Week 8. Problem set

1. Perform Heap-Sort [CLRS, §6.4] on the following input array:

2	3	0	4	7	1	5	8	6

   Show the state of the array after each call to Max-Heapify from the Heap-Sort procedure (solution must have 8 arrays).

2. A d-ary heap is similar to a binary heap, except non-leaf nodes have d children instead of 2 children (except the last non-leaf node, which is allowed to have fewer children). Adjust the array representation and the efficient implementations of Max-Heapify and Build-Max-Heap. **Write down** the generalized pseudocode of Max-Heapify and Build-Max-Heap for d-ary heaps for any d. Then, perform a variation of Heap-Sort [CLRS, §6.4] using a 4-ary heap on the following input array:

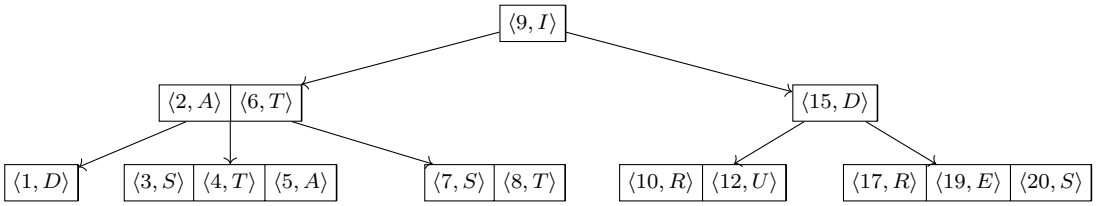
2	3	0	4	7	1	5	8	6

   Show the state of the array after each call to Max-Heapify from the Heap-Sort procedure (solution must contain 8 arrays and the pseudocode for generalized Max-Heapify and Build-Max-Heap).

3. Insert the $\langle \mathsf{key}, \mathsf{value} \rangle$ items into an empty B-tree [CLRS, §18] with minimum degree $t = 2$:

   (a) $\langle 32, I\rangle$, $\langle 17, I\rangle$, $\langle 9, X\rangle$, $\langle 21, C\rangle$, $\langle 11, E\rangle$
   (b) $\langle 2, A\rangle$, $\langle 28, E\rangle$, $\langle 36, G\rangle$, $\langle 3, E\rangle$, $\langle 4, T\rangle$
   (c) $\langle 18, B\rangle$, $\langle 13, E\rangle$, $\langle 6, T\rangle$, $\langle 7, R\rangle$, $\langle 37, I\rangle$
   (d) $\langle 26, L\rangle$, $\langle 33, N\rangle$, $\langle 20, U\rangle$, $\langle 24, I\rangle$, $\langle 30, D\rangle$

   Show the state of the tree after every 5 insertions. Depict each tree as a sequence of arrays for each layer. For example, consider this B-tree:



   The tree above must be depicted as follows:

(layer 1) $[\langle 9, I\rangle]$
(layer 2) $[\langle 2, A\rangle | \langle 6, T\rangle]$ $[\langle 15, D\rangle]$
(layer 3) $[\langle 1, D\rangle]$ $[\langle 3, S\rangle | \langle 4, T\rangle | \langle 5, A\rangle]$ $[\langle 7, S\rangle | \langle 8, T\rangle]$ $[\langle 10, R\rangle | \langle 12, U\rangle]$ $[\langle 17, R\rangle | \langle 19, E\rangle | \langle 20, S\rangle]$

References

[CLRS] Cormen, T.H., Leiserson, C.E., Rivest, R.L. and Stein, C., 2022. *Introduction to algorithms, Fourth Edition*. MIT press.

[GTG] M. T. Goodrich, R. Tamassia, and M. H. Goldwasser. *Data Structures and Algorithms in Java*. WILEY 2014.