

# KTH Challenge 2014 Solutions

April 13, 2014

# Jury

- Erik Aas (KTH)
- Oskar Werkelin Ahlin (Spotify)
- Per Austrin (KTH)
- Andreas Lundblad (Oracle)
- Ulf Lundström (KTH/Stanford)
- Lukáš Poláček (KTH/Spotify), head of jury
- Marc Vinyals (KTH)

# F – Falling Mugs

## Problem

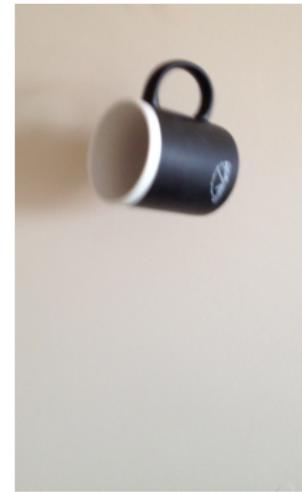
Find  $n_1$  and  $n_2$  such that  $n_2^2 - n_1^2 = D$ .

## Solution

- Want to solve:

$$(n + x)^2 - n^2 = 2xn + x^2 = D$$

- Note that  $x \leq \sqrt{D}$
- Try each  $x$ , solve for  $n$  in  $O(\sqrt{D})$  time
- Or try both for  $O(D^{3/2})$  time
- Or look at equation and conclude that we can take  $x \in \{1, 2\}$  for constant time



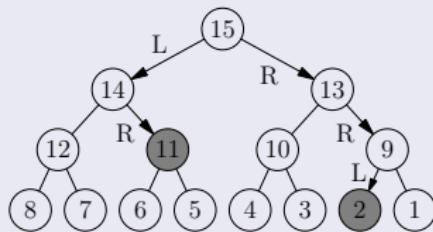
281 submissions, 48 correct, first at 0:12:05.

# A – Numbers on a tree

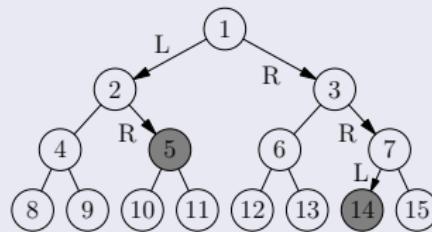
## Problem

Find the label of a node described by a path from the root

## Solution



(a) Original tree



(b)  $16 - x$

Replacing number  $x$  with  $2^{H+1} - x$  leads to the classic tree labelling.

121 submissions, 65 correct, first at 0:05:46.

# C – Cow Crane

## Problem

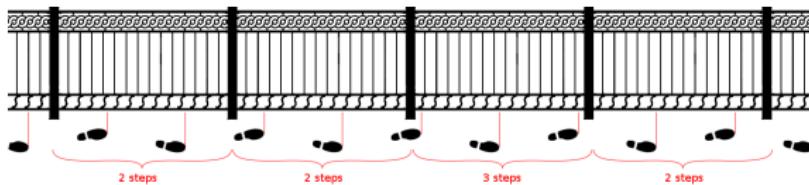
Check if the two cows can be picked up and moved such that they are placed at the new position before their meal.

## Solution

- You might want to temporarily drop off a cow at a certain position.
- Try all possible movements of the crane, check if one of them works.
- Or do it greedily - a couple of cases to think about to get it right.

Statistics: 88 submissions, 37 correct, first at 0:47:52.

# I – Count von Walken's Fence



## Problem

Check if distance between poles is feasible

## Solution

- Starting point  $0 < x < 1$  from first fence post
- For each  $i$  we get bounds on  $x$

$$x + \sum_{j=1}^i c_j < D \cdot i < 1 + x + \sum_{j=1}^i c_j$$

- Check lower bound smaller than upper bound

# G – Intercept

## Problem

Find vertices crossed by all shortest paths



## Solution

- Construct graph of shortest paths (Dijkstra)
- Find articulation vertices

## Solution

- Count how many shortest paths reach a vertex
- Too large number? Hash it!

58 submissions, 8 correct, first at 1:06:26.

# B – Absurdistan Roads II

## Problem

Calculate the probability that a particular random graph is connected



## Solution

- There are  $(n - 1)^n$  different graphs
- Let  $c_k$  be the number of connected graphs of size  $k$ .
- We have

$$c_k = (k - 1)^k - \sum_{i=2}^{k-2} c_i \binom{k-1}{i-1} (k-i-1)^{k-i}$$

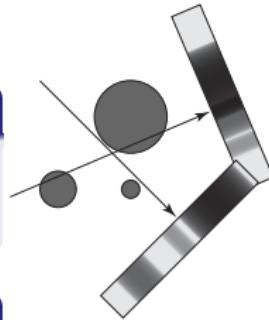
- The sum counts the number of disconnected graphs. Index  $i$  is the size of the component containing node 1.

9 submissions, ?? correct, first at 0:19:42.

# D – Tomosynthesis

## Problem

Find largest range of angles where no circles overlap.



## Solution

- Find overlap range for each pair of circles.
- Go through the sorted list of the starts and ends of these intervals to find ranges of no overlap.

Statistics: 2 submissions, 2 correct, first at 1:06:02.

# H – Radar

## Problem

Find the smallest  $z$  within  $y_i$  of  $x_i$  modulo  $m_i$  for  $i = 1, 2, 3$ .



## Solution

- Smallest  $z$  will have  $z \equiv x_i - y_i \pmod{m_i}$  for some  $i = 1, 2, 3$
- Try all  $O(y^2)$  possibilities for the remaining two equations
- Solve modular congruences with Chinese Remainder Theorem
- Special case:  $z = 0!$

40 submissions, ?? correct, first at 2:37:50.

# E – Pizza Problems

## Problem

Find choice of toppings so that everyone gets  $> 1/3$  of their wishes, assuming it is possible to make everyone get  $\geq 2/3$  of their wishes.



## Solution

- Flip choice for a random unsatisfied wish for some friend who is not yet happy.
- With probability  $\geq 1/2$  this takes us one step closer to a solution.
- Such a random walk converges in  $O(\#\text{toppings}^2)$  steps with high probability.

(Generalization of Papadimitriou's 2-Sat algorithm.)

7 submissions, ?? correct, first at ???.

# This was fun! When is the next contest?

- We train every two weeks at KTH, check [www.csc.kth.se/contest](http://www.csc.kth.se/contest).
- Next training on Thursday April 24 at 17:15 in Orange.
- Nordic Championships in October, North-western Europe qualifier in November.
- Plenty of other online competitions every week.
- Subscribe to our calendar and RSS feed.

# Boot camp June 6 – June 8

- 3 days on Möja in the archipelago.
- Lectures,  
trainings and fun activities.
- By invitation only.
- Also camp for Swedish  
IOI team, Linköping University and  
FAU-Erlangen Germany.



Photo by The U.S. Army

# Guide To Programming Contests

- <http://contest-wiki.csc.kth.se/>
- Written by Lukáš.
- The first training program for programming contests.
- Well received in the contest community.