# Real-time prediction of bus arrival using joint models for vehicle and road state

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NEW ZEALAND

SCIENCE
DEPARTMENT OF STATISTICS

# Overview

1. A quick motivation
2. Two real-time models: vehicle (particle filter) & road (Kalman filter)
3. Predicting arrival times

# What's wrong with the current[1] system?

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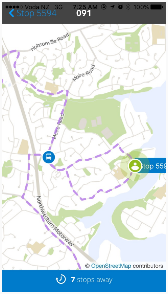

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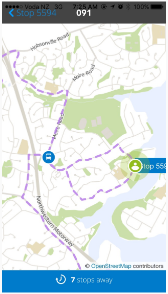


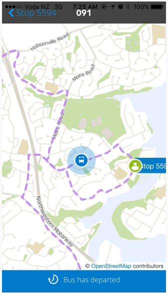

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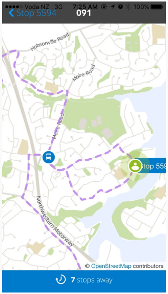

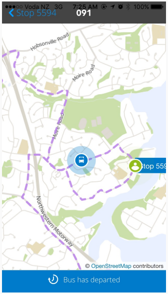

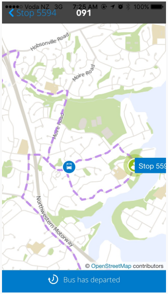

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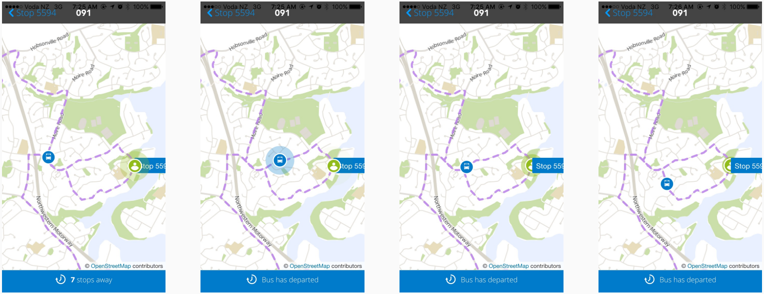
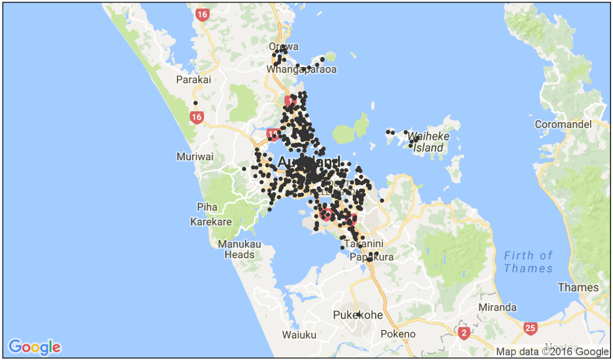


# What's wrong with the current[1] system?

- Prediction inaccuracy
- Prone to errors
- Recent modelling frameworks don't make use of all real-time vehicle data



# Vehicle State Model

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**Goal:** use observations of bus location (GPS) . . .

$$\mathbf{Y}_k = \begin{bmatrix} \phi_k \\ \lambda_k \\ t_k \end{bmatrix} = \begin{bmatrix} \text{latitude (degrees)} \\ \text{longitude (degrees)} \\ \text{timestamp} \end{bmatrix}$$

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... to infer **unobservable vehicle state** ...

$$\mathbf{X}_k = \begin{bmatrix} d_k \\ v_k \\ s_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \text{distance into trip (meters)} \\ \text{velocity/speed } (ms^{-1}) \\ \text{last visited stop} \\ \vdots \end{bmatrix}$$

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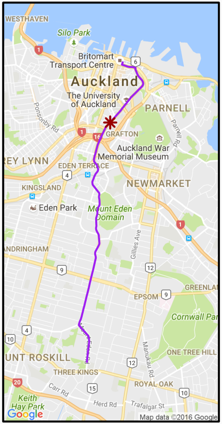
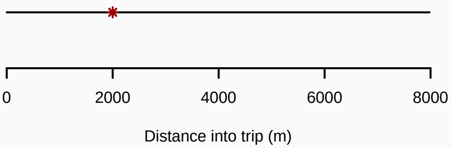
... in real time.

# Vehicle State Model

$\mathbf{Y}_k$




$\mathbf{X}_k$ (first component, $d_k$)




Example: Route 274, Britomart to Three Kings

# Vehicle State Model: Particle Filter

- Represent $\mathbf{X}_k$ by a sample of point-estimates (particles) $\mathbf{x}_k^{(i)}$
  1. generate sample of plausible vehicle state predictions
  2. remove predictions no longer plausible, given observation
- Flexible modeling framework, fewer assumptions
- Better coverage of possible states (**multimodality**)
- Intuitive likelihood function

# Vehicle State Model: Particle Filter

**Step 1: predict**

- Start with vehicle state at previous observation, $\mathbf{X}_{k-1} = \{\mathbf{x}_{k-1}^{(i)} : i = 1, \ldots, N\}$

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1. Add system noise

$$v_k^{(i)} \sim \mathcal{N}_T(v_{k-1}^{(i)}, \sigma_v^2), \qquad 0 \leq v_k^{(i)} \leq \mathbf{v}_{\max}$$

$\mathbf{v}_{\max} \approx$ road speed limit

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$$\mathbf{x}_k^{(i)} = f(\mathbf{x}_{k-1}^{(i)}, \sigma_v^2, \ldots)$$

1. Add system noise
2. Move particles along route (Law of Motion)

$$d_k^{(i)} = d_{k-1}^{(i)} + (t_k - t_{k-1}) v_k^{(i)}$$

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2. Move particles along route (Law of Motion)
3. What about intermediate stop(s)?
   - Does the particle stop? $p_{s_k}^{(i)} \sim \mathrm{Bernoulli}(\pi_{s_k})$
   - If so, for how long? $\bar{t}_{s_k} \sim \mathcal{E}(\tau_{s_k})$

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    - **Dwell time** $= p_{s_k}^{(i)}(\gamma + \bar{t}_{s_k})$

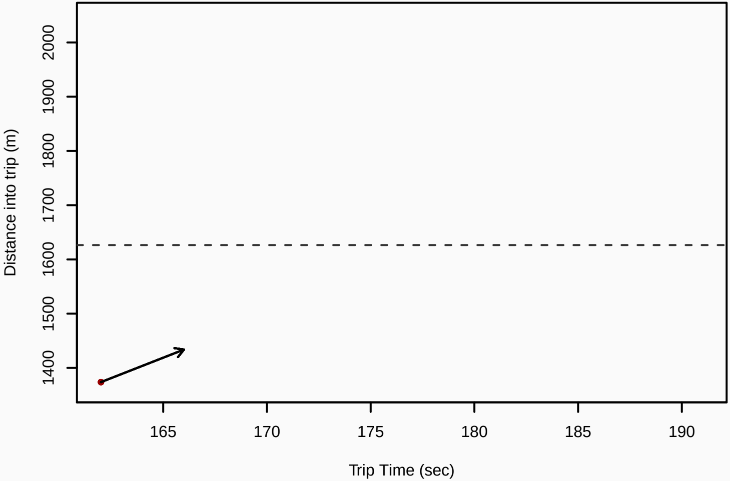
      $\gamma$ = minimum dwell time (deccelerate/accelerate, open/close doors)

      $\bar{t}_{s_k}$ = passengers on/off

# Vehicle State Model: Particle Filter

**Step 1: predict**

Example ($N = 10$ particles)



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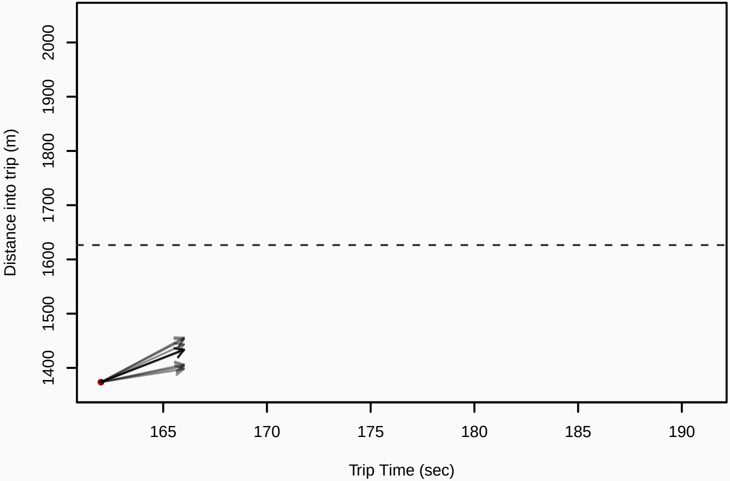

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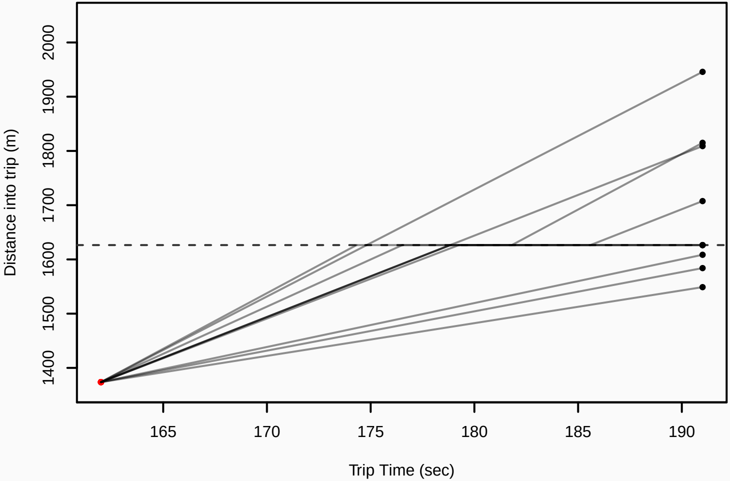

