Attractors in Quark-Gluon Plasma Dynamics

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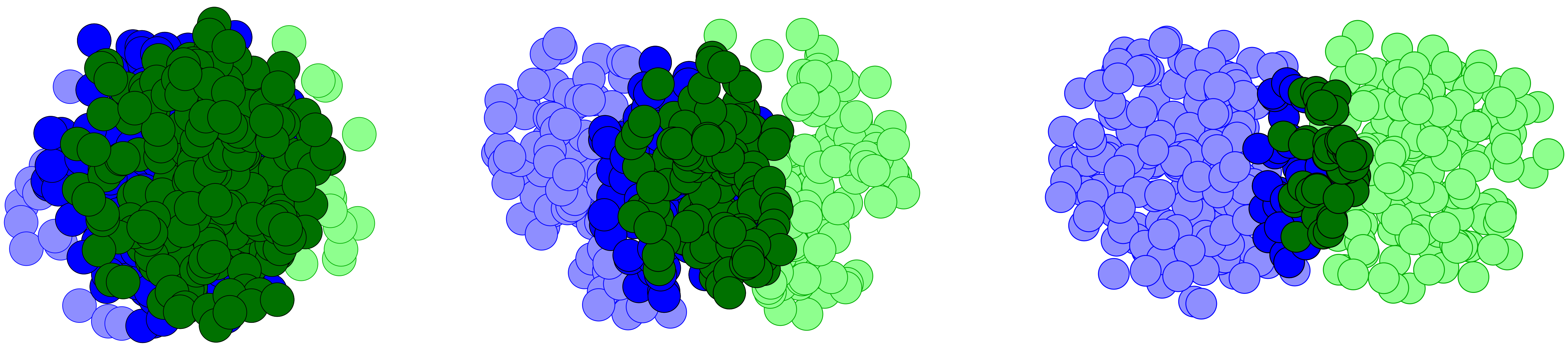
Ultra-relativistic heavy-ion collisions

and quark-gluon plasma



- Collisions of nuclei of Pb, Au, …
- Longitudinal expansion dominates initially
- Almond-shaped collision region leads to a pressure asymmetry
- Measured hadron spectra reveal memory of the initial state
- QGP behaves as a fluid rather than a gas
- Evolution of QGP is successfully modelled by Fluid Dynamics

QGP and Fluid Dynamics
the early-time puzzle

- Fluid Dynamics is an effective near-equilibrium description, anchored in the appropriate microscopic theory.
- The reduction of complexity at sufficiently late times is the hallmark of the approach to equilibrium.
- The success of hydrodynamic models in QGP dynamics suggests a rapid reduction of complexity also at early times.

QGP and pre-hydrodynamic attractors

- Pre-hydrodynamic attractors originate from the specific kinematic conditions of heavy-ion collisions (initial dominance of the longitudinal expansion)
- They have been identified in diverse dynamical settings:
  - Hydrodynamic models
  - Kinetic theory (weakly-coupled quasiparticles)
  - Strongly-coupled supersymmetric Yang-Mills theory (AdS/CFT)
- They provide a simple, semi-analytic picture of how information contained in the initial state is relayed to the freeze-out stage
- The partial loss of memory of initial conditions can be understood in terms of non-hydrodynamic modes

Perturbations of equilibrium
at the linearised level

- Systems perturbed out of equilibrium typically return to it:

$$\mathscr{L}\delta\Phi = 0 \qquad \Longrightarrow \qquad \delta\Phi \sim e^{-i\omega t + ikz}$$

- Hydrodynamic (long-lived, long-wavelength) modes such as

$$\omega = c_s k - \frac{i\Gamma_s}{2} k^2 + \ldots$$

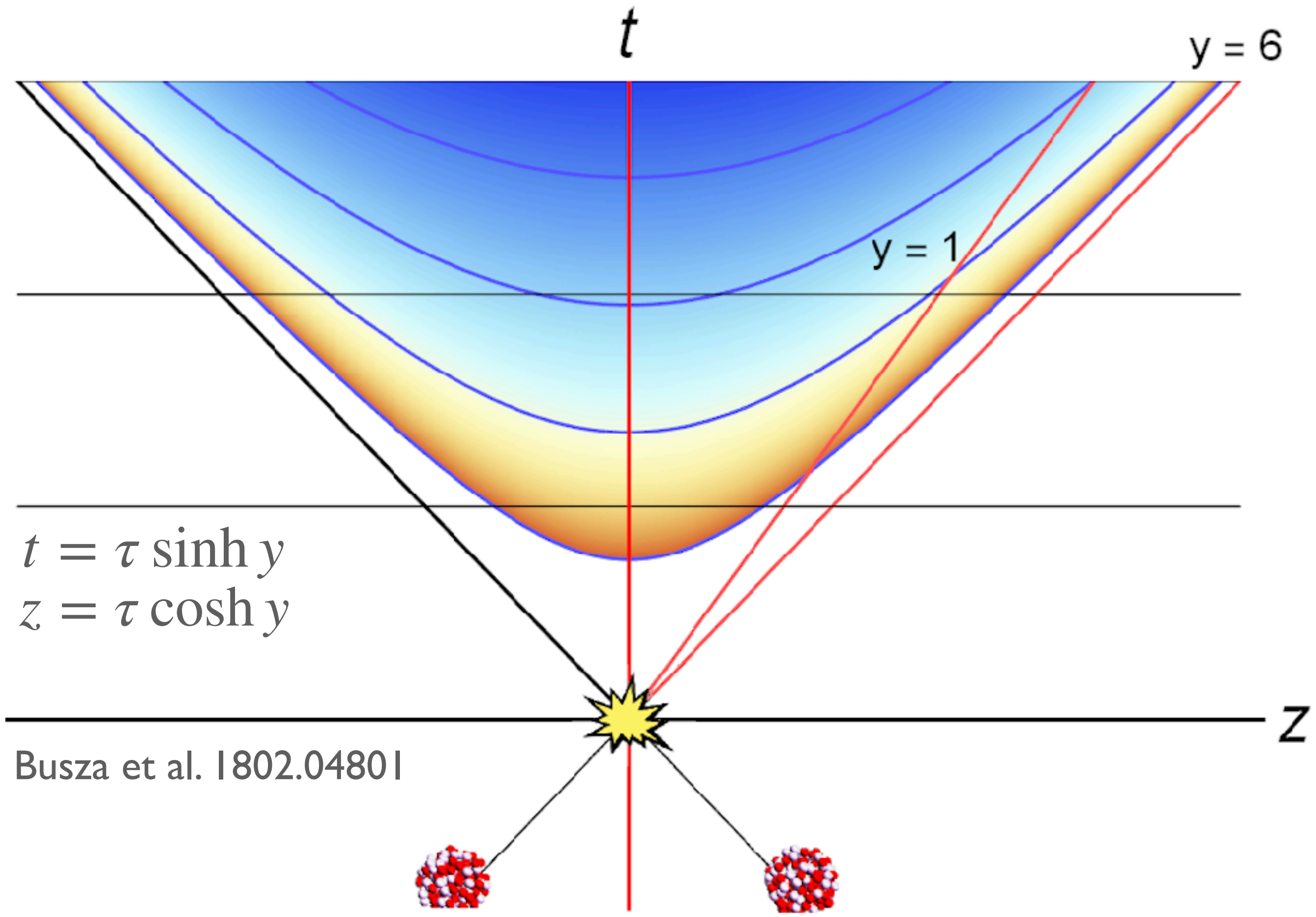
- Non-hydrodynamic (transient) modes such as

