

# Artin Algebra

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# 1 Introduction

## 1.1 Theorem

**Theorem**

this is

## 1.2 Corollary

**Corollary**

## 1.3 Proposition

**Proposition**

## 1.4 Theorem

**Lemma**

## 1.5 Box

## 1.6 Definition

**Definition**

## 2 Groups

### 2.1 Groups and subgroups

#### 2.1.1 group

##### Group

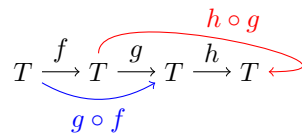
A group is a set together with a law of composition that has the following properties:

- $\forall a, b, c \in G, (ab)c = a(bc)$
- $\exists 1 \in G, \forall a \in G, \text{ s.t. } a1 = 1a = a$
- $\forall a \in G, \exists b \in G, \text{ s.t. } ab = ba = 1$

order of group it's the number of elements it contains.

$$|G| = \text{number of elements}$$

Here are some intuition for associative law:



##### Law of Cancellation

$a, b, c \in G$ , if  $ab = ac$  or  $ba = ca$ , then  $b = c$   
if  $ab = b$  or  $ba = b$ , then  $a = 1$

It needs to be mentioned that  $a$  must be invertible.

#### 2.1.2 permutation

##### permutation

A permutation of a set  $S$  is a bijective map  $p$  from a set  $S$  to itself:

$$p : S \rightarrow S$$

for example,  $p$  is a permutation over set  $\{1, 2, 3, 4\}$ :

i	1	2	3	4	5
$p(i)$	3	5	4	1	2

Since  $p(3) = 4, P(4) = 1, p(1) = 3$ , we have  $(3 \ 4 \ 1)$ . Similarly, we have  $(2 \ 5)$ . Thus we can write  $P$  as  $(2 \ 5)(3 \ 4 \ 1)$ .

The product of 2 perturbations can be written as:  $qp = [(1 \ 4 \ 5 \ 2)] \circ [(2 \ 5)(3 \ 4 \ 1)] = (3 \ 5 \ 1)$ . (hint:  $qp(3) = q(3) \circ p(3)$ , for  $p: 3 \rightarrow 4$ , for  $q: 4 \rightarrow 5$ , thus for  $pq, 3 \rightarrow 5$ )

### 2.1.3 symmetric group

#### symmetric group

The group of permutations of the set of indices  $\{1, 2, \dots, n\}$ , denoted by  $S_n$

Attention, a symmetric group is a group of mapping, working on a set of mapping.

In  $S_2$ , one of elements(2!) must be 1, then we call it  $\{1, g\}$ , then we have a table  $gg, 1g, g1, 11$  only  $gg$  is not defined. Assume  $gg = g$ , then according to law of cancellation, we have  $g = 1$ , it's forbidden, thus we only have  $gg = 1$ .

In  $S_3$ , let  $x = (1 \ 2 \ 3)$  and  $y = (1 \ 2)$ , then we have:

$$x^3 = 1 \quad y^2 = 1 \quad yx = x^2y \quad xy = yx^2 \quad (1)$$

thus we can eliminate some perturbations and get all of them in a distinctive way:

$$S_3 = \{1, x, x^2, y, xy, x^2y\} \quad (2)$$

This is an important example, since  $xy \neq yx$ , it's the smallest group whose law of composition is not commutative.

### 2.1.4 Subgroup

#### subgroup

A subset  $H$  of group  $G$  is a subgroup if it has the following properties:

- Closure: If  $a$  and  $b$  are in  $H$ , then  $ab$  are in  $H$
- Identity:  $1$  is in  $H$
- Inverse: if  $a$  is in  $H$  then  $a^{-1}$  is in  $H$