Extracting Problem Structure with LLMs for Optimized SAT Local Search*

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Abstract

Local search preprocessing makes Conflict-Driven Clause Learning (CDCL) solvers faster by providing high-quality starting points and modern SAT solvers have incorporated this technique into their preprocessing steps. However, these tools rely on basic strategies that miss the structural patterns in problems. We present a method that applies Large Language Models (LLMs) to analyze Python-based encoding code. This reveals hidden structural patterns in how problems convert into SAT. Our method automatically generates specialized local search algorithms that find these patterns and use them to create strong initial assignments. This works for any problem instance from the same encoding type. Our tests show encouraging results, achieving faster solving times compared to baseline preprocessing systems.

1 Introduction

Local search preprocessing guides CDCL solvers to faster solutions through better starting points. Balint and Manthey [[2013]] showed that preprocessing improves CDCL-based SAT-solving performance. Current SAT solvers like CaDiCaL [Biere, 2019; Biere et al., 2020; Biere et al., 2024] and CryptoMiniSat [Soos, 2020] have adopted this approach in their preprocessing phase. These tools apply basic strategies that work well for random problems but miss critical patterns in structured instances. SAT encodings of real problems contain inherited patterns from graph layouts, data connections, and domain-specific rules. The transformation to Conjunctive Normal Form (CNF) obscures these patterns. Current local search methods skip these structures in favor of general approaches. This paper addresses these limitations by introducing a framework that leverages LLMs to generate local search strategies tailored to encoding structures, enabling solvers to take advantage of these patterns for improved performance. Our research addresses three questions:

- 1. How can LLMs analyze PySAT [Ignatiev *et al.*, 2024] code to interpret how problem structure translates to SAT clauses?
- 2. How can we create local search strategies that recognize and exploit these encoding patterns?
- 3. What performance gains do structure-aware preprocessing methods achieve versus standard approaches?

Our method applies LLMs to read and interpret the PySAT code that converts structured problems to SAT. The model spots high-level constructs like graph connections, path constraints, or counting

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limits and then builds these insights into a specialized local search procedure. This procedure targets the encoding rather than specific instances, making it applicable across all instances encoded this way. This use of LLMs moves beyond generic applications of AI tools by integrating them directly into the SAT solving workflow. While existing research highlights using LLMs for interactive code generation, such as in GitHub Copilot [Jiang *et al.*, 2024], our approach addresses the challenges associated with fully automated code generation for specialized tasks like SAT preprocessing. Significantly, the strategies we generate are encoding-specific but apply across problem instances, addressing a key gap in SAT-solving techniques.

An automated pipeline tests and corrects each local search procedure. The framework systematically generates, validates, and refines a diverse set of local search strategies through a two-phase process: a gathering phase to encourage diversity and a refinement phase to improve quality. This method automates aspects of algorithm design while maintaining performance guarantees, reducing the reliance on manual, domain-specific effort.

We evaluated our method on multiple structured problems, including Directed Feedback Vertex Set, Bounded Depth Decision Trees, and Treewidth. Our results demonstrate the effectiveness of using LLMs to generate encoding-specific preprocessing algorithms. While challenges remain in adapting these algorithms to encoding requirements, the approach provides a reproducible solution for improving SAT preprocessing through automation. The LLM-generated algorithms effectively exploit encoding structure, with several variants outperforming baselines on hard instances. For Directed Feedback Vertex Set specifically, our methods solved 12 additional instances beyond conventional SAT solvers while maintaining performance on easier cases, demonstrating the potential for LLMs to enhance SAT solving through automated, problem-aware preprocessing.

Supplementary Material Code and instances are available on Zenodo [Schidler and Szeider, 2025]

2 Background and Related Work

2.1 SAT solving

The propositional satisfiability problem (SAT) takes a propositional formula and asks if there exists a variable assignment that makes the formula true, i.e., that *satisfies* the formula. Modern complete SAT solvers use Conflict-Driven Clause Learning (CDCL) [Silva and Sakallah, 1996; Marques-Silva *et al.*, 2021; Fichte *et al.*, 2023] and search for a satisfying assignment or proves that no such assignment exists.

A common way of using SAT solvers is by *encoding* other problems into SAT, i.e., representing a problem instance in propositional logic such that the satisfiability/unsatisfiability of the formula corresponds to a Yes/No answer for the original problem; and a satisfying variable assignment corresponds to a solution of the original problem. An example would be encoding whether a given graph admits a proper k-coloring, where a satisfying assignment can be translated into a k-coloring. An encoding requires an *encoding scheme* that translates instances from the original problem into propositional logic and thereby creates the actual encoding of an instance. In our example, it would be an algorithm that produces for a given graph and integer k an encoding that is satisfiable if and only if the graph admits a k-coloring.

2.2 Local Search Algorithms for SAT

The GSAT algorithm [Selman *et al.*, 1992] introduced the basic local search approach for SAT. GSAT picks the variable flip that leads to the most satisfied clauses. WalkSAT [Selman *et al.*, 1994] operates by randomly selecting an unsatisfied clause and then choosing a variable within that

clause to flip. These algorithms sparked numerous variations: Novelty [McAllester *et al.*, 1997] considers the time since a variable's last flip when selecting moves. Adaptive Novelty+ [Hoos, 2002] automatically tunes its noise parameter during search. ProbSAT [Balint and Schöning, 2012] uses probability distributions based on make and break values to choose variables. YalSAT [Biere, 2014] combines ideas from several algorithms with additional restart strategies.

2.3 Local Search in CDCL Solvers

Modern CDCL solvers use local search during preprocessing to find promising initial assignments. CaDiCaL [Biere, 2019; Cai *et al.*, 2022; Biere *et al.*, 2024] implements a variant of ProbSAT in its rephasing procedure. The solver stores good assignments found during search and uses them as starting points after restarts. This hybrid approach performs particularly well on random and hard combinatorial instances by combining the systematic nature of CDCL with local search's ability to find solutions in satisfiable regions quickly. The integration of local search into CDCL brings two main benefits: better initial assignments can guide the solver toward solutions faster, and local search patterns can inform restart strategies. However, current implementations use generic local search methods that don't account for problem structure.

