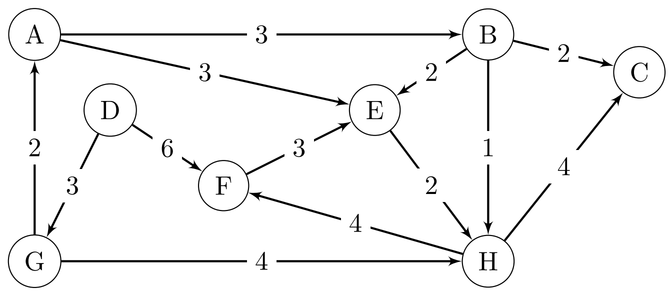
Week 13. Problem set

1. Run Edmonds-Karp algorithm [CLRS, § 24.2] on the given network:
   (a) Identify the *source* and the *sink* of the network.
   (b) For every iteration of the algorithm
      i. write down the *augmenting path* (sequence of vertices),
      ii. show the *residual network* **after** the iteration
   (c) Write down the *maximum flow* value after the last iteration.
   (d) Show that the flow is maximum by demonstrating a *minimum cut* of the network (as two sets of vertices).



2. Suppose that in addition to edge capacities, a flow network $\mathcal{N} = \langle V, E \rangle$ has *vertex capacities*: each vertex $v \in V$ has a limit $\ell(v)$ on how much flow can pass through v.
   (a) Show how to transform $\mathcal{N}$ into an equivalent flow network $\mathcal{N}' = \langle V', E' \rangle$ without vertex capacities, such that maximum flow in $\mathcal{N}'$ has the same value as the maximum flow in $\mathcal{N}$.
   (b) What is the size of V'?
   (c) What is the size of E'?

3. Consider a flow network $\mathcal{N}$ with integer capacities and known maximum flow.
   (a) Suppose the capacity of a single edge $\langle u, v \rangle$ is increased by 1. Describe a $O(V+E)$ algorithm to update the maximum flow.
   (b) Suppose the capacity of a single edge $\langle u, v \rangle$ is decreased by 1. Describe a $O(V+E)$ algorithm to update the maximum flow.

References

[CLRS] Cormen, T.H., Leiserson, C.E., Rivest, R.L. and Stein, C., 2022. *Introduction to algorithms, Fourth Edition*. MIT press.