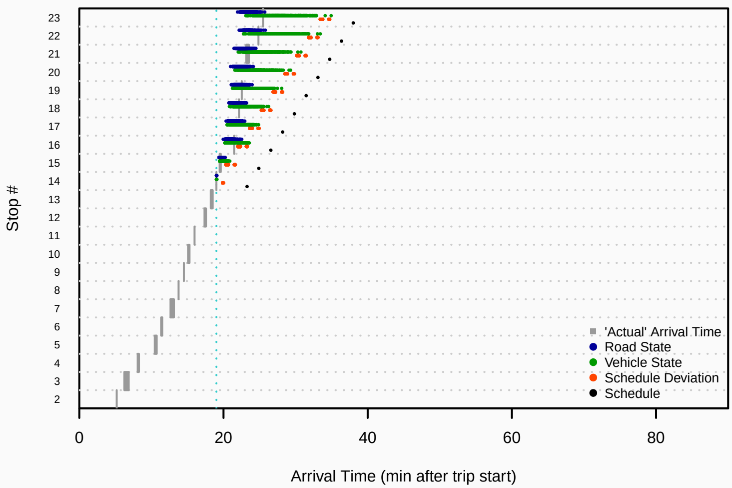

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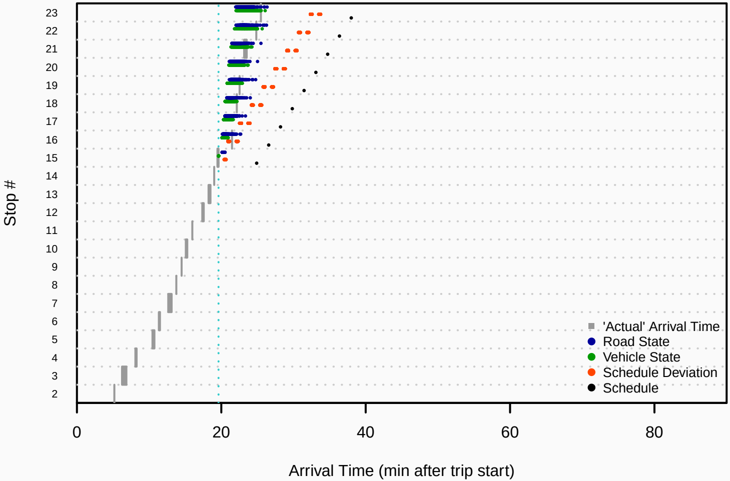

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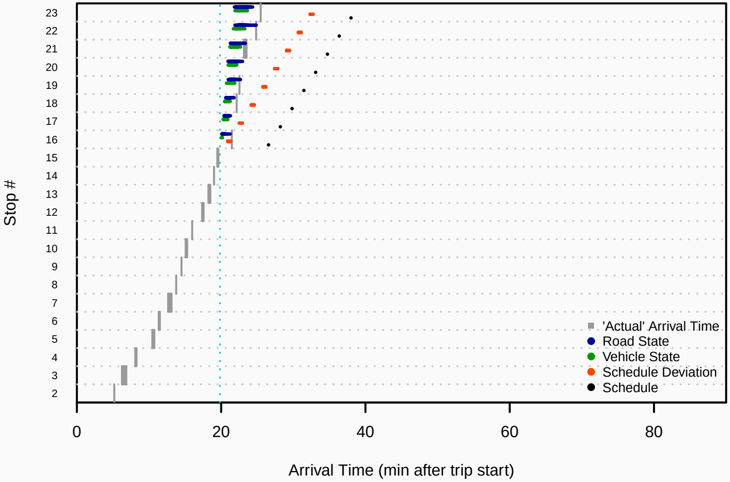

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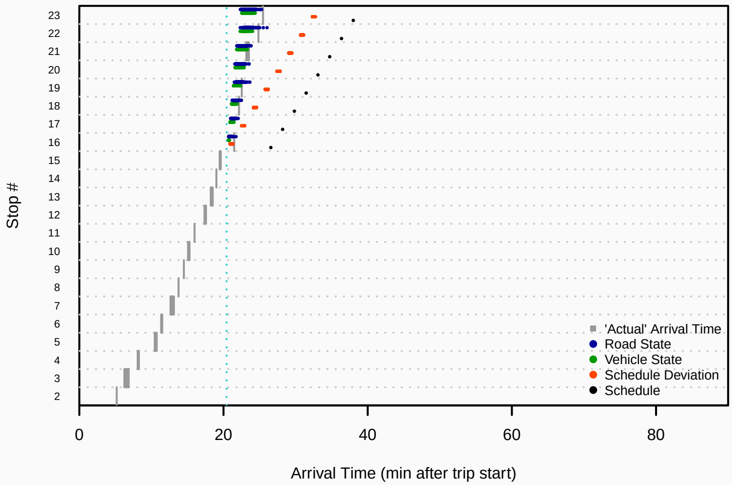