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**Step 2: update**

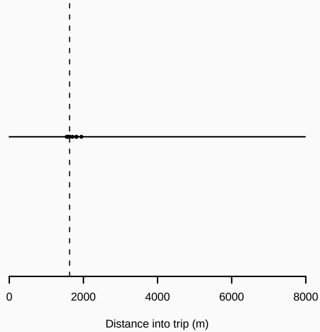
- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2)$

# Vehicle State Model: Particle Filter

## Step 2: update

- Likelihood function $\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2)$
  - Transform particles onto flat plane

$$\mathbf{z}_k^{(i)} = g(h(\mathbf{x}_k^{(i)}) | \mathbf{Y}_k)$$



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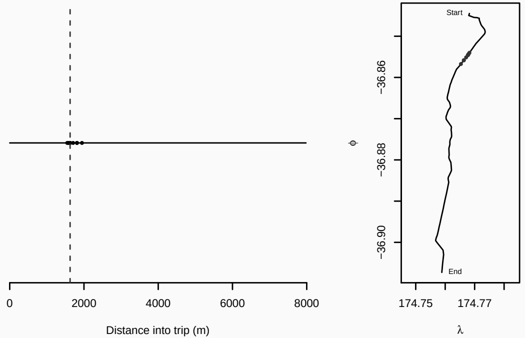
$h$: measurement function (distance into trip $\rightarrow$ lat/lon)



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## Step 2: update

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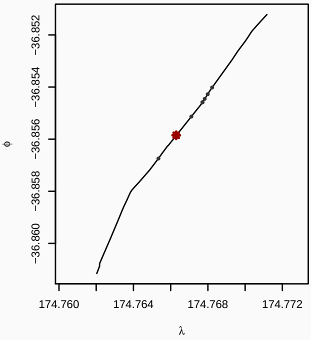
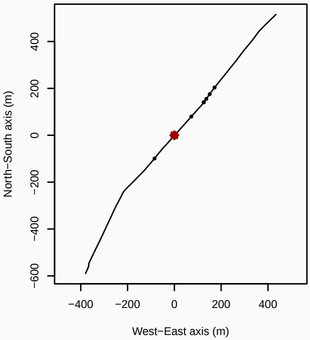
$$\mathbf{z}_k^{(i)} = g(h(\mathbf{x}_k^{(i)}) | \mathbf{Y}_k)$$

$g(\cdot | \mathbf{Y}_k)$: projection centered on $\mathbf{Y}_k$, 1 unit = 1 meter in all directions



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- Likelihood function $\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
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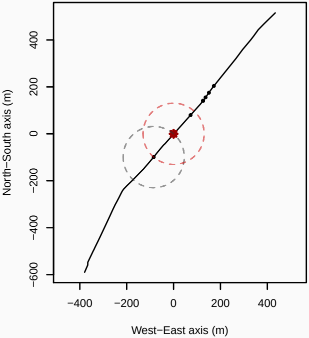
$$\mathbf{Y}_k|\mathbf{z}_k^{(i)}, \sigma_y^2 \sim \mathbf{z}_k^{(i)}|\mathbf{Y}_k, \sigma_y^2 \sim \mathcal{N}_2(\mathbf{0}, \sigma_y^2 I_2) \qquad (\sigma_y^2 = \text{GPS error})$$



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$$\mathbf{Y}_k | \mathbf{z}_k^{(i)}, \sigma_y^2 \sim \mathbf{z}_k^{(i)} | \mathbf{Y}_k, \sigma_y^2 \sim \mathcal{N}_2(\mathbf{0}, \sigma_y^2 I_2) \qquad (\sigma_y^2 = \text{GPS error})$$

  - For each particle

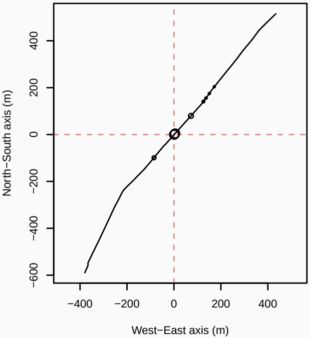
$$\ell(\mathbf{Y}_k | \mathbf{x}_k^{(i)}, \sigma_y^2, h, g) \propto e^{-\frac{1}{2\sigma^2}\left((\mathbf{z}_k^{(i)})^T \mathbf{z}_k^{(i)}\right)}$$

# Vehicle State Model: Particle Filter

**Step 2: update**

- Likelihood function $\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
- Compute weights

$$w_i = \frac{\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)})}{\sum_{j=1}^{N} \ell(\mathbf{Y}_k|\mathbf{x}_k^{(j)})}$$



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- Likelihood function $\ell(\mathbf{Y}_k|\mathbf{x}_k^{(i)}, \sigma_y^2, h, g)$
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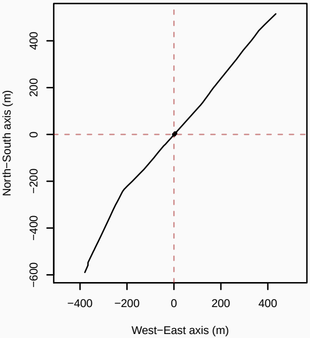

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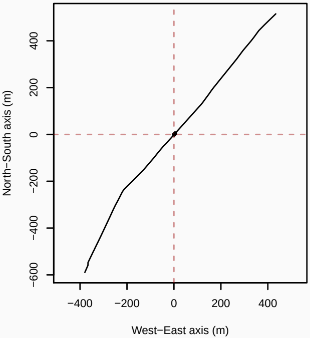
  $\Rightarrow$ keep particles plausible given data



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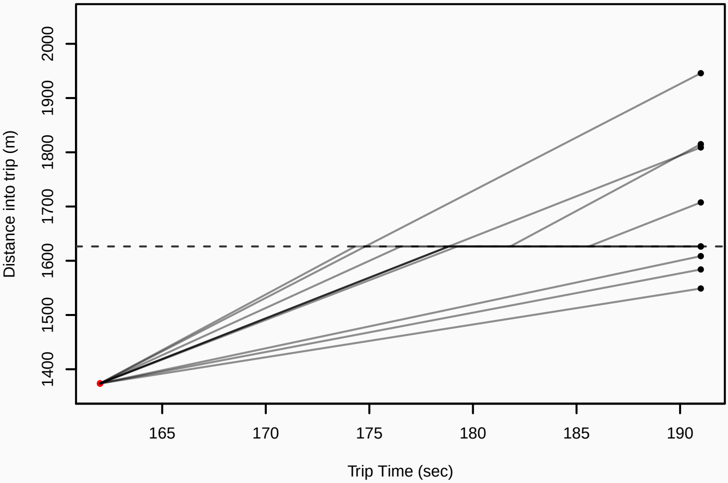

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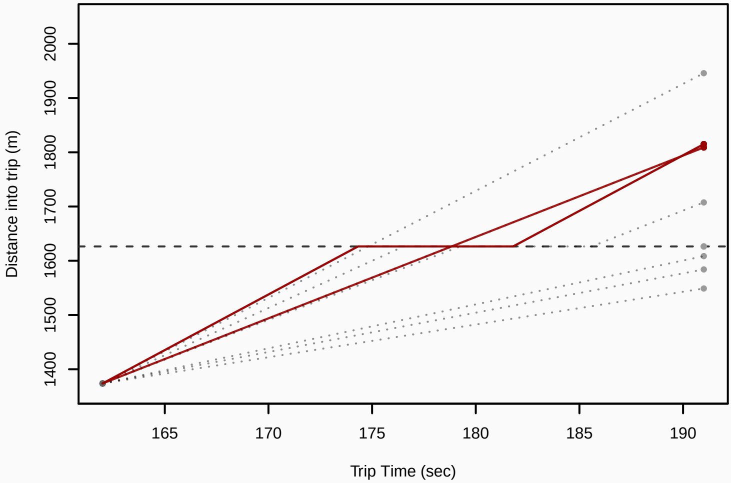

# Road State Model

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1. Particle filter $\Rightarrow$ speed estimates for a given bus
2. Identify segments of road common to multiple routes
3. Estimate speed along road segments using all busses