2.4 Machine Learning and SAT solving

Machine learning has enhanced SAT solving in multiple ways. Deep neural networks can learn variable selection policies that speed up SAT solving [Selsam and Bjørner, 2019]. A recent line of work leverages graph neural networks to guide SAT local search [Yolcu and Póczos, 2019]. LLMs have recently shown success in algorithm generation. AlphaCode [Li *et al.*, 2022] and CodeGen [Nijkamp *et al.*, 2023] can generate correct implementations of algorithms. For constraint satisfaction, system StreamLLM generates streamlining constraints using LLM-calls [Voboril *et al.*, 2025; Voboril *et al.*, 2024], and MCP-Solver, on the other hand, allows interactive calls to a constraint solver within an LLM chat via the Model Context Protocol [Szeider, 2024]. However, using LLMs to analyze and improve algorithms is still a new direction. Related to our approach is work that uses LLMs to optimize compiler passes [Cummins *et al.*, 2023]. For our target problems, SAT encodings remain competitive with specialized algorithms.

The work in this paper differs from this related work in three fundamental ways: (i) we use LLMs to generate specialized local search algorithms, not general SAT solvers, (ii) we focus on problem structure, not instance-specific features, and (iii) we provide runtime and correctness guarantees.

3 Problem Statement

We consider the problem of automatically finding problem specific local search approaches that perform well in conjunction with CDCL SAT solvers. Existing local search approaches have two drawbacks. First, these algorithms are usually general purpose algorithms that do not consider, or know of, the encoding scheme that has been used to create the formula. We expect that special considerations for the encoding scheme boosts the performance of local search. Second, it has been shown empirically that local search often struggles to find satisfying assignments on its own [Li and Li, 2012; Cai and Zhang, 2021; Cai et al., 2022]. As discussed in the previous section, hybrid approaches that combine local search and CDCL solvers often achieve better results on hard combinatorial instances than either paradigm on its own [Cai et al., 2022]. Hence, local search methods specifically designed for hybrid approaches that are specific to the encoding scheme promise an improved performance on these hard instances.

Creating specialized algorithms is a time-consuming effort and is often focused on well-known approaches. This focus is necessary, as manually creating specialized algorithms with a variety of

approaches is usually infeasible. Hence, we evaluate if automatically generating prototypes using large language models (LLMs) is feasible, such that these prototypes implement a wide variety of different approaches.

We evaluate the performance of a local search function by setting the *default phases* of the CDCL solver to the assignment found by the local search function. The default phase of a variable is either true or false and determines which value the CDCL solver assigns the variable whenever the solver chooses a value for the variable. In the best case, the local search finds a satisfying assignment, in which case the CDCL solver will terminate almost immediately. Further, the closer the local search assignment is to a satisfying assignment, the faster the CDCL solver is expected to solve the instance. We then measure how long the SAT solver takes to solve an instance. This can be compared against the SAT solver using no local search or other local search functions.

We apply our approach to three carefully chosen combinatorial problems with increasing encoding complexity: Graph Coloring with its straightforward CNF representation, Directed Feedback Vertex Set with its more sophisticated reachability-based encoding, and Bounded Depth Decision Trees with its highly complex encoding that captures intricate decision paths. This selection allows us to systematically explore the capabilities and limitations of our approach as the underlying SAT encodings become more intricate.

Graph Coloring (Coloring) Coloring is one of Karp's original 21 NP-complete problems [Karp, 1972]. Given a graph G=(V,E) and a positive integer k, we need to assign each vertex one of k different colors such that adjacent vertices are not monochromatic. The classic encoding scheme is comparatively simple [Gelder, 2008], checking whether G permits a k-coloring requires $O(|V| \cdot k)$ many clauses.

Directed Feedback Vertex Set (DFVS) Given a directed graph G=(V,A) and a non-negative integer k, the goal is to find a set of vertices with cardinality at-most k, such that after removing these vertices, the directed graph is acyclic. There are different ways to encode this into SAT [Janota et al., 2017]. We use an encoding scheme that encodes reachability, which requires $O(|V|^3)$ many clauses. Further, additional complexity is introduced by the cardinality constraint used to limit the number of removed vertices.

Bounded Depth Decision Trees (BDDT) Given a labeled dataset with numerical features and a non-negative integer k, the goal is to find a decision tree with depth at-most k that correctly predicts the label (or class) for the whole dataset. We use the encoding scheme by Shati *et al.* [[2021]], which is fairly complicated as it has to encode, among other things, every possible path through a tree of depth k.

Next, we discuss our approach for generating these local search functions.

4 Methodology

In this section, we describe our approach that is sketched in Figure 1. The process consists of two phases, a *Gathering Phase* and a *Refinement Phase*. We will describe each part of the process, starting with the input, the encoding scheme.

4.1 Standardizing Encoding Schemes

There are many ways to implement encoding schemes. Since we are defining a general approach for generating local search methods, we need a standardized way of representing encoding schemes. We express our encoding schemes in Python using the popular framework PySAT [Ignatiev *et al.*,

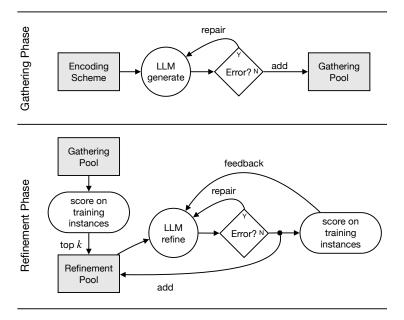


Figure 1: Schematic of the whole approach from the encoding scheme to pool of refined local search functions. From this pool we take the top-performing functions for the evaluation on test instances.

2018]. With PySAT, one can express the encoding scheme in terms of understandable—both by a human and an LLM—programming constructs. Further, PySAT abstracts away cardinality constraints behind a single function.

