# 3D Drawing Visibility & Rasterization

CS559 - Spring 2018

Lectures 9-11

March 20-27, 2018

- Put a 3D primitive in the World Modeling
- Figure out what color it should be Shading
- Position relative to the Eye
   Viewing / Camera Transformation
- 4. Get rid of stuff behind you/offscreen Clipping
- 5. Figure out where it goes on screen

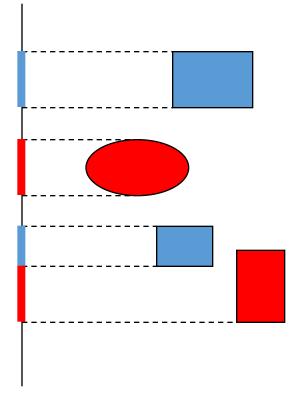
  Projection (sometimes called Viewing)
- 6. Figure out if something else blocks it Visibility / Occlusion
- 7. Draw the 2D primitive
  Rasterization (convert to Pixels)

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#### **Orthographic Projection**

Projection = transformation that reduces dimension

Orthographic = flatten the world onto the film plane

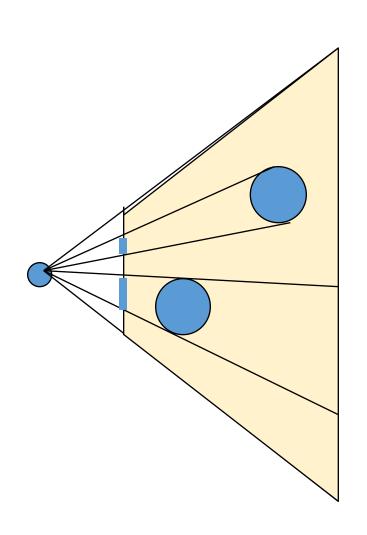


#### Perspective Projection

Eye point Film plane Frustum

Simplification
Film plane centered with respect to eye
Sight down Z axis

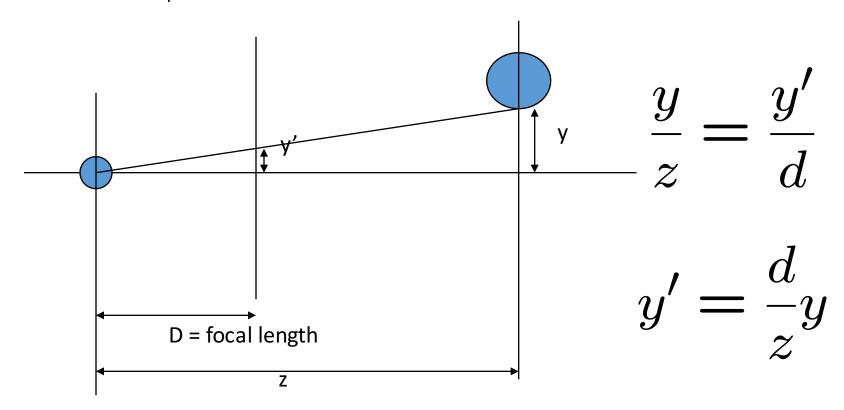
• Can transform world to fit



#### **Basic Perspective**

Similar Triangles

Warning = using d for focal length (like book)
F will be "far plane"



#### Use Homogeneous coordinates!

Use divide by w to get perspective divide

Issues with simple version:

Font / back of viewing volume

Need to keep some of Z in Z (not flatten)

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z \end{bmatrix} = \begin{bmatrix} x/z \\ y/z \\ z/z = 1 \\ 1 \end{bmatrix}$$

#### Simplest Projective Transform

$$\begin{pmatrix} dx \\ dy \\ 1 \\ z \end{pmatrix} = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

After the divide by w...

Note that this is dx/z, dy/z (as we want) Note that z' is 1/z (we can't keep Z)

Fancier forms scale things correctly

### The real perspective matrix

N =near distance, F =far distance

Z = n put on front plane, z=f put on far plane

$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

## Shirley's Perspective Matrix

After we do the divide, we get an unusual thing for z – it preserves order, keeps n&f

$$\mathbf{P}\mathbf{x} = \mathbf{P} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \frac{x \frac{n}{z}}{y \frac{n}{z}} \\ n + f - \frac{fn}{z} \\ 1 \end{bmatrix}$$

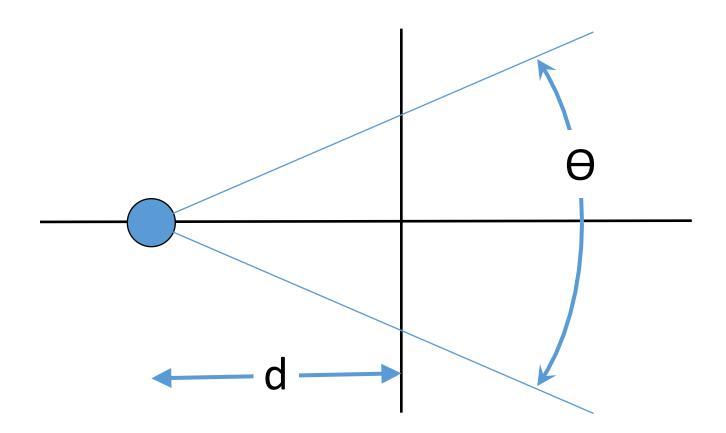
#### The TWGL perspective matrix

perspective(fov, aspect, zNear, zFar)

 $\rightarrow$  {Mat4}

fov = field of view (specify focal length)
aspect ratio (witdth of image)
assuming height is 1
[-zNear,-zFar] remapped to [-1,+1]

# Field of View



- Put a 3D primitive in the World Modeling
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- 6. Figure out if something else blocks it Visibility / Occlusion
- 7. Draw the 2D primitive

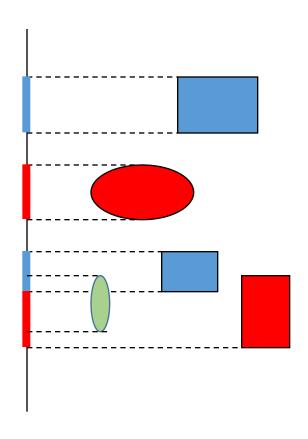
  Rasterization (convert to Pixels)

# Visibility: What objects do you see?

What objects are offscreen?
To avoid drawing them
(generally called clipping)
What objects are blocked?
Need to make things look solid

Assumes we have "filled" primitives Triangles, not lines

# Now we're in Screen Coordinates with depth



#### Bad ideas...

Last drawn wins sometimes object in back what you seen depends on ...

Wireframe (nothing blocks anything)
hard to see what's going on if complex

#### How to make objects solid

Physically-Based Analytic Geometry

Object-space methods (order)
Image-space methods (store per pixel)

## Painter's Algorithm

Order the objects

Draw stuff in back first

Stuff in front blocks stuff in back

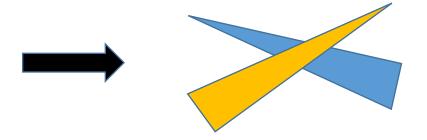
#### Simple version

Pick 1 point for each triangle Sort by this one point (this is OK for P4)

# What about triangles that ... Intersect? Overlap?

Need to divide triangles that intersect (if you want to get it right)

A triangle can be in front of and behind



#### Downsides of Painters Algorithm

Need to sort

O(n log n)

need all triangles (not immediate)

Dealing with intersections = lots of triangles

Need to resort when the camera moves

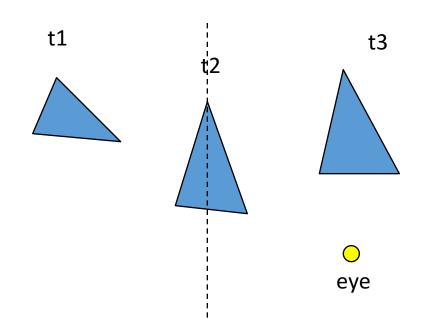
#### **Binary Space Partitions**

Fancy data structure to help painters algorithm

Stores order from any viewpoint

A plane (one of the triangles) divides other triangles

Things on same side as eye get drawn last



T2 divides into groups
T3 is on same side of eye

### Using a BSP tree

Recursively divide up triangles

Traverse entire tree
Draw farther from eye subtree
Draw root
Draw closer to eye subtree
Always O(n) to traverse

(since we explore all nodes)
No need to worry about it being balanced

#### Building a BSP tree

Each triangle must divide other triangles Cut triangles if need be

Goal in building tree: minimize cuts

#### Painters Problem 2: Overdraw

All triangles get drawn

Even if something else will cover it

Depth Complexity = # of things at each pixel

Inefficient, uses lots of memory bandwidth

#### **Z-Buffer**

An image space approach

Hardware visibility solution
Throw memory at the problem

Every pixel stores color and depth

## Z-buffer algorithm

Clear all pixels to "farthest value" (-inf)

```
for each triangle for each pixel if new Z > \text{old } Z: // in front write new color and Z
```

### Simple

```
The only change to triangle drawing: test Z before writing pixels
```

```
writeColor(@pixel) becomes:
    readZ(@pixel)
    test
    writeZandColor(@pixel)
```

#### Notice...

Order of triangles usually doesn't matter

#### Except...

If the Z is equal, we have a tie

We can decide if first or last wins

Either way, order matters

Z-Fighting

# **Z-Fighting**

Z Equal? Order matters

Z Really close? random numerical errors cause flips

#### **Z-Resolution**

Remember – we don't have real Z we have 1/Z (bunches resolution)

Old days: integer Z-buffer was a problem
Nowadays: floating point Z-buffers
Z-resolution less of an issue
Keep near and far close

#### **Transparent Objects**

Draw object in back

Draw transparent object in front

But...

Draw transparent object in front

Draw object in back (Z-buffer prevents)

#### Overdraw

Still drawing all objects – even unseen

Can save writes if front objects first

Early z-test...

Avoid computing pixel color if it will fail z-test

## Using the Z buffer

Give polygons in any order (except...)
Use a Z-Buffer to store depth at each pixel

Things that can go wrong:

Near and far planes matter

Culling tricks can be problematic

You may need to turn the Z-buffer on

Don't forget to clear the Z-Buffer!

# Culling

**Quickly** determine that things cannot be seen – and avoid drawing them

Must be faster to rule things out than to draw them

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# A Quick Word on Shading (for P4)

Color of triangle depends...

Color per triangle (OK for P4)

Color per vertex

Color per pixel

## Lighting basics

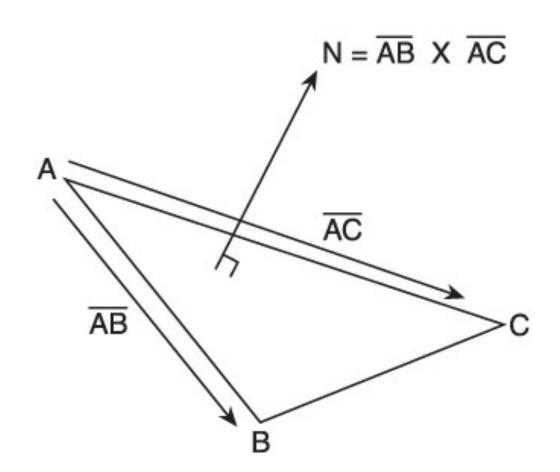
To simulate light, we need to know where the triangle is in the world

```
Global Effects (other objects) reflections, shadows, ...
```

Local Effects (how the light bounces off) shininess, facing the light, ...

## **Local Geometry**

Normal Vector – sticks "out" of the triangle



## Transforming Normal Vectors

Transform triangle, re-compute the normal or...

Normal is transformed by the inverse transpose of the transform

If the triangle is transformed by M The normal is tranformed by  $(M^{-1})^T$ 

## **Inverse Transpose?**

Yes – ask me offline for detailed proof (the book just asserts it as fact)

For a rotation, the inverse is the transpose  $M = (M^{-1})^T$ 

But only for rotations ...

## What can I use a normal vector for?

Simplest lighting: Diffuse Shading

If surface is pointing towards light, it gets more light

brightness ~= N • L

N = unit normal vector

L = unit light direction vector

# Simple things for P4

High noon...

$$C' = ( \frac{1}{2} + \frac{1}{2} N \cdot [0,1,0] ) C$$

Top and bottom...

$$C' = ( \frac{1}{2} + \frac{1}{2} abs(N \cdot [0,1,0])) C$$

Make sure N is a unit vector!

## Program 4

Just like P3 (transform points) but...

- 1. Draw Triangles (solids)
- 2. Compute Normals (and shade)
- 3. Store triangles in a list and sort Painter's Algorithm Visibility

# What coordinate system to compute lighting in?

Window (Screen)

Normalized Device – [-11] Projection loses normals

Camera / Eye

Camera space is OK

World

World space is good

Object . . .