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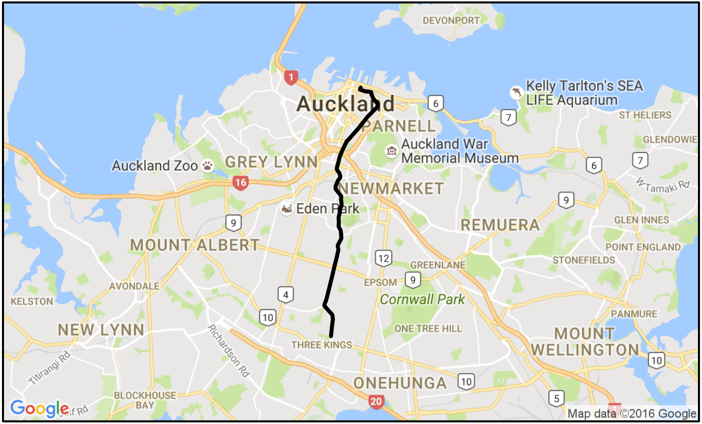
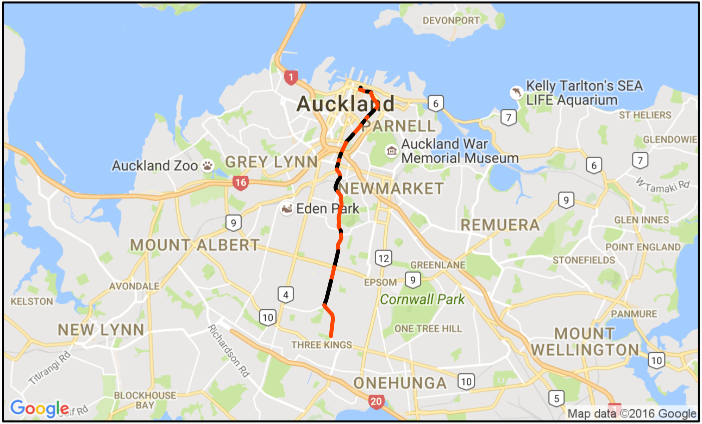
# Road State Model

2. Identify segments of road common to multiple routes



# Road State Model

2. Identify segments of road common to multiple routes
   $\Rightarrow$ between (major) intersections



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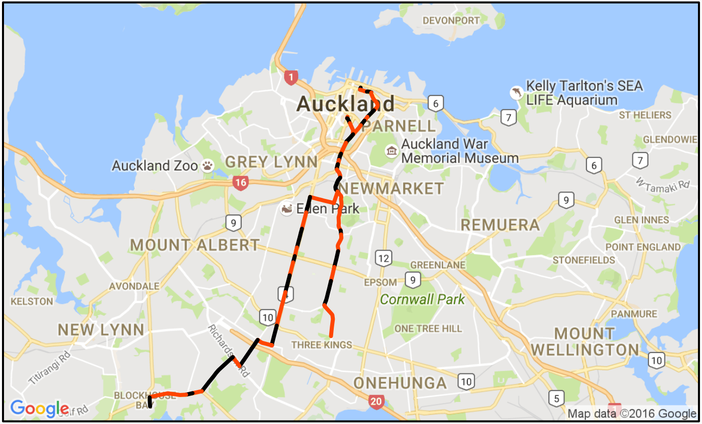

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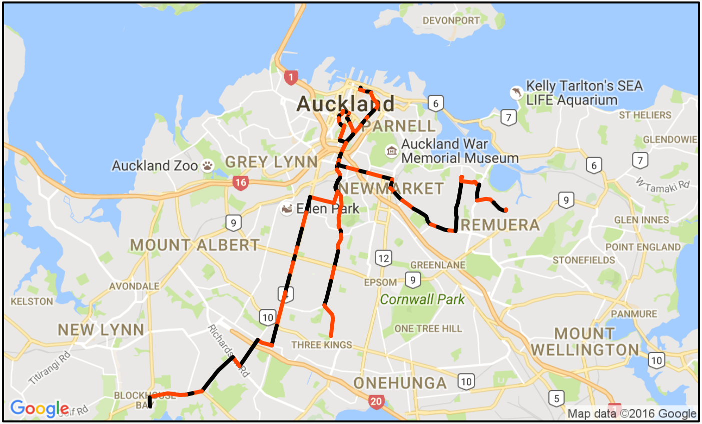

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3. Estimate speed along road segments using <u>all busses</u>

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$\Rightarrow$ **Kalman filter**

**Road state:** mean speed for all $M$ road segments at time $t_\ell$

$$\boldsymbol{\nu}_\ell = [\nu_{1\ell}\ \nu_{2\ell}\ \cdots\ \nu_{M\ell}]^T$$

with associated covariance matrix

$$\Xi_\ell = \begin{bmatrix} \xi_{1\ell} & 0 & \cdots & 0 \\ 0 & \xi_{2\ell} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi_{M\ell} \end{bmatrix}$$

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3. Estimate speed along road segments using all busses

⇒ **Kalman filter**

- no complex model necessary (Normal distribution adequate)

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3. Estimate speed along road segments using all busses

⇒ **Kalman filter**

- no complex model necessary (Normal distribution adequate)
- updated using particle filter estimates

$$\mathbf{v}_\ell = \boldsymbol{\nu}_\ell + \mathbf{r}_\ell$$

  - $\mathbf{v}_\ell$: mean speed of particles
  - $\mathbf{r}_\ell \sim \mathcal{N}(0, \hat{\mathbf{R}}_\ell)$, $\hat{\mathbf{R}}_\ell$: variance of particle speeds

# Predicting Arrival Time

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1. Schedule
2. Schedule deviation (AT?)
3. Vehicle state
4. Road state

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Some notation:

- $S_j^t$ = scheduled arrival time at stop $j$
- $\hat{A}_j$ = (predicted) arrival time at stop $j$
- $\tilde{T}_{s_k}^a, \tilde{T}_{s_k}^d$ = observed arrival/departure delay at last stop (from Auckland Transport's API)

# Prediction Arrival Time

1. **Schedule**
   - $\hat{A}_j = S_j^t$
   - Baseline for other predictors

# Prediction Arrival Time

## 2. Schedule deviation

- $\hat{A}_j = \begin{cases} S_j^t + \tilde{T}_{s_k}^d & \text{if departed stop } s_k \\ S_j^t + \tilde{T}_{s_k}^a & \text{if not departed stop } s_k \end{cases}$
- **OR** use particle estimates of arrival/departure delay, $\tilde{A}_{s_k}^{(i)}$ and $\tilde{D}_{s_k}^{(i)}$

# Prediction Arrival Time

3. **Vehicle state**

   - $S_j^d =$ distance along route of stop $j$

# Prediction Arrival Time

## 3. Vehicle state

- $S_j^d$ = distance along route of stop $j$
- $\hat{A}_j = t_k + \frac{S_j^d - d_k}{v_k}$

# Prediction Arrival Time

**3. Vehicle state**

- $S_j^d$ = distance along route of stop $j$
- $\hat{A}_j{}^{(i)} = t_k + \frac{S_j^d - d_k{}^{(i)}}{v_k{}^{(i)}}$
- Prediction for each particle

# Prediction Arrival Time

**3. Vehicle state**

- $S_j^d =$ distance along route of stop $j$
- $\hat{A}_j{}^{(i)} = t_k + \frac{S_j^d - d_k{}^{(i)}}{v_k{}^{(i)}} + \sum_{\ell=s_k+1}^{j-1} p_\ell^{(i)}(\gamma + \bar{t}_\ell^{(i)})$
- Prediction for each particle
- Allow for dwell time uncertainty

# Prediction Arrival Time

**4. Road state**

- $r_k$ = route segment index
  $R_b$ = distance along route of start of segment $b$

# Prediction Arrival Time

**4. Road state**

- $r_k =$ route segment index
  $R_b =$ distance along route of start of segment $b$
- $\hat{A}_j = t_k +$

# Prediction Arrival Time

## 4. Road state

- $r_k$ = route segment index
  $R_b$ = distance along route of start of segment $b$
- $\hat{A}_j = t_k + \frac{R_{s_k+1} - d_k}{\nu_{s_k}} +$
- travel time until end of current segment

# Prediction Arrival Time

**4. Road state**

- $r_k =$ route segment index
  $R_b =$ distance along route of start of segment $b$
- $\hat{A}_j = t_k + \frac{R_{s_k+1} - d_k}{\nu_{s_k}} + \sum_{b \in B^\star} \frac{R_{b+1} - R_b}{\nu_b} +$
- travel time through intermediate segments ($B^\star$)

# Prediction Arrival Time

## 4. Road state

- $r_k$ = route segment index
  $R_b$ = distance along route of start of segment $b$
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- travel time along segment $b'$ to stop $j$

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- For each particle, sample $v_b^{(i)} \sim \mathcal{N}(\nu_b, \xi_b)$

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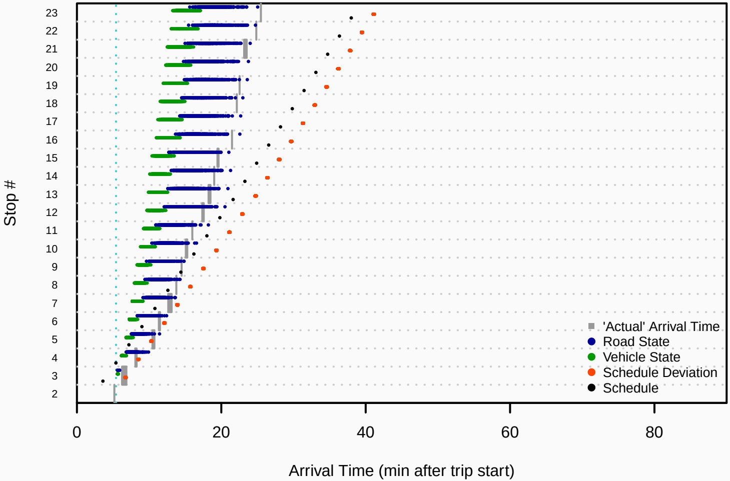
  $+ \sum_{\ell = s_k + 1}^{j-1} p_\ell^{(i)} (\gamma + \bar{t}_\ell^{(i)})$
- For each particle, sample $v_b^{(i)} \sim \mathcal{N}(\nu_b, \xi_b)$
- Allow for dwell time uncertainty