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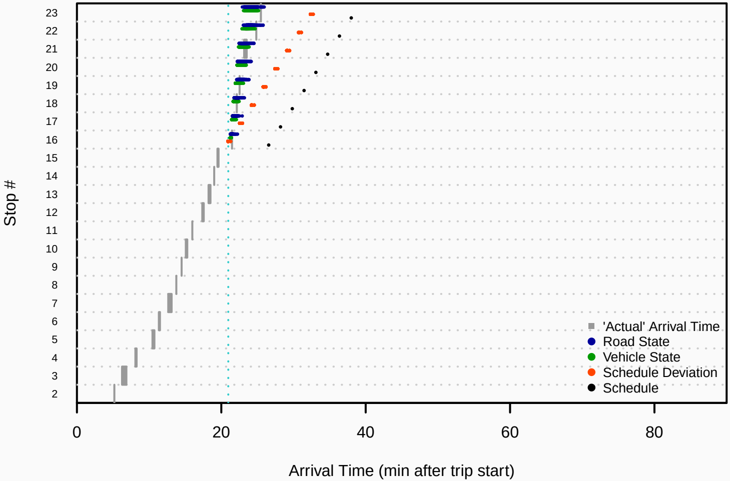

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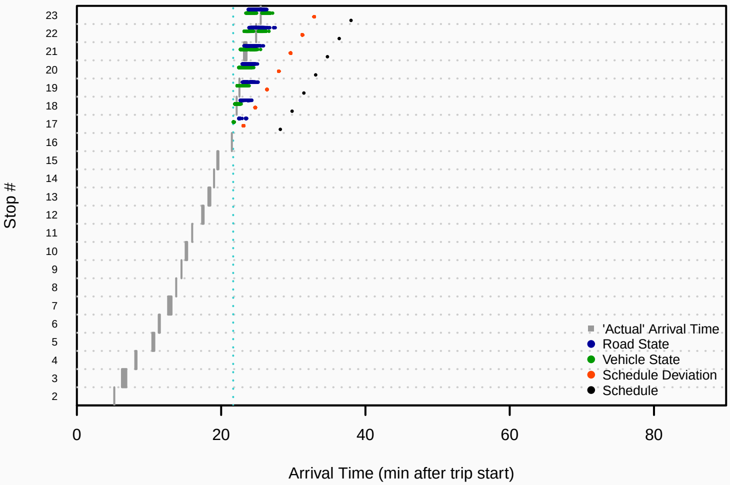

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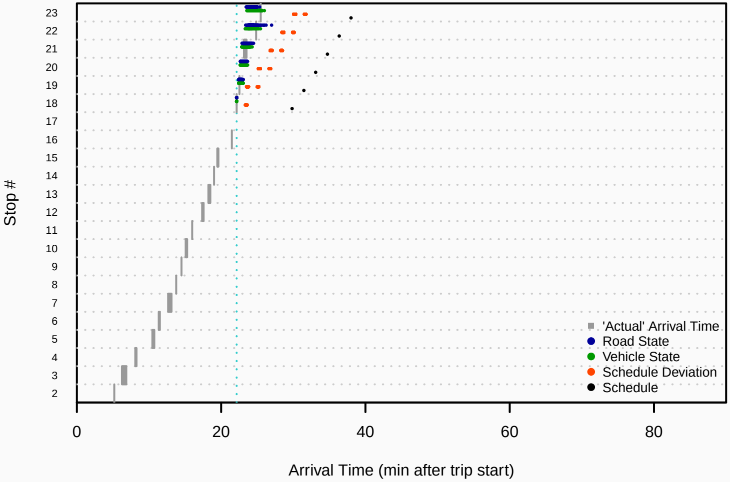