Lights attached to objects?

Local

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## Where are we going next...

We've made a graphics pipeline

Triangles travel through steps... get turned into shaded pixels

How do we use the hardware to make this go fast...

#### Rasterization

Figure out which pixels a primitive "covers"

Turns primitives into pixels

#### Rasterization

Let the low-level library take care of it Let the hardware take care of it

Writing it in software is different than hardware

Writing it today (with cheap floating point) is different than a few years ago

#### Rasterization

```
Input: primitive (in screen coords)
```

```
Output list of pixels "covered"
```

## What primitives

**Points** 

Lines

Triangles

Generally build other things from those Approximate curves

## **Rasterizing Points**

Easy! 1 pixel - and we know where

#### Issues:

```
What if we want different sizes?

(points smaller than a pixel?)

Discretization?

(pixels are an integer grid)
```

## Welcome to the world of Aliasing

The real world is (effectively) continuous

Our models are continuous

Our displays are discrete

This is a deep problem – we'll come back

## Do I care about Aliasing?

**Jaggies** 

Crawlies

Things not moving smoothly

Can't make small things

Can't put things where you want

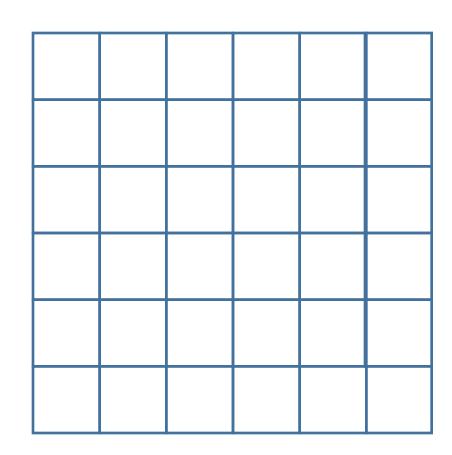
Errors add up to weird patterns

(or, simply, Yes)

Little Square Model

Not preferred

Simpler to start

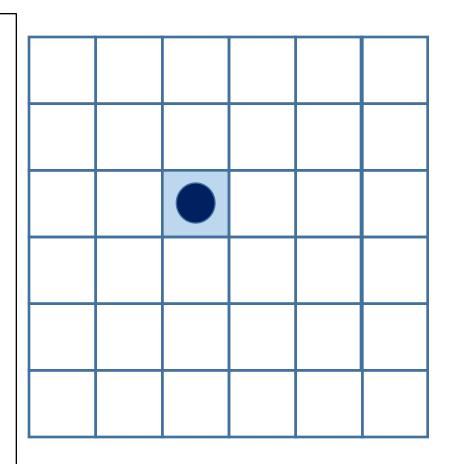


Simple Drawing

Pick the:

Nearest pixel (center)

Fill the pixel



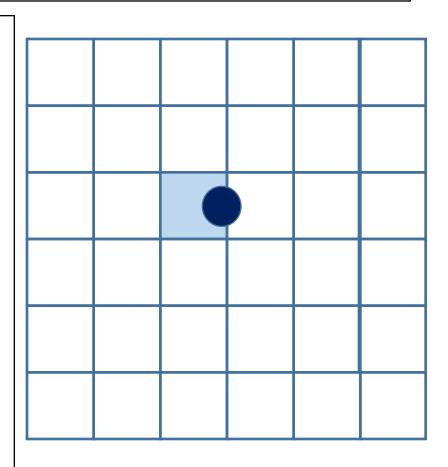
Simple Drawing

Pick the:

Nearest pixel (center)

(cover multiple pixels)

Fill the pixel



Simple Drawing

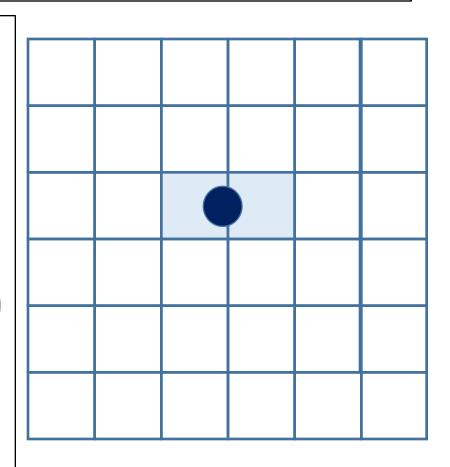
Pick the:

Nearest pixel (center)

(cover multiple pixels)

Fill the pixel

(partially fill pixel)



## Dealing with Aliasing?

## Simple:

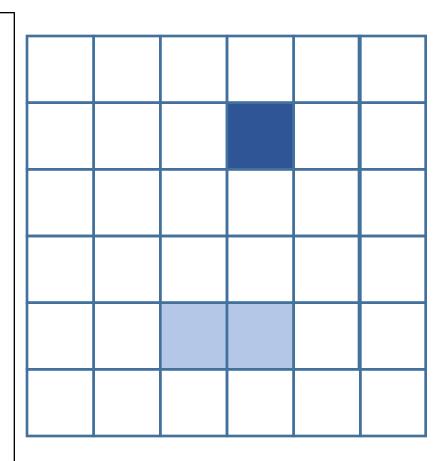
Aliased (jaggies, ...)

Crisp

#### **Anti-Aliased:**

Less aliased

Blurry



## Other Anti-Aliasing Issues

Z-Buffer is a binary choice

Partially filling can be a problem

Depends on lots of other stuff

Really elegant math! Good theory!

#### Lines

#### Historical:

Vector graphics hardware Simulate with "new" pixel-based (CRT)

## Brezenham's Algorithm (1960s)

Integer only line drawing No divisions

# Today?

Floating point is cheap Division isn't too expensive

Make lines into degenerate triangles

# Triangles (Polygons)

The really important primitive

Determine which pixels are covered Also do interpolation (UV, color, W, depth)

Scan conversion
Generically used as a term for rasterization
An old algorithm that isn't used by hardware
Not to be confused with Scanline rendering
Related, but deals with whole scenes

## Scan Conversion Algorithm

Idea:

Scan top to bottom "walk edges" (active edge list) Scan left to right Active Edges (for this scanline) Brezenham's Alg (or equiv) to get begin/end Change active list at vertex

### Scan-Conversion

#### Cool

- Simple operations, very simple inner loops Works for arbitrary polygons (active list management tough)
- No floating point (except for interpolation of values)

#### Downsides

Very serial (pixel at a time) / can't parallelize Inner loop bottle neck if lots of computation per pixel

# How does the hardware do it? (or did it last I learned about it)

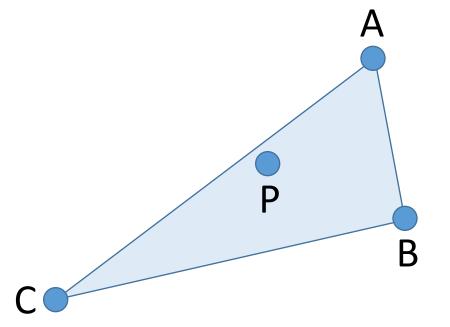
Find a box around the triangle For each pixel in the box compute the barycentric coordinates check if they are inside the triangle Do pixels in parallel (in hardware) otherwise, really wasteful Barycentric coordinates are useful

## **Barycentric Coordinates**

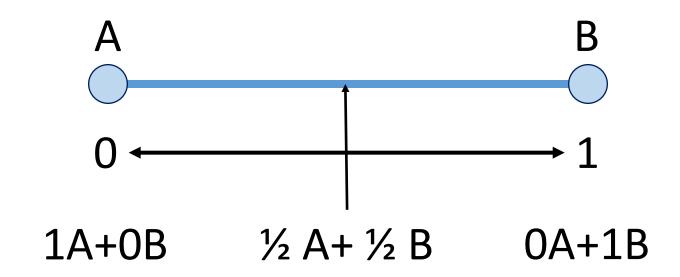
Any point in the plane is a convex combination of the vertices of the triangle

P=
$$\alpha$$
A+ $\beta$ B+ $\gamma$ C  
α+ $\beta$ + $\gamma$ =1

Inside triangle  $0 \le \alpha$ ,  $\beta$ ,  $\gamma \le 1$ 



## Linear Interpolation



Interpolative coordinate (t)  $0 \le t \le 1$  then in line segment

## Dealing with Aliasing?

## Simple:

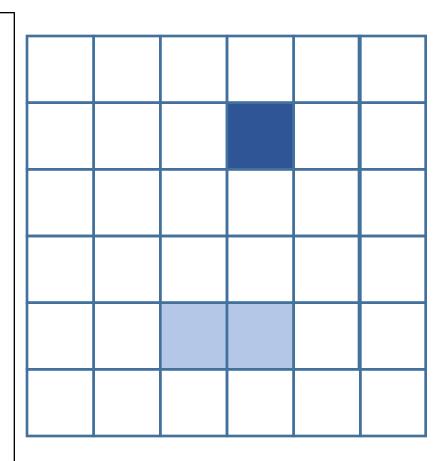
Aliased (jaggies, ...)

Crisp

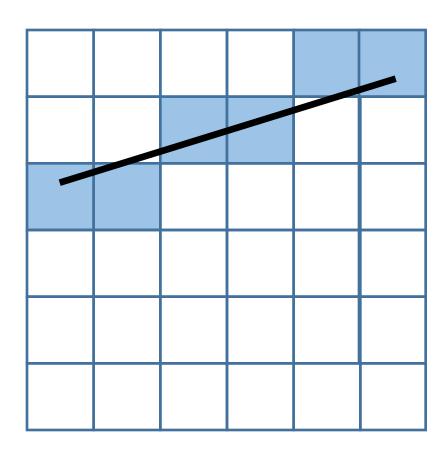
#### **Anti-Aliased:**

Less aliased

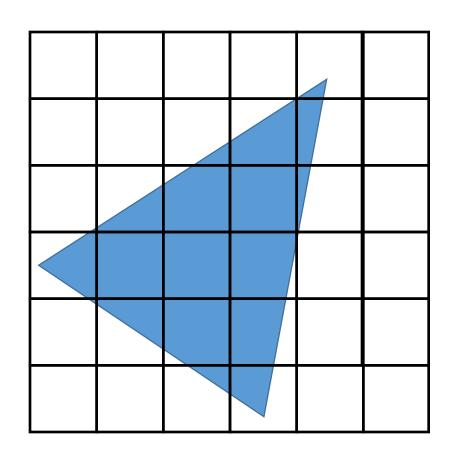
Blurry



## Lines



# Triangles

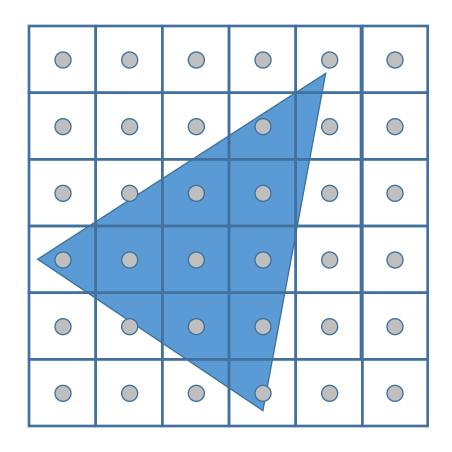


### Hardware Rasterization

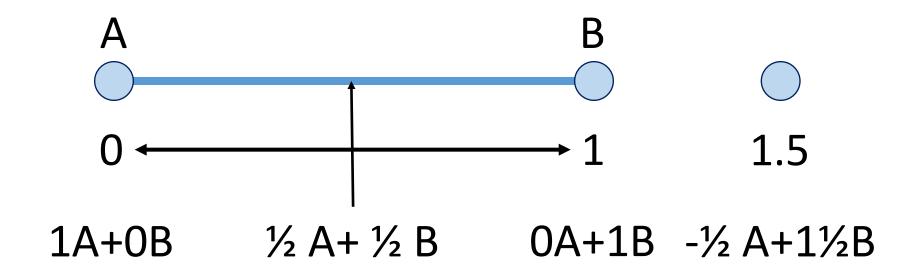
## For each point:

Compute barycentric coords

Decide if in or out



# Linear Interpolation P = (1-t) A + t B (t is the coord)



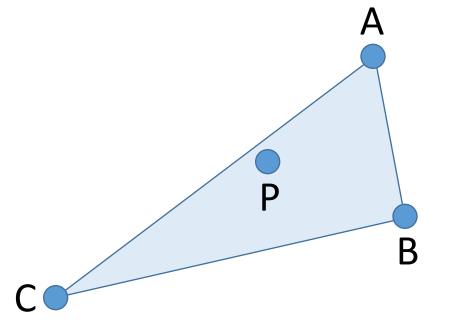
Interpolative coordinate (t) P = (1-t) A + t B $0 \le t \le 1$  then in line segment