# Predicting Arrival Time



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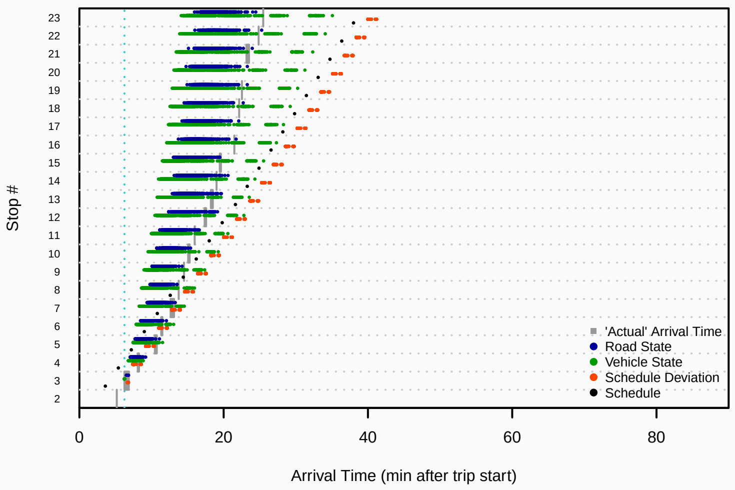

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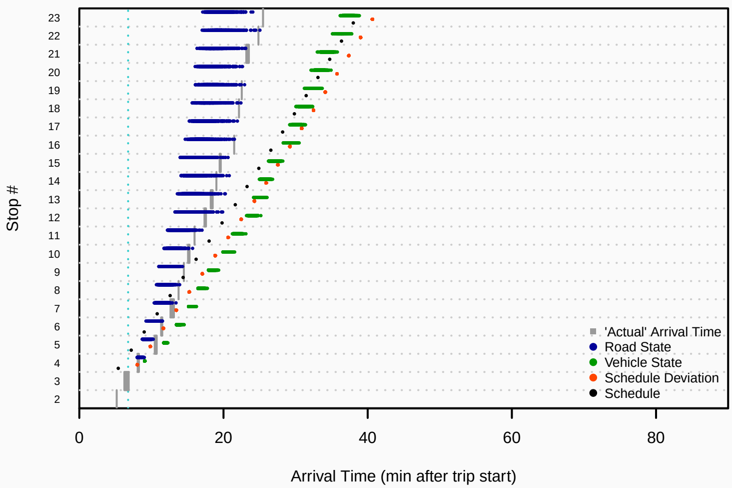

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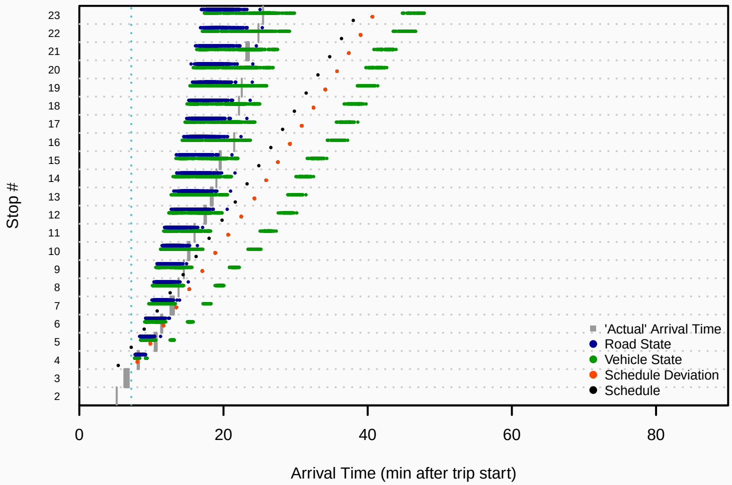

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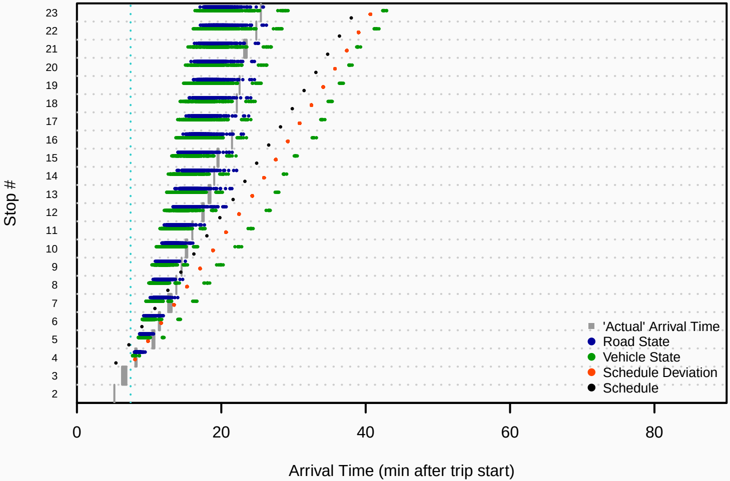

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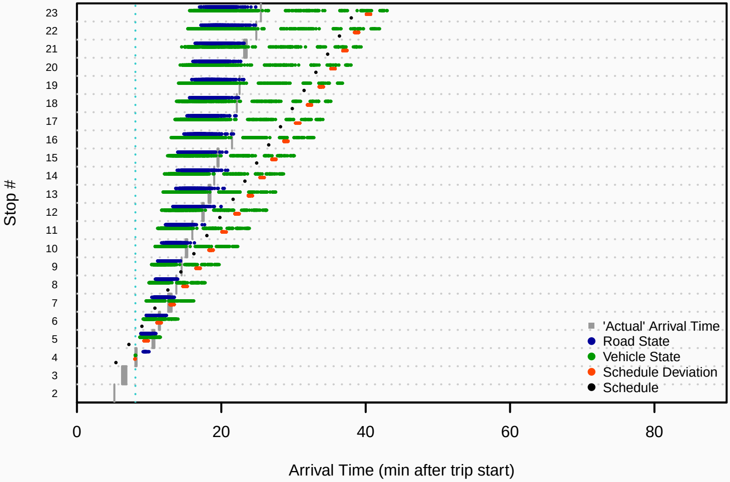

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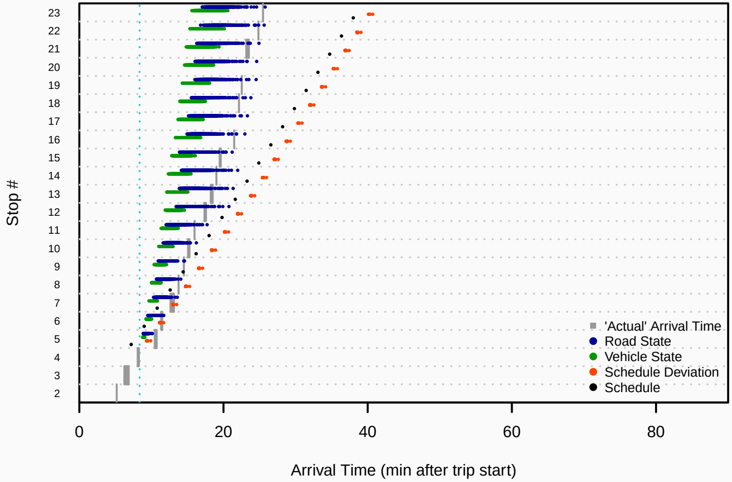

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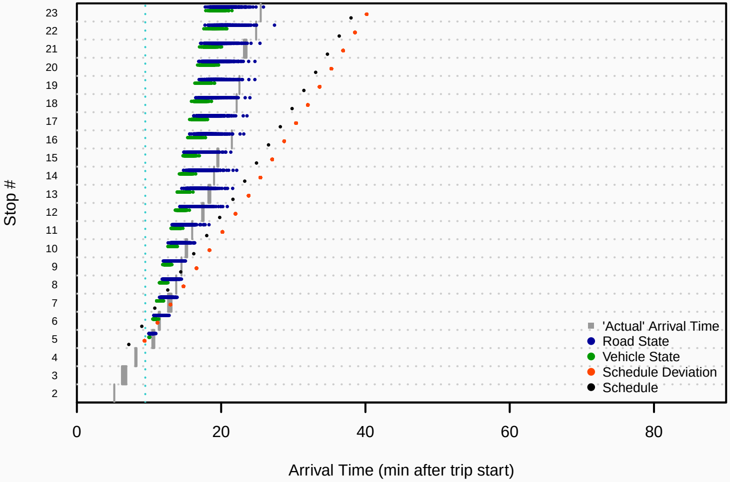

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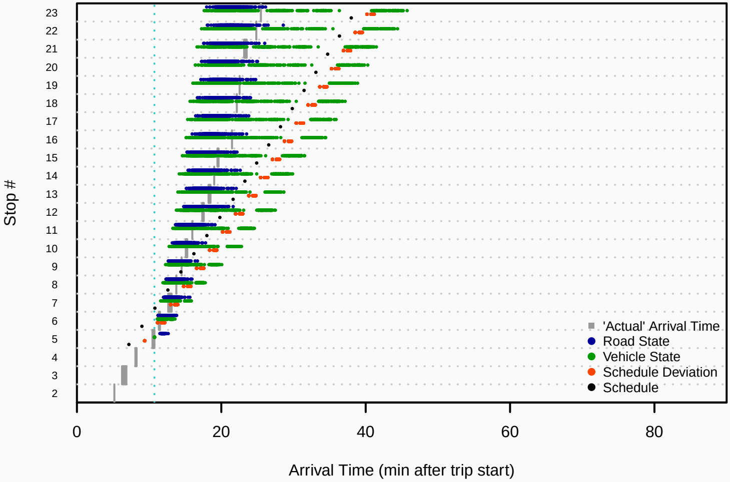