4.2 Gathering Phase

We provide the LLM with the PySAT encoding scheme, without stating the problem it encodes or any information on the instances we will test the local searches on. The LLM is then requested to return a local search function that meets the following specifications:¹

- Performs local search for the encoding.
- Has a specific function name and takes as input the original instance, the encoded instance as PySAT objects, and a timeout.
- Returns a (partial) variable assignment where a Boolean value can be accessed using the variable identifier.
- Must return within the specified timeout.
- Is encouraged to use the input instance and the structure of the encoding,
- · as well as novel approaches.

Whenever we receive a local search function, we verify its correctness ("Error?" in Figure 1). We verify that the local search function can be executed, finishes within the timeout, and returns a (partial) assignment by running the local search function on an easy instance for half a minute. Whenever an error is detected, the LLM is provided the error and, when applicable, the line the

¹The prompts are given in the supplementary material.

error occurred in. The LLM is then asked to repair the issue. This repair cycle is repeated, if necessary, until either the code passes the verification, or a defined limit is reached.

Every local search function we find is added to the prompt's context, such that the LLM does not repeat it or similar functions. We use an increasing temperature to encourage increasingly creative ideas.

4.3 Scoring

Since we aim to generate a wide variety of local search functions, we require a method to automatically compare their performance. Given a local search function, we run it on a set of training instances ("score on training instances" in Figure 1). For each instance we let the local search find an assignment and pass it to a SAT solver that tries to solve the instance based on the provided assignment. This allows us to rank a set of local search functions. All local search functions that caused a runtime error are ranked last and we use the following criteria for the remaining functions, lower values are better.

- The number of instances where the local search function did not return an assignment within the timeout.
- 2. The number of instances with a SAT solver timeout.
- 3. The average runtime over successful SAT solver calls.

4.4 Refinement Phase

In this phase, we focus on improving the existing local search functions found in the previous phase. We pick the top searches from the gathering phase and process them one by one. Therefore, refinement runs are independent of each other. We start from the original request and search function as context and ask the LLM to vary the function, while not changing the overall idea. Each version is scored as described above and the LLM receives feedback about the function's relative performance to the previous version, where a SAT runtime change is only considered significant if it the average runtime changed by more than 10%. Depending on the function's performance being better or worse, the LLM is requested to either continue with similar refinements, or revert and try a different approach. In case there is no significant difference, the LLM is prompted to perform a bigger change.

After completing the refinement phase, we pick the top local search functions for our final Test Evaluation.

5 Experimental Evaluation

In this section, we evaluate the quality the generated local search functions. Further, we also explore how well the different parts of our approach contribute to this quality.²

5.1 Setup

We use the OpenAI models o1-mini-2024-09-12 and gpt-4o-2024-11-20, as well as Anthrophic claude-3-5-sonnet-20241022. We run gathering and refinement on a MacBook M1 and the Test Evaluation on servers with two AMD EPYC 7402 CPUs having 24 cores running at 2.80GHz. Each run has a memory limit of 128 GB. We use Cadical 1.9.5 as the SAT solver and PySAT 1.8.dev13.

²Code and Results are in the Supplementary Materials.

We consider one separate set of benchmarks for each of our three encoding schemes. Since we encode optimization problems, we require an upper bound for the encoding, e.g., we need some good k to encode the decision problem, if a graph allows a k-coloring. We find an upper bound differently for each problem and describe the procedures subsequently. We split the benchmarks for each problem instances into two sets. *training instances* are instances solved by the SAT solver alone in between 10 and 60 seconds and are used to quickly determine the quality of local search functions. *test instances* are the instances that take longer than one minute to solve.

Graph Coloring We consider 257 graphs from TreewidthLib³ and DIMACS⁴. For each instance, we establish an upper bound using DSATUR [Brélaz, 1979] and then improve this bound for ten hours using the SAT encoding. Further, whenever we know a better upper bound from literature [Sun *et al.*, 2021], we decrease the upper bound by one to obtain a hard instance. This results in 10 training and 38 testing instances.

DFVS We follow the evaluation in [Kiesel and Schidler, 2023] and use 513 graphs from PACE 2022 [Großmann *et al.*, 2022], ICCMA⁵, and random graphs according to Zhou [[2016]]. We establish the optimal solution using DAGer [Kiesel and Schidler, 2022; Kiesel and Schidler, 2023]. We discard any instance not solvable by DAGer. This results in 18 training instances and 124 test instances.

BDDT We consider 69 datasets used in related work on optimal decision trees [Bessiere *et al.*, 2009; Olson *et al.*, 2017; Narodytska *et al.*, 2018; Verwer and Zhang, 2019; Avellaneda, 2020; Schidler and Szeider, 2024]. We run the SAT encoding on each instance for ten hours, starting with an initial bound of 10, as the encoding becomes too large with higher bounds. We decrease the bound with every solution we find. The training set consists of 9 instances and and the test set contains 32 instances.

We evaluate a local search function on the training instances as described in Section 4.3. We run the local search function with a timeout of 60 seconds but only count it is a violation of the timeout if no assignment is returned within 120 seconds. The one minute timeout is used, as Python is quite slow and shorter timeouts would make it harder to distinguish the performance of the local search functions. The SAT solver is run for at most 120 seconds. Since we know that the training instances are solvable within one minute, 120 seconds allows for enough variance in the runtime.

5.2 Comprehension

The LLMs do not receive any information on the purpose of the PySAT encoding schemes. Hence, it is an interesting question if they understand the encodings. We evaluate this by asking the three LLM models to explain the PySAT encoding schemes. All three models perform similarly on this task. They correctly identify the Graph Coloring and BDDT encoding scheme in detail, including the concept of the encoding scheme and the semantics of the schemes's variables. All three models can explain the parts of the DFVS encoding scheme. However, they fail to identify its overall purpose.

5.3 Gathering Phase

The LLM models generate the local search functions with varying speed during the Gathering Phase. The difference in performance is also observable in the Refinement Phase, where it matters less as the evaluation on the training instances takes much longer than the LLM queries.

³The graphs were kindly provided by Fichte *et al.* [[2017]].