#### **Barycentric Coordinates**

Any point in the plane is a convex combination of the vertices of the triangle

P=
$$\alpha$$
A+ $\beta$ B+ $\gamma$ C  
α+ $\beta$ + $\gamma$ =1

Inside triangle  $0 \le \alpha$ ,  $\beta$ ,  $\gamma \le 1$ 



### Barycentric Coords are Useful!

Every point in plane has a coordinate  $(\alpha \beta \gamma)$  such that:  $\alpha + \beta + \gamma = 1$ 

Easy test inside the triangle

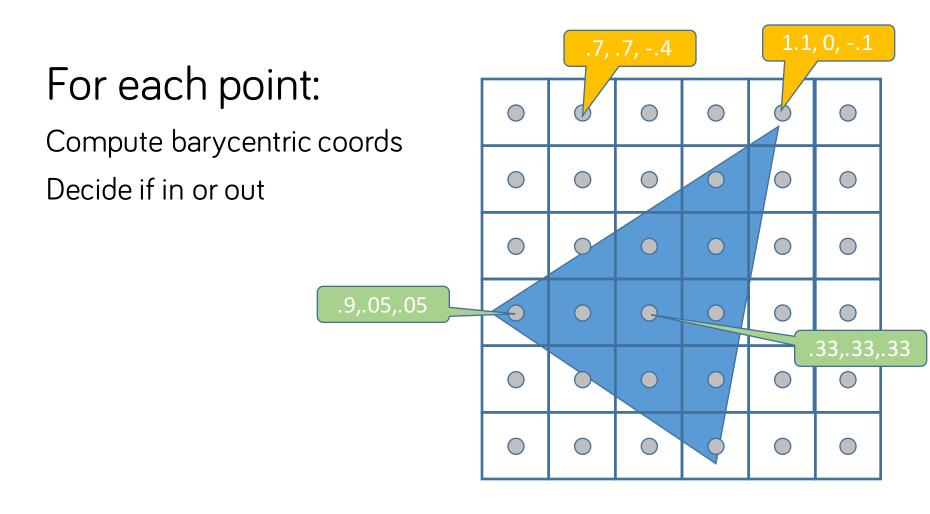
$$0 \leftarrow \alpha, \beta, \gamma \leftarrow 1$$

Interpolate values across triangles

$$\mathbf{x}_p = \alpha \mathbf{x}_1 + \beta \mathbf{x}_2 + \mathbf{y} \mathbf{x}_3$$

$$\mathbf{c}_p = \alpha \mathbf{c}_1 + \beta \mathbf{c}_2 + \mathbf{y} \mathbf{c}_3$$

#### Hardware Rasterization



#### Wasteful?

Can do all points in parallel

We want the coordinates (coming soon)

Does the right things for touching triangles Each point in 1 triangle

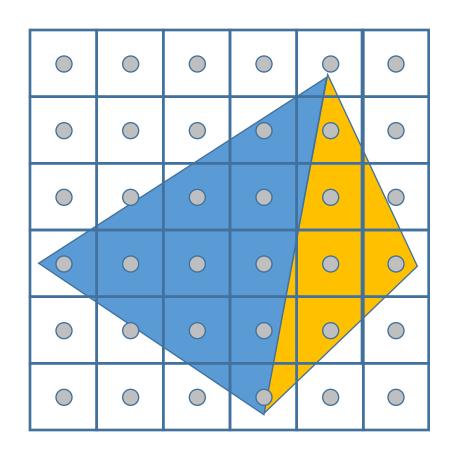
#### Hardware Rasterization

Each center point in one triangle

If we choose consistently for "on-the-edge" cases

Over simplified version:

$$0 \le a,b,c < 1$$



#### Note

Triangles are independent

Even in rasterization

(they are independent throughout process)

#### The steps of 3D graphics

Model objects (make triangles) Transform (find point positions) Shade (lighting - per tri / vertex) Transform (projection) Rasterize (figure out pixels) Shade (per-pixel coloring) Write pixels (with Z-Buffer test)

#### A Pipeline

Triangles are independent

Vertices are independent

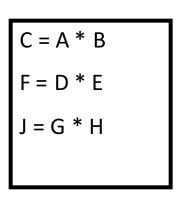
Pixels (within triangles) are independent

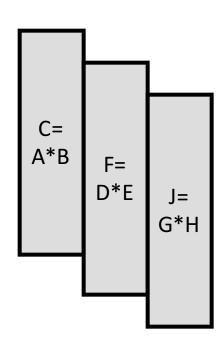
(caveats about sharing for efficiency)

Don't need to finish 1 before start 2 (might want to preserve finishing order)

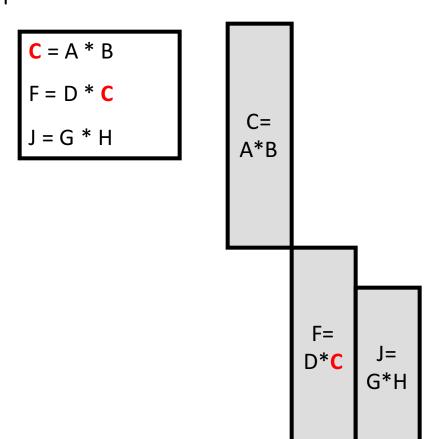
#### Pipelining in conventional processors

Start step 2 before step 1 completes





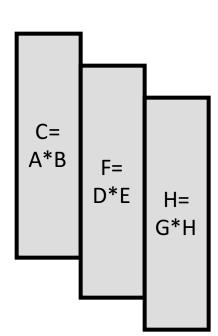
Unless step 2 depends on step 1 Pipe Stall



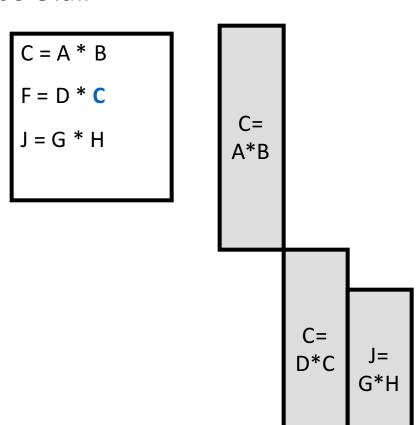
# Triangles are independent! No stalls! (no complexity of handling stalls)

Start step 2 before step 1 completes

C = A \* B F = D \* E J = G \* H



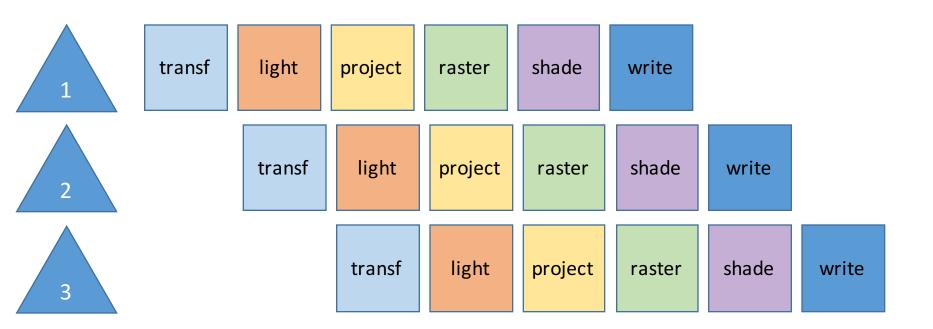
Unless step 2 depends on step 1 Pipe Stall



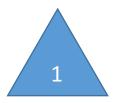
# A Pipeline



# A Pipeline



# Vertices are independent Parallelize!



transf
transf







shade

write

#### **Parallelization**

```
Vertex operations
     split triangles / re-assemble
     compute per-vertex not per-triangle
Pixel (fragment) operations
     lots of potential parallelism
     less predictable
Use queues and caches
```

## Why do we care?

This is why the hardware can be fast

It requires a specific model Hardware implements this model

The programming interface is designed for this model. You need to understand it.

#### Some History...

Custom Hardware (pre-1980) rare, each different Workstation Hardware (early 80s-early 90s) increasing features, common methods Consumer Graphics Hardware (mid 90s-) cheap, eventually feature complete Programmable Graphics Hardware (2002-)

### Graphics Workstations 1982-199X

Implemented graphics in hardware

Providing a common abstraction set

Fixed function – it was built into the hardware

## Silicon Graphics (SGI)

Stanford Research Project 1980 Spun-off to SGI (company) 1982

The Geometry Engine

4x4 matrix mutiply chip
approximate division

Raster engine (Z-buffer)



The 4D-2XO series

4 processors (240)

Different graphics

1988 - GT/GTX

1990 - VGX



### Why do you care?

- It's the first time the abstractions were right later stuff adds to it
  It's where the programming model is from
- it was IrisGL before OpenGL
- It's the pipeline at it's essense we'll add to it, not take away

#### The Abstractions

```
Points / Lines / Triangles
Vertices in 4D
Color in 4D (RGBA = transparency)
Per-Vertex transform (4x4 + divide by w)
Per-Vertex lighting
Color interpolation
Fill Triangle
Z-test (and other tests)
Double buffer (and other buffers)
```

#### What's left to add?

All of this was in software in the 80s

1990 - texture

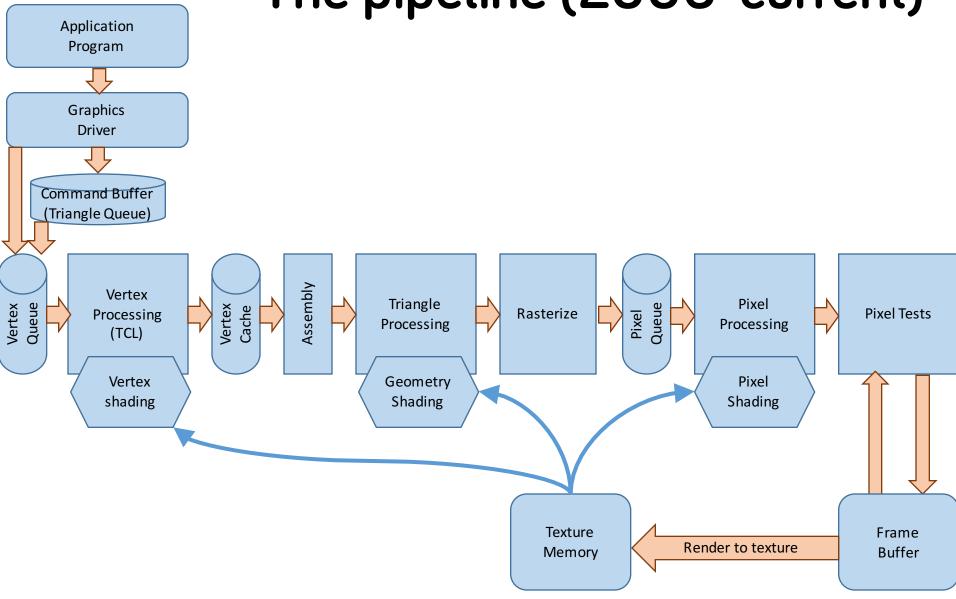
1992 - multi-texture (don't really need)