$$\omega = -\frac{i}{\tau_R} + \ldots$$

- A causal theory with viscosity must include some number of such transient (non-hydrodynamic) excitations

Attractors in Bjorken flow

in conformal models



Busza et al. 1802.04801

$$(T^{\mu}_{\nu}) = \mathrm{diag}(-\mathcal{E}, \mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_T)$$

Expressed in terms of 2 functions of proper time:

$$\mathcal{P}_L \equiv \frac{\mathcal{E}}{3}\left(1 - \frac{2}{3}\mathcal{A}\right), \quad \mathcal{P}_T \equiv \frac{\mathcal{E}}{3}\left(1 + \frac{1}{3}\mathcal{A}\right)$$

Conservation of the energy-momentum tensor:

$$\frac{d \log T}{d \log w} = \frac{\mathcal{A} - 6}{\mathcal{A} + 12}$$

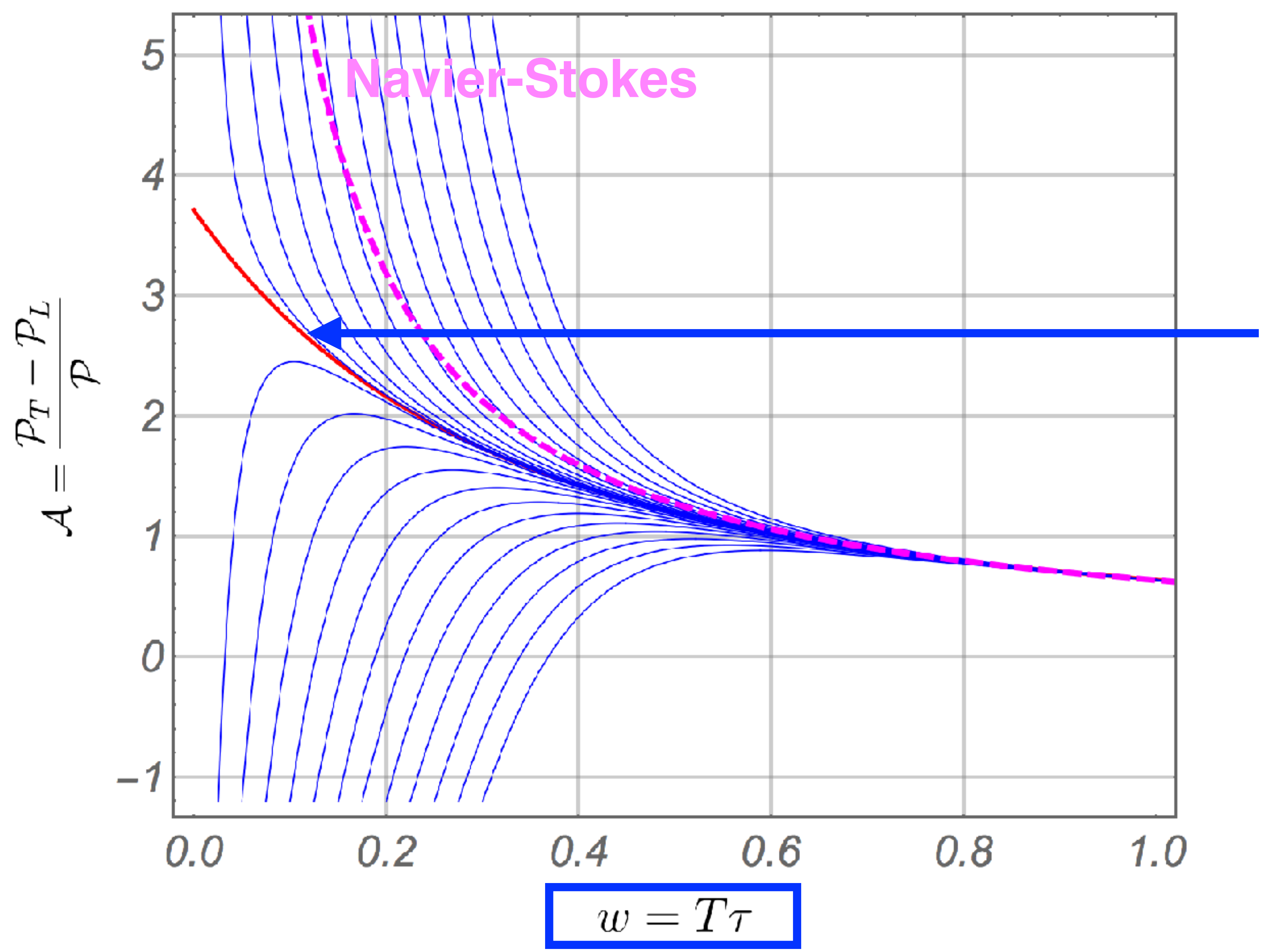
$$\mathcal{E}(\tau) \sim T(\tau)^4, \quad w \equiv \tau T$$