⁴https://sites.cc.gatech.edu/dimacs10/

⁵https://argumentationcompetition.org/2021/

The speed of finding the local search functions is mainly determined by two factors: (i) how fast the LLM model can come up with new searches despite large contexts, and, (ii) how many repair cycles the LLM model requires to fix code issues, and how long these cycles take (which relates to (i)). Generating one local search function can take between a second and several minutes, mostly depending on how many local search functions we already generated—and are, therefore, in the context—and how well the LLM model performs regarding (i). We observe that GPT o1-mini and Claude Sonnet do not slow down much with larger contexts, while the speed of GPT 4o significantly decreases with larger contexts.

The initial version returned by the LLMs is rarely without issues and repair cycles are required for all LLM models. However, Claude Sonnet and GPT 40 often require several iterations and may not be able to fix their code within ten tries, while GPT o1-mini is able to fix issues in fewer iterations. Consequently, GPT o1-mini can generate the 50 searches in the Gathering Phase the fastest, Claude Sonnet is fast but requires more tokens, and GPT 40 takes the longest time to complete the Gathering Phase.

5.4 Refinement Phase

We pick the top five local search functions from each LLM model for the Refinement Phase. This results in 15 *Base* local search versions for each problem. We refine each of these local searches 19 times, to obtain a total of 20 versions per local search, which results in 100 local searches per LLM model. After the first ten refinements (versions 2–11), we re-encourage the LLM to use the structure of the encoding (versions 12–20). We refer to versions 2–11 as *Refined* versions and version 12–20 as *Structure* versions.

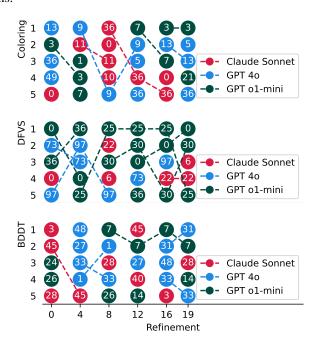


Figure 2: The top five local search functions for different problems over different refinement iterations.

Figure 2 shows the relative ranking of the different local search functions at different points in the refinement phase. This phase indeed achieves significant changes over time, with the ranking constantly changing. A closer look reveals that the results can indeed change significantly in the

course of a single refinement. The LLMs also rarely fail to revert a worsening refinement and are, therefore, able to incorporate our feedback from the training runs.

5.5 Test Instance Performance

In this evaluation, the base local search functions and their refinements are run on the test instances. Since it is infeasible to evaluate all refinements of all local search functions, we focus on the Base version, the best Refined version, and the best Structure version. The best version is determined by the performance on the training instances. We run the SAT solver without any local search on all instances for one hour as a reference. We evaluate a local search function on a test instance by running it with a timeout of 15 minutes and then run the SAT solver using the local search's assignment for up to one hour. We use a timeout of 15 minutes and do not decrease the SAT solver timelimit, as we expect a good native implementation, e.g., in C++, would run orders of magnitudes faster than the Python prototypes. The results of the best local search function from each LLM model are in Table 1.6

We are interested in two metrics: (i) how many instances are solved by the SAT solver within the timeout when using a local search function, and (ii) how many instances can be solved within the timeout using the local search function that the SAT solver alone could not solve. The first metric establishes a general usefulness of the local search function, while the second metric establishes whether the local search function is useful for hard instances.

	Base		Refined		Structure	
Method	Solved	New	Solved	New	Solved	New
Coloring (38 insta	ances)					
SAT	8	-	-	-	-	_
Claude Sonnet 36	5 4	0	5	0	8	3
GPT 4o 49	6	1	7	0	5	0
GPT o1-mini 7	3	0	12	7	3	0
DFVS (124 instar	nces)					
SAT	61	-	-	-	-	_
Claude Sonnet 6	38	5	46	6	47	6
GPT 4o 73	46	6	44	6	50	8
GPT o1-mini 30	60	10	61	12	58	11
BDDT (32 instan	ces)					
SAT	16	-	-	-	_	_
Claude Sonnet 3	14	0	7	0	8	1
GPT 4o 31	10	1	7	0	8	0
GPT o1-mini 7	10	1	13	1	13	1

Table 1: Results for the best local search functions from each LLM model. *Solved* indicates how many instances the method solved and *New* how many of those the SAT solver alone could not solve. *Base* is the initial version found in the Gathering Phase, *Refined* is the best version in the first 10 refinements, and *Structure* is the best version in the last 9 refinements.

The results differ for each problem. It is hard to find hard but solvable instances for graph coloring. Most instances are either very easy or very hard. Hence, most of our instances stay unsolved by any method. Overall, several instances that can be solved without a local search function

⁶The results for all 15 Base versions are in the supplementary material.

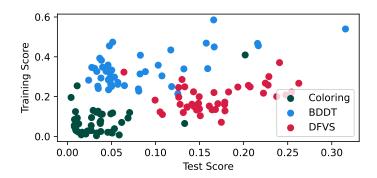


Figure 3: Local search function scores on the training instances and test instances. Each marker represents one local search function.

are not solved with it. This indicates that the local search's assignment is not close enough to a satisfying solution. This does not necessarily imply that the hybrid approach performs worse, as the SAT solver alone can also find satisfying assignments by chance. Hence, local search functions should not be used on instances that do not require them. However, the best local search function achieves more solved instances than the SAT solver alone, showing that the hybrid approach is an improvement. This local search function would not have been found without the refinement phase, showing that gathering functions alone is not sufficient for good results. Overall, already the prototypes created by the LLM look encouraging for hard instances, we expect an efficient implementation to achieve even better results.

The local searches for DFVS achieve overall relatively better results than the local searches for graph coloring. There is also a clear difference in performance between the different LLM models, with GPT o1-mini finding the best-performing local search functions, while Claude Sonnet struggles to find good local search functions. While the average over all local search functions looks better for DFVS than for graph coloring, the best function can only help solve as many instances as the SAT solver alone. As with graph coloring, several instances go unsolved whenever local search used, although the SAT solver alone can solve them. The results show that the best local search function can aid the SAT solver on several hard instances. As in the case of graph coloring, refinement is necessary to find the best performing local search.