1998 (2000) – programmable shading

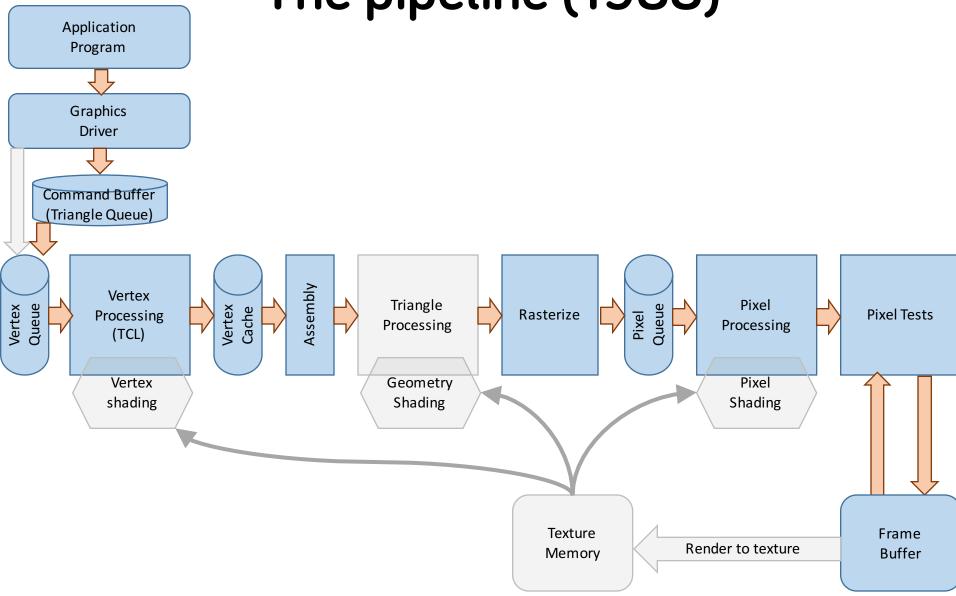
2002 – programmable pipelines

2005 - more programmability

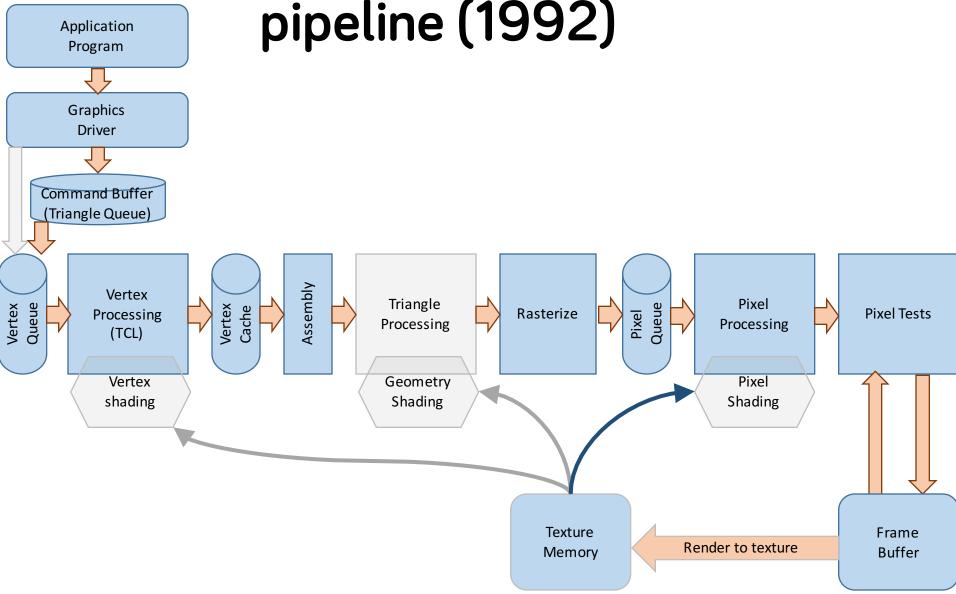
### The pipeline (2006-current)

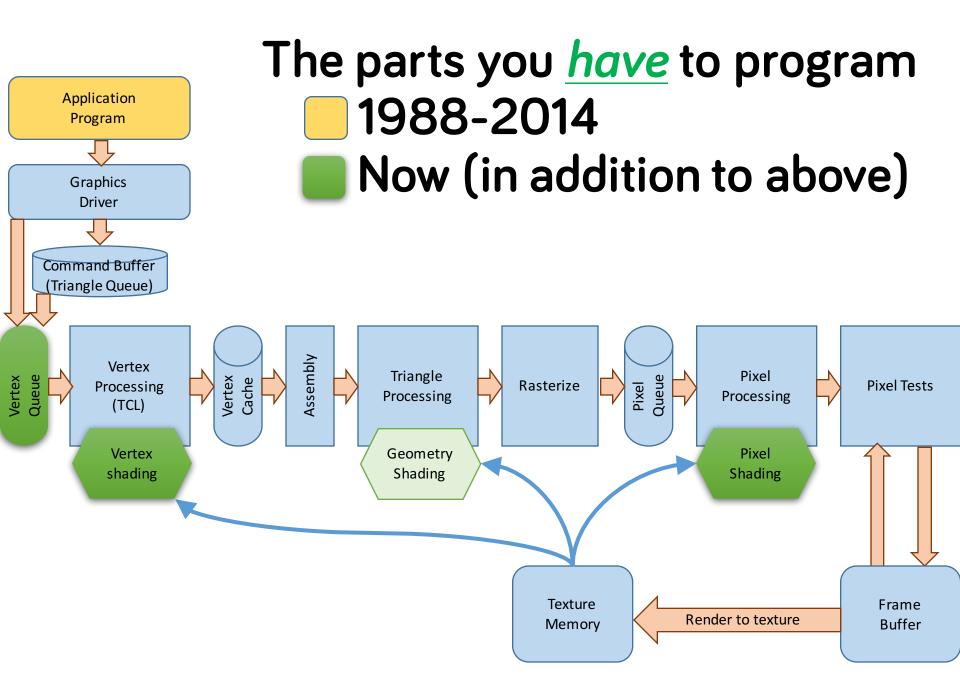


## The pipeline (1988)



# The full fixed-function pipeline (1992)





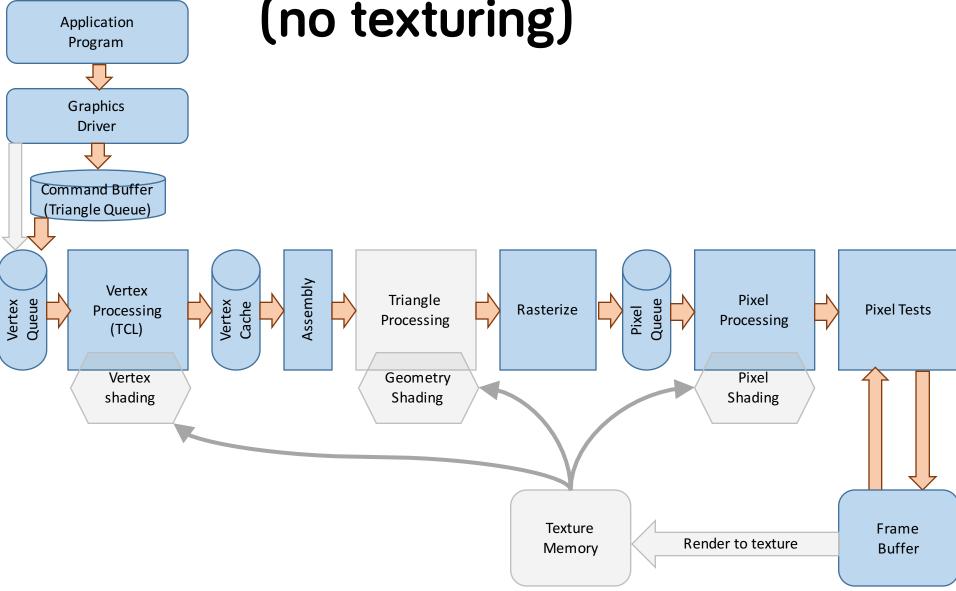
# A Triangle's Journey

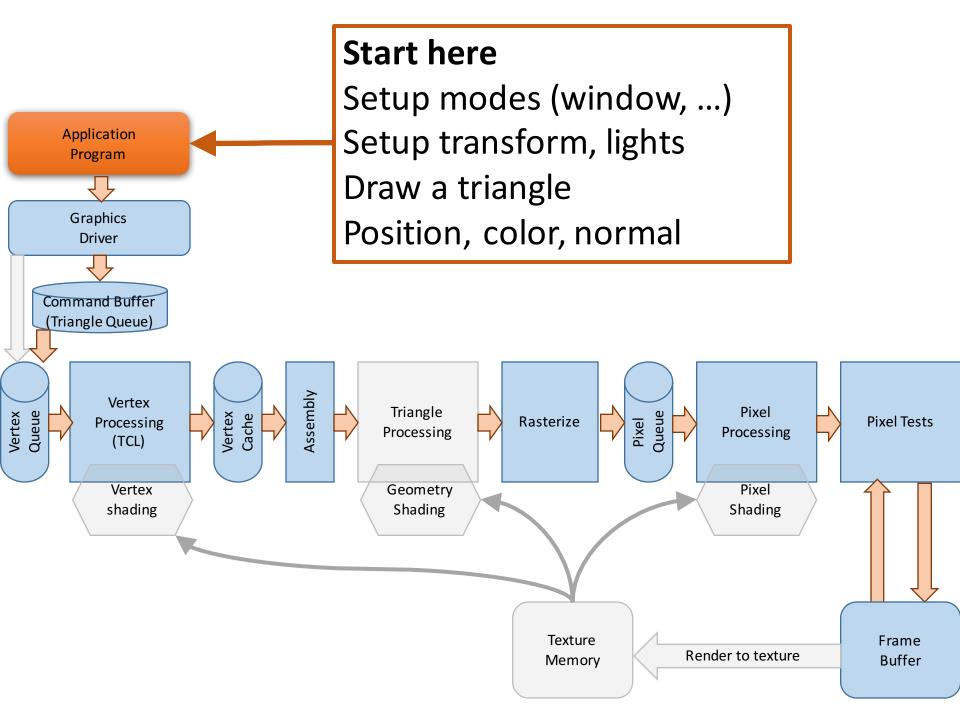
# Things to observe as we travel through the pipeline...

and what could we do to avoid it

What does each stage do?
What are its inputs and output?
important for programmability
Why would it be a bottleneck?

# The pipeline (1988) (no texturing)





#### Drawing a triangle

```
Modes per triangle
     which window, how to fill, use z-buffer, ...
Data per-vertex
     position
     normal
     color
     other things (texture coords)
```

#### Per Vertex?

Modes per triangle

which window, how to fill, use z-buffer, ...

Data per-vertex

position

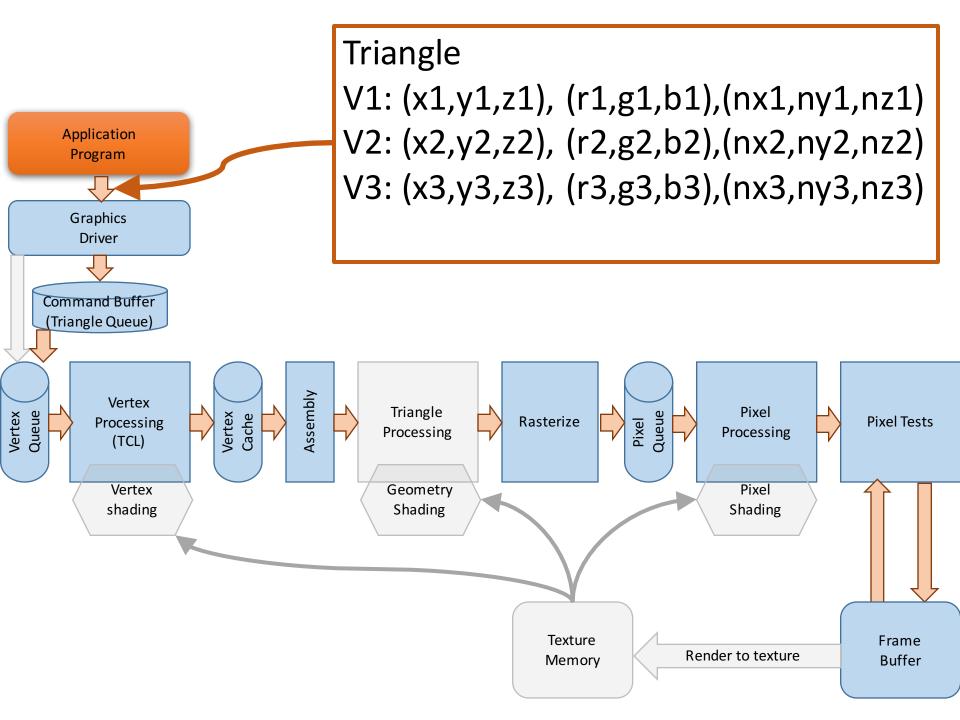
normal ← allow us to make non-flat color ← allows us to interpolate