- The conservation equation requires a single integration constant
- Remaining initial data is contained in $\mathcal{A}(w)$
- In many models $\mathcal{A}(w)$ follows a universal attractor

The attractor in Bjorken flow
in conformal Mueller-Israel-Stewart theory

$$C_\tau\left(1+\frac{\mathcal{A}}{12}\right)\mathcal{A}' + \frac{C_\tau}{3w}\mathcal{A}^2 = \frac{3}{2}\left(\frac{8C_\eta}{w} - \mathcal{A}\right)$$

The pressure anisotropy satisfies this first order ODE, where
$C_\eta \equiv \eta/s, \quad C_\tau \equiv \tau_R T$



An attractor connects the early, far-from-equilibrium domain to the hydrodynamic region at late times

There is a rapid reduction of complexity initially, followed by a period of more moderate loss of memory

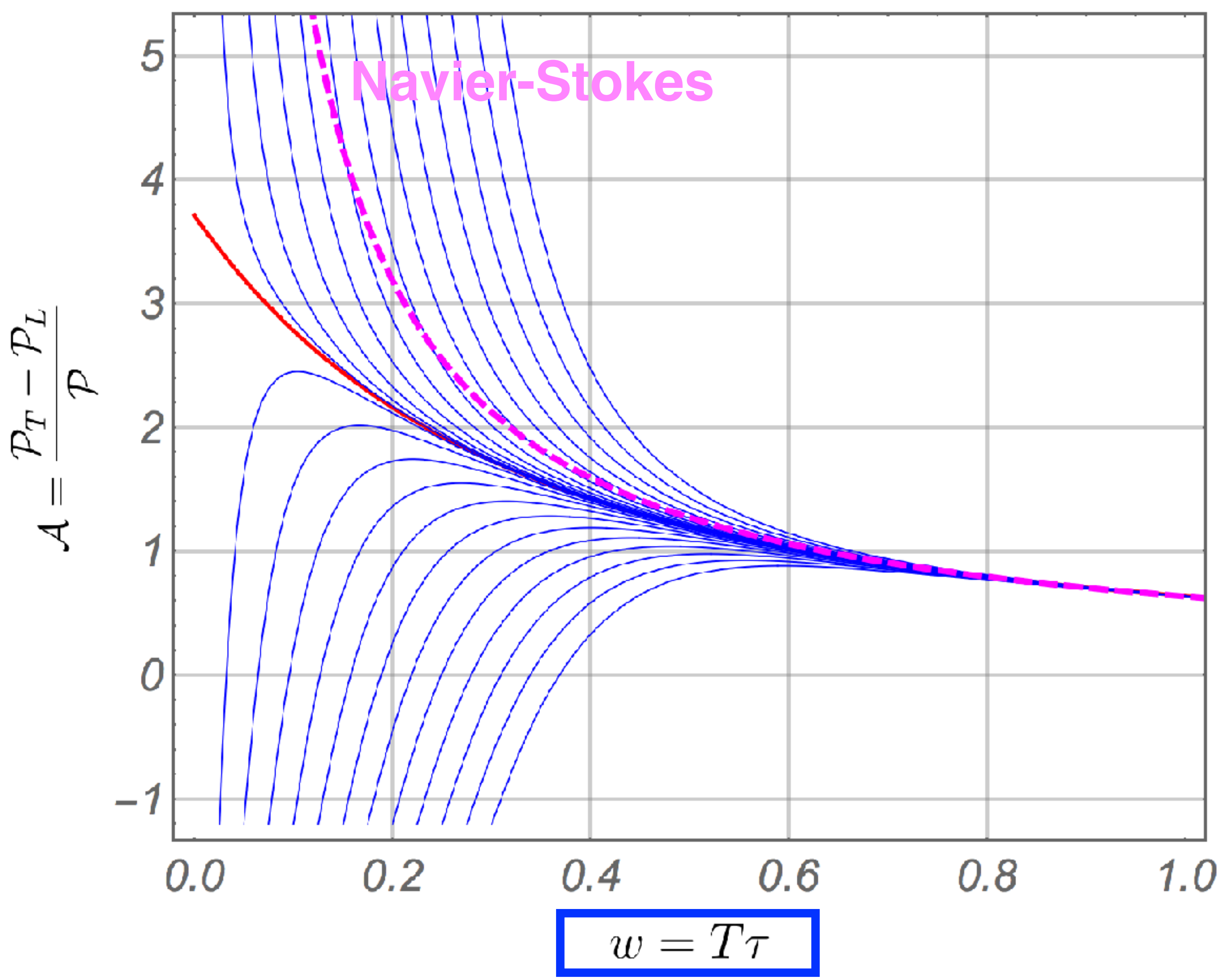
Solutions starting off the attractor reach its vicinity even if the pressure anisotropy is large so the system is still far from equilibrium.