BDDT is the problem the LLMs struggled most with. No matter which local search function is used, the SAT solver alone can solve more instances. It is not known how many additional instances can reasonably be expected to be solvable by a SAT solver. Hence, the local search functions might perform much better in case the test instances contained more hard but solvable instances. One of the best local search functions can aid the SAT solver to solve instance *objectivity* in 2483 seconds, which was reported by Schidler and Szeider [[2024]] as not solvable within six hours. This raises our confidence in the quality of the local search functions.

5.6 Discussion

Our results show that it is indeed possible to automatically generate and evaluate local search prototypes using LLMs. In the last part of our evaluation we want to discuss aspects of our approach that are of interest for adapting our approach to other problems.

5.6.1 Test and Training Correlation

The automatic evaluation relies on some automated way of ranking the different local search functions. Since large scale tests for each of the many generated functions are infeasible, we rely on

the training instances as an approximation of performance on the interesting test instances. An important question is, how well does the test performance approximate the training performance? We explore this using a score that is based on the time it takes the SAT solver to solve the instance with the assignment provided by the local search. The score for a local search function on a specific instance is the fastest time over all local search functions divided by the time for the scored local search function. Hence, the faster the result, the closer to one the score, where timeouts are counted as score 0. Figure 3 reports the average score over all instances.

The correlation is strongest for DFVS, becomes weaker for BDDT, and is almost non-present for coloring. DFVS has the most training instances and the training instances come from several different sources, giving the training set diversity. In contrast, coloring instances tend to be either easy or hard and few instances fit into the 10 to 60 second range. This leads to many test instance from the same source and low diversity. BDDT also has fewer test instance than DFVS, but the datasets are very different from each other.

The selection of training instances is, therefore, very important for finding the best performing local search functions. Nonetheless, the coloring results show that our approach can still pick out very good local search functions, which is also indicated by the very high scoring marks in Figure 3.

5.6.2 Code Diversity

We manually reviewed the code generated by the LLMs in an effort to judge how much the code varies, as well as the overall quality of the code. Due to the large number of generated local search functions—over 900—we cannot review all the code in detail. Hence, we focus on trends within the code.

The LLMs manage to create prototypes and these prototypes are rarely performance-optimized. An example is the scoring function present in all of the local search functions we reviewed. The scoring function determines how good an assignment is and is often implemented by iterating over all clauses and checking if they are satisfied. The scoring function is then called whenever the assignment changes, or even for each considered change to the assignment. In case of methods like WalkSAT that flip one variable per iteration, this is a major performance bottleneck. We address this specific issue via prompt and the LLMs can often fix it, but performance bottlenecks that are specific to single local search functions cannot be addressed via generic prompt. Hence, we expect that the local search functions can be improved even more with a performance-oriented implementation. Further, potential issues should be assessed before the Gathering Phase by generating a small set of initial local search functions and manually reviewing them. These issues can then be addressed directly, as with the scoring function bottleneck.

The LLMs usually provide their implementation goal in the comments. According to these comments, the local search functions implement over 50 different meta-heuristics, like

genetic algorithms, tabu search, simulated annealing, harmony search, ant colony optimization, agent-based optimization, cellular automaton based optimization, swarm optimization, great deluge, firefly, bee colony optimization, multi-agent-based optimization, market-based optimization, quantum-based binary optimization.

Further, every LLM model also tries WalkSAT in some iteration for every problem. Interestingly, Claude Sonnet often tries to combine different meta-heuristics, while the GPT models try to implement a single approach. This single approach closely resembles pseudocode in local search functions generated by GPT 40, while those generated by GPT 01-mini have clear adaptations due to our prompts. These adaptations are also observable in the Claude Sonnet generated functions. This great variety is encouraging for sampling promising prototypes. Unfortunately, the comments often don't match the semantics of the code, making a manual review necessary to verify what the code actually does.

Issues are usually filtered away due to bad performance on the training set. Surprisingly, even wrong implementations can sometimes deliver good results. Hence, we found several such issues in even the top-performing local search functions. Consequently, there is room for even better performance when efficiently implementing the prototypes, but a proper analysis of the code is necessary beforehand.

5.6.3 Analysis of Generated Search Strategies

In this last part, we analyze the code of the best local search functions and report the most interesting ideas. For a variable flip, we define its *conflict score* as S-U, where S is the number of unsatisfied clauses that become satisfied and U is the number of satisfied clauses that become unsatisfied. Higher scores indicate better flips.

Graph Coloring The graph coloring encoding is comparatively simple. Consequently, the LLMs often generate local search functions for graph coloring that run on the graph, and only when returning, convert the best coloring into a variable assignment. This works well and the best function we found works with this principle. The best local search function that uses the SAT encoding implements tabu search and in each iteration performs the best—according to the conflict score—variable flip among 20 random variables. Whenever the improvements stagnate, the search uses the 20 random variables whose flips lead to an improvement most often. The initial assignment is created by assigning colors to nodes in order if decreasing node degree, choosing the color that causes the fewest monochromatic edges.

DFVS The DFVS PySAT encoding scheme is an interesting case. As described in Section 5.2, this scheme is the only one that the LLMs do not fully comprehend. This leads to strange effects, where several local search functions are able to extract the upper bound d from the encoding, but misunderstand the polarity of the variables inside the cardinality constraint. This leads to the local search function seemingly excluding d many nodes from the graph, while actually excluding all but d many nodes.

The best local search function is a straightforward WalkSAT implementation. Other good local search functions use greedy heuristics based on node degree or node centrality to pick the best variable flips. While there are many well-performing local search functions, there is a lack of problem specific functionality, and where it is present, it is wrong, as with the polarity issue stated above. Overall, the lesson-learned from this PySAT case, is the importance of the LLM model's understanding of the encoding scheme.

BDDT The best local search functions for BDDT are highly adapted to the PySAT encoding scheme. The best function for BDDT is generated by Claude Sonnet. It correctly identifies the variables that must be assigned, and ignores those variables that are implied by unit propagation. The remaining variables are separated in levels, corresponding to the depth of the respective node in the tree. During initialization, the function assigns each node a random feature and picks a threshold from the middle of the dataset. This is reflected in the assignment such that all corresponding constraints are satisfied. The search is performed level by level. Whenever the search on one level stagnates, it moves to the next level, cycling back to the root level if necessary. The search itself is performed by first picking a random unsatisfied clause that contains a variable from the current level and then picks the variable with the highest conflict score from the clause .

