INNOPOLIS UNIVERSITY

Week 3. Problem set

1. Consider a hash table with 13 slots and the hash function $h(k) = (k^2 - k + 11) \mod 13$. Show the state of the hash table after inserting the keys (in this order)

   $$6,\ 28,\ 19,\ 15,\ 20,\ 33,\ 12,\ 17,\ 10,\ 13,\ 3,\ 39$$

   with collisions resolved by *linear probing* [CLRS, §11.4].

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Key													

2. Consider a dictionary that maps an integer (e.g. group number) and a short[1] string (e.g. student email) to a number (e.g. grade), and is implemented as a hashtable T of hashtables:

   - for an integer k, if T contains k, then $T[k]$ is a hashmap with string keys
   - for a string s, if $T[k]$ contains s, then $T[k][s]$ is a floating point number

   Compute average case time complexities of successful and unsuccessful search in this hashtable of hashtables with the different possible combinations of chaining and open addressing, assuming

   (a) *independent uniform hashing* for hashtables with separate chaining [CLRS, §11.2]

   (b) *independent uniform permutation hashing* for hashtables with open addressing [CLRS, §11.4]

   (c) for the hashtable T, load factor α and size n

   (d) for the hashtables $T[k]$, average load factor β and average size m

	Successful search	Unsuccessful search
T uses separate chaining and each $T[k]$ uses separate chaining		
T uses separate chaining and each $T[k]$ uses open addressing		
T uses open addressing and each $T[k]$ uses separate chaining		
T uses open addressing and each $T[k]$ uses open addressing		

3. **(+0.5% extra credit)** Compute the average case time complexity of the following algorithm:

```
1  secret(M, k):
2    Q := new linked queue
3    while M.hasKey(k) and k > 0
4      item := M.get(k)
5      k := min(item.value, k) - 1
6      if k == item.value - 1
7        Q.offer(item)
8    while not Q.isEmpty()
9      M.remove(Q.poll())
10   return k
```

   M is a hashtable of size n with load factor α that resolves collisions by *separate chaining* [CLRS, §11.2]. The answer **must** use O-notation and may depend on n, k, and α. Assume *independent uniform hashing* and worst case for the contents (values) of the input map M.

   Justify your answer (3–5 sentences). Detailed proof is not required.

[1] Assume that strings have a relatively small constant maximum size.

References

[CLRS] Cormen, T.H., Leiserson, C.E., Rivest, R.L. and Stein, C., 2022. *Introduction to algorithms, Fourth Edition*. MIT press.