The attractor – the late time asymptotic view

in conformal MIS

$$\mathscr{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0} \frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}}$$

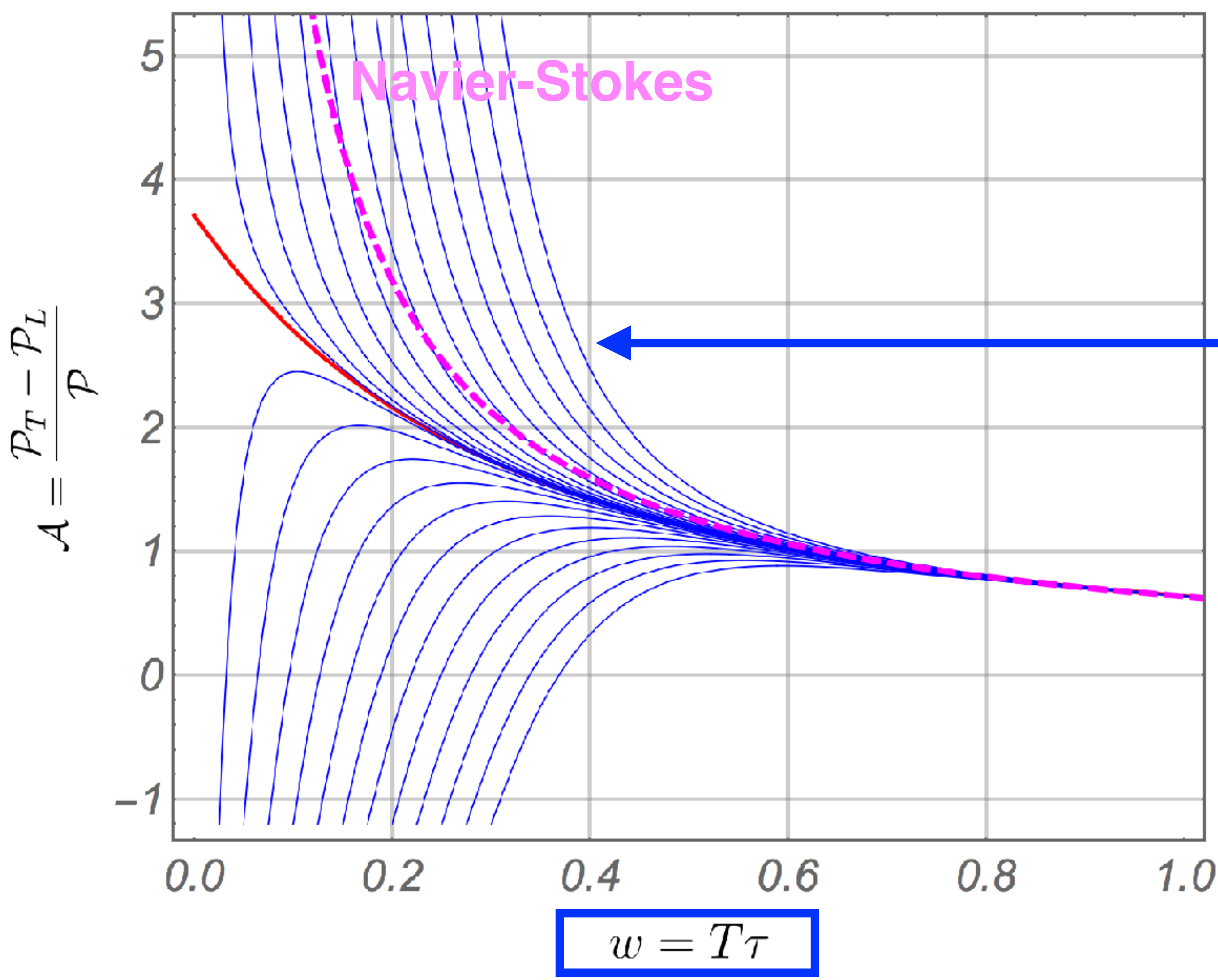
The expansion coefficients do not depend on initial conditions



At asymptotically late times there is no memory of the initial conditions

The attractor – the transseries view
in conformal MIS

$$\mathscr{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0} \frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}} + \underbrace{\left(\sigma\, w^{\frac{C_\eta}{C_\tau}} e^{-\frac{3}{2C_\tau}w}\right) \sum_{n\geq 0} \frac{a_n^{(1)}}{w^n}}_{\text{1st transseries sector}} + \ldots$$