Another local search method separates the variables in different layers, based on the features they encode, thresholds, or classes. The search then adapts the assignment layer by layer, fixing the assignment of variables that occur together with the most variables outside the layer first. This process is repeated and whenever search stagnates, the order of layers is changed.

Overall, the quality and specificity of the local search functions for BDDT shows how well the LLMs can adapt the code to even complicated encoding schemes, as long as the LLM model understands them.

6 Conclusion

We demonstrated that it is possible to automatically generate effective local search algorithms by having LLMs analyze SAT encoding schemes. Our key innovation lies in targeting the encoding methodology rather than specific problem instances, allowing the generated strategies to work across all problems sharing the same encoding pattern. This scheme-centric approach produced diverse search strategies whose performance correlated with the LLMs' comprehension of the underlying encoding structures.

While we focused on LLM's, exploring distilled variants could make the generation process more computationally feasible. Our diversity measures in the exploration phase could also be enhanced by leveraging model embeddings to quantify the distinctness of generated strategies better. Finally, while computationally more demanding, the next generation of reasoning models could enable deeper encoding comprehension and more sophisticated search strategies, further expanding the possibilities of automated algorithm generation.

References

- [Avellaneda, 2020] Florent Avellaneda. Efficient inference of optimal decision trees. In *Proceedings of AAAI 2020*. AAAI Press, 2020.
- [Balint and Manthey, 2013] Adrian Balint and Norbert Manthey. Boosting the performance of SLS and CDCL solvers by preprocessor tuning. In *POS@SAT*, volume 29 of *EPiC Series in Computing*, pages 1–14. EasyChair, 2013. DOI: https://doi.org/10.29007/28WW
- [Balint and Schöning, 2012] Adrian Balint and Uwe Schöning. Choosing probability distributions for stochastic local search and the role of make versus break. In *Theory and Applications of Satisfiability Testing SAT 2012 15th International Conference, Trento, Italy, June 17-20, 2012. Proceedings*, volume 7317 of *Lecture Notes in Computer Science*, pages 16–29. Springer, 2012. DOI: https://doi.org/10.1007/978-3-642-31612-8_3
- [Bessiere *et al.*, 2009] Christian Bessiere, Emmanuel Hebrard, and Barry O'Sullivan. Minimising decision tree size as combinatorial optimisation. In *Proceedings of CP 2009*, pages 173–187, Berlin, Heidelberg, 2009. Springer Berlin Heidelberg.
- [Biere et al., 2020] Armin Biere, Katalin Fazekas, Mathias Fleury, and Maximilian Heisinger. CaDiCaL, Kissat, Paracooba, Plingeling, and Treengeling entering the SAT competition 2020. In Proceedings of SAT Competition 2020: Solver and Benchmark Descriptions, pages 51–53, 2020.
- [Biere et al., 2024] Armin Biere, Tobias Faller, Katalin Fazekas, Mathias Fleury, Nils Froleyks, and Florian Pollitt. Cadical 2.0. In Computer Aided Verification 36th International Conference, CAV 2024, Montreal, QC, Canada, July 24-27, 2024, Proceedings, Part I, volume 14681 of Lecture Notes in Computer Science, pages 133–152. Springer, 2024. DOI: https://doi.org/10.1007/978-3-031-65627-9_7
- [Biere, 2014] Armin Biere. Yet another local search solver and Lingeling and friends entering the SAT Competition 2014. In *Proc. of SAT Competition 2014 Solver and Benchmark Descriptions*, volume B-2014-2 of *Department of Computer Science Series of Publications B*, pages 39–40. University of Helsinki, 2014.

- [Biere, 2019] Armin Biere. CaDiCaL at the SAT race 2019. In *Proceedings of SAT Race 2019: Solver and Benchmark Descriptions*, pages 8–9, 2019.
- [Brélaz, 1979] Daniel Brélaz. New methods to color the vertices of a graph. *Commun. ACM*, 22(4):251–256, apr 1979. URL: https://dl.acm.org/doi/10.1145/359094.359101
- [Cai and Zhang, 2021] Shaowei Cai and Xindi Zhang. Deep cooperation of CDCL and local search for SAT. In Theory and Applications of Satisfiability Testing - SAT 2021 - 24th International Conference, Barcelona, Spain, July 5-9, 2021, Proceedings, volume 12831 of Lecture Notes in Computer Science, pages 64–81. Springer, 2021. DOI: https://doi.org/10.1007/978-3-030-80223-3_6
- [Cai *et al.*, 2022] Shaowei Cai, Xindi Zhang, Mathias Fleury, and Armin Biere. Better decision heuristics in CDCL through local search and target phases. *J. Artif. Intell. Res.*, 74:1515–1563, 2022. DOI: https://doi.org/10.1613/JAIR.1.13666
- [Cummins *et al.*, 2023] Chris Cummins, Volker Seeker, Dejan Grubisic, Mostafa Elhoushi, Youwei Liang, Baptiste Rozière, Jonas Gehring, Fabian Gloeckle, Kim M. Hazelwood, Gabriel Synnaeve, and Hugh Leather. Large language models for compiler optimization. *CoRR*, abs/2309.07062, 2023. DOI: https://doi.org/10.48550/ARXIV.2309.07062
- [Fichte et al., 2017] Johannes Klaus Fichte, Neha Lodha, and Stefan Szeider. SAT-based local improvement for finding tree decompositions of small width. In *Theory and Applications of Satisfiability Testing SAT 2017 20th International Conference, Melbourne, VIC, Australia, August 28 September 1, 2017, Proceedings*, volume 10491 of *Lecture Notes in Computer Science*, pages 401–411. Springer, 2017. DOI: https://doi.org/10.1007/978-3-319-66263-3_25
- [Fichte *et al.*, 2023] Johannes Klaus Fichte, Daniel Le Berre, Markus Hecher, and Stefan Szeider. The silent (r)evolution of SAT. *Commun. ACM*, 66(6):64–72, 2023. DOI: https://doi.org/10.1145/3560469
- [Gelder, 2008] Allen Van Gelder. Another look at graph coloring via propositional satisfiability. *Discret. Appl. Math.*, 156(2):230–243, 2008. DOI: https://doi.org/10.1016/J.DAM.2006.07.016
- [Großmann *et al.*, 2022] Ernestine Großmann, Tobias Heuer, Christian Schulz, and Darren Strash. The PACE 2022 parameterized algorithms and computational experiments challenge: Directed feedback vertex set. In *Proceedings of IPEC 2022*, volume 249 of *LIPIcs*, pages 26:1–26:18. Schloss Dagstuhl Leibniz-Zentrum für Informatik, 2022. DOI: https://doi.org/10.4230/LIPICS. IPEC.2022.26
- [Hoos, 2002] Holger H. Hoos. An adaptive noise mechanism for walksat. In *Proceedings of the Eighteenth National Conference on Artificial Intelligence and Fourteenth Conference on Innovative Applications of Artificial Intelligence, July 28 August 1, 2002, Edmonton, Alberta, Canada, pages 655–660.* AAAI Press / The MIT Press, 2002. URL: http://www.aaai.org/Library/AAAI/2002/aaai02-098.php
- [Ignatiev *et al.*, 2018] Alexey Ignatiev, Antonio Morgado, and Joao Marques-Silva. PySAT: A Python toolkit for prototyping with SAT oracles. In *SAT*, pages 428–437, 2018. DOI: https://doi.org/10.1007/978-3-319-94144-8_26
- [Ignatiev et al., 2024] Alexey Ignatiev, Zi Li Tan, and Christos Karamanos. Towards universally accessible SAT technology. In 27th International Conference on Theory and Applications of Satisfiability Testing, SAT 2024, August 21-24, 2024, Pune, India, volume 305 of LIPIcs, pages 16:1–16:11. Schloss Dagstuhl Leibniz-Zentrum für Informatik, 2024. DOI: https://doi.org/10.4230/LIPICS.SAT.2024.16