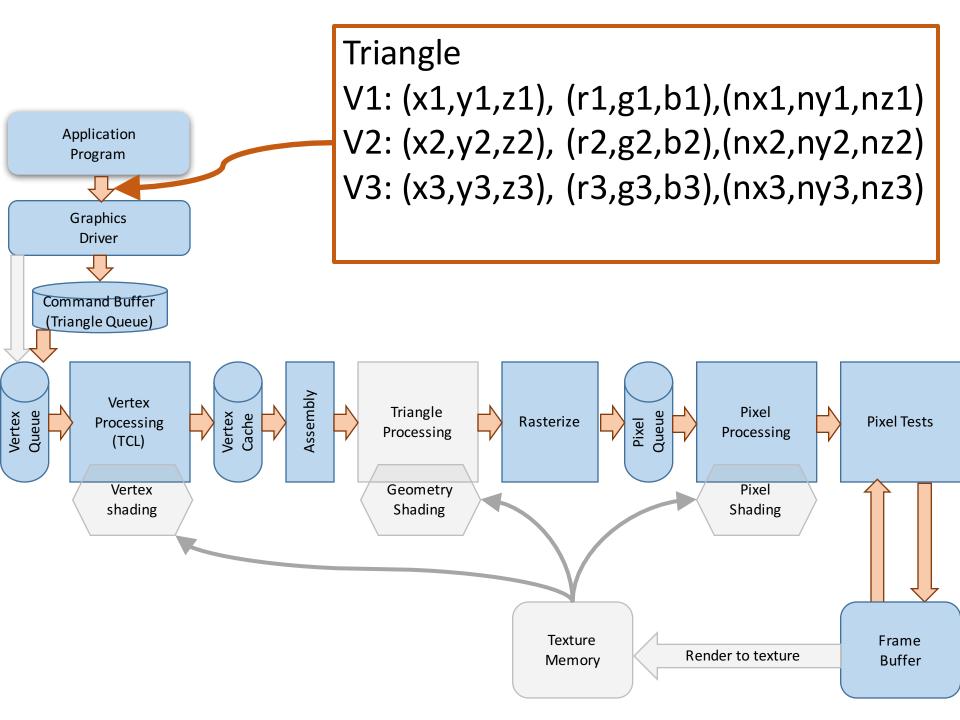
other things (texture coords)

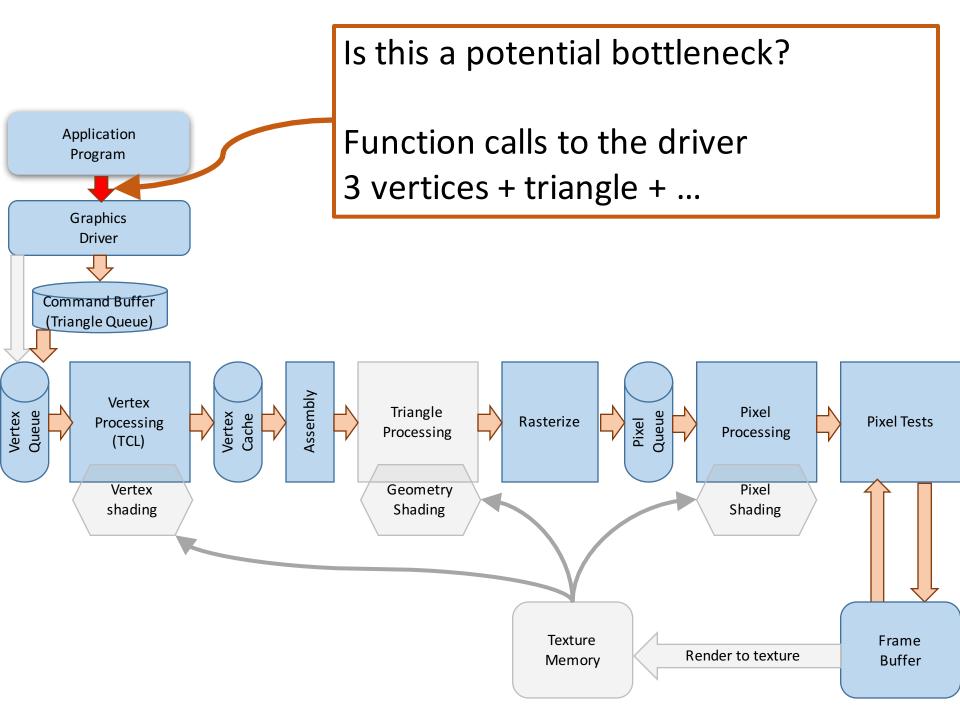
#### Per-Vertex not Per-Triangle

Allows sharing vertices between triangles

Or make all the vertices the same (color, normal, ...) to get truly flat







## Old style OpenGL

```
begin(TRIANGLE);
c3f(r1,q1,b1);
n3f(nx1,ny1,nz1);
v3f(x1,y1,z1);
c3f(r2,q2,b2);
n3f(nx2,ny2,nz2);
v3f(x2,y2,z2);
c3f(r3,q3,b3);
n3f(nx3,ny3,nz3);
v3f(x3,y3,z3);
end(TRIANGLE);
```

11 function calls35 arguments pushed

Old days:

This is a lot less than the number of pixels!

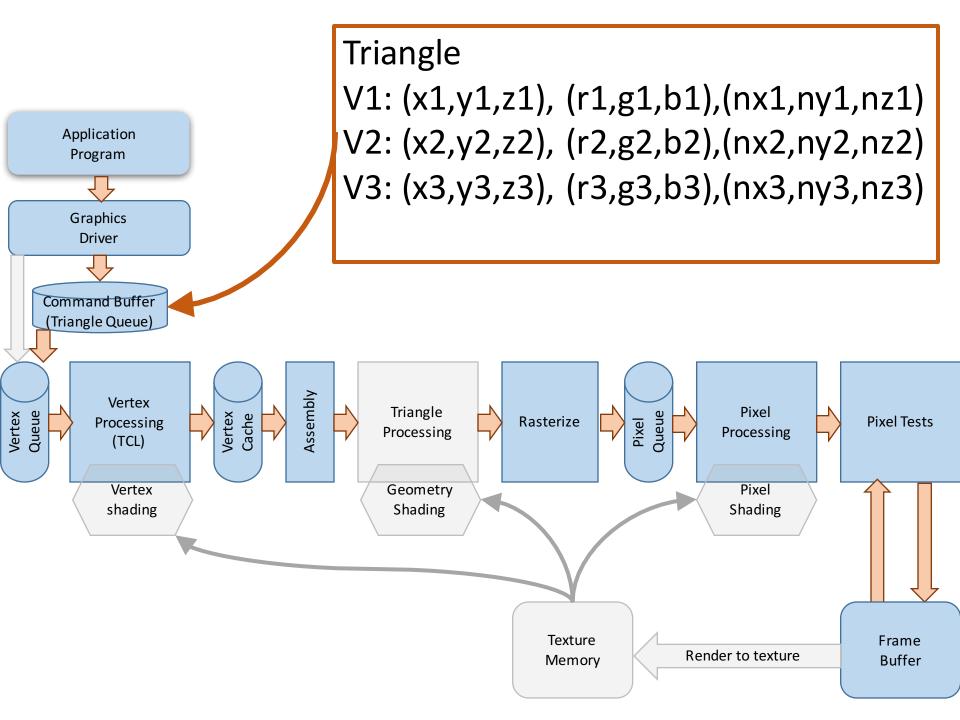
Nowadays:

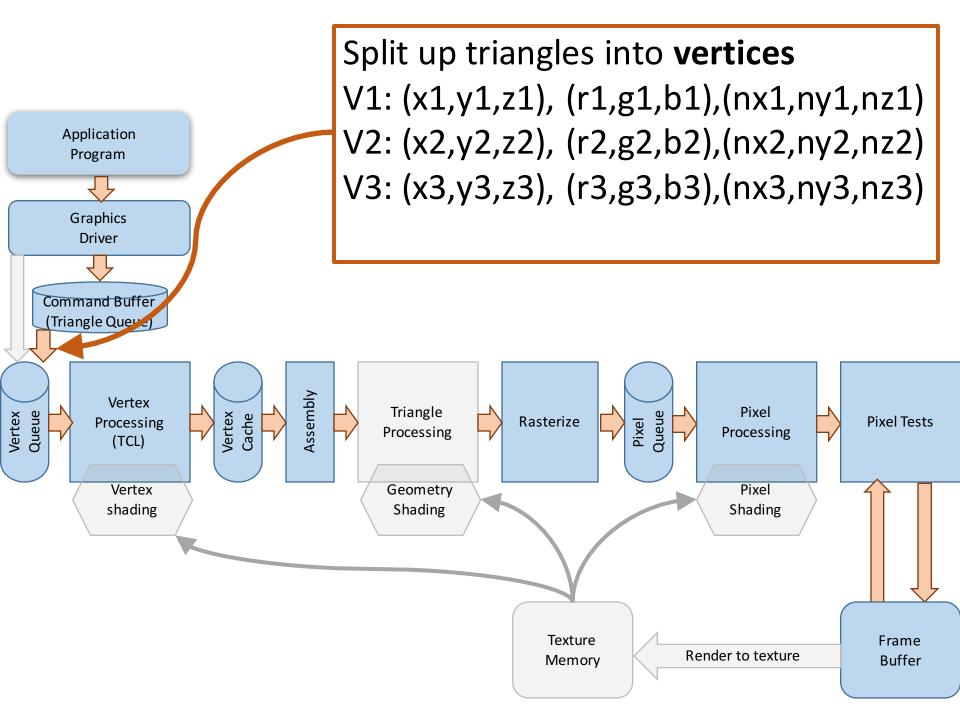
Just the memory access swamps the process

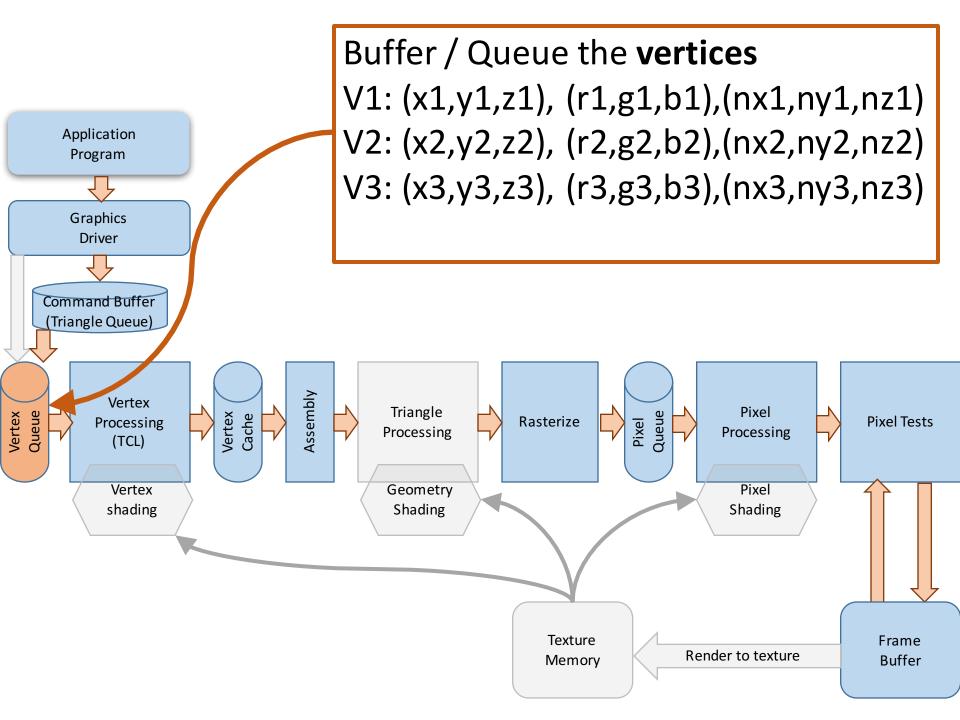
### Coming Soon...