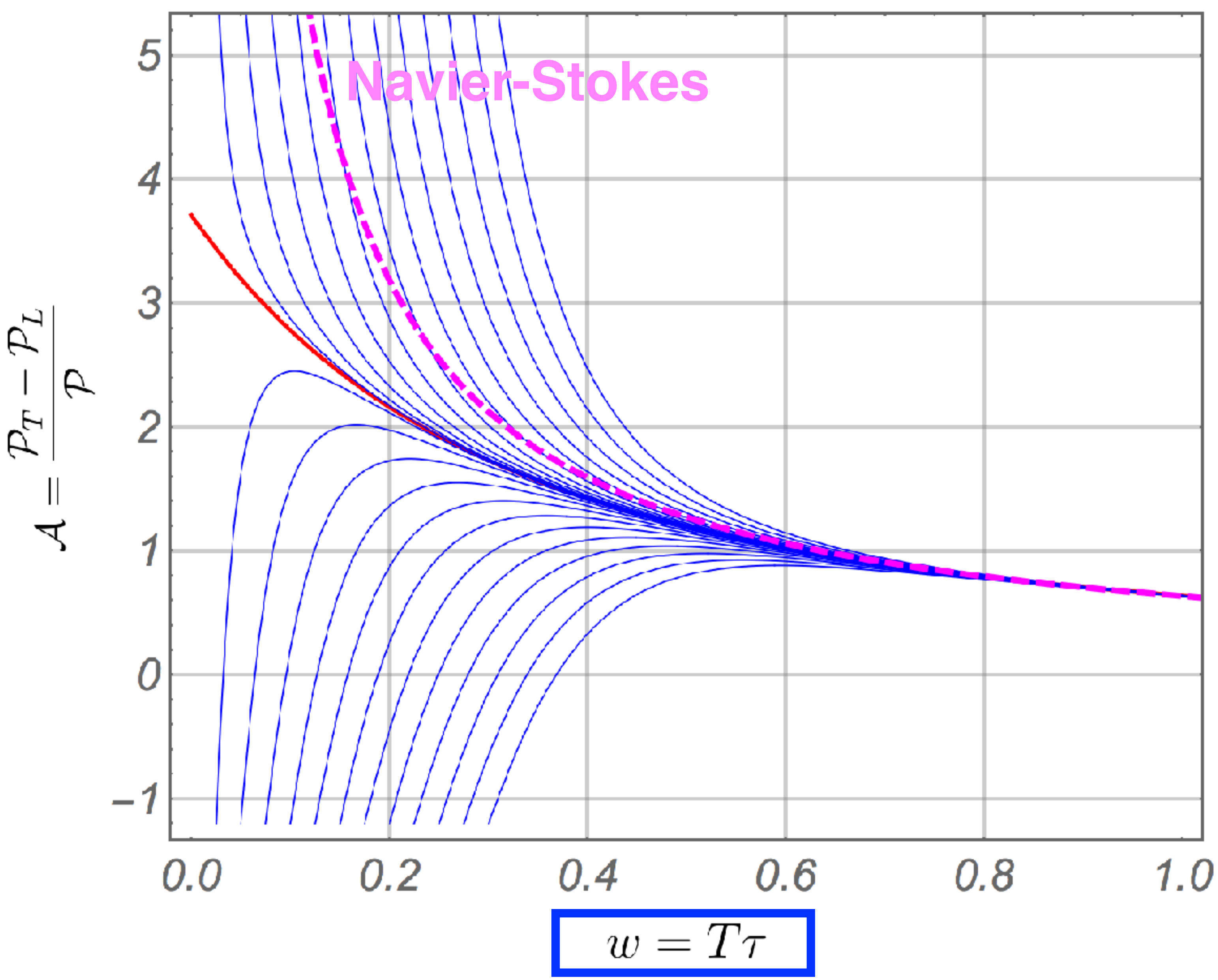


Different initial conditions are captured by exponentially-suppressed corrections to the asymptotic gradient series through the transseries parameter.

The scale of the exponential damping is set by the relaxation time, which is the non-hydrodynamic mode frequency.

The attractor – three stages

- Expansion-dominated early-time stage
- Pre-hydrodynamic stage (non-hydrodynamic mode decay)
- Asymptotic (hydrodynamic) stage



The expansion-dominated stage
depends weakly on model parameters
which points to its kinematic origin

The pre-hydrodynamic stage
depends on both the model parameters
and the initial state:
this is where freeze-out takes place

The asymptotic stage
is independent of initial conditions

Transverse dynamics as perturbations

a semi-analytic extension of the Bjorken model

- Most of the interesting physics involves transverse dynamics
- Dependence on transverse coordinates can be incorporated by linearising around the Bjorken attractor:

$$T(\tau, \mathbf{x}) = T(\tau) + \delta T(\tau, \mathbf{x}) = T(\tau)\Big(1 + \delta\hat{T}(\tau, \mathbf{x})\Big)$$

- Fourier modes

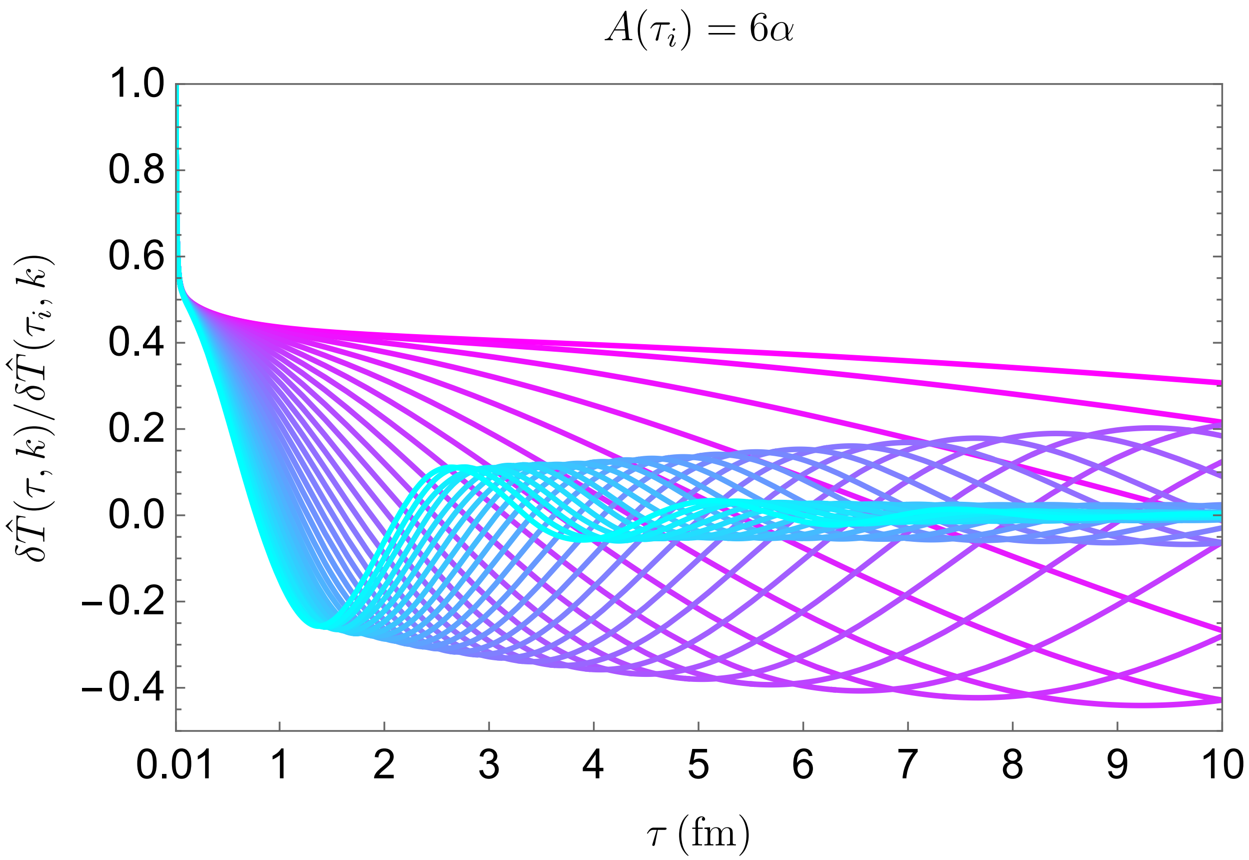
$$\hat{\phi}(\tau, \mathbf{x}) = \int \frac{d^2k}{(2\pi)^2}\, e^{i\mathbf{k}\cdot\mathbf{x}}\hat{\phi}(\tau, \mathbf{k})$$