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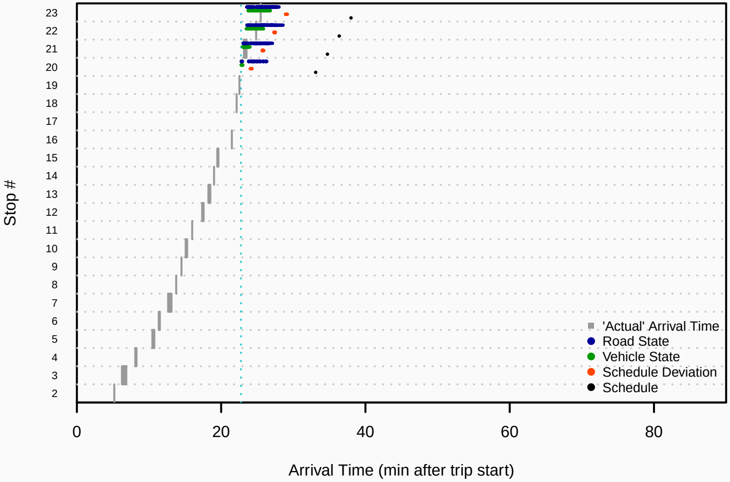

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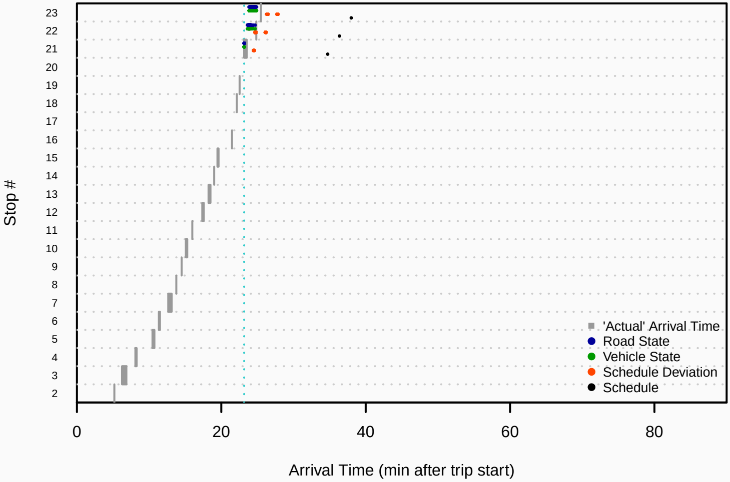

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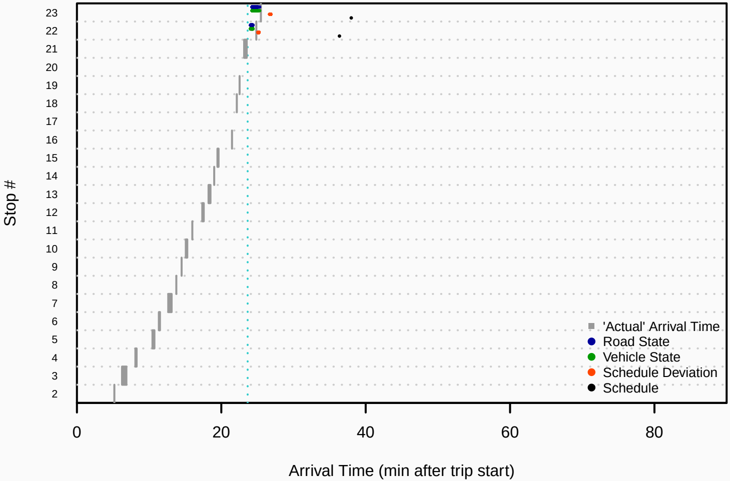

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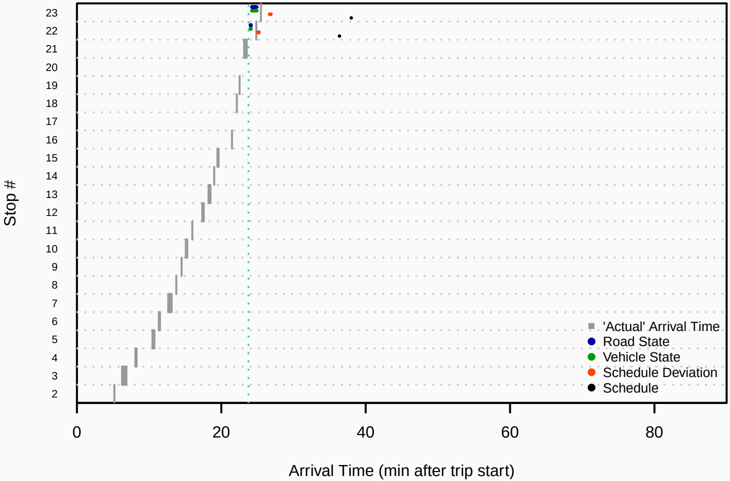