Block transfers of data

Data for lots of triangles moved as a block Try to draw groups of triangles







## **Buffering Vertices**

#### Old Days:

Vertex processing expensive

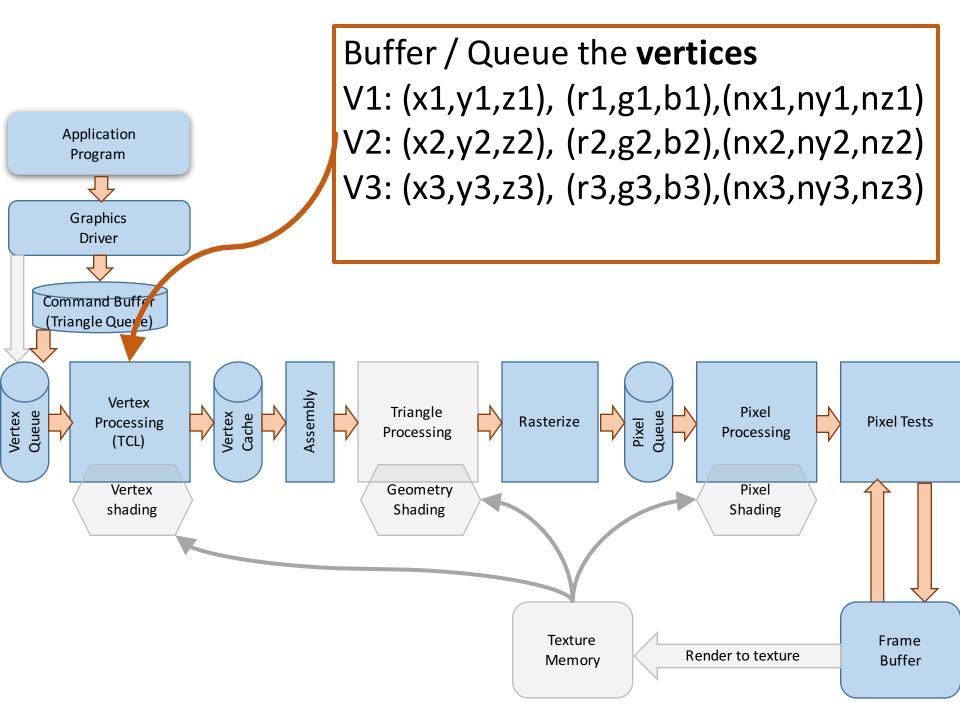
Try to maximize re-use

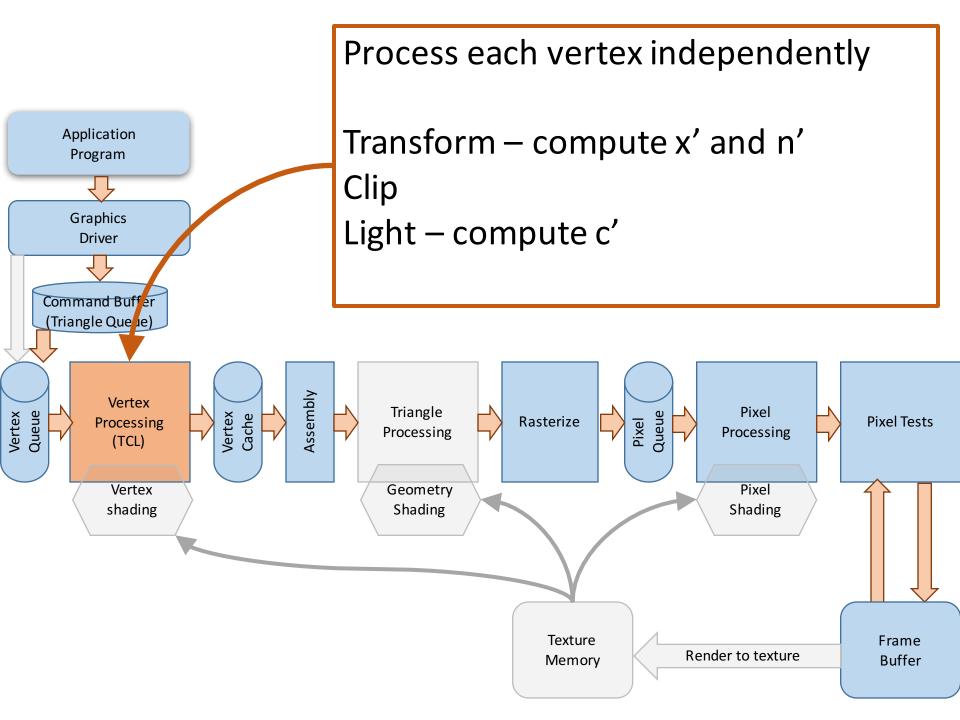
Process once an use for many triangles

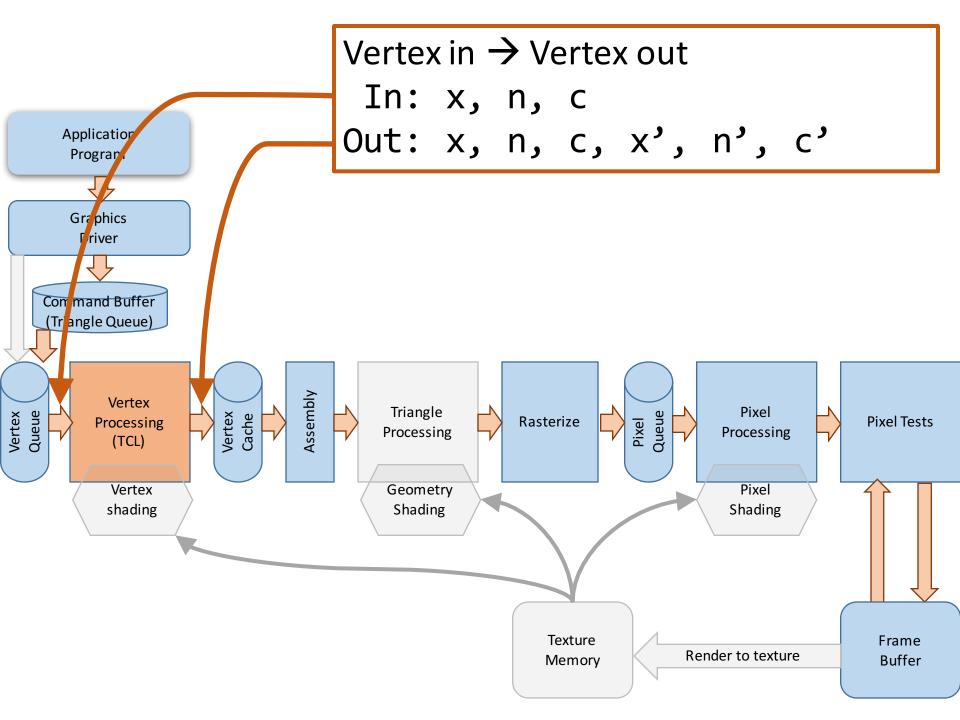
### Nowadays

Getting vertex to hardware is expensive

Process vertices in parallel







## **Vertex Processing**

Just adds information to vertices

Computes transformation
screen space positons, normals
Computes "lighting"
new colors

(in the old days, clipping done here hence TCL)

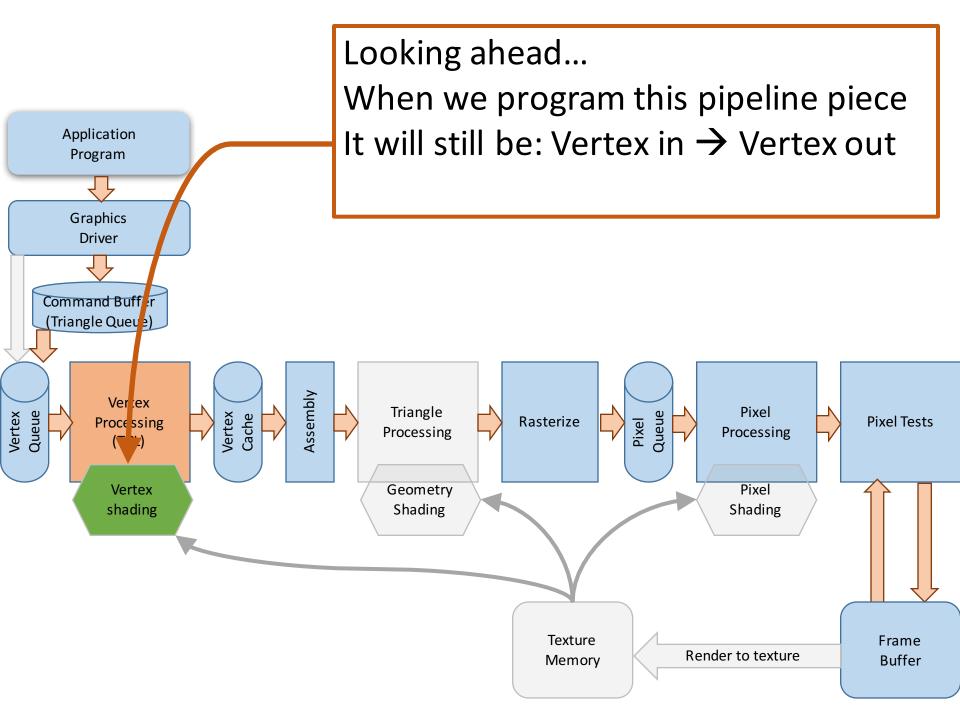
# Vertex Processing: Each vertex is independent

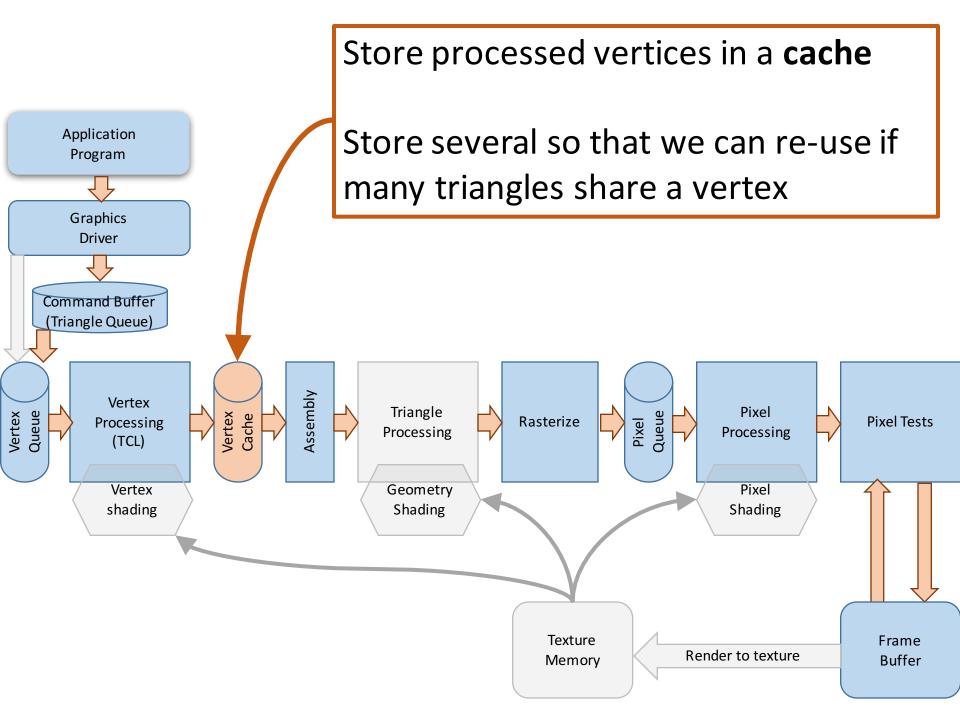
Inputs are:

vertex information for **this vertex** any "global" information current transform, lighting, ...

Outputs are:

vertex information for this vertex





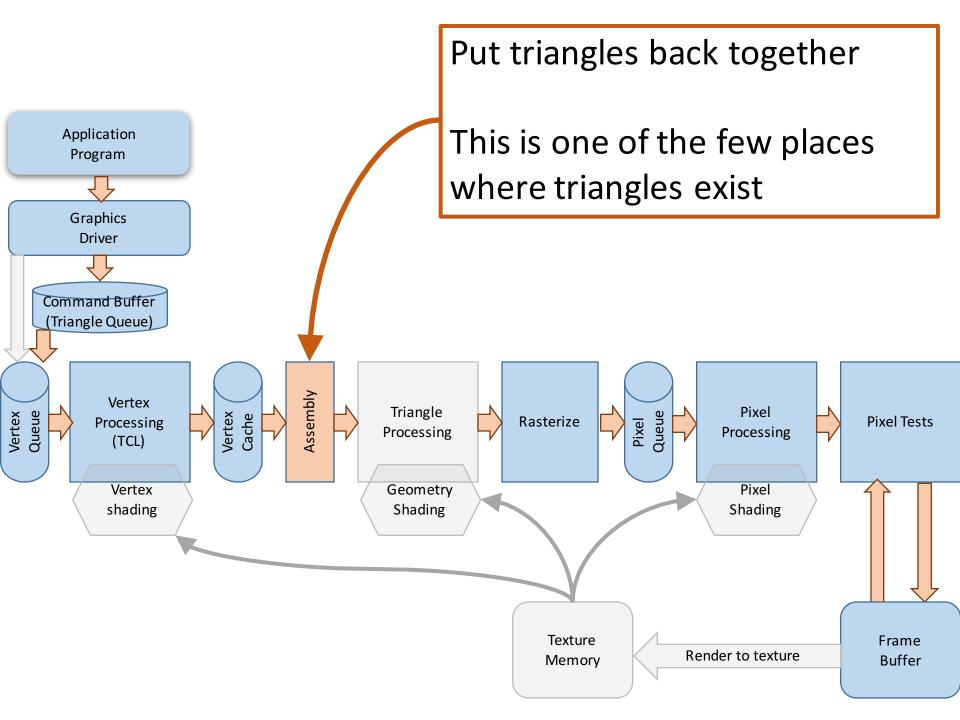
## **Vertex Caching**

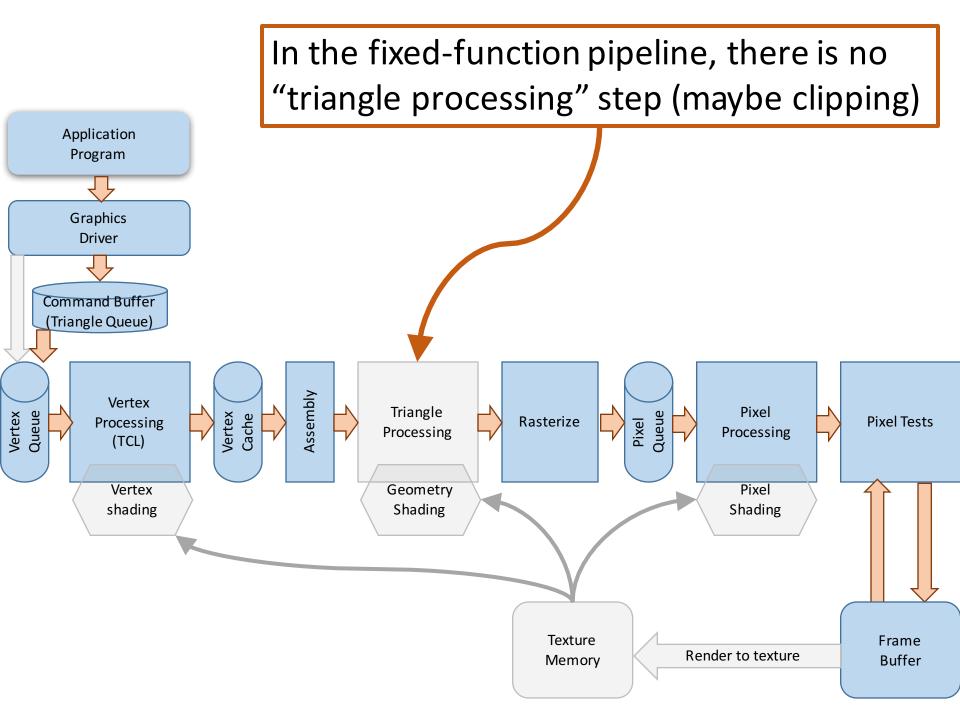
Old days:

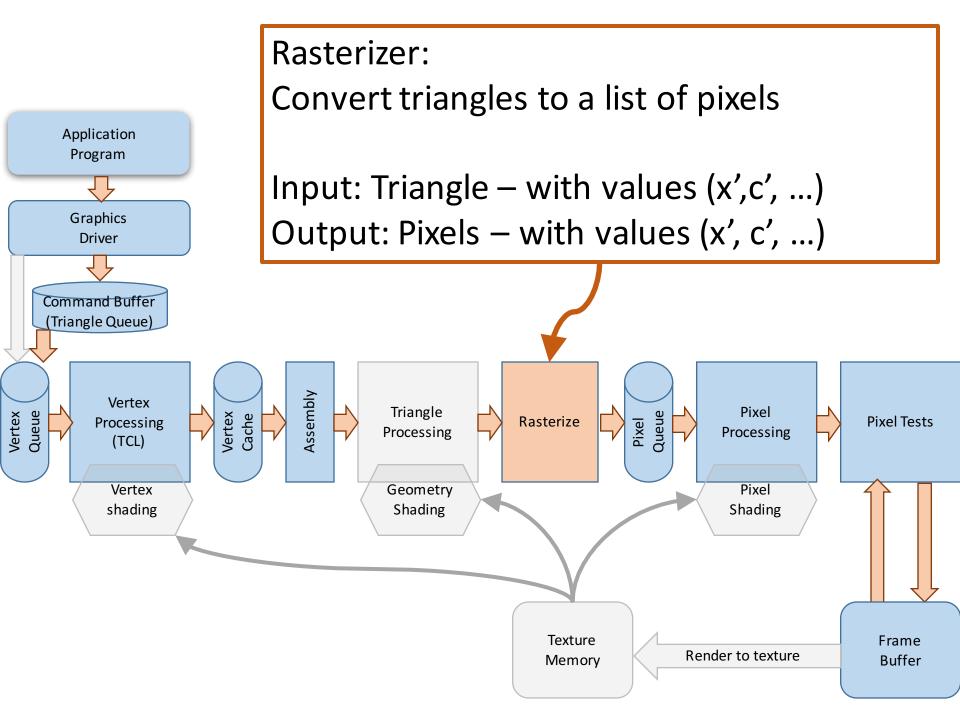
Big deal, important for performance

Now:

Not even sure that it's always done







## Pixels or Fragments

I am using the terms interchangably (actually, today I am using pixel) Technically... Pixel = a dot on the screen Fragment = a dot on a trianglemight not become a pixel (fails z-test) might only be part of a pixel

## Where do pixel values come from?

Each vertex has values

Each pixel comes from 3 vertices

Pixels interpolate their vertices' values

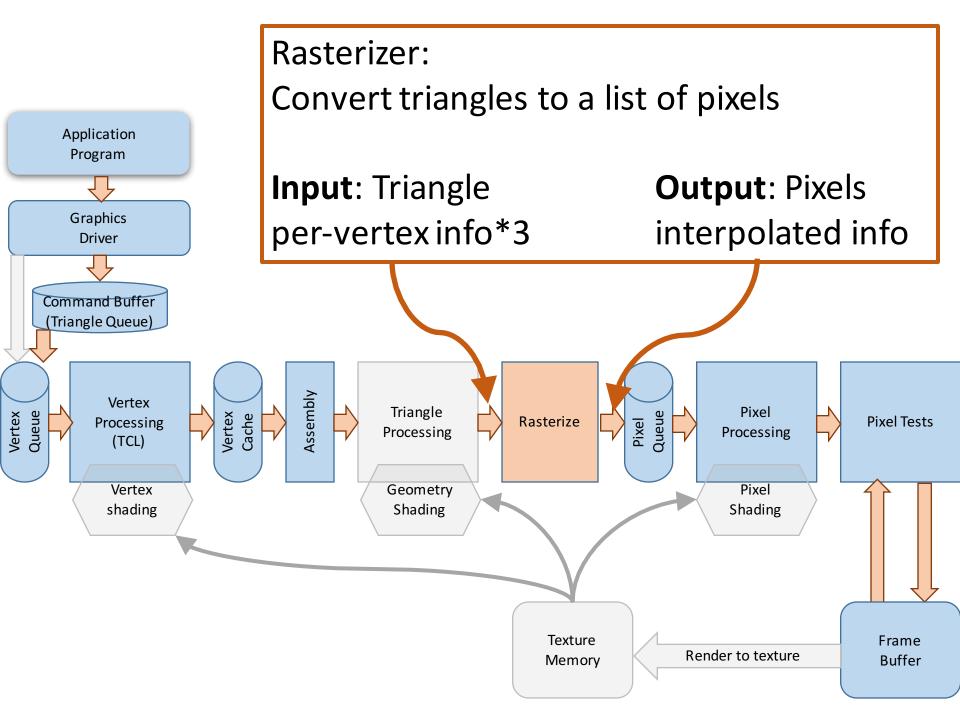
Barycentric interpolation

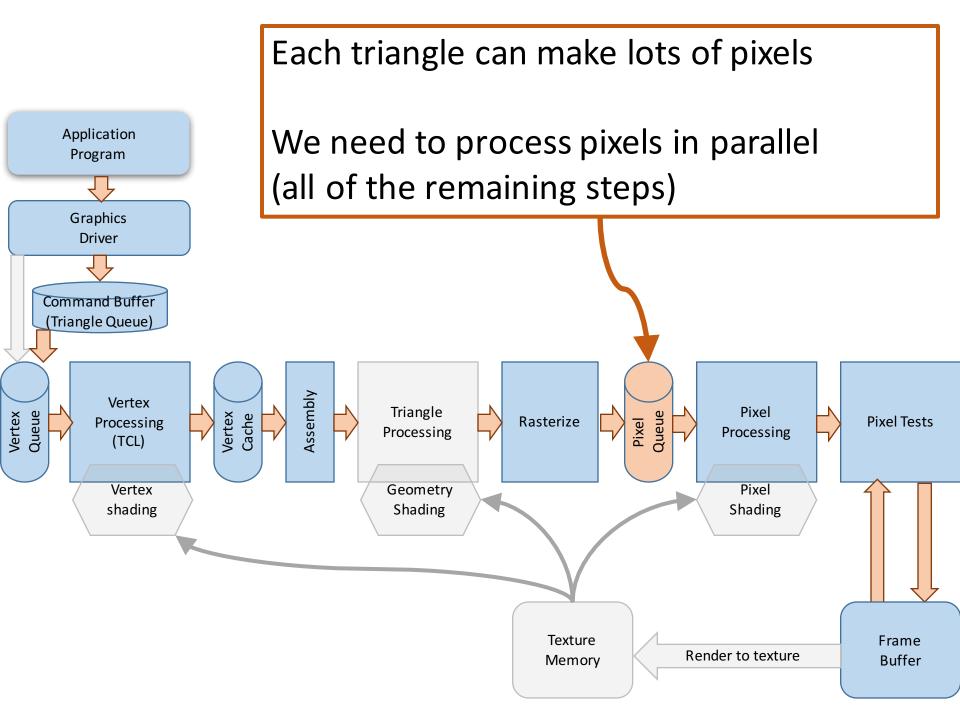
All values (in a pixel) are interpolated

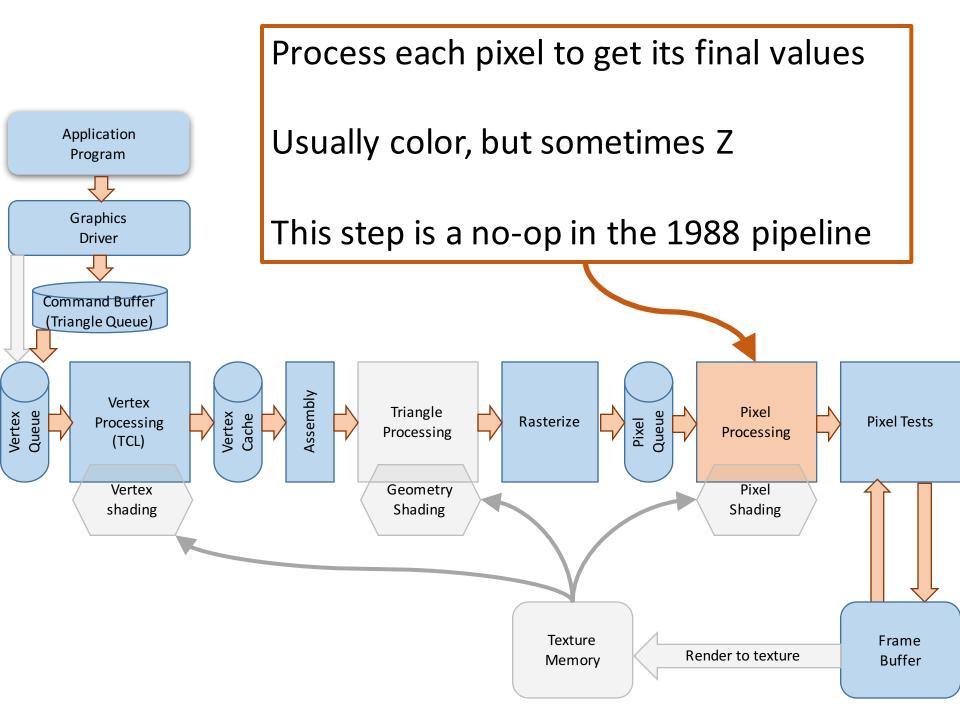
## Each triangle is separate

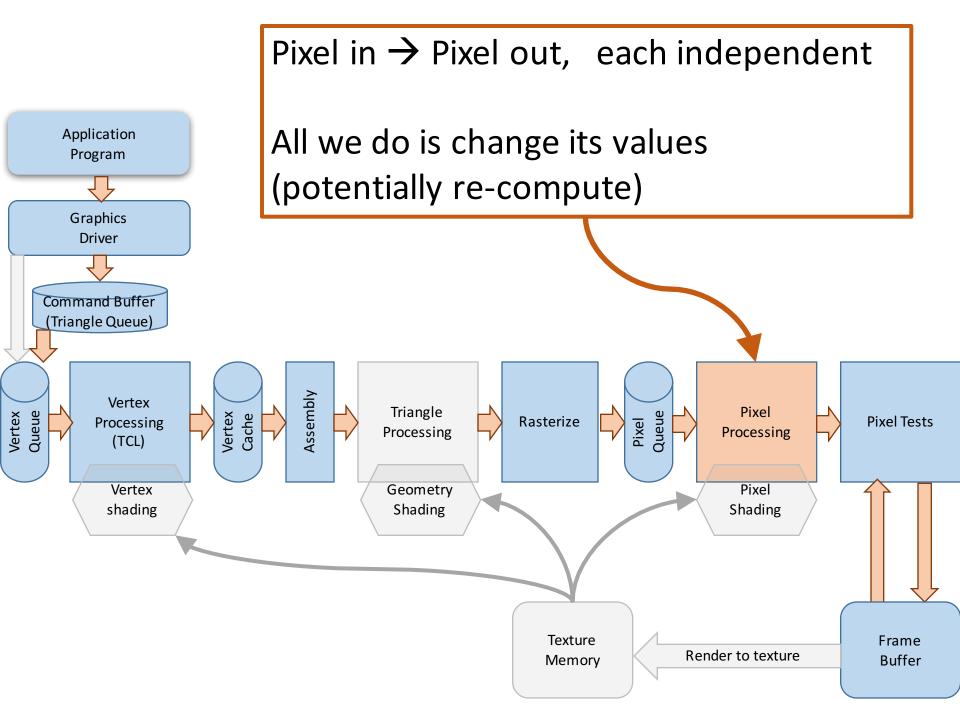
Careful processing of edges so no cracks

