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This signal at a time t is the sum of scaled ExmpMG (exponentially modified gaussian) at every time step

$$s(t) = \sum_{i=0}^{T/\Delta t} \sum_{j=0}^{N} GAexpmg(t-i)$$

With $A \sim AmpDistrib(1, \sigma_A^2)$, and $N \sim Pois(\lambda_N)$
We can start by calculating the expected value of a random point ζ

$$s(\zeta) = \sum_{i=0}^{T/\Delta t} \sum_{j=0}^{N} GAexpmg(\zeta - i - \mu)$$

With μ, the offset of the Exmpg

$$E(s(\zeta)) = E[\sum_{i=0}^{T/\Delta t} \sum_{j=0}^{N} GAexpmg(\zeta - i - \mu)] = \sum_{i=0}^{T/\Delta t} E[\sum_{j=0}^{N} GAexpmg(\zeta - i - \mu)]$$

$$= G \sum_{i=0}^{T/\Delta t} E(N)E(A)E[expmg(\zeta - i - \mu)]$$

$$= G \sum_{i=0}^{T/\Delta t} \lambda_N expmg(\zeta - i - \mu)$$

$$= G f_{NSB} \sum_{i=0}^{T/\Delta t} \Delta t expmg(\zeta - i - \mu)$$

$$= G f_{NSB} J_1$$

With $J_1 = \int_{-\infty}^{\infty} expmg(x)dx$ the area of a single PE

Same thing with the variance

$$Var(s(\zeta)) = Var[\sum_{i=0}^{T/\Delta t} \sum_{j=0}^{N} GAexpmg(\zeta - i - \mu)]$$

Since there is no covariance :

$$= \sum_{i=0}^{T/\Delta t} Var[\sum_{j=0}^{N} GAexpmg(\zeta - i - \mu)]$$

$$= \sum_{i=0}^{T/\Delta t} \Big(E[N]Var[GAexpmg(\zeta - i - \mu)] + Var(N)E[GAexpmg(\zeta - i - \mu)]^2 \Big)$$

$$= \Delta t f_{NSB} \sum_{i=0}^{T/\Delta t} \Big(Var[GAexpmg(\zeta - i - \mu)] + E[GAexpmg(\zeta - i - \mu)]^2 \Big)$$

$$= f_{NSB} G^2 (\sigma_A^2 + 1) \sum_{i=0}^{T/\Delta t} \Delta t expmg(\zeta - i - \mu)^2$$

$$= f_{NSB} G^2 (\sigma_A^2 + 1) J_2$$

With $J_2 = \int_{-\infty}^{\infty} expmg(x)^2 dx$
Thus we have

$$\frac{Var(s(\zeta))}{E(s(\zeta))} = \frac{f_{NSB} G^2 (\sigma_A^2 + 1) J_2}{G f_{NSB} J_1} = G \frac{(\sigma_A^2 + 1) J_2}{J_1}$$

Measurement We call $E'(s) = E(s) - \xi$
We define

$$\eta \equiv \frac{Var(s)}{E'(s) G} = \frac{(\sigma_A^2 + 1) J_2}{J_1}$$

The integral of the pulse shape can be calculated and yields

$$J_1 = A e^{\lambda^2 \sigma^2}$$

$$J_2 = \frac{A^2 \lambda}{2} e^{\lambda^2 \sigma^2}$$

Then the gain is obtained with

$$\frac{Var(s)}{E'(s)} \frac{J_1}{J_2 (\sigma_A^2 + 1)} = G$$