- Linearised MIS equations: a set of 6 coupled ODEs for each **k**
- Initial states provide initial conditions for the modes

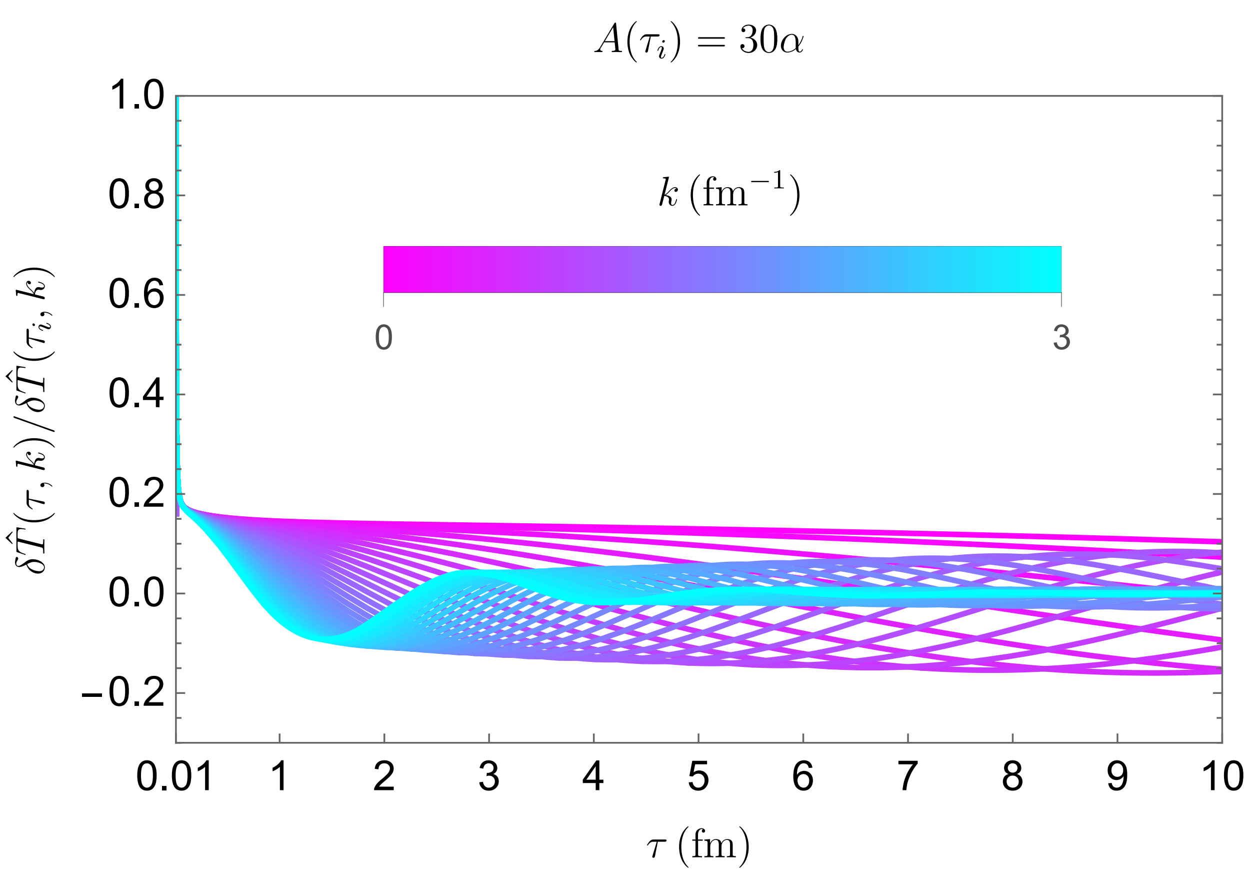
An, MS 2312.07703

Transverse dynamics as perturbations

stability of perturbations around the attractor



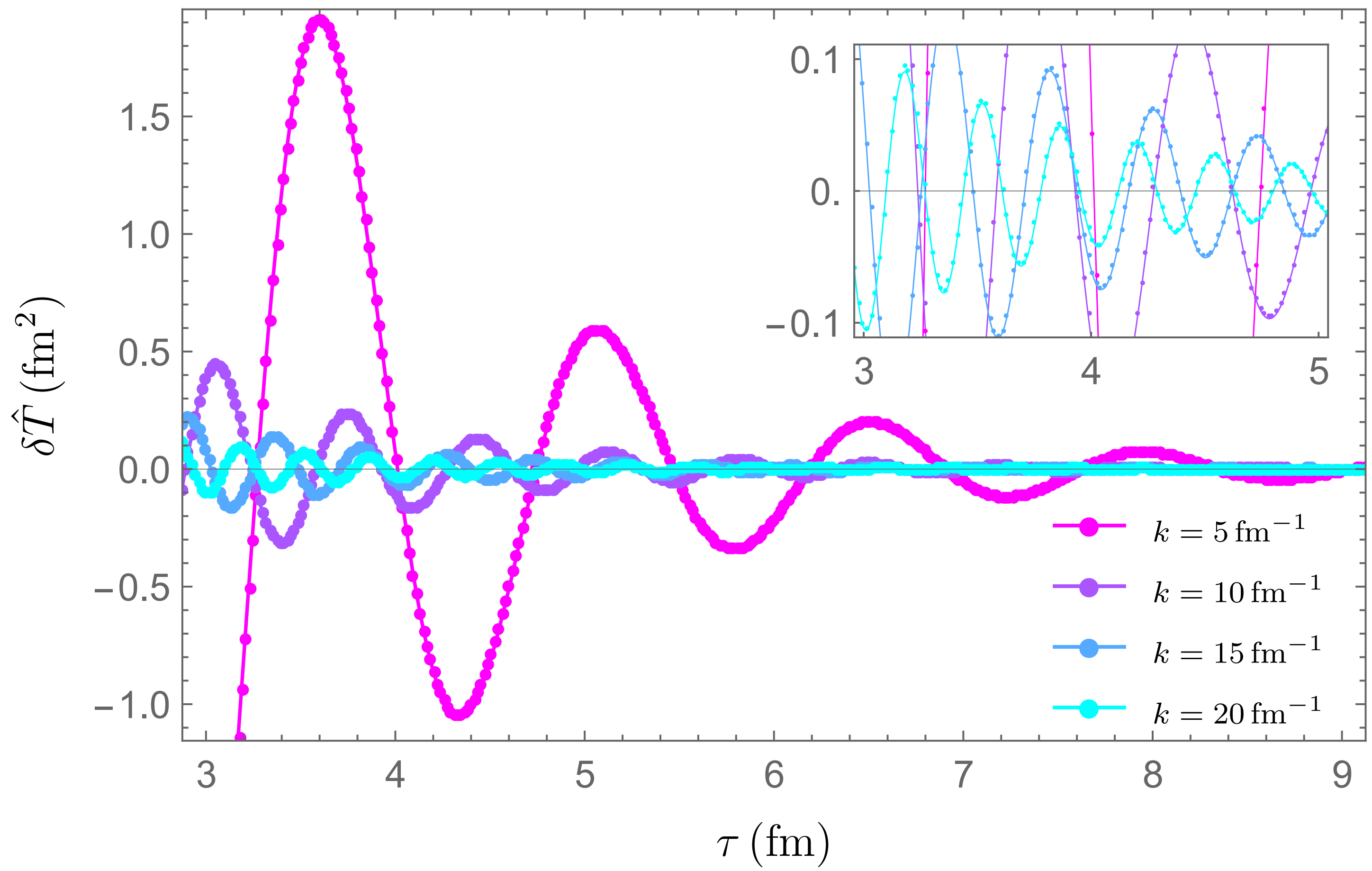
Perturbations initialised on the attractor



Perturbations initialised off the attractor

Transverse dynamics as perturbations