- [Janota et al., 2017] Mikolás Janota, Radu Grigore, and Vasco Manquinho. On the quest for an acyclic graph. In *Proceedings of the 24th RCRA International Workshop on Experimental Evaluation of Algorithms for Solving Problems with Combinatorial Explosion 2017, Bari, Italy, November 14-15, 2017*, volume 2011 of *CEUR Workshop Proceedings*, pages 33–44. CEUR-WS.org, 2017. URL: https://ceur-ws.org/Vol-2011/paper4.pdf
- [Jiang *et al.*, 2024] Juyong Jiang, Fan Wang, Jiasi Shen, Sungju Kim, and Sunghun Kim. A survey on large language models for code generation. *CoRR*, abs/2406.00515, 2024. DOI: https://doi.org/10.48550/ARXIV.2406.00515
- [Karp, 1972] Richard M. Karp. Reducibility among combinatorial problems. In *Complexity of Computer Computations* 1972, The IBM Research Symposia Series, pages 85–103. Plenum Press, New York, 1972. DOI: https://doi.org/10.1007/978-1-4684-2001-2-9
- [Kiesel and Schidler, 2022] Rafael Kiesel and André Schidler. PACE solver description: Dager cutting out cycles with maxsat. In *Proceedings of IPEC 2022*, volume 249 of *LIPIcs*, pages 32:1–32:4. Schloss Dagstuhl Leibniz-Zentrum für Informatik, 2022. DOI: https://doi.org/10.4230/LIPICS.IPEC.2022.32
- [Kiesel and Schidler, 2023] Rafael Kiesel and André Schidler. A dynamic MaxSAT-based approach to directed feedback vertex sets. In *Proceedings of the Symposium on Algorithm Engineering and Experiments, ALENEX 2023, Florence, Italy, January 22-23, 2023*, pages 39–52. SIAM, 2023. DOI: https://doi.org/10.1137/1.9781611977561.CH4
- [Li and Li, 2012] Chu Min Li and Yu Li. Satisfying versus falsifying in local search for satisfiability (poster presentation). In *Theory and Applications of Satisfiability Testing SAT 2012 15th International Conference, Trento, Italy, June 17-20, 2012. Proceedings*, volume 7317 of *Lecture Notes in Computer Science*, pages 477–478. Springer, 2012. DOI: https://doi.org/10.1007/978-3-642-31612-8_43
- [Li et al., 2022] Yujia Li, David H. Choi, Junyoung Chung, Nate Kushman, Julian Schrittwieser, Rémi Leblond, Tom Eccles, James Keeling, Felix Gimeno, Agustin Dal Lago, Thomas Hubert, Peter Choy, Cyprien de Masson d'Autume, Igor Babuschkin, Xinyun Chen, Po-Sen Huang, Johannes Welbl, Sven Gowal, Alexey Cherepanov, James Molloy, Daniel J. Mankowitz, Esme Sutherland Robson, Pushmeet Kohli, Nando de Freitas, Koray Kavukcuoglu, and Oriol Vinyals. Competition-level code generation with alphacode. CoRR, abs/2203.07814, 2022. DOI: https://doi.org/10.48550/ARXIV.2203.07814
- [Marques-Silva et al., 2021] João Marques-Silva, Inês Lynce, and Sharad Malik. Conflict-driven clause learning SAT solvers. In *Handbook of Satisfiability Second Edition*, volume 336 of *Frontiers in Artificial Intelligence and Applications*, pages 133–182. IOS Press, 2021. DOI: https://doi.org/10.3233/FAIA200987
- [McAllester et al., 1997] David A. McAllester, Bart Selman, and Henry A. Kautz. Evidence for invariants in local search. In Proceedings of the Fourteenth National Conference on Artificial Intelligence and Ninth Innovative Applications of Artificial Intelligence Conference, AAAI 97, IAAI 97, July 27-31, 1997, Providence, Rhode Island, USA, pages 321–326. AAAI Press / The MIT Press, 1997. URL: http://www.aaai.org/Library/AAAI/1997/aaai97-050.php
- [Narodytska et al., 2018] Nina Narodytska, Alexey Ignatiev, Filipe Pereira, and Joao Marques-Silva. Learning optimal decision trees with SAT. In *Proceedings of IJCAI 2018*, pages 1362–1368. ijcai.org, 7 2018. DOI: https://doi.org/10.24963/ijcai.2018/189
- [Nijkamp *et al.*, 2023] Erik Nijkamp, Bo Pang, Hiroaki Hayashi, Lifu Tu, Huan Wang, Yingbo Zhou, Silvio Savarese, and Caiming Xiong. Codegen: An open large language model for code with multi-turn program synthesis. In *ICLR*. OpenReview.net, 2023.