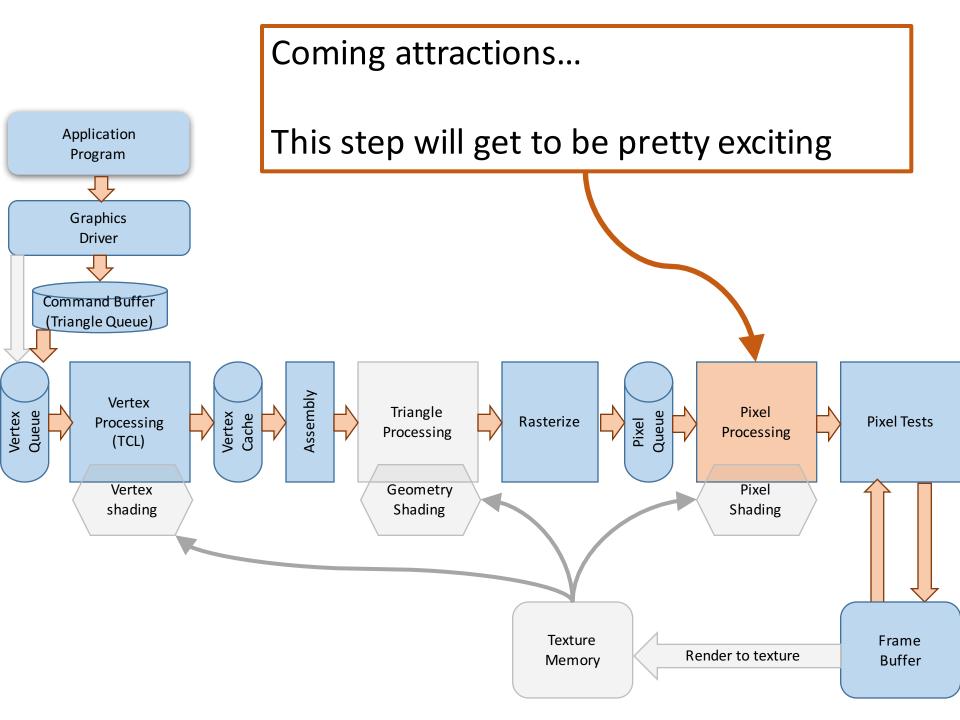
1 triangle  $\rightarrow$  many pixels











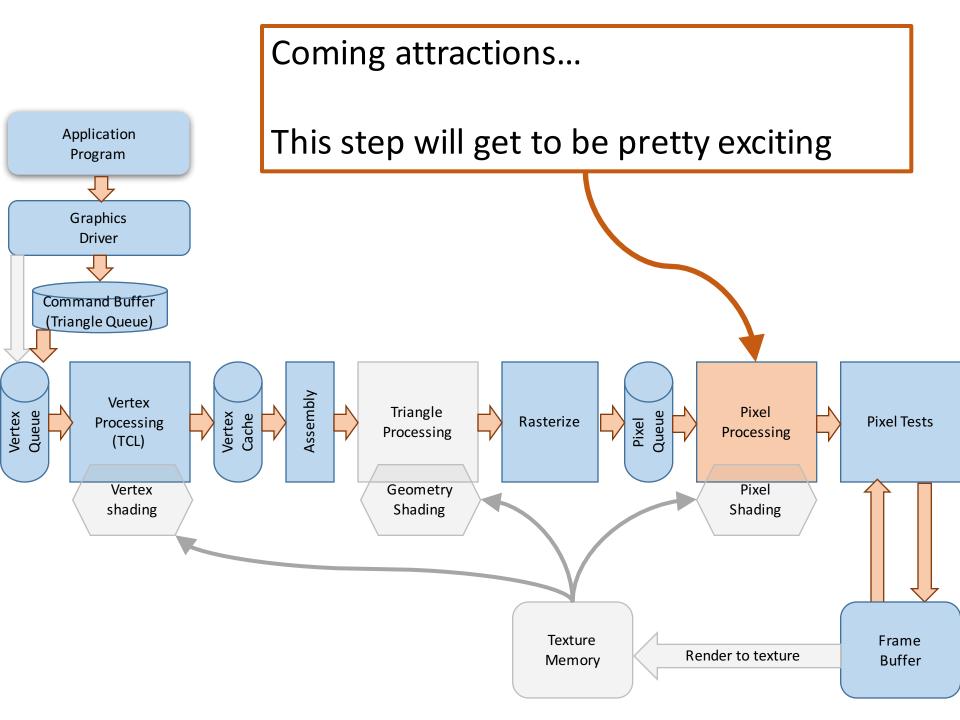
## Pixel Processing Ground Rules

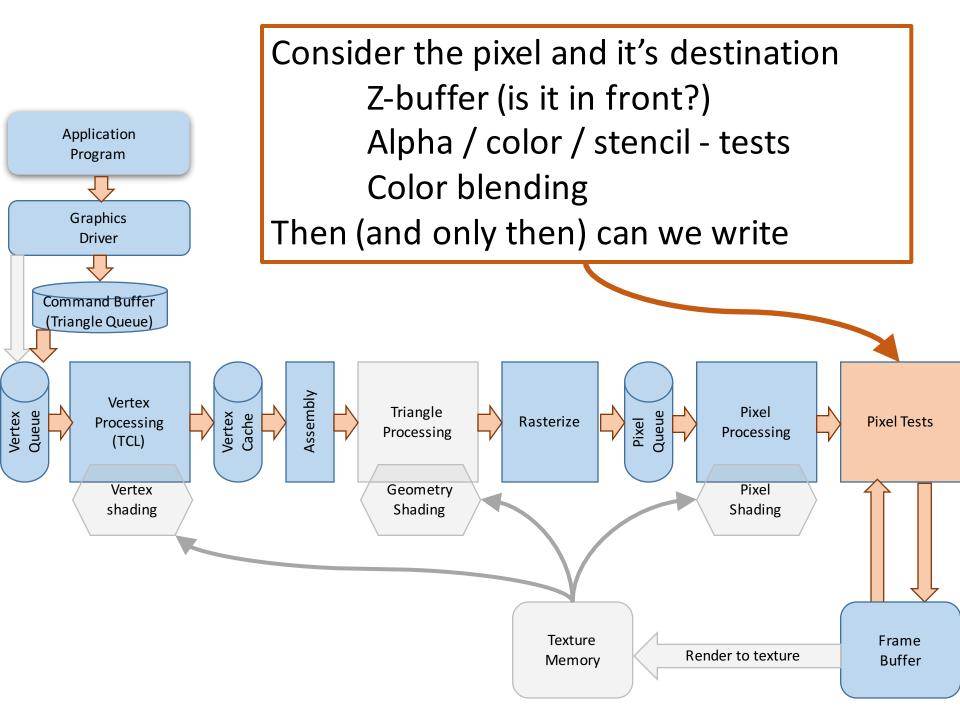
Pixels are independent

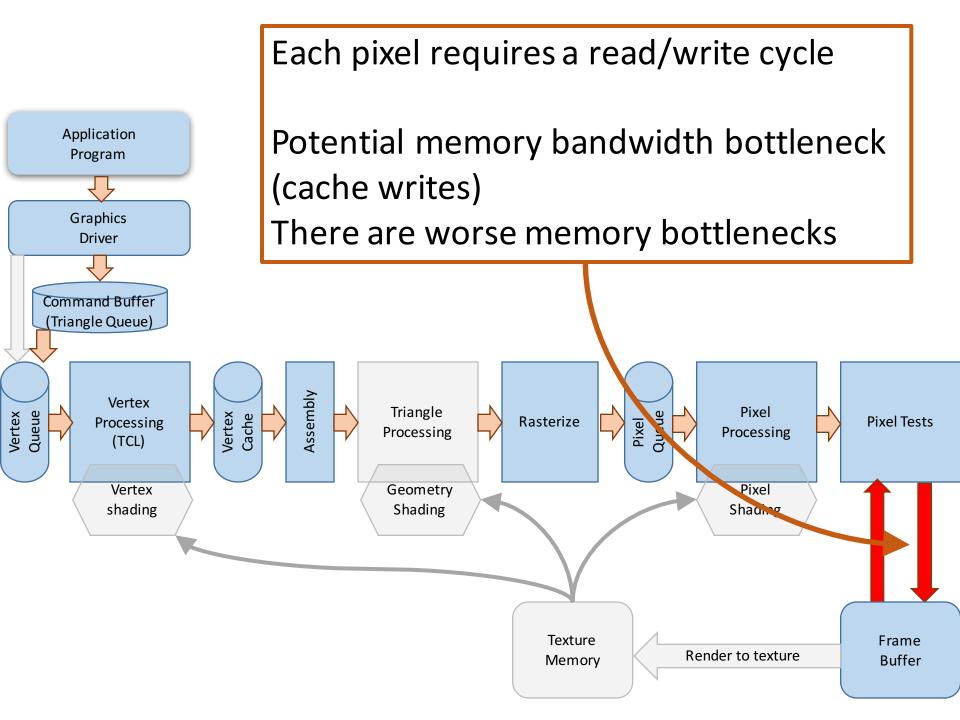
Pixel in  $\rightarrow$  Pixel out

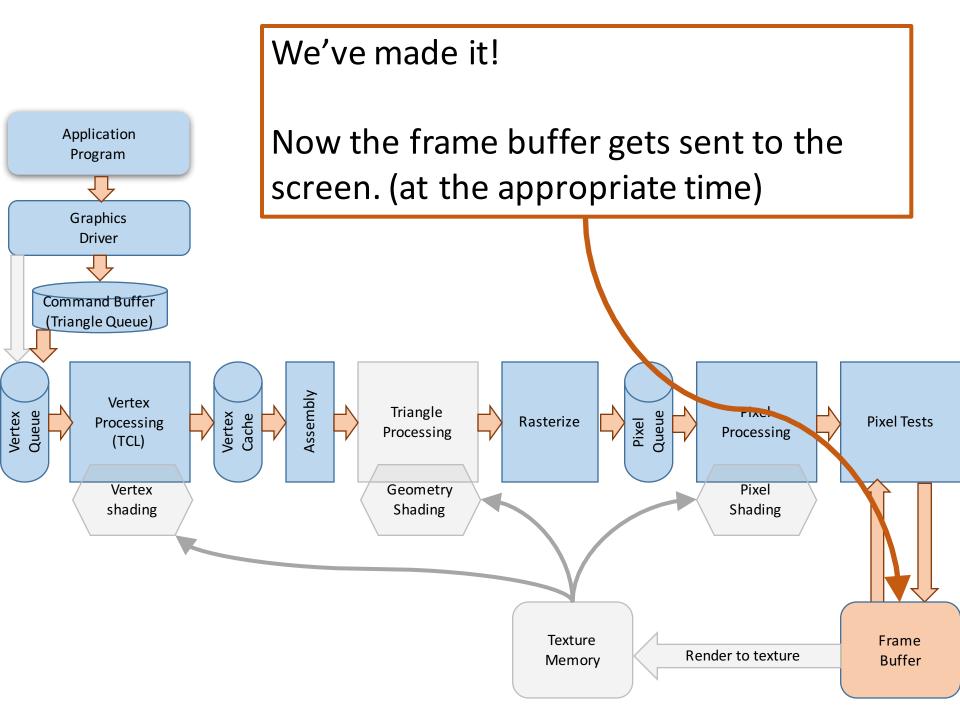
Changing its position (x,y) makes it a different pixel (so you can't)

Can change other values Or "reject"









#### What if we didn't make it...

Suppose the triangle's pixels are occluded Removed by the z-buffer