late time asymptotics



$$\delta\hat{T} = \sum_{i=1}^{4} \sigma_i (\Lambda\tau)^{\beta_i} e^{-i\omega_i\tau - A_i(\Lambda\tau)^{2/3}}(1+\ldots)$$

Different initial conditions are reflected by the amplitudes which determine the physics at freeze-out time

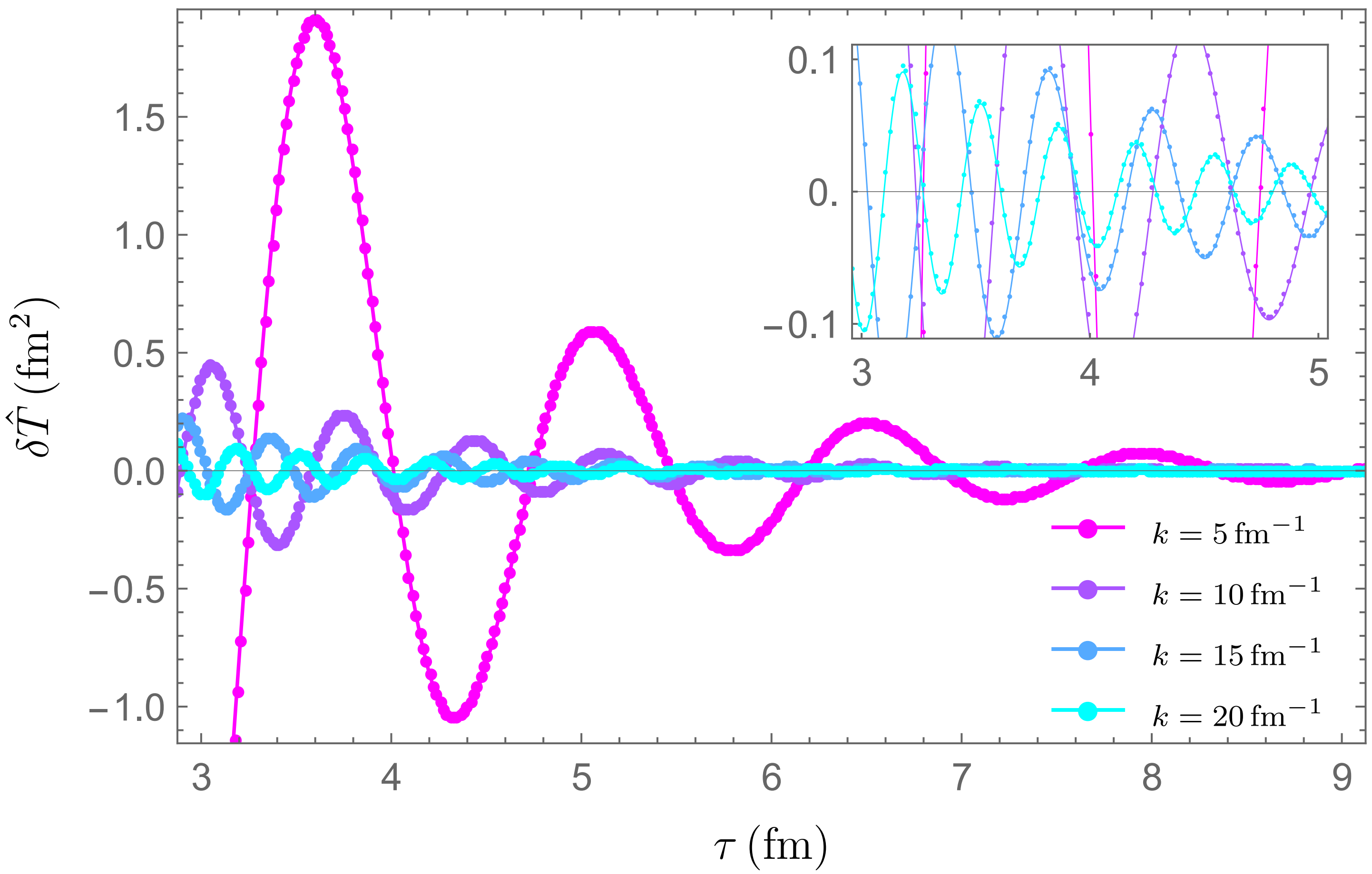
$$A_1 = A_2 = \frac{\alpha^2}{C_\tau c_\infty^2}, \quad A_3 = \frac{3}{2C_\tau}, \quad A_4 = \frac{1}{2C_\tau c_\infty^2}, \quad A_5 = A_6 = \frac{3}{4C_\tau},$$