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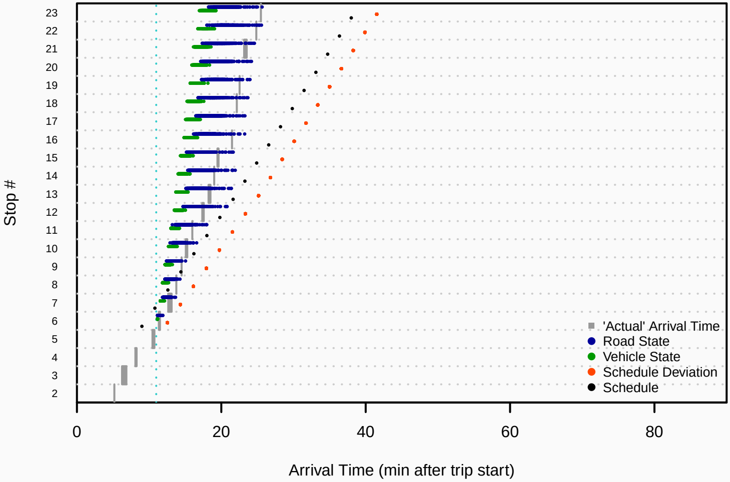

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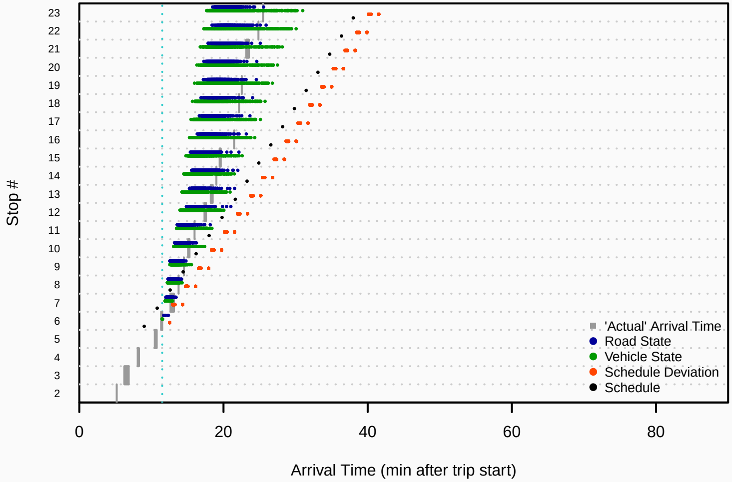

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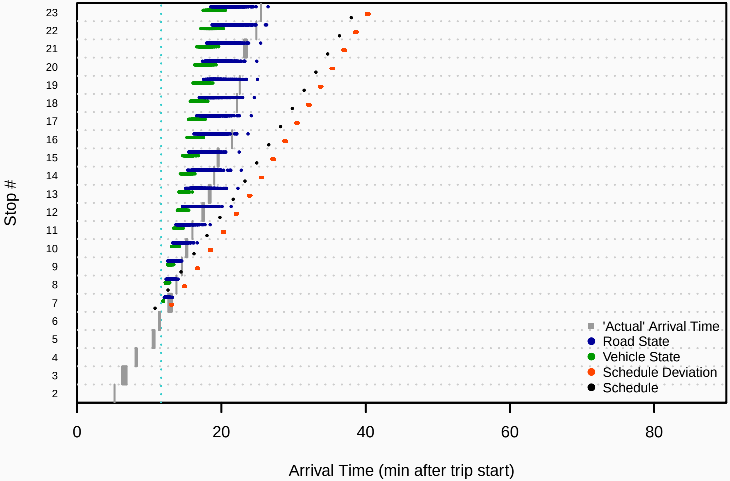

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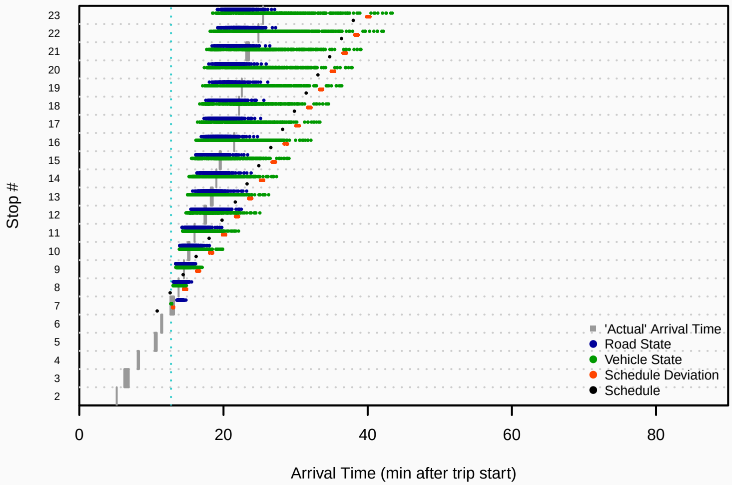

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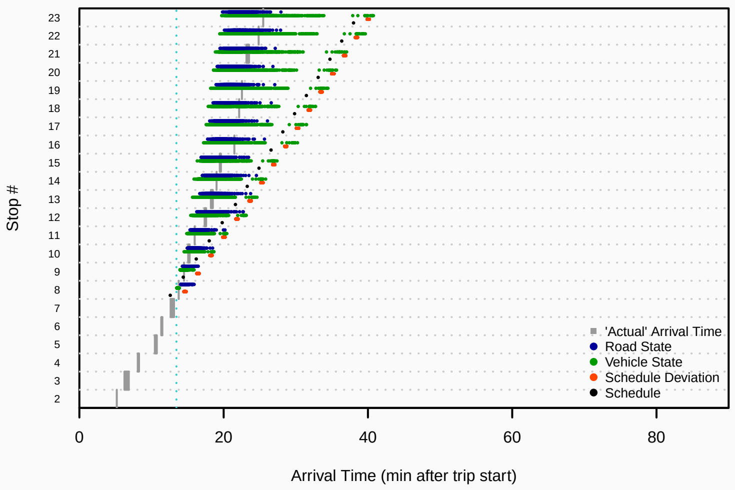

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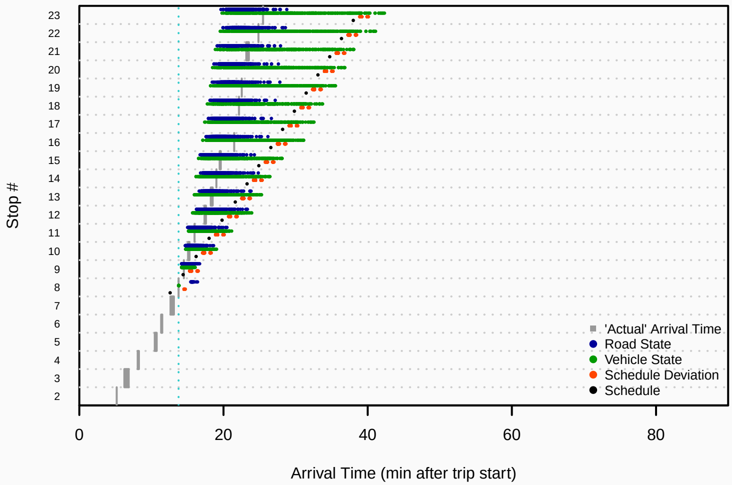

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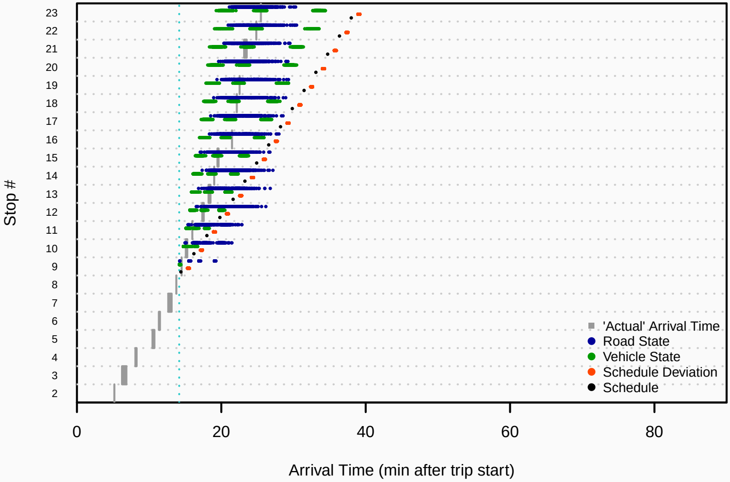

