Aufgabe 2

$|\varphi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$

1) $|\varphi_1\rangle = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\ 0\end{pmatrix} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}0\\ 1\end{pmatrix} =$

$= \frac{1}{2\sqrt{2}}\begin{pmatrix}1\cdot 1 + 1\cdot 0\\ 1\cdot 1 + (-1)\cdot 0\end{pmatrix} + \frac{\sqrt{3}}{2\sqrt{2}}\begin{pmatrix}1\cdot 0 + 1\cdot 1\\ 1\cdot 0 + (-1)\cdot 1\end{pmatrix} =$

$= \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} + \frac{\sqrt{3}}{2\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix} = \frac{1}{2\sqrt{2}}\left(\begin{pmatrix}1\\ 0\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\right) + \frac{\sqrt{3}}{2\sqrt{2}}\left(\begin{pmatrix}1\\ 0\end{pmatrix} - \begin{pmatrix}0\\ 1\end{pmatrix}\right) =$

~~$|\varphi_2\rangle = \frac{\sqrt{3}}{2\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{\sqrt{3}}{2\sqrt{2}}\begin{pmatrix}-1\\ 1\end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} =$~~

$= \frac{1}{2\sqrt{2}}\left(|0\rangle + |1\rangle\right) + \frac{\sqrt{3}}{2\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

2) $|\varphi_2\rangle = \frac{\sqrt{3}}{2\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{\sqrt{3}}{2\sqrt{2}}\begin{pmatrix}-1\\ 1\end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} =$

$= \frac{\sqrt{3}}{2\sqrt{2}}\left(\begin{pmatrix}0\\ 1\end{pmatrix} - \begin{pmatrix}1\\ 0\end{pmatrix}\right) + \frac{1}{2\sqrt{2}}\left(\begin{pmatrix}1\\ 0\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\right) = \frac{\sqrt{3}}{2\sqrt{2}}\left(|1\rangle - |0\rangle\right) + \frac{1}{2\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

3) $|\varphi_3\rangle = \frac{\sqrt{3}}{2\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}-1\\ 1\end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}\cdot\frac{1}{\sqrt{2}} =$

$= \frac{\sqrt{3}}{4}\begin{pmatrix}1\cdot(-1) + 1\cdot 1\\ 1\cdot(-1) + (-1)\cdot 1\end{pmatrix} + \frac{1}{4}\begin{pmatrix}1\cdot 1 + 1\cdot 1\\ 1\cdot 1 + (-1)\cdot 1\end{pmatrix} =$

$= \frac{1}{4}\begin{pmatrix}2\\ 0\end{pmatrix} + \frac{\sqrt{3}}{4}\begin{pmatrix}0\\ -2\end{pmatrix} = \frac{1}{4}\cdot 2\begin{pmatrix}1\\ 0\end{pmatrix} + \frac{\sqrt{3}}{4}(-2)\begin{pmatrix}0\\ 1\end{pmatrix} =$

$= \frac{1}{2}|0\rangle - \frac{\sqrt{3}}{4}|1\rangle$

2) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\cdot 0 + 1\cdot 1 & 1\cdot 1 + 1\cdot 0\\ 1\cdot 0 + (-1)\cdot 1 & 1\cdot 1 + (-1)\cdot 0\end{pmatrix}\cdot\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} =$

$= \begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \begin{pmatrix}1\cdot 1 + 1\cdot 1 & 1\cdot 1 + 1\cdot(-1)\\ -1\cdot 1 + 1\cdot 1 & -1\cdot 1 + 1\cdot(-1)\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix} = \frac{1}{2}\cdot 2\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} =$

$= \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

(Zeilen: $\frac{1}{2}$, $\frac{1}{2}$)

3) Entweder 0 oder 1
Wahrscheinlichkeit $= \frac{1}{4}$ Wahrscheinlichkeit $= \frac{3}{4}$