$$\omega_1 = -\omega_2 = c_\infty k\left[1 + \frac{2\alpha^2}{3c_\infty^2}\left(2C_\tau(1-\alpha^2) - \frac{(1+\alpha^2)\Lambda^2}{C_\tau^2 c_\infty^4 k^2}\right)(\Lambda\tau)^{-2/3}\right], \quad \omega_3 = \omega_4 = 0$$

$$c_\infty \equiv \sqrt{\frac{1}{3}\left(1 + 4\frac{C_\eta}{C_\tau}\right)}, \qquad \alpha \equiv \sqrt{\frac{C_\eta}{C_\tau}}$$

Transverse dynamics as perturbations

late time asymptotics



$$\delta\hat{T} = \sum_{i=1}^{4} \sigma_i (\Lambda\tau)^{\beta_i} e^{-i\omega_i\tau - A_i(\Lambda\tau)^{2/3}} (1 + \ldots)$$

Large wave vector modes are damped more strongly than small wavelength modes

$$\beta_1 = \beta_2 = \frac{1}{54 c_\infty^4}\left(1 + 8\alpha^2 + 64\alpha^4 + 32\alpha^6 + \frac{4\alpha^2\Lambda^2}{C_\tau^3 c_\infty^4 k^2}\right),$$

$$\beta_3 = -\frac{2}{3}(1-\alpha^2),$$

$$\beta_4 = \frac{2\alpha^2}{27 c_\infty^4}\left(1 - 16\alpha^2 - \frac{2\Lambda^2}{C_\tau^3 c_\infty^4 k^2}\right)$$

Transverse dynamics as perturbations

freeze-out and flow

- Flow is usually quantified in terms of coefficients in the expansion

$$\frac{dN(p_\perp,\phi)}{p_\perp dp_\perp d\phi dy} = v_0(p_\perp)\left(1+\sum_{n=1}^{\infty} 2v_n(p_\perp)\cos(n\phi)\right)$$

- These coefficients are can be expressed in terms of transverse averages of the perturbations:

$$v_0(\hat{p}_\perp) = \frac{m_\perp \tau_f}{(2\pi)^3}\Sigma_\perp \left[F_0 + F_1\langle\delta\hat{T}\rangle_\perp + F_{11}\langle\delta\hat{T}\delta\hat{T}\rangle_\perp + \frac{1}{2}\hat{p}_\perp^2\left(F_3\langle\delta\hat{\pi}_{ii}\rangle_\perp + F_{13}\langle\delta\hat{\pi}_{ii}\delta\hat{T}\rangle_\perp + F_{22}\langle\delta u_i\delta u_i\rangle_\perp\right)\right],$$

$$v_2(\hat{p}_\perp) = \frac{\hat{p}_\perp^2\left(F_3\langle\delta\hat{\pi}_{11}-\delta\hat{\pi}_{22}\rangle_\perp + F_{13}\langle(\delta\hat{\pi}_{11}-\delta\hat{\pi}_{22})\delta\hat{T}\rangle_\perp + F_{22}\langle\delta u_1^2-\delta u_2^2\rangle_\perp\right)}{4(F_0+F_1\langle\delta\hat{T}\rangle_\perp + F_{11}\langle\delta\hat{T}\delta\hat{T}\rangle_\perp) + 2\hat{p}_\perp^2\left(F_3\langle\delta\hat{\pi}_{ii}\rangle_\perp + F_{13}\langle\delta\hat{\pi}_{ii}\delta\hat{T}\rangle_\perp + F_{22}\langle\delta u_i\delta u_i\rangle_\perp\right)},$$

- Elliptic flow originates entirely from the exponentially-suppressed corrections which are still not negligible at freeze-out

Summary

- The physics of QGP has lead to foundational issues in the field of relativistic fluid dynamics
- The special kinematics characteristic of heavy-ion collisions leads to pre-hydrodynamic attractors
- Almost all physical observables in heavy ion physics can be interpreted as transseries corrections to the Bjorken attractor