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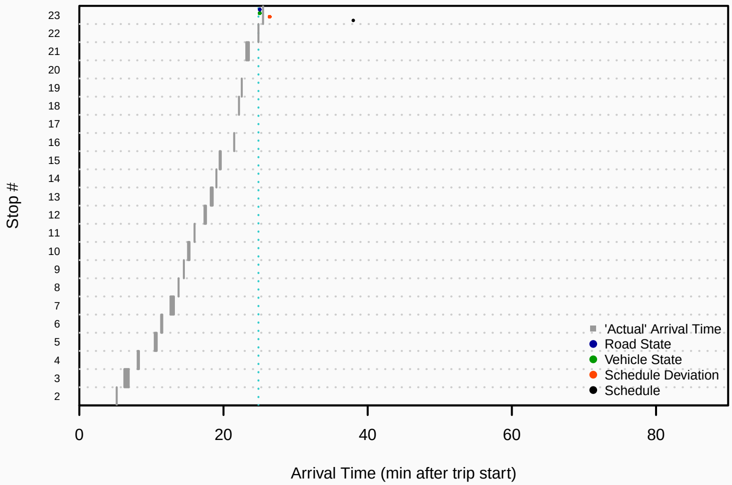

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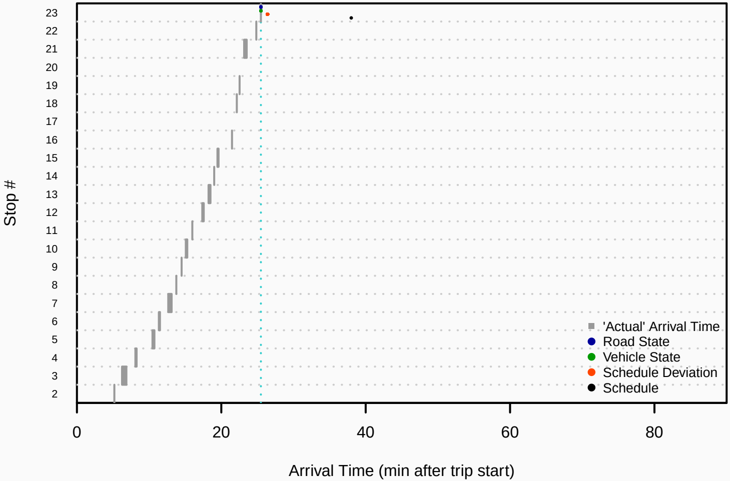

Predicting Arrival Time

Conclusions:

- **Schedule:** ...

Predicting Arrival Time

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Predicting Arrival Time

Conclusions:

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Predicting Arrival Time

Conclusions:

- **Schedule:** ...
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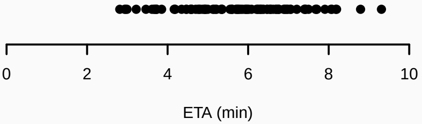
Predicting Arrival Time: Intervals

How do we communicate estimate + uncertainty to commuters?



[1] arguably

Predicting Arrival Time: Intervals

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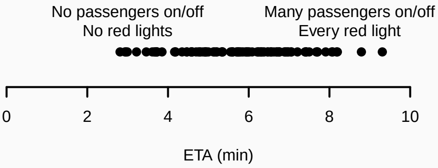

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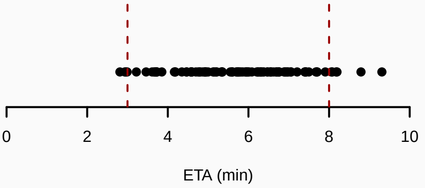
Predicting Arrival Time: Intervals

How do we communicate estimate + uncertainty to commuters?

⇒ **Prediction intervals**



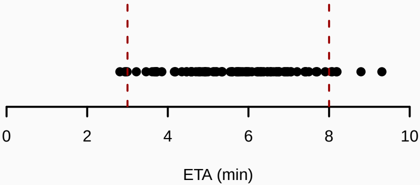
[1] arguably

Predicting Arrival Time: Intervals

How do we communicate estimate + uncertainty to commuters?

⇒ **Prediction intervals**

- easy to compute from particle sample
- intuitive[1]: ETA 6 min (mean) versus ETA 3–8 min
- Biased to reduce chance of missing bus



[1] arguably

What's Next?

- Add more routes
  - $\Rightarrow$ automate intersection detection
- Historical data to estimate parameters
  - $\Rightarrow$ Dwell times, stopping probabilties
  - $\Rightarrow$ Segment speed covariance matrix (including off-diagonals)
  - $\Rightarrow$ Model wait time at intersections
- Scale up: <u>ALL routes/busses</u>
  - $\Rightarrow$ computational speed
  - $\Rightarrow$ run in real-time
- Selection of "best" quantiles for prediction intervals
- ...

Thank you!

Questions?