- [Olson *et al.*, 2017] Randal S. Olson, William La Cava, Patryk Orzechowski, Ryan J. Urbanowicz, and Jason H. Moore. PMLB: a large benchmark suite for machine learning evaluation and comparison. *BioData Mining*, 10(1):36, Dec 2017. DOI: https://doi.org/10.1186/s13040-017-0154-4
- [Schidler and Szeider, 2024] André Schidler and Stefan Szeider. SAT-based decision tree learning for large data sets. *J. Artif. Intell. Res.*, 80:875–918, 2024. DOI: https://doi.org/10.1613/JAIR.1. 15956
- [Schidler and Szeider, 2025] André Schidler and Stefan Szeider. Extracting problem structure with LLMs for optimized SAT local search, January 2025. DOI: https://doi.org/10.5281/zenodo. 14732109
- [Selman et al., 1992] Bart Selman, Hector J. Levesque, and David G. Mitchell. A new method for solving hard satisfiability problems. In *Proceedings of the 10th National Conference on Artificial Intelligence, San Jose, CA, USA, July 12-16, 1992*, pages 440–446. AAAI Press / The MIT Press, 1992. URL: http://www.aaai.org/Library/AAAI/1992/aaai92-068.php
- [Selman et al., 1994] Bart Selman, Henry A. Kautz, and Bram Cohen. Noise strategies for improving local search. In *Proceedings of the 12th National Conference on Artificial Intelligence, Seattle, WA, USA, July 31 August 4, 1994, Volume 1*, pages 337–343. AAAI Press / The MIT Press, 1994. URL: http://www.aaai.org/Library/AAAI/1994/aaai94-051.php
- [Selsam and Bjørner, 2019] Daniel Selsam and Nikolaj S. Bjørner. Guiding high-performance SAT solvers with unsat-core predictions. In Mikolas Janota and Inês Lynce, editors, *Theory and Applications of Satisfiability Testing SAT 2019 22nd International Conference, SAT 2019, Lisbon, Portugal, July 9-12, 2019, Proceedings*, volume 11628 of *Lecture Notes in Computer Science*, pages 336–353. Springer, 2019. DOI: https://doi.org/10.1007/978-3-030-24258-9_24
- [Shati *et al.*, 2021] Pouya Shati, Eldan Cohen, and Sheila A. McIlraith. SAT-based approach for learning optimal decision trees with non-binary features. In *CP*, volume 210 of *LIPIcs*, pages 50:1–50:16. Schloss Dagstuhl Leibniz-Zentrum für Informatik, 2021. DOI: https://doi.org/10. 4230/LIPICS.CP.2021.50
- [Silva and Sakallah, 1996] João P. Marques Silva and Karem A. Sakallah. GRASP a new search algorithm for satisfiability. In Proceedings of the 1996 IEEE/ACM International Conference on Computer-Aided Design, ICCAD 1996, San Jose, CA, USA, November 10-14, 1996, pages 220–227. IEEE Computer Society / ACM, 1996. DOI: https://doi.org/10.1109/ICCAD.1996.569607
- [Soos, 2020] Mate Soos. Cryptominisat 5.6.8. In *Proceedings of SAT Competition 2020: Solver and Benchmark Descriptions*, pages 37–38, 2020.
- [Sun et al., 2021] Wen Sun, Jin-Kao Hao, Yuhao Zang, and Xiangjing Lai. A solution-driven multilevel approach for graph coloring. *Appl. Soft Comput.*, 104:107174, 2021. DOI: https://doi.org/10.1016/J.ASOC.2021.107174
- [Szeider, 2024] Stefan Szeider. MCP-solver: Integrating language models with constraint programming systems. *CoRR*, abs/2501.00539, 2024. DOI: https://doi.org/10.48550/ARXIV.2501.00539
- [Verwer and Zhang, 2019] Sicco Verwer and Yingqian Zhang. Learning optimal classification trees using a binary linear program formulation. In *Proceedings of AAAI 2019*, pages 1625–1632. AAAI Press, 2019. DOI: https://doi.org/10.1609/aaai.v33i01.33011624
- [Voboril *et al.*, 2024] Florentina Voboril, Vaidyanathan Peruvemba Ramaswamy, and Stefan Szeider. Generating streamlining constraints with large language models. *CoRR*, abs/2408.10268, 2024. DOI: https://doi.org/10.48550/ARXIV.2408.10268

[Voboril et al., 2025] Florentina Voboril, Vaidyanathan Peruvemba Ramaswamy, and Stefan Szeider. Realtime generation of streamliners with large language models. NSE 2025, the First International Workshop on Neuro-Symbolic Software Engineering (May 3, 2025), affiliated with ICSE 2025, the IEEE/ACM International Conference on Software Engineering, 2025.

[Yolcu and Póczos, 2019] Emre Yolcu and Barnabás Póczos. Learning local search heuristics for boolean satisfiability. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada, pages 7990–8001, 2019. URL: https://proceedings.neurips.cc/paper/2019/hash/12e59a33dea1bf0630f46edfe13d6ea2-Abstract.html

[Zhou, 2016] Hai-Jun Zhou. A spin glass approach to the directed feedback vertex set problem. Journal of Statistical Mechanics: Theory and Experiment, 2016(7):073303, 2016. URL: https://doi.org/10.1088/1742-5468/2016/07/073303