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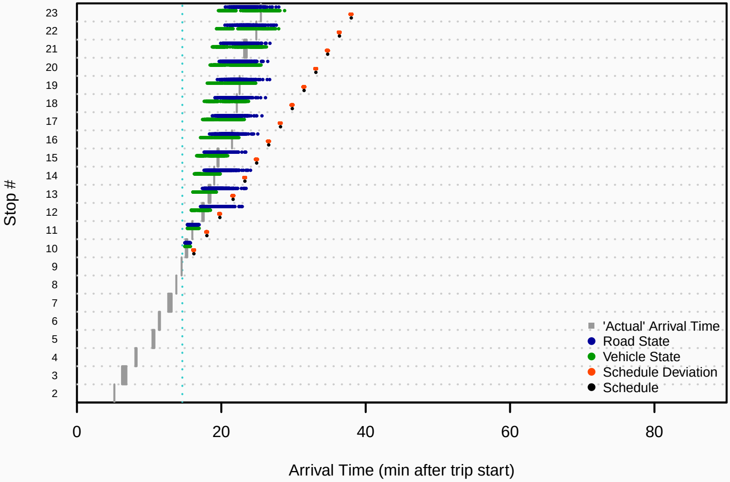

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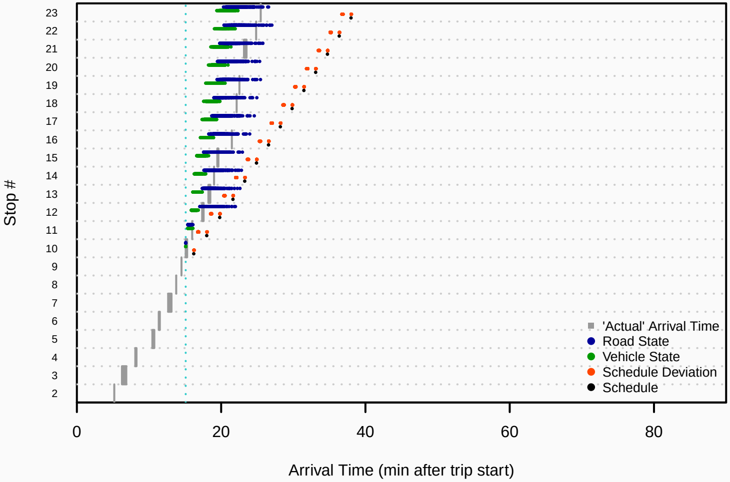

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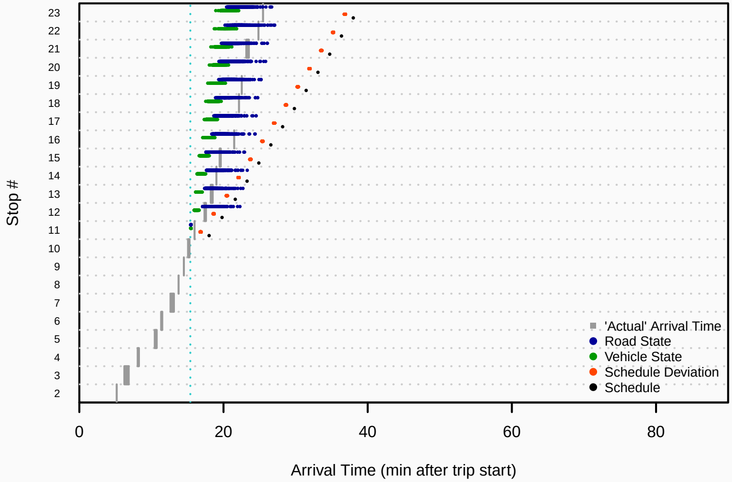

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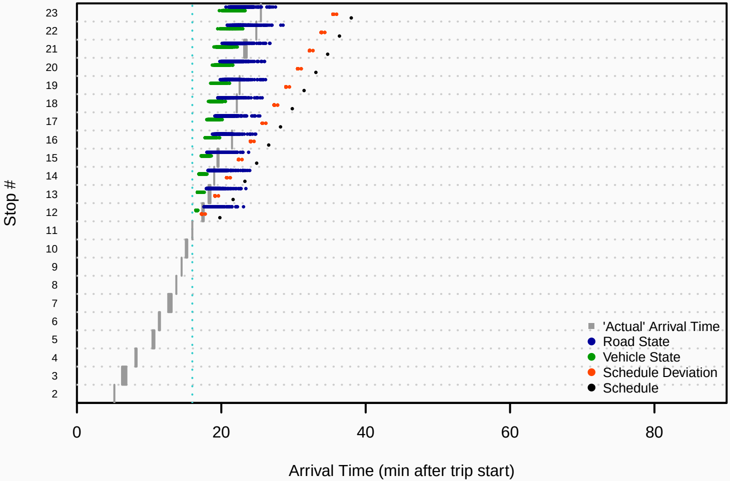

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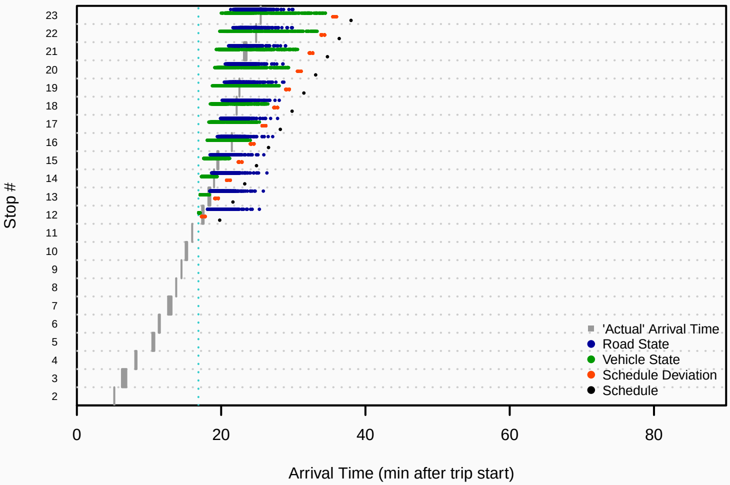

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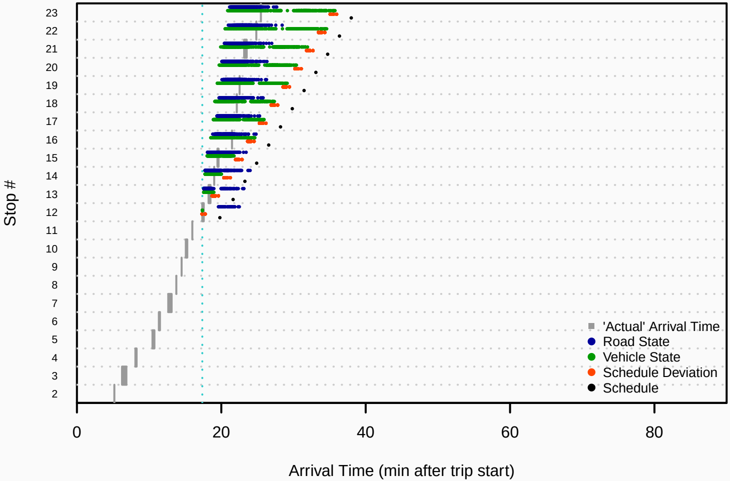

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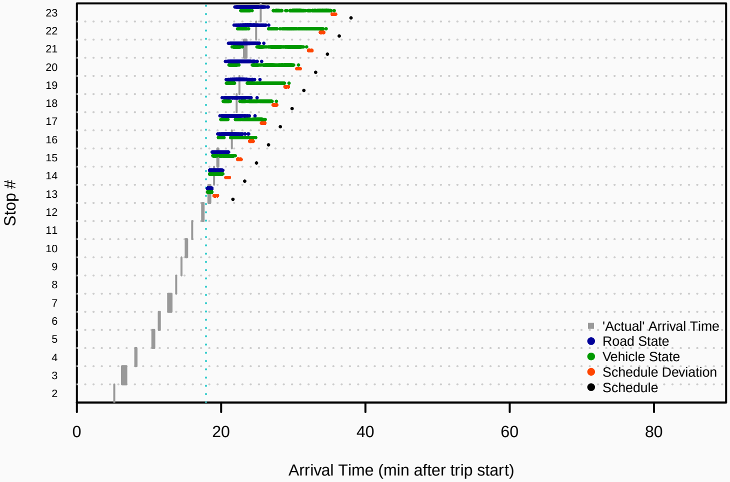

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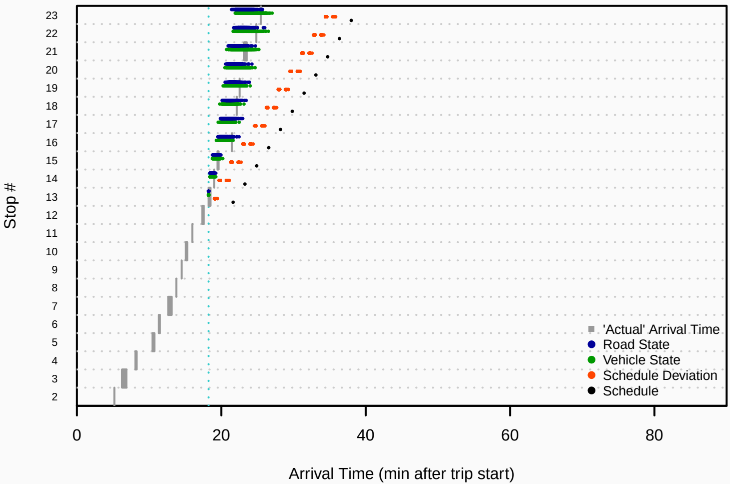

