41. $x_0=2 \quad y_0=3$

$x_1=7 \quad y_1=2$

$x_2=10 \quad y_2=4$

a) $Ax=y$; $A=\begin{pmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{pmatrix}$; $y=\begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix}$

$Ax=y$; $\begin{pmatrix} 1 & 2 & 4 \\ 1 & 7 & 49 \\ 1 & 10 & 100 \end{pmatrix}\begin{pmatrix} t_0 \\ t_1 \\ t_2 \end{pmatrix}=\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

$\begin{cases} t_0+2t_1+4t_2=3 \\ t_0+7t_1+49t_2=2 \\ t_0+10t_1+100t_2=4 \end{cases}$ $\begin{cases} 3-2t_1-4t_2+7t_1+49t_2=2 \\ t_0+10t_1+100t_2=4 \end{cases}$

$\begin{cases} 5t_1+45t_2=-1 \\ 8t_1+96t_2=1 \end{cases}$ $-120t_2=-13$; $t_2=\frac{13}{120}$

$5t_1+45\cdot\frac{13}{120}=-1$; $t_1=-\frac{47}{40}$

$t_0=3-2\cdot\left(-\frac{47}{40}\right)-4\cdot\frac{13}{120}=\frac{59}{12}$

$x=\begin{pmatrix} t_0 \\ t_1 \\ t_2 \end{pmatrix}=\begin{pmatrix} \frac{59}{12} \\ -\frac{47}{40} \\ \frac{13}{120} \end{pmatrix}$; $M(x)=\frac{59}{12}-\frac{47}{40}x+\frac{13}{120}x^2$

b) $l_i(t)=\dfrac{\prod_{k=1,k\neq i}^{n}(t-t_k)}{\prod_{k=1,k\neq i}^{n}(t_i-t_k)}$, $i=1,\dots,n$;

$l_0(x)=\dfrac{(x-7)(x-10)}{(2-7)(2-10)}=\dfrac{(x-7)(x-10)}{40}$

$l_1(x)=\dfrac{(x-2)(x-10)}{(7-2)(7-10)}=\dfrac{(x-2)(x-10)}{15}$

$l_2(x)=\dfrac{(x-2)(x-7)}{(10-2)(10-7)}=\dfrac{(x-2)(x-7)}{24}$;

$P(x)=y_0l_0(x)+y_1l_1(x)+y_2\dots$

$=3\cdot\dfrac{(x-7)(x-10)}{40}-2\cdot\dfrac{(x-2)(x\dots}{15}$

$+\dfrac{(x-2)(x-7)}{6}$;

c) a)

b)

d) Aus c) sehen wir, dass $M(x) = P(x)$:

$\cancel{M(x)}$

$M(3) = \frac{213}{90} = \frac{71}{30} \approx 2,367$

$M(5) = \frac{7}{4} = 1,75$

$M(8) = \frac{29}{20} = 1,45$

42) $x_0 = -1 \quad y_0 = 0$

$x_1 = 1 \quad y_1 = 2$

$x_2 = 2 \quad y_2 = 4$

a) $\pi_j(x) := \prod_{k=1}^{j-1} (t - t_k), \; j = 1, \dots, n$

$p_{n-1}(x) = t_1 + t_2(x - x_1) + t_3(x - x_1)(x - x_2) + \dots$

$\dots + t_n(x - x_1)(x - x_2)\dots(x - x_{n-1})$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & x_1 - x_0 & 0 \\ 1 & x_2 - x_0 & (x_2 - x_0)(x_2 - x_1) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix}$$

$$Ax = y: \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} t_0 \\ t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}; \quad t_0 = 0, \; t_1 = 1, \; t_2 = \frac{1}{3}$$

$p_2(x) = 0 + 1(x+1) + \frac{1}{3}(x+1)(x-1)$

b)

c) $p_2(0) = 1 \cdot 1 + \frac{1}{3} \cdot 1 \cdot (-1) = \frac{2}{3}$

43) $x_0 = -1,\ y_0 = 0$

$x_1 = 1 \quad y_1 = 2$

$x_2 = 2 \quad y_2 = 4$

a) $f[x_1, x_2 \dots, x_k] = \dfrac{f[x_2, x_3 \dots, x_3] - f[x_1, x_2 \dots x_{k}]}{x_k - x_1}$

$f[x_0] = f[-1] = y_0 = 0$

$f[x_1] = f[1] = 2;\ f[x_0, x_1] = \dfrac{f[x_1] - f[x_0]}{x_1 - x_0} = \dfrac{2-0}{1+1}$

$f[x_2] = 4;\ f[x_1, x_2] = \dfrac{f[x_2] - f[x_1]}{x_2 - x_1} = 2;\ f[x_0, x_1, x_2] =$

$= \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \dfrac{2-1}{2+1} = \frac{1}{3}$

b) $x_3 = 3;\ y_3 = 2$

$f[x_3] = 2;\ f[x_2, x_3] = \dfrac{f[x_3] - f[x_2]}{x_3 - x_2} = \dfrac{2-4}{1} = -2$

$f[x_1, x_2, x_3] = \dfrac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \dfrac{-2-2}{2} = -2$

$f[x_0, x_1, x_2, x_3] = \dfrac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} =$

$= \dfrac{-2 - \frac{1}{3}}{3+1} = -\dfrac{7}{12}$

$p_3(x) = (x+1) + \frac{1}{3}(x+1)(x-1) - \frac{7}{12}(x+1)(x-1)(x-2)$

c) $p_3(0) = \frac{2}{3} - \frac{7}{12}(1)(-1)(-2) = -\frac{1}{2}$

d)

46.

$$f(x)=\begin{cases} a(x-2)^2+b(x-1)^3 & x\in(-\infty;1] \\ c(x-2)^2 & x\in[1,3] \\ d(x-2)^2+e(x-3)^3 & x\in[3,\infty) \end{cases}$$

$f(1)=1 \quad f'''(0)=6 \quad f'''(4)=6$

$a(1-2)^2+b(1-1)^3=c(1-2)^2$

$a\cdot(-1)^2 = c\cdot(-1)^2=1;\ a,c=1$

1) $f'(x)=2ax-4a+3bx^2-6bx+3b$

$f''(x)=2a+6bx-6b$

$f'''(x)=6b=6;\ b=1$

3) $f'''(x)=6e=6;\ e=1$

$f(3)=d(3-2)^2+(3-3)^2=(3-2)^2;\ d=1$

$$f(x)=\begin{cases}(x-2)^2+(x-1)^3 & x\in(-\infty;1] \\ (x-2)^2 & x\in[1;3] \\ (x-2)^2+(x-3)^3 & x\in[3;\infty)\end{cases}$$

überprüfen:

$f(1)=(1-2)^2+(1-1)^3=(1-2)^2$ V

$f(3)=(3-2)^2=(3-2)^2+(3-3)^2$ V

$$f'(x)=\begin{cases}2(x-2)+3(x-1)^2 & x\in(-\infty;1] \\ 2(x-2) & x\in[1;3] \\ 2(x-2)+3(x-3)^2 & x\in[3;\infty)\end{cases}$$

$f'(1)=2(1-2)+3(1-1)^2=2(1-2)$ V

$f'(3)=2(3-2)=2(3-2)+3(3-3)^2$ V

$$f''(x)=\begin{cases}2+6(x-1) & x\in(-\infty;1] \\ 2 & x\in[1;3] \\ 2+6(x-3) & x\in[3;\infty)\end{cases}$$

$f''(1)=2+6(1-1)=2$ V; $f''(3)=2=2+6$