

Artin Algebra

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1 Introduction

1.1 Theorem

Theorem

this is

1.2 Corollary

Corollary

1.3 Proposition

Proposition

1.4 Theorem

Lemma

1.5 Box

1.6 Definition

Definition

2 matrix

2.1 matrix multiplication

Definition

$M(T)$:matrix of a linear map;
notation: $M(T, (v_1, \dots, v_n), (w_1, \dots, w_m))$

$$Tv_k = A_{1,k}w_1 + \dots + A_{m,k}w_m$$

$$A_{m,n} = \begin{pmatrix} A_{1,1} & A_{1,2} & \dots & A_{1,n} \\ A_{2,1} & A_{2,2} & \dots & A_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m,1} & A_{m,2} & \dots & A_{m,n} \end{pmatrix}$$

for $A = M(T, (u_1, \dots, u_p), (v_1, \dots, v_n))$, $C = M(S, (v_1, \dots, v_n), (w_1, \dots, w_m))$, thus A is a n by p matrix, and C is a m by n matrix. We can derive the form of Matrix multiplication via mapping as followed:

$$(CA)u_k = C(Au_k) \quad (1)$$

$$= C(A_{1,k}v_1 + \dots + A_{n,k}v_n) \quad (2)$$

$$= A_{1,k}Cv_1 + \dots + A_{n,k}Cv_n \quad (3)$$

$$= \sum_{i=1}^n A_{i,k}Cv_i \quad (4)$$

$$= \sum_{i=1}^n A_{i,k} \sum_{j=1}^m C_{j,i}w_j \quad (5)$$

$$= \sum_{j=1}^m \sum_{i=1}^n A_{i,k}C_{j,i}w_j \quad (6)$$

then we get $(CA)_{j,k} = \sum_{i=1}^n C_{j,i}A_{i,k}$

2.2 upper triangle matrix

3 Groups

3.1 Groups and subgroups

3.1.1 group

Group

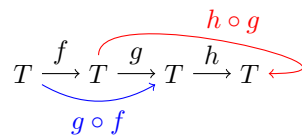
A group is a set together with a law of composition that has the following properties:

- $\forall a, b, c \in G, (ab)c = a(bc)$
- $\exists 1 \in G, \forall a \in G, \text{ s.t. } a1 = 1a = a$
- $\forall a \in G, \exists b \in G, \text{ s.t. } ab = ba = 1$

order of group it's the number of elements it contains.

$$|G| = \text{number of elements}$$

Here are some intuition for associative law:



Law of Cancellation

$a, b, c \in G$, if $ab = ac$ or $ba = ca$, then $b = c$
if $ab = b$ or $ba = b$, then $a = 1$

It needs to be mentioned that a must be invertible.

3.1.2 permutation

permutation

A permutation of a set S is a bijective map p from a set S to itself:

$$p : S \rightarrow S$$

for example, p is a permutation over set $\{1, 2, 3, 4\}$:

i	1	2	3	4	5
p(i)	3	5	4	1	2

Since $p(3) = 4, P(4) = 1, p(1) = 3$, we have $(3 \ 4 \ 1)$. Similarly, we have $(2 \ 5)$. Thus we can write P as $(2 \ 5)(3 \ 4 \ 1)$.

The product of 2 perturbations can be written as: $qp = [(1 \ 4 \ 5 \ 2)] \circ [(2 \ 5)(3 \ 4 \ 1)] = (3 \ 5 \ 1)$. (hint: $qp(3) = q(3) \circ p(3)$, for $p \ 3 \rightarrow 4$, for $q \ 4 \rightarrow 5$, thus for $pq, 3 \rightarrow 5$)

3.1.3 symmetric group

symmetric group

The group of permutations of the set of indices $\{1, 2, \dots, n\}$, denoted by S_n

Attention, a symmetric group is a group of mapping, working on a set of mapping.

In S_2 , one of elements $(2!)$ must be 1, then we call it $\{1, g\}$, then we have a table $gg, 1g \ g1 \ 11$ only gg is not defined. Assume $gg = g$, then according to law of cancellation, we have $g = 1$, it's forbidden, thus we only have $gg = 1$.

In S_3 , let $x = (1 \ 2 \ 3)$ and $y = (1 \ 2)$, then we have:

$$x^3 = 1 \quad y^2 = 1 \quad yx = x^2y \quad xy = yx^2 \quad (7)$$

thus we can eliminate some perturbations and get all of them in a distinctive way:

$$S_3 = \{1, x, x^2, y, xy, x^2y\} \quad (8)$$

This is an important example, since $xy \neq yx$, it's the smallest group whose law of composition is not commutative.

3.1.4 Subgroup

subgroup

A subset H of group G is a subgroup if it has the following properties:

- Closure: If a and b are in H , then ab are in H
- Identity: 1 is in H
- Inverse: if a is in H then a^{-1} is in H