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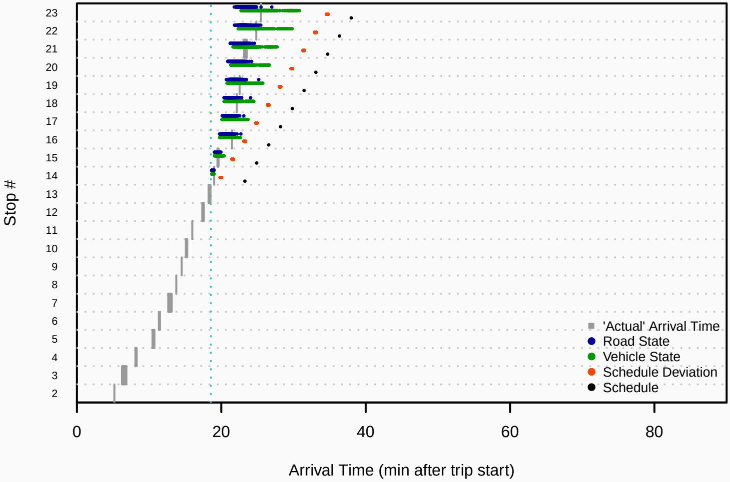

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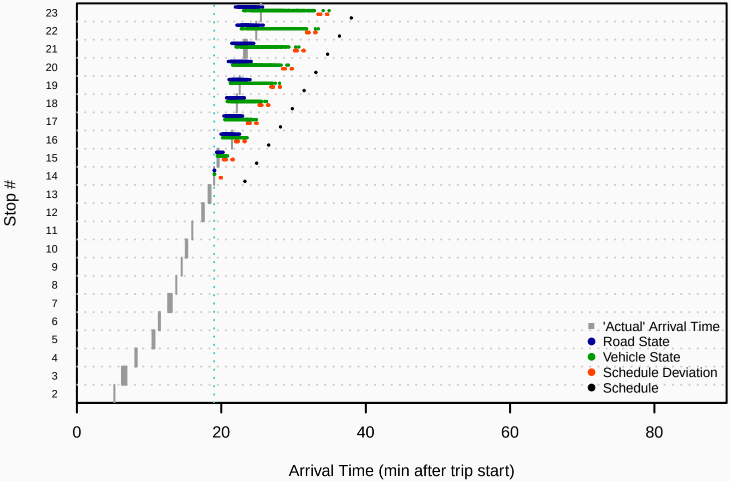

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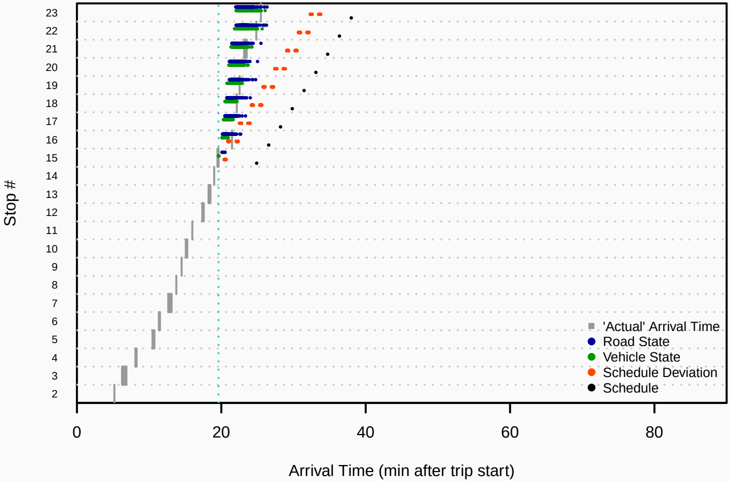

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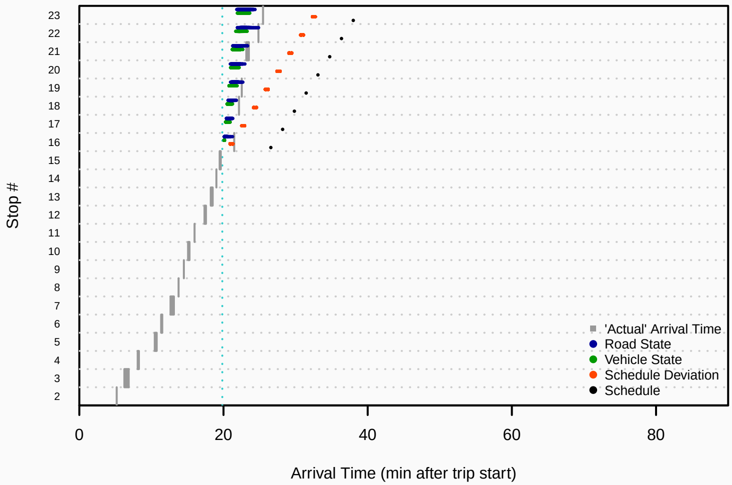

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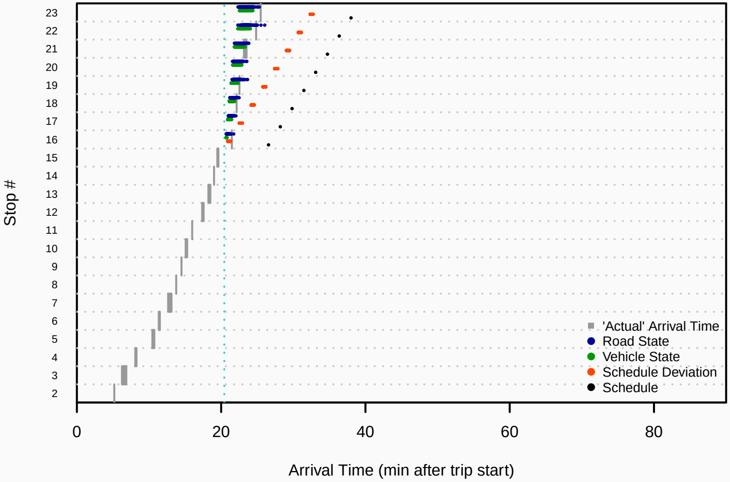

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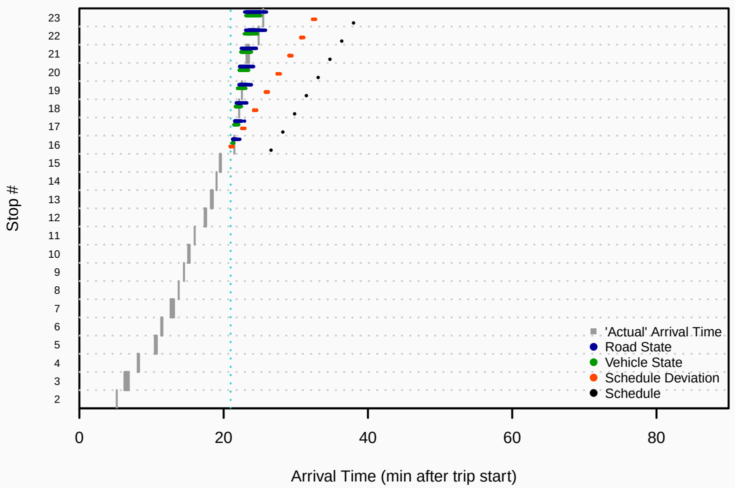

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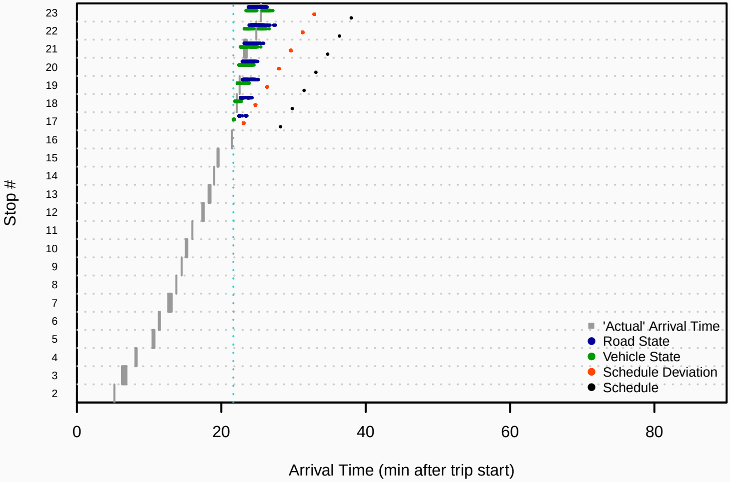

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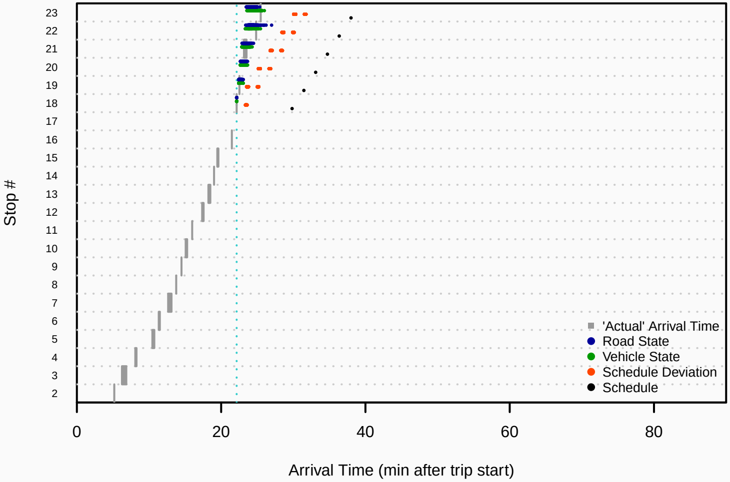

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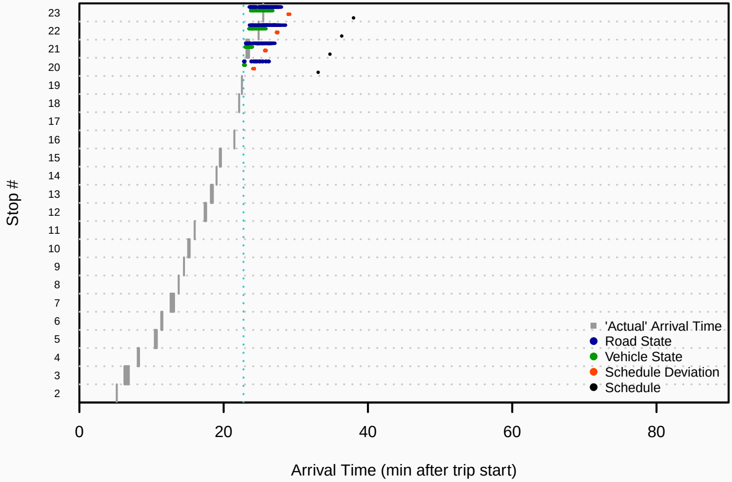

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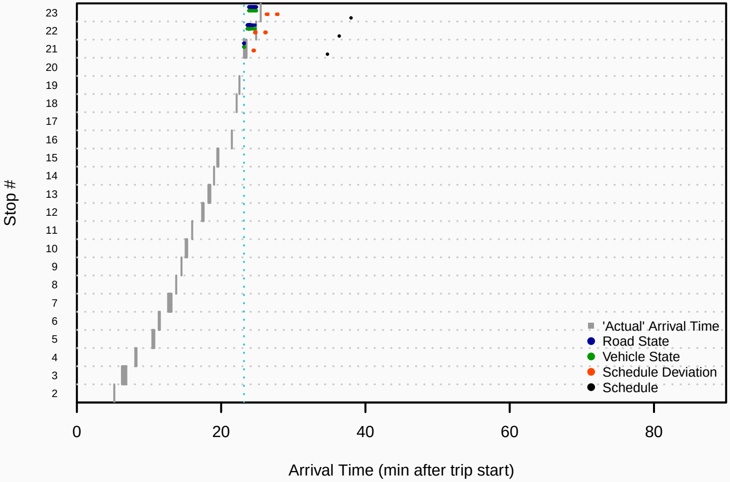

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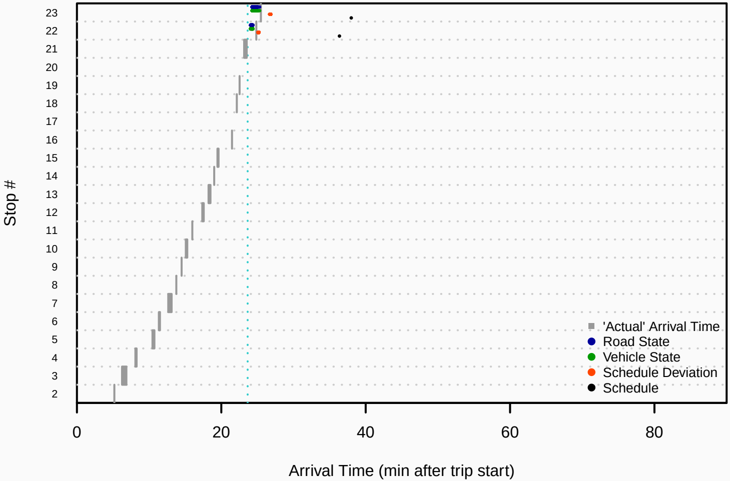

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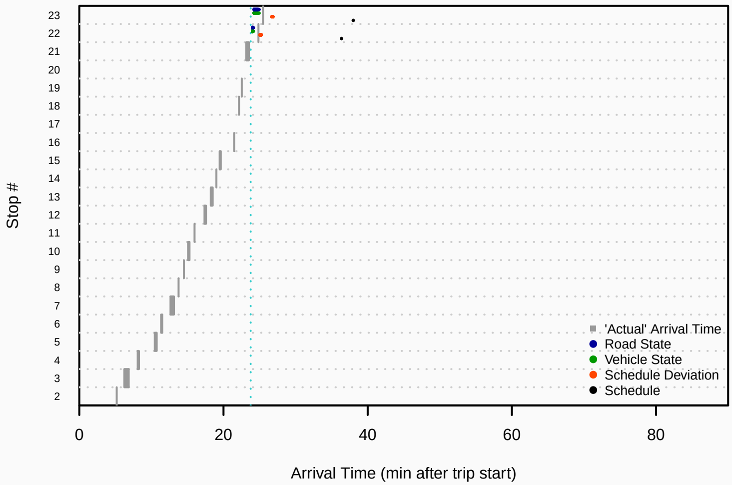

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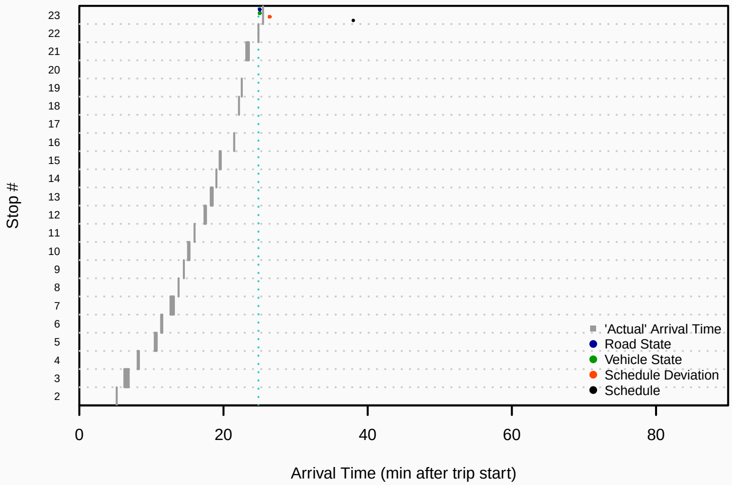

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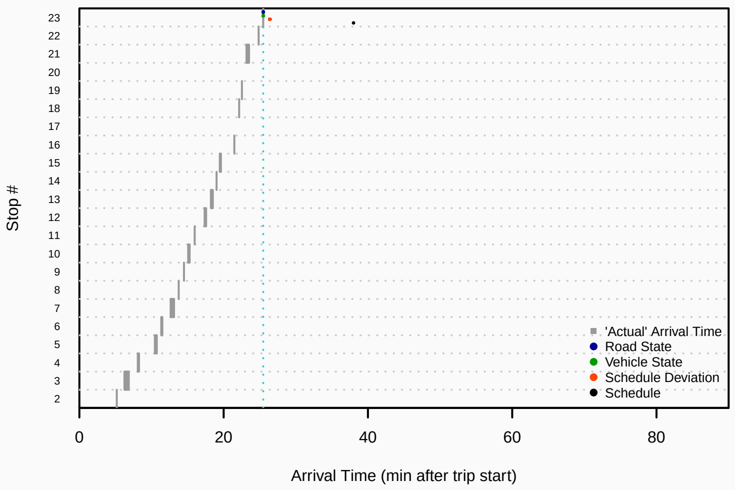

# Predicting Arrival Time

Conclusions:

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# Predicting Arrival Time

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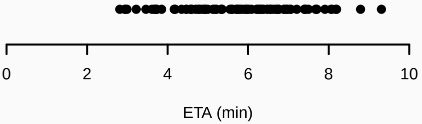
# Predicting Arrival Time: Intervals

How do we communicate estimate + uncertainty to commuters?




[2] arguably

# Predicting Arrival Time: Intervals

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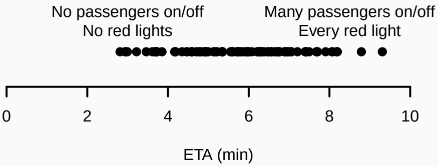
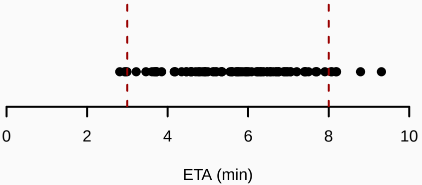



[2] arguably

# Predicting Arrival Time: Intervals

How do we communicate estimate + uncertainty to commuters?

$\Rightarrow$ **Prediction intervals**



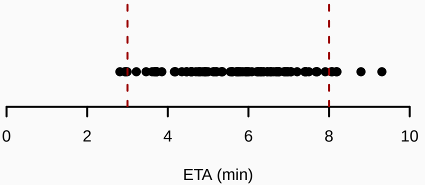

[2] arguably

# Predicting Arrival Time: Intervals

How do we communicate estimate + uncertainty to commuters?

⇒ **Prediction intervals**

- easy to compute from particle sample
- intuitive[2]: ETA 6 min (mean) versus ETA 3–8 min
- Biased to reduce chance of missing bus




[2] arguably

## What's Next?

- Add more routes
  - $\Rightarrow$ automate intersection detection
- Historical data to estimate parameters
  - $\Rightarrow$ Dwell times, stopping probabilties
  - $\Rightarrow$ Segment speed covariance matrix (including off-diagonals)
  - $\Rightarrow$ Model wait time at intersections
- Scale up: ALL routes/busses
  - $\Rightarrow$ computational speed
  - $\Rightarrow$ run in real-time
- Selection of "best" quantiles for prediction intervals
- ...

**Thank you!**

**Questions?**