Week 1. Problem set

1. Compute asymptotic worst case time complexity of the following algorithm (see pseudocode conventions in [CLRS, §2.1]). You **must** use Θ-notation. For justification, provide execution cost and frequency count for each line in the body of the `secret` procedure. Optionally, you may provide the details for the computation of the running time $T(n)$ for worst case scenario. Proof for the asymptotic bound is not required for this exercise.

```
1  /* A is a 0-indexed array,
2   * n is the number of items in A */
3  secret(A, n):
4    for i = 1 to n
5      s := 1
6      j := i - 1
7      while j < n and A[j] ≥ A[i-1]
8        j := j + s
9        s := s + 2
10     exchange A[i-1] with A[min(j, n - 1)]
```

Line	Execution Cost	Frequency Count
1	—	—
2	—	—
3	—	—
4		
5		
6		
7		
8		
9		
10		

2. Indicate, for each pair of expressions (A, B) in the table below whether $A = O(B)$, $A = o(B)$, $A = \Omega(B)$, $A = \omega(B)$, or $A = \Theta(B)$. Write your answer in the form of the table with *YES* or *NO* written in each cell:

A	B	$A = O(B)$	$A = o(B)$	$A = \Omega(B)$	$A = \omega(B)$	$A = \Theta(B)$
$1 + \sin(n)$	$\left(1 + \frac{1}{n}\right)^{\frac{1}{n}}$					
$\sqrt[5]{n}$	$\log_3^2 n$					
n^2	$\log_2^n n$					
$\left(1 + \frac{1}{10^{100}}\right)^n$	$n^{(10^{100})}$					
$\log_2\left((n+1)!\right)$	$n \log_2 n$					

3. Let f and g be functions from positive integers to positive reals. Assume $f(n) > n$ for $n > 2025$. Using the formal definition of asymptotic notation [CLRS, §3.2], prove *formally* that

$$\min(f(n) - \sqrt{n}, g(n) + n) = O(f(n) + g(n))$$

References

[CLRS] Cormen, T.H., Leiserson, C.E., Rivest, R.L. and Stein, C., 2022. *Introduction to algorithms, Fourth Edition